

Optics and Modern Physics

CHAPTER 1

Geometrical Optics: Part 1

1.1–1.78

Introduction	1.1
Nature of Objects and Images	1.1
Types of Objects	1.1
Types of Images	1.1
Reflection of Light	1.2
Regular Reflection	1.2
Diffused Reflection	1.2
Laws of Reflection	1.2
Vector Form of the Law of Reflection	1.3
Reflection from a Plane Surface: Plane Mirror	1.3
Image Formation from Plain Mirror	1.3
Field of View	1.5
Effect of Rotation of Plane Mirror	1.7
Angle of Deviation	1.7
Finding Angle of Deviation in Case of Several Mirrors	1.8
Images Formed by Two Plane Mirrors	1.8
Relation Between Velocity of Object and Image	1.9
<i>Concept Application Exercise 1.1</i>	1.11
Reflection from a Curved Surface	1.11
Spherical Mirrors	1.11
Sign Convention: Cartesian Convention	1.13
Rules for Ray Diagrams	1.13
Position, Size and Nature of Image Formed by Spherical Mirrors	1.13
Mirror Formula	1.13
Magnification	1.14
Magnification from a Concave Mirror	1.14
Magnification from a Convex Mirror	1.14
Image Formation	1.15
Nature of Image Formed by Concave Mirror	1.15
Nature of Image Formed by Convex Mirror	1.15

Object Distance (u) vs. Image Distance (v) Graph	1.16
Cutting of a Mirror	1.16
<i>Concept Application Exercise 1.2</i>	1.22
Images Formed by Two Mirrors	1.23
Relation Between Object and Image Velocity	1.24
Graphical Method of Determining the Focal Length of a Concave Mirror	1.26
<i>Concept Application Exercise 1.3</i>	1.27
Refraction of Light	1.28
Refractive Index	1.28
Laws of Refraction	1.28
Principle of Reversibility of Light Rays	1.28
Vector Representation of a Light Ray	1.32
Critical Angle and Total Internal Reflection	1.32
Deviation of Ray Due to Refraction	1.33
Apparent Shift of an Object Due to Refraction	1.36
<i>Concept Application Exercise 1.4</i>	1.39
Refraction Through a Parallel Slab	1.40
Lateral Displacement of Emergent Beam Through a Glass Slab	1.41
Refraction Across Multiple Slabs	1.42
Slab and Mirror Combined	1.43
Refraction in a Medium with Variable Refractive Index	1.46
Measurement of Refractive Index of a Liquid by a Concave Mirror	1.48
Measurement of Refractive Index of a Liquid by a Travelling Microscope	1.48
<i>Concept Application Exercise 1.5</i>	1.49
Prism	1.50
Condition of No Emergence	1.50
Condition of Grazing Emergence	1.50
Condition of Maximum Deviation	1.51
Condition of Minimum Deviation	1.51
Thin Prisms	1.53
Dispersion of Light	1.53
Deviation Without Dispersion	1.54
Dispersion Without Deviation	1.54
<i>Concept Application Exercise 1.6</i>	1.55
<i>Exercises</i>	1.56
<i>Single Correct Answer Type</i>	1.56
<i>Multiple Correct Answers Type</i>	1.64
<i>Linked Comprehension Type</i>	1.68
<i>Matrix Match Type</i>	1.69
<i>Numerical Value Type</i>	1.71
<i>Archives</i>	1.73
<i>Answers Key</i>	1.77

1

Geometrical Optics: Part 1

INTRODUCTION

Light is a form of radiant energy, that is, energy emitted by excited atoms or molecules which can cause the sensation of vision in a normal human eye.

The branch of physics which deals with the phenomena of light is called optics. The two branches of Optics are as follows:

Geometrical optics: This deals with the study of light in which light is considered as moving along a straight line as a ray. A ray of light gives the direction of propagation of light. When light meets a surface which separates two media, reflection and refraction take place. An image or an array of images may be formed due to this.

Physical optics: It deals with the theories regarding the nature of light and provides an explanation for the different phenomena in light, such as reflection, refraction, interference, diffraction, polarization, and rectilinear propagation.

NATURE OF OBJECTS AND IMAGES

Ray optics primarily deals with determining the position and nature of the image formed when an object is placed in front of an optical element. Before we proceed further, let us clearly define what is an object and what is an image.

TYPES OF OBJECTS

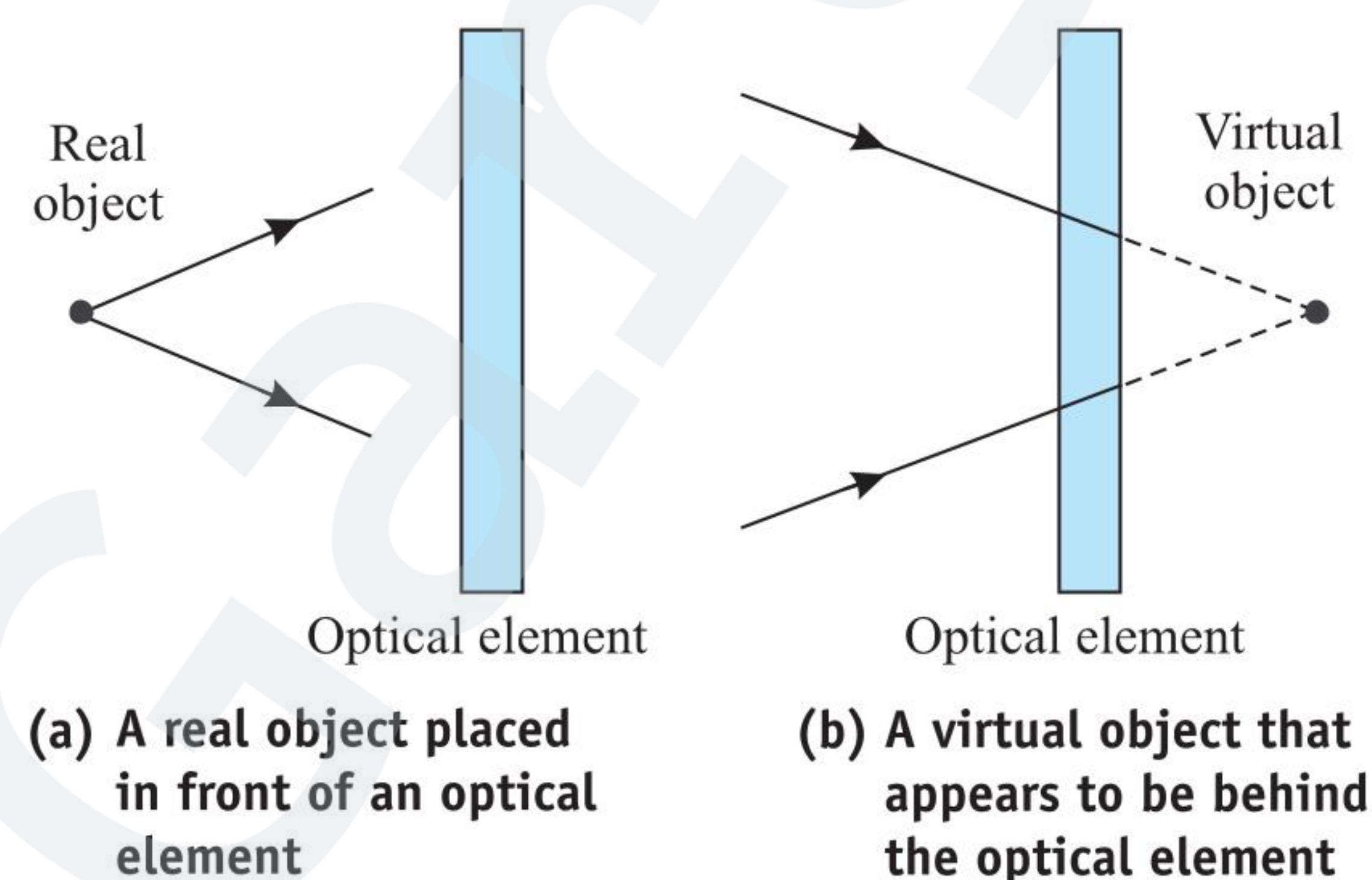
An object is a source of light rays that are incident on an optical element. An object may be a point object or an extended object. Since extended objects can be modeled as a collection of points, we only need to study point objects. Objects are of two kinds: real objects and virtual objects.

Real Object

An object is real if two or more incident rays actually emanate or seem to emanate from a point. Figure (a) is a typical example of a real object. In this case, two rays emanate from the object and are incident on the optical element and the object is actually present. Hence, it is called a real object.

Virtual Object

Now, consider a converging set of rays as shown in Fig. (b). If not intercepted the rays will meet at a point. However, if the rays are intercepted by an optical element placed as shown in the figure, then the point of convergence is a virtual point behind the optical element. This point is called the virtual object for the optical element.



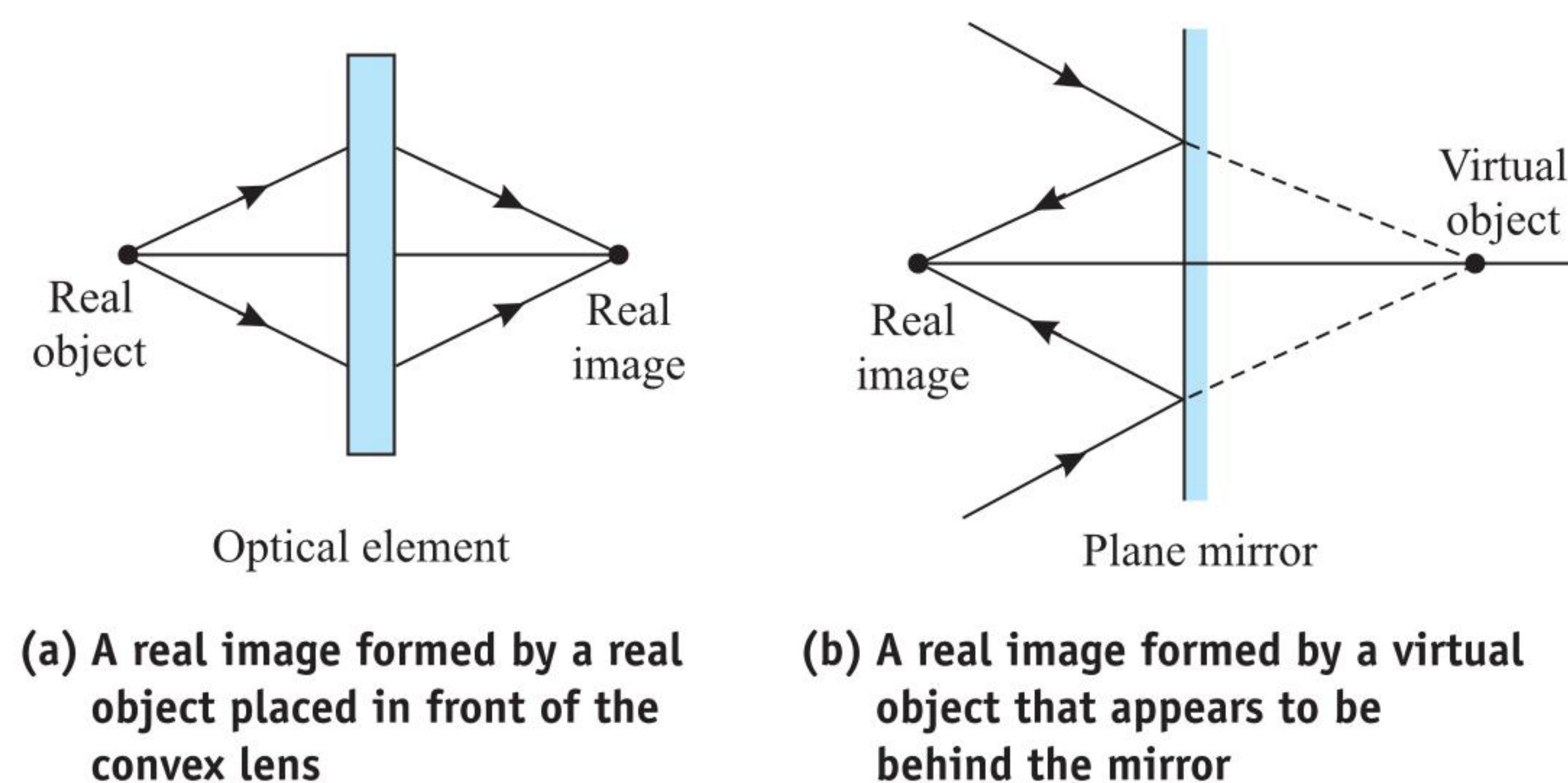
So, an object is real when two rays emanate or diverge from a point, and an object is virtual when two incident rays seem to converge to that point.

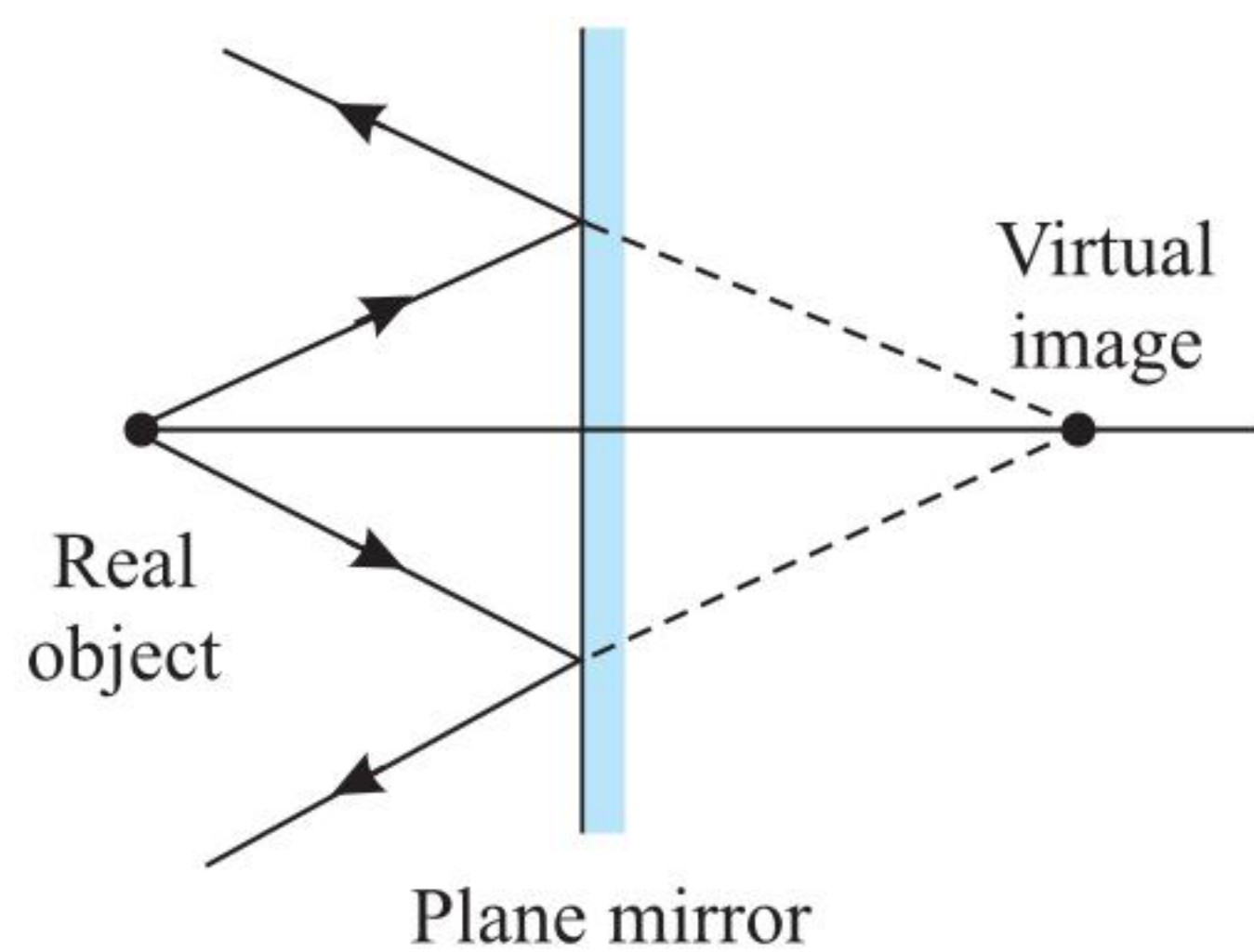
TYPES OF IMAGES

An image is the point of convergence or apparent point of divergence of rays after they interact with a given optical element. An object provides rays that will be incident on an optical element. The optical element reflects or refracts the incident light rays which then meet at a point to form an image. As in the case of objects, images too can be real or virtual.

Real Image

Real images are formed when the reflected or refracted rays actually meet or converge to a point. If a screen is placed at that point, a bright spot will be visible on the screen. Thus, a real image can be captured on a screen. Examples of real images are shown in Figs. (a) and (b). Note that in the former the object is real while in the latter the object is virtual. Thus, both real and virtual objects can form real images.





(c) A virtual image formed by a real object in front of the mirror

Virtual Image

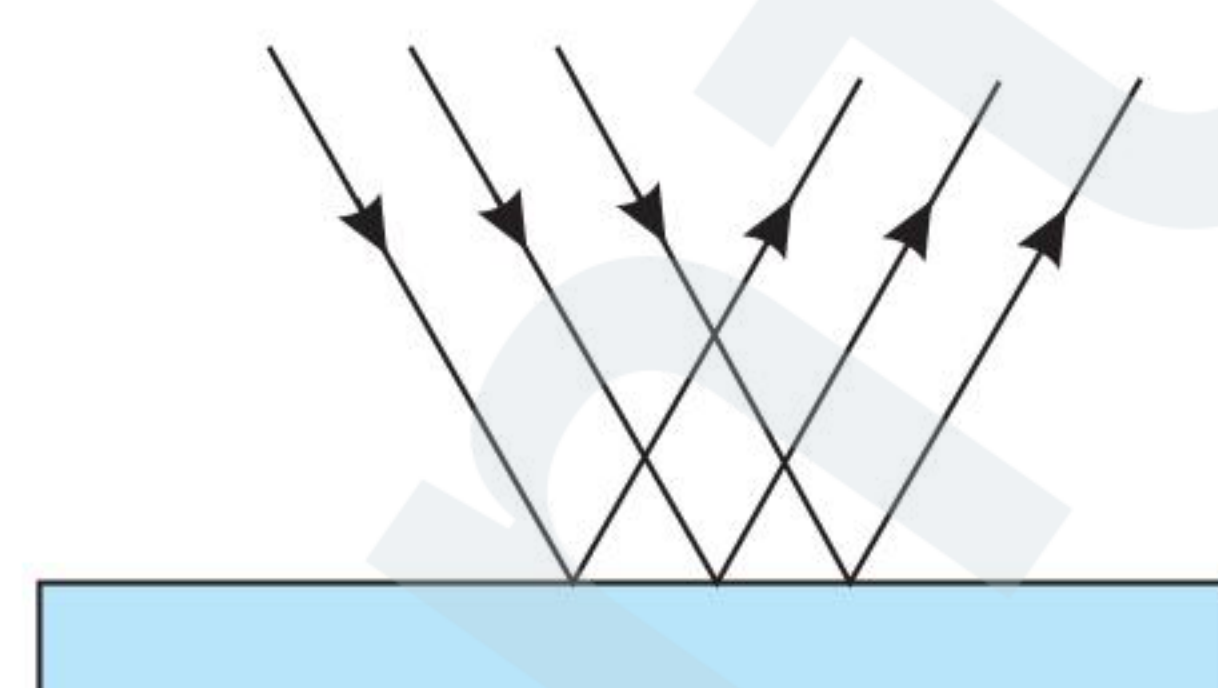
When light rays, after interacting with the optical element, actually meet at a point the image formed is a real image. However, if the rays do not meet at a point but appear to emanate from a point, then a virtual image is formed. Consider the case of an object placed in front of a plane mirror. Here, the two reflected rays will never meet at any point. However, if we extend the reflected rays backwards, they appear to emanate from a point. This point is the virtual image of the object. As we will see later in the section on lenses, it is possible for a virtual image to be formed by a virtual object as well. Thus, we can conclude that both virtual and real images can be formed by either real or virtual objects depending on the optical element.

REFLECTION OF LIGHT

When light rays strike the boundary of two media such as air and glass, a part of light is turned back into the same medium. This is called **reflection of light**.

REGULAR REFLECTION

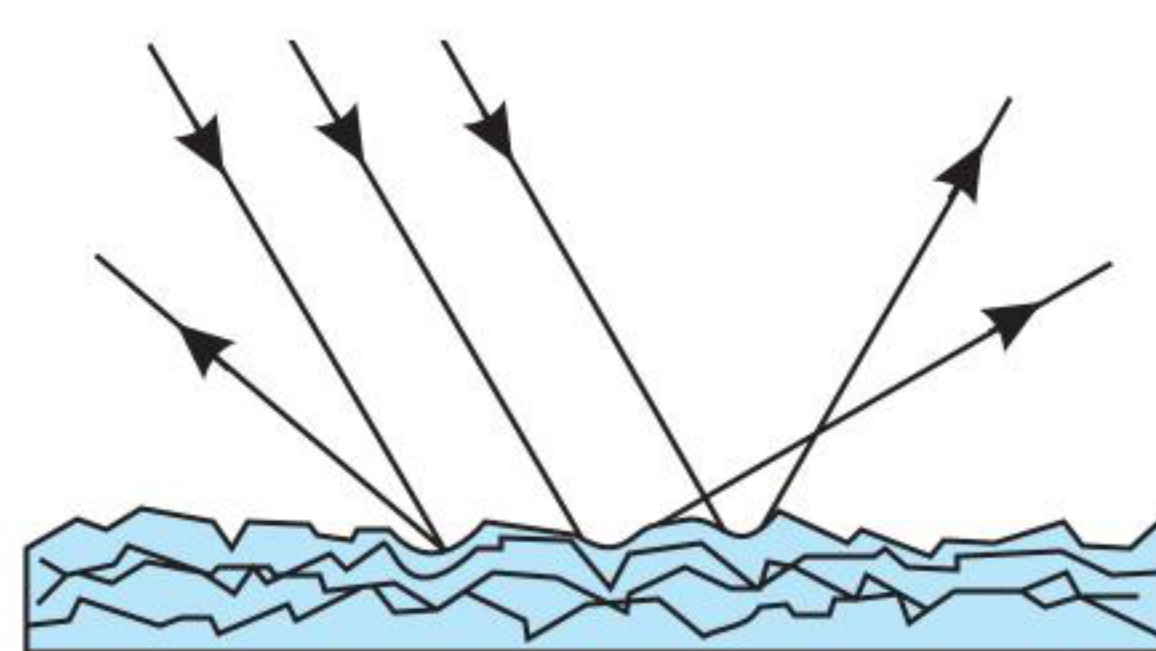
When the reflection takes place from a perfect plane surface it is called **regular reflection** [see Fig. (a)]. In this case, the reflected light has large intensity in one direction and negligibly small intensity in other directions.



(a) Regular reflection

DIFFUSED REFLECTION

When the surface is rough, we do not get a regular behaviour of light. Although at each point light ray gets reflected irrespective of the overall nature of surface, difference is observed because even in a narrow beam of light there are many rays which are reflected from different points of surface. It is quite possible that these rays may move in different directions due to irregularity of the surface. This process enables us to see an object from any position. Such a reflection is called as **diffused reflection** [see Fig. (b)]. For example, reflection from a wall, from a newspaper, etc. This is why you cannot see your face in the newspaper and on the wall.

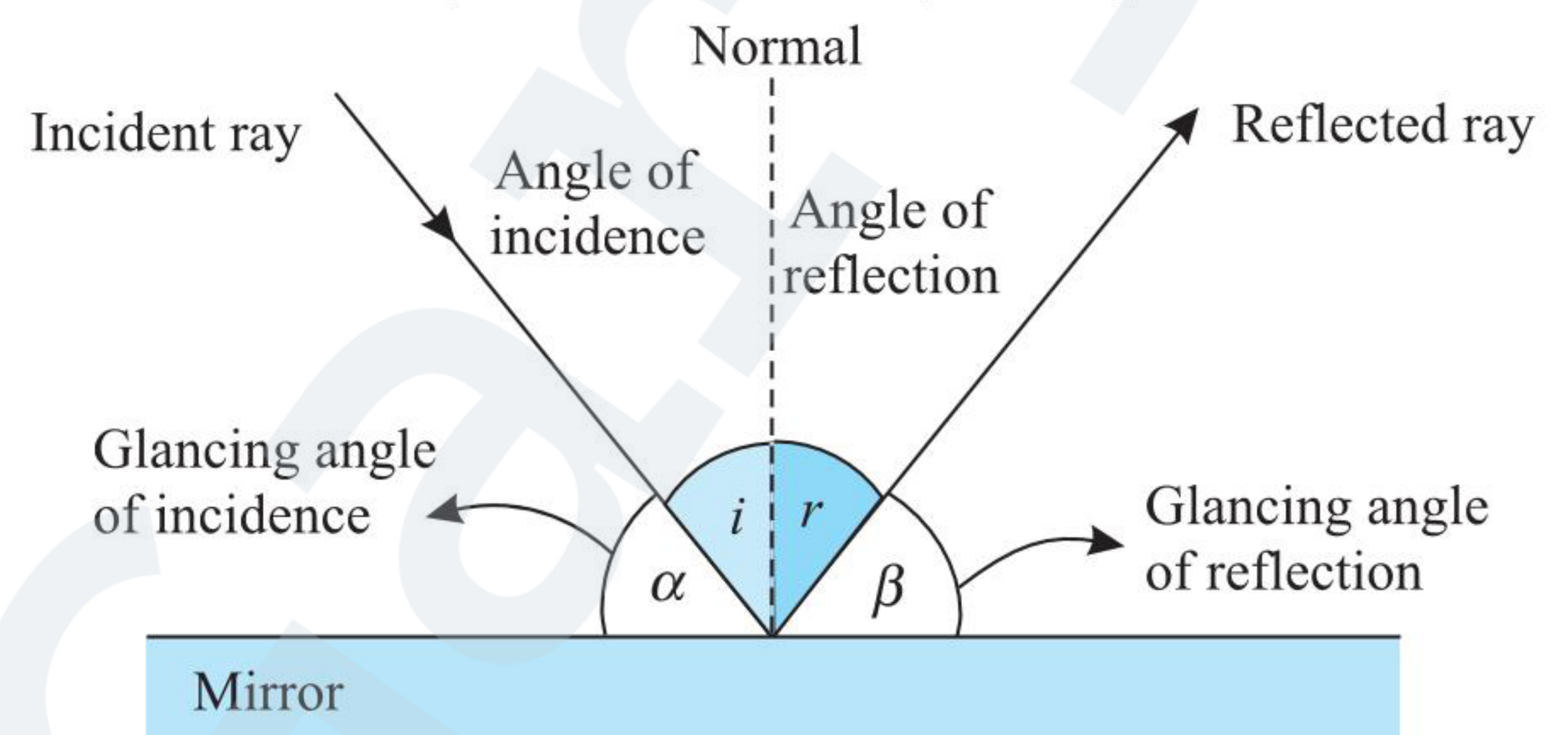


(b) Diffused reflection

LAWS OF REFLECTION

Reflection of light is the process of deflecting a beam of light in the same medium. It has been found experimentally that rays undergoing reflection follow two laws called the Laws of reflection:

1. The incident ray, the reflected ray, and the normal at the point of incidence lie in the same plane. This plane is called the **plane of incidence (or plane of reflection)**.
2. The **angle of incidence** (the angle between normal and the incident ray) and the **angle of reflection** (the angle between the reflected ray and the normal) are equal, i.e., $\angle i = \angle r$.



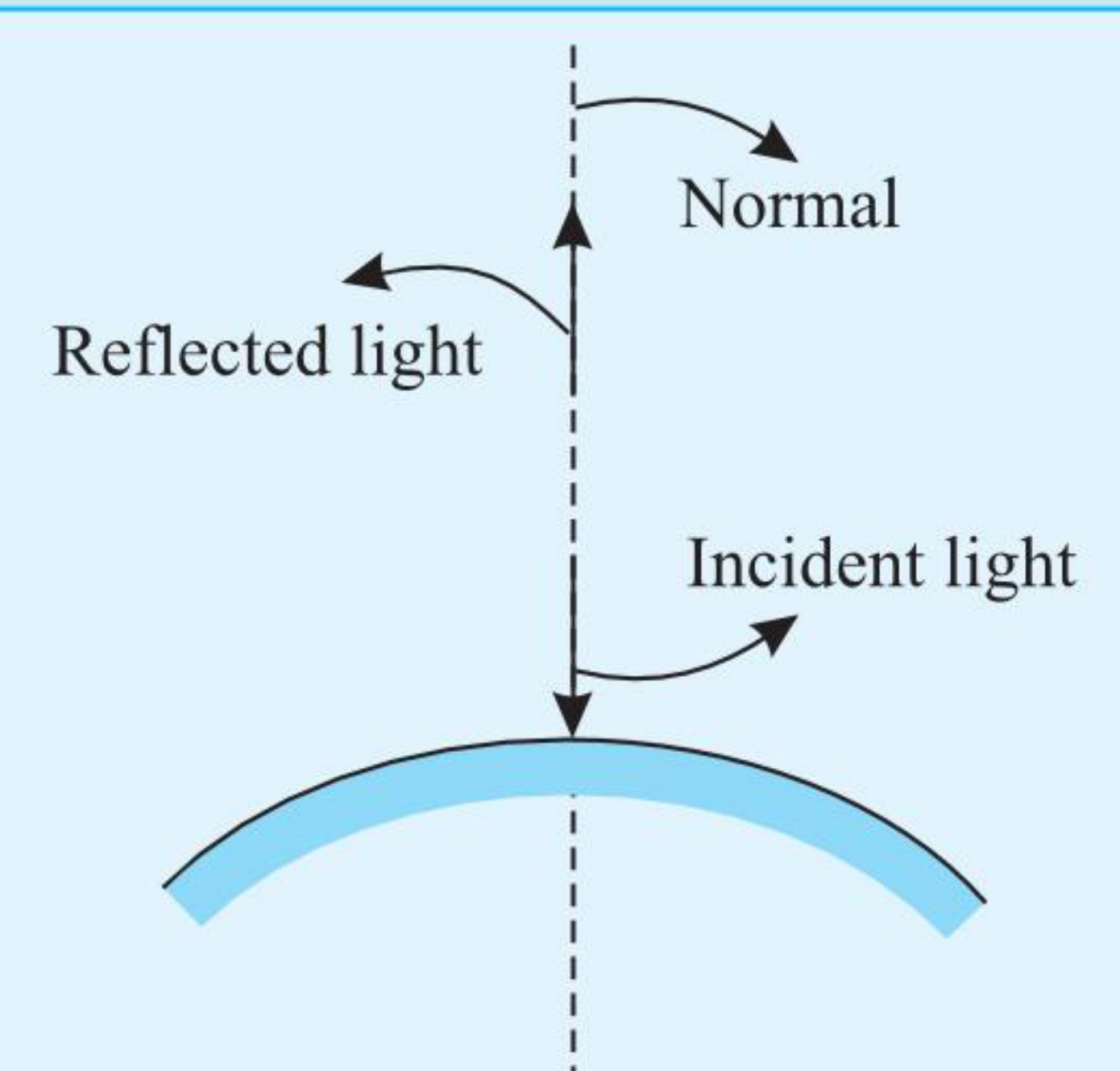
Important Points:

- The angle between an incident ray (on a plane of surface) and the surface is called the **glancing angle of incidence**. Simply it is the angle made by the incident ray with the plane of the surface (on which the light is incident)
- The angle between a surface and reflected ray is called **glancing angle of reflection**. Simply it is the angle made by the reflected light with the plane of the surface (from which the light is reflected)
- In case of reflection,
Glancing angle of incidence (α) = Glancing angle of reflection (β)

Special Cases

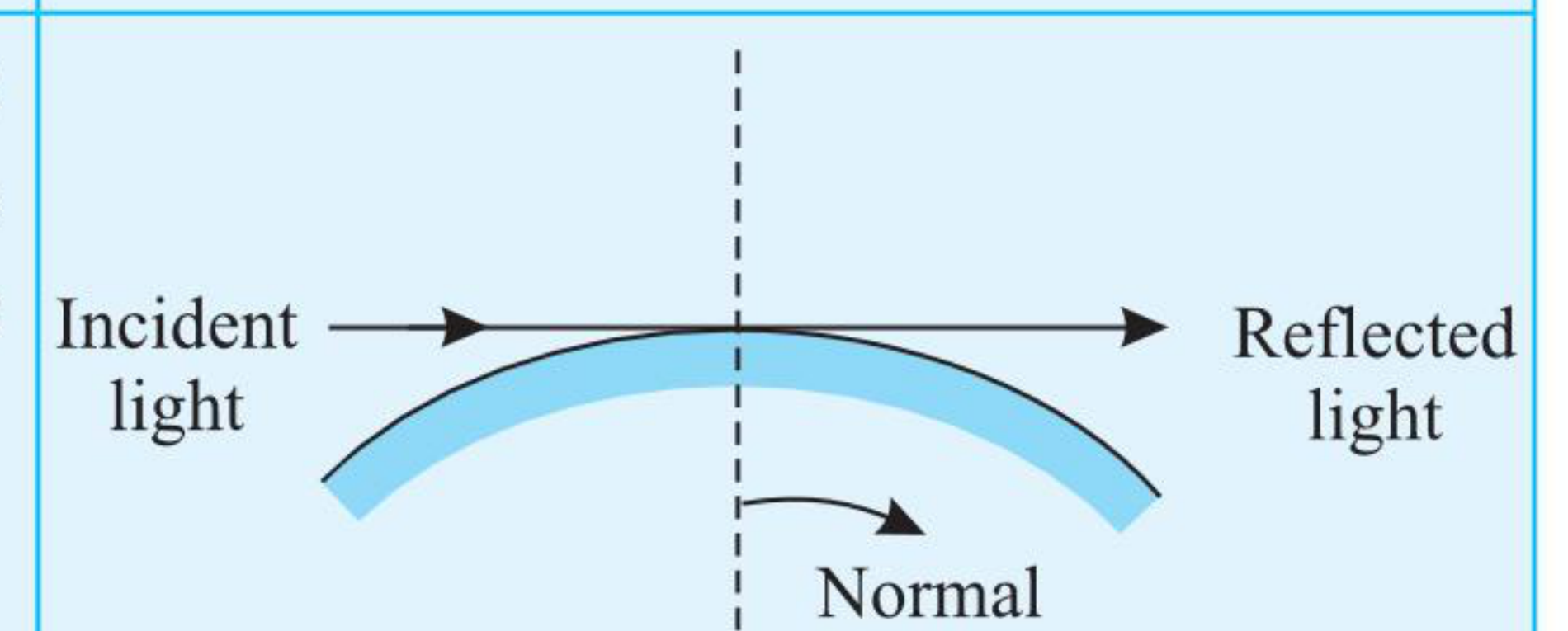
Normal incidence:

In case light is incident normally
 $i = r = 0$



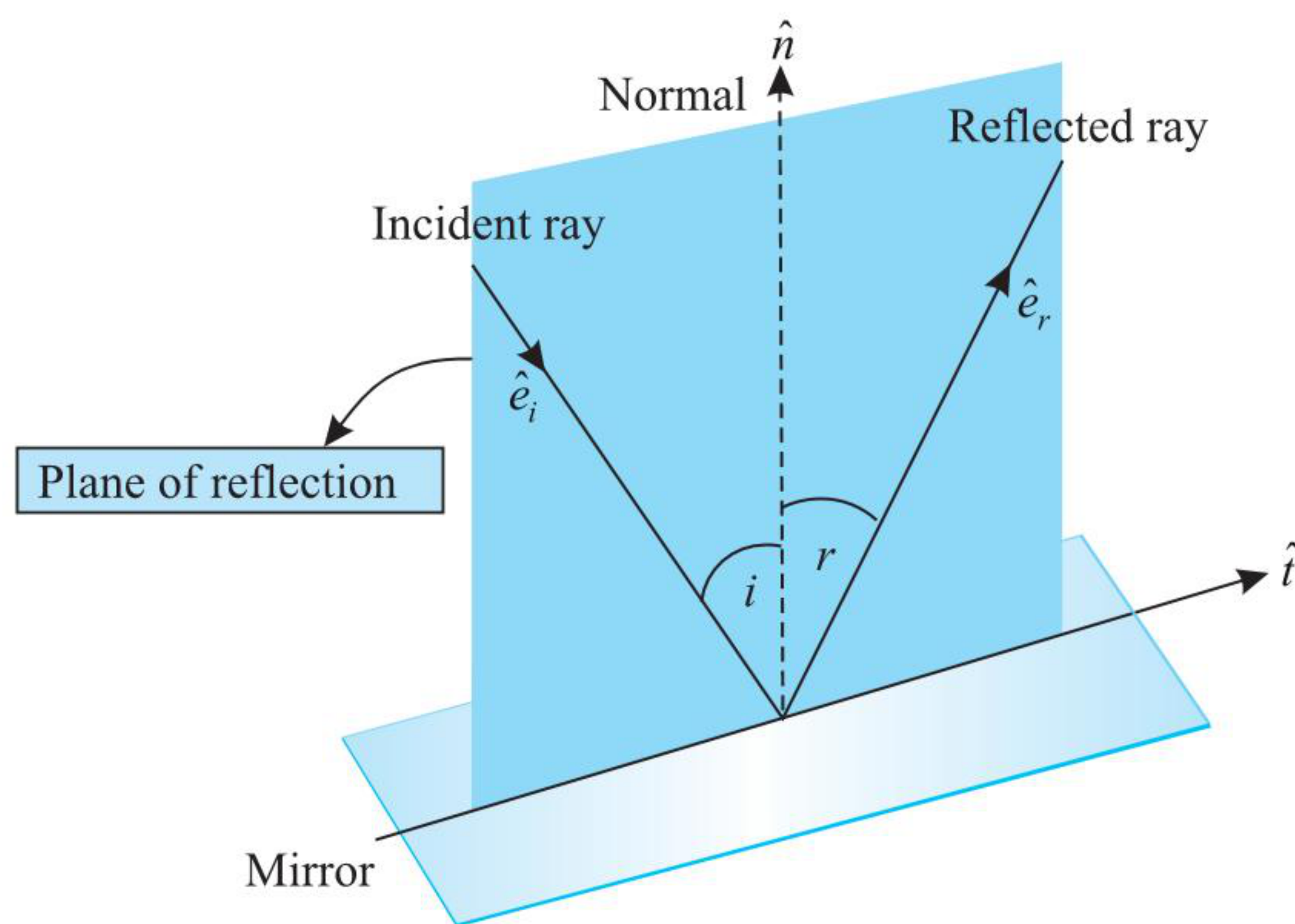
Grazing incidence:

In case light strikes the reflecting surface tangentially
 $i = r = 90^\circ$



VECTOR FORM OF THE LAW OF REFLECTION

Let, $\hat{e}_i$ = unit vector along the incident ray,
 $\hat{e}_r$ = unit vector along the reflected ray,
 $\hat{t}$ = unit vector along tangential direction,
 $\hat{n}$ = unit vector along outward normal



Let, $\angle i = \angle r = \angle \theta$

$$\hat{e}_i = \sin \theta \hat{t} - \cos \theta \hat{n} \quad \dots(i)$$

$$\hat{e}_r = \sin \theta \hat{t} + \cos \theta \hat{n} \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$\hat{e}_r - \hat{e}_i = 2 \cos \theta \hat{n} \quad \dots(iii)$$

Also, $\hat{e}_i \cdot \hat{n} = (1)(1) \cos(\pi - \theta) = -\cos \theta$

By putting the value of $\cos \theta$ in Eq. (iii),

$$\hat{e}_r - \hat{e}_i = -2(\hat{e}_i \cdot \hat{n}) \hat{n}$$

The unit vector along the reflected ray, $\hat{e}_r = \hat{e}_i - 2(\hat{e}_i \cdot \hat{n}) \hat{n}$

It is the vector form of the law of reflection.

ILLUSTRATION 1.1

A ray of light is incident on a plane mirror along a vector $a\hat{i} + b\hat{j} - c\hat{k}$. The normal on incidence point is along $\frac{\hat{i} + \hat{j}}{\sqrt{2}}$. Find a unit vector along the reflected ray.

Sol. Unit vector in the direction of incident ray

$$\hat{e}_i = \frac{a\hat{i} + b\hat{j} - c\hat{k}}{\sqrt{a^2 + b^2 + c^2}} \text{ and we are given the unit vector in the direction of normal } \hat{n} = \frac{\hat{i} + \hat{j}}{\sqrt{2}}.$$

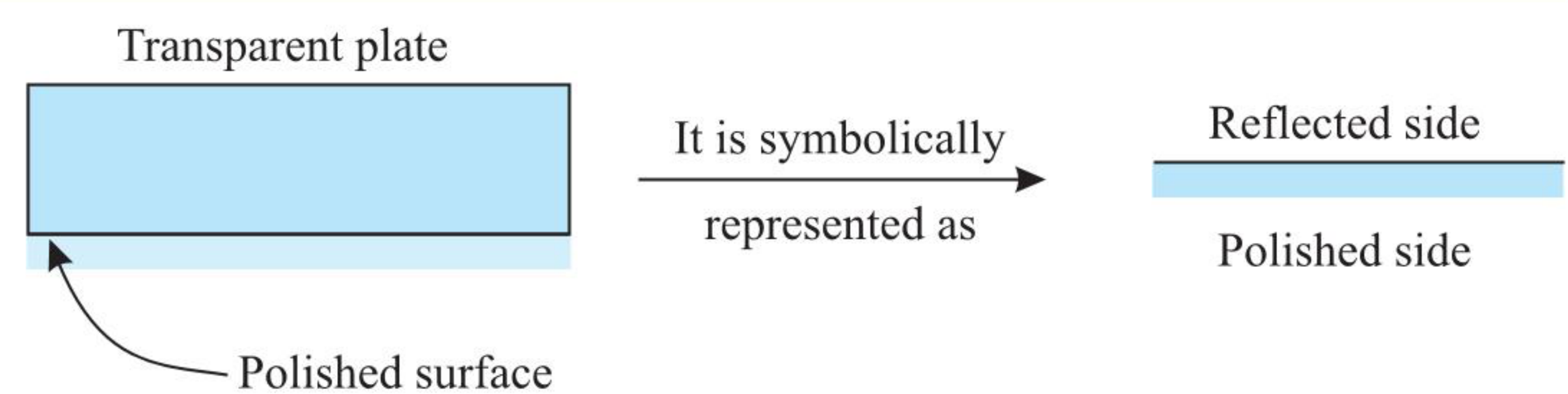
The unit vector along the reflected ray, $\hat{e}_r = \hat{e}_i - 2(\hat{e}_i \cdot \hat{n}) \hat{n}$

$$\therefore \hat{e}_r = \frac{a\hat{i} + b\hat{j} - c\hat{k}}{\sqrt{a^2 + b^2 + c^2}} - 2 \left[\frac{a+b}{\sqrt{a^2 + b^2 + c^2} \sqrt{2}} \right] \frac{\hat{i} + \hat{j}}{\sqrt{2}}$$

$$\Rightarrow \hat{e}_r = \frac{1}{\sqrt{a^2 + b^2 + c^2}} [-b\hat{i} - a\hat{j} - c\hat{k}]$$

REFLECTION FROM A PLANE SURFACE: PLANE MIRROR

A plane mirror is formed by polishing one surface of a plane thin glass plate (see figure). It is also said to be silvered on one side.

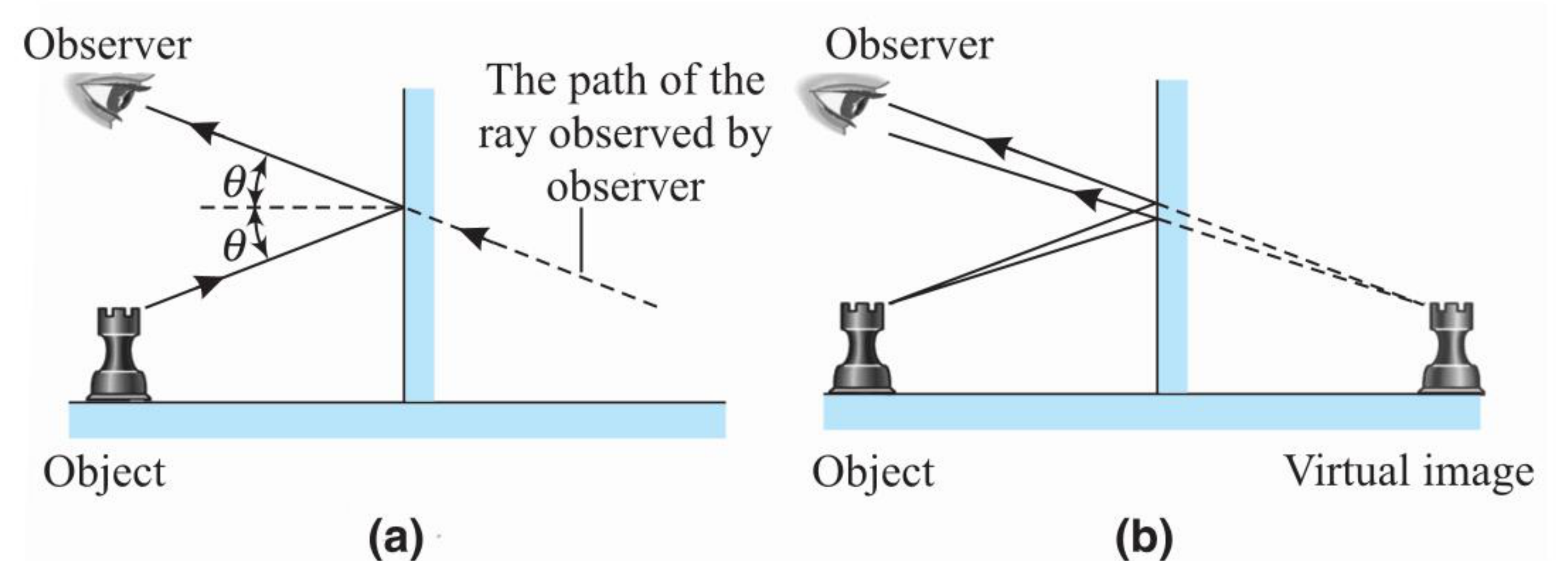


A beam of parallel rays of light, incident on a plane mirror, will get reflected as a beam of parallel reflected rays.

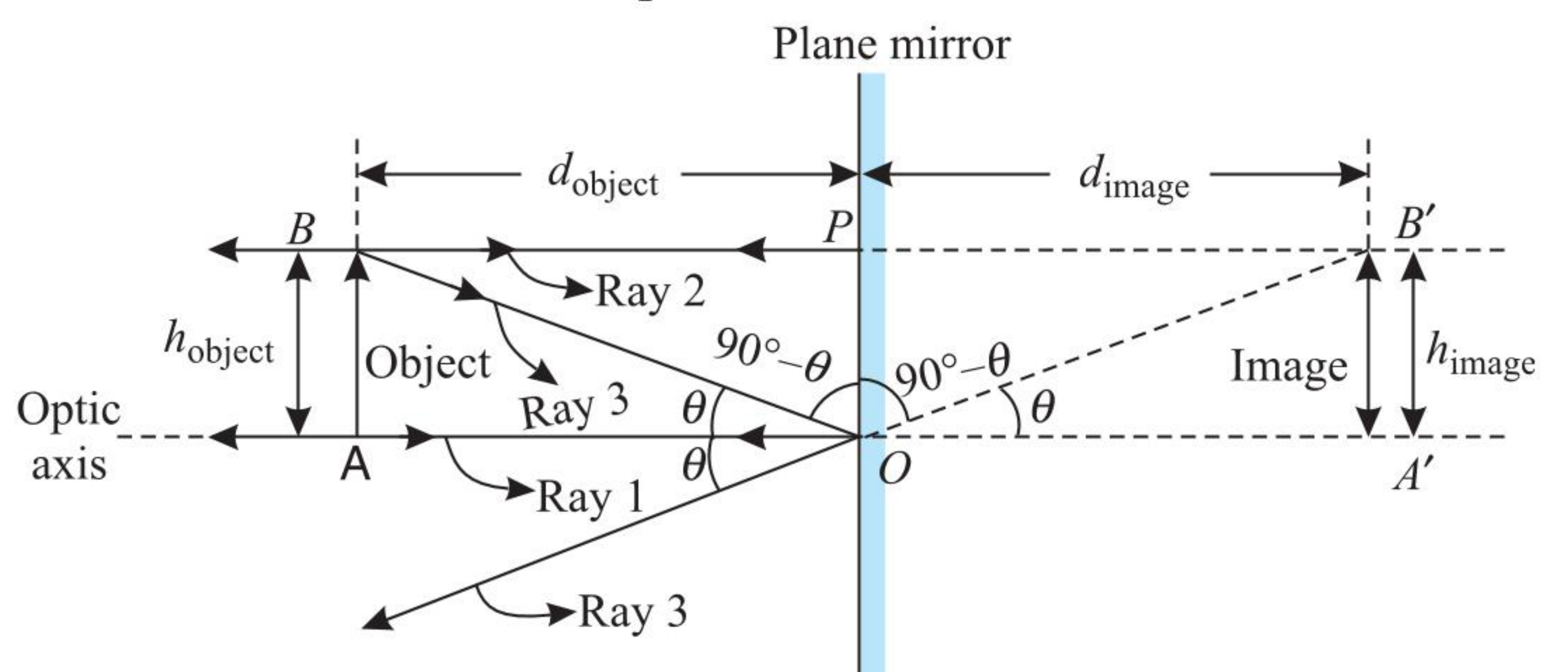
IMAGE FORMATION FROM PLAIN MIRROR

If you stand in front of a plain mirror, you will see an image of yourself that appears to be behind the mirror. This perception occurs because your brain assumes that light rays are reaching to your eyes have traveled in straight lines with no change in direction. Thus, if light rays appear to originate at a point (say behind the mirror), your eye observes a source of light at that point, whether or not an actual light source is there. This type of image, from which the light does not emanate, is referred to as a **virtual image**. By their nature, virtual images cannot be displayed on a screen.

Now let us observe, why an image appears to originate from behind a plane mirror. In Fig. (a), you can observe a light ray travelling from the top of an object. After reflection from the mirror (angle of reflection equals angle of incidence) the ray enters the eye of the observer. To the observer, it appears that the ray originates from behind the mirror, somewhere back along the dashed line. In fact, here the rays going in all directions leave each point on the object, but only a small bundle of such rays is intercepted by the eye. In Fig. (b), you can observe a bundle of two rays leaving the top of the object. All the rays that leave a given point on the object, no matter what angle θ they have when they strike the mirror, appear to originate from a corresponding point on the image behind the mirror (see the dashed lines in Fig. (b)).



For image construction, let us place an object (an arrow) with height h_{object} a distance d_{object} from the mirror. The object is oriented so that the tail of the arrow is on the **optic axis**, which is defined as a normal to the plane of the mirror.



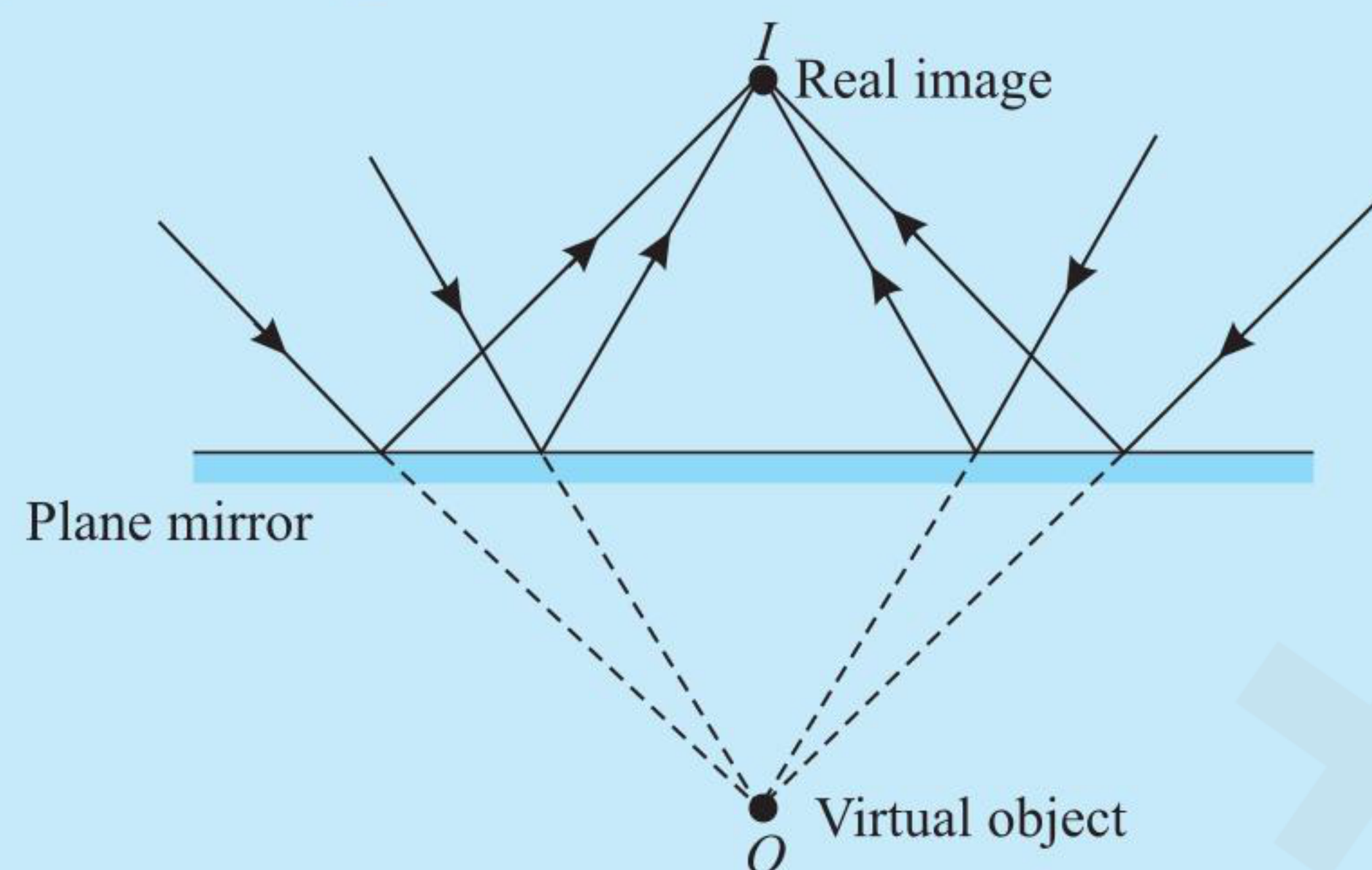
1. The first light ray (Ray-1), starts from the bottom of the arrow along the optic axis. This ray is reflected directly back on itself.

- The second light ray (Ray-2), starts from the top of the arrow parallel to the optic axis and is reflected directly back on itself.
- The third ray (Ray-3), starts from the top of the arrow, strikes the mirror where the optic axis intersects the mirror, and is reflected with an angle equal to its angle of incidence.

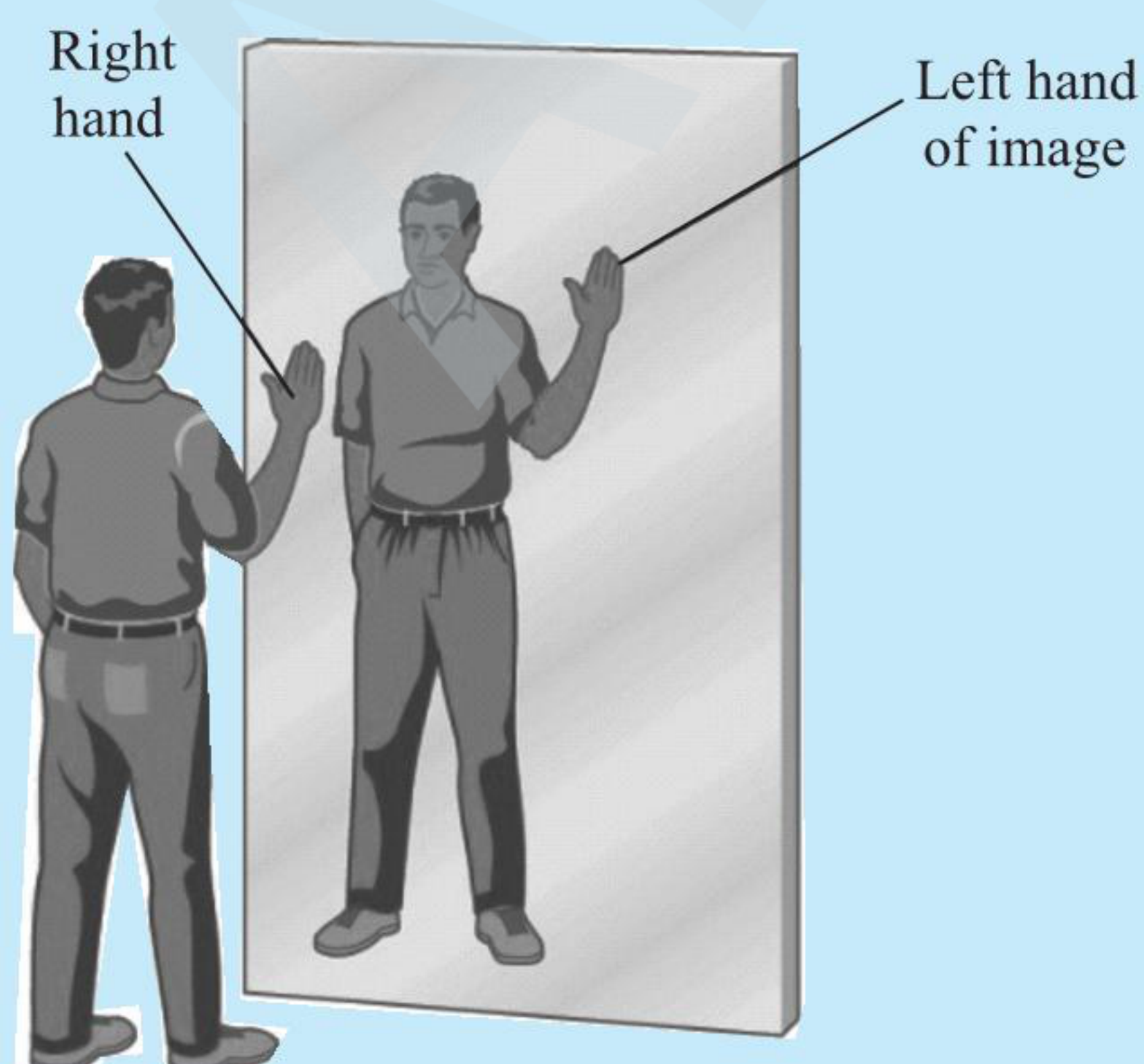
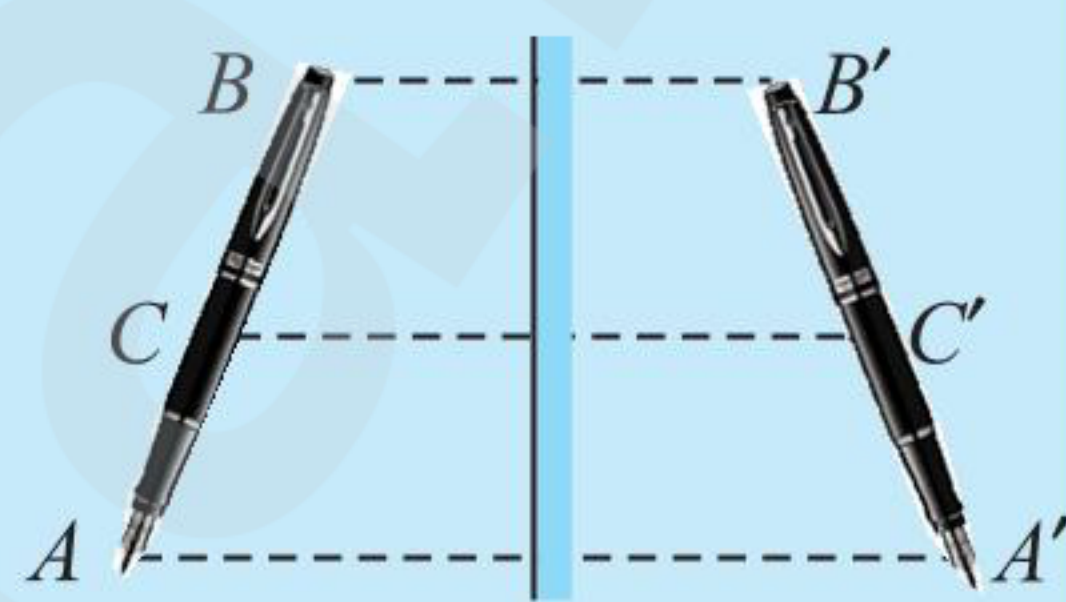
The extrapolations of ray-2 and ray-1 intersect at the point where the top of the image forms (see figure). It turns out that all rays from the arrow tip that hit the mirror—not just the two rays shown here—extrapolate such that they intersect at the image. Thus, the mirror produces a virtual image on the opposite side of the mirror. This image has a height h_{image} and is located a distance d_{image} to the right of the mirror. In figure the triangle BOP is congruent with the triangle $B'OP$. Thus, $h_{\text{image}} = h_{\text{object}}$ i.e. the size of the image is the same as that of the object and $|d_{\text{image}}| = |d_{\text{object}}|$ i.e. distance of object from mirror is equal to the distance of image from the mirror.

Important Points:

- For a real object the image is virtual and for a virtual object the image is real.



- An extended object like AB shown in figure can be considered as a combination of infinite number of point objects from A to B . Image of every point object will be formed individually and thus infinite images will be formed. A' will be image of A , C' will be image of C , B' will be image of B , etc. All point images together form an extended image. Thus, extended image is formed of an extended object.
- The image is upright, if the extended object is placed parallel to the plane mirror (see figure).



- The image is inverted, if the extended object lies perpendicular to the plane mirror (see figure).

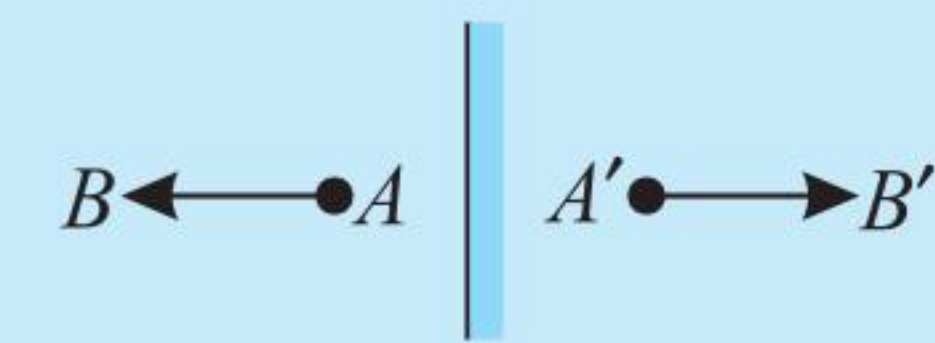
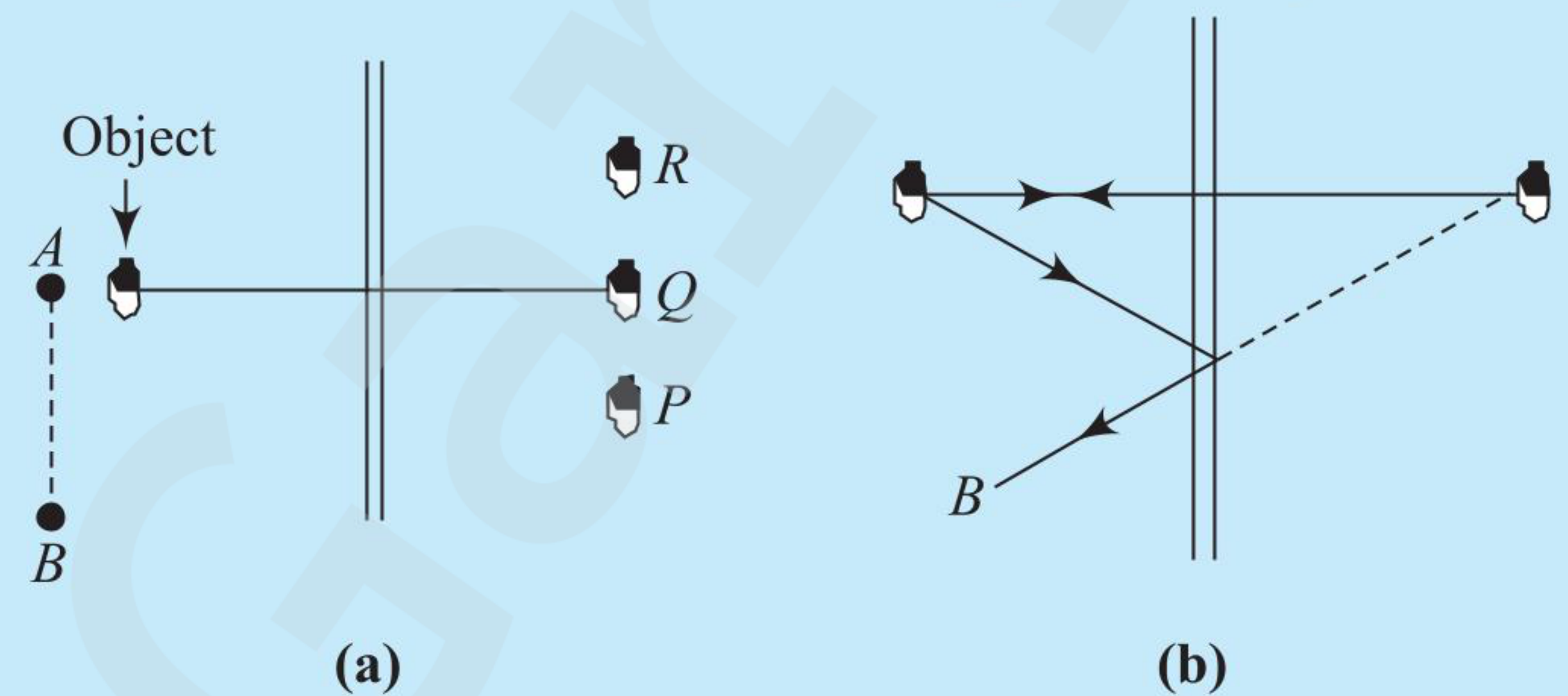


ILLUSTRATION 1.2

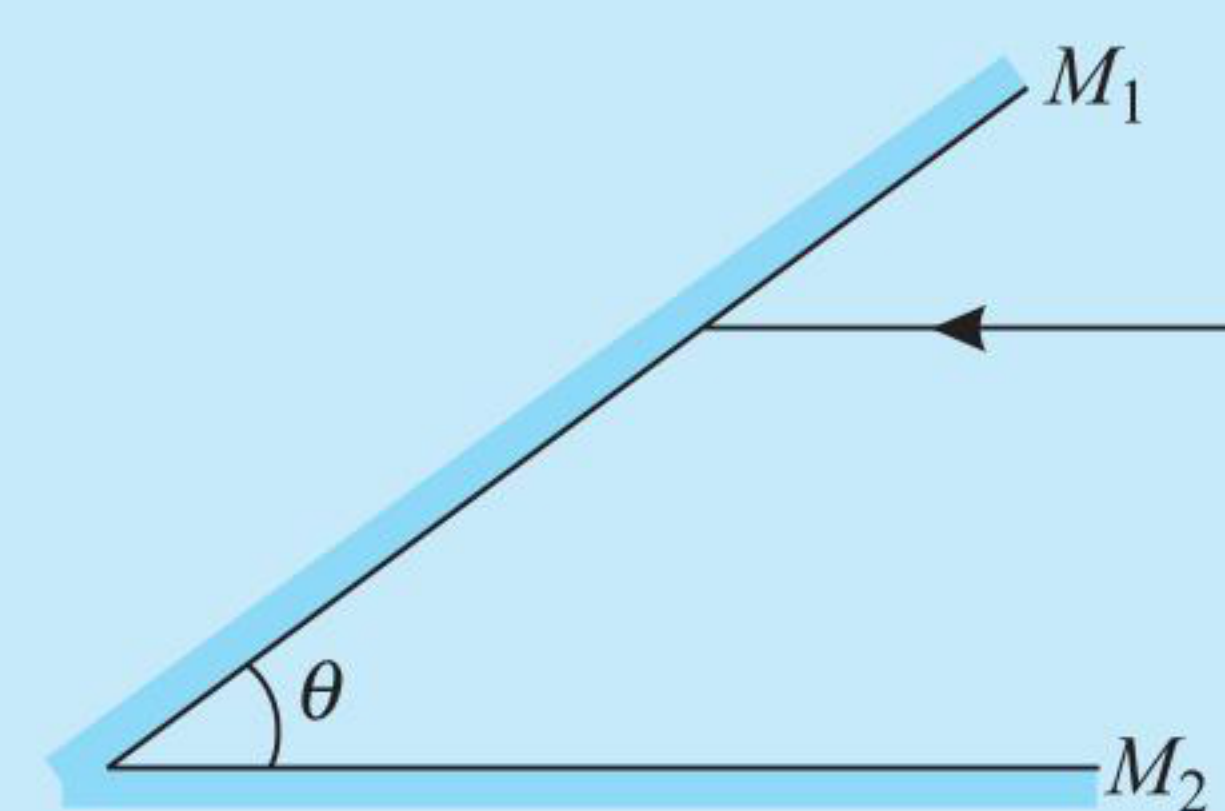
Figure (a) shows an object placed in front of a plane mirror. P , Q and R are the three positions where the image of object may be seen. Observer A is able to see the image at position Q . Where does the observer B see the image of the object?



Sol. Observer B also observes the image of the object at the position Q which is explained by the ray diagram in Fig. (b). The position of image will be independent of the position of the observer.

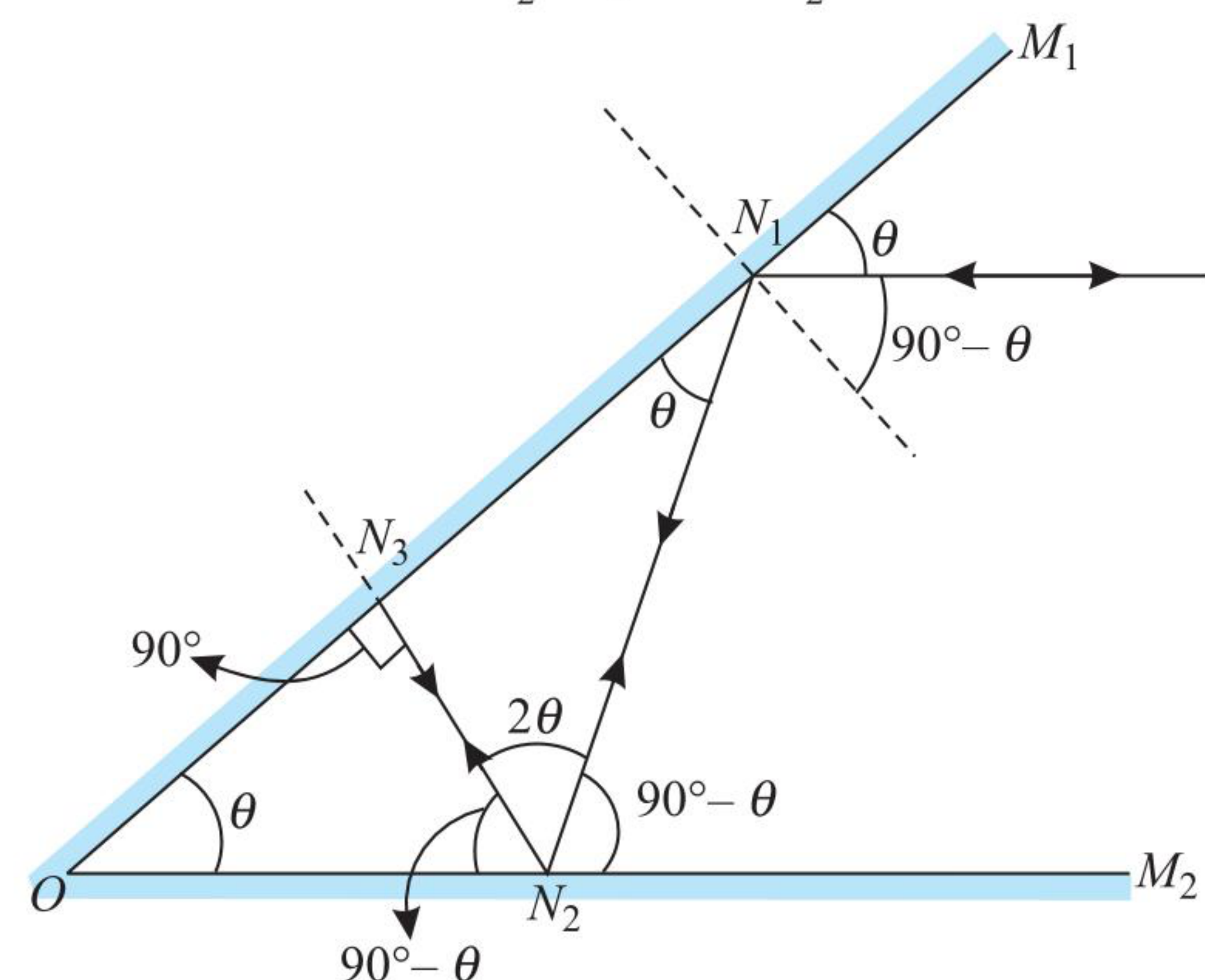
ILLUSTRATION 1.3

Two plane mirrors M_1 and M_2 are inclined at angle θ . A light ray incident travelling parallel to the surface of mirror M_2 and incident on mirror M_1 . What should be the value of θ so that light ray retraces its path after third reflection.



Sol. If light ray retraces its path, it should strike one of the mirrors normally. In the question it is given that after third reflection the ray retraces its path. It means, the ray should be incident on mirror M_1 normally after reflection from mirror M_2 .

Initially the ray strikes the mirror M_1 at point N_1 . Then after 1st reflection it strikes mirror M_2 at point N_2 .



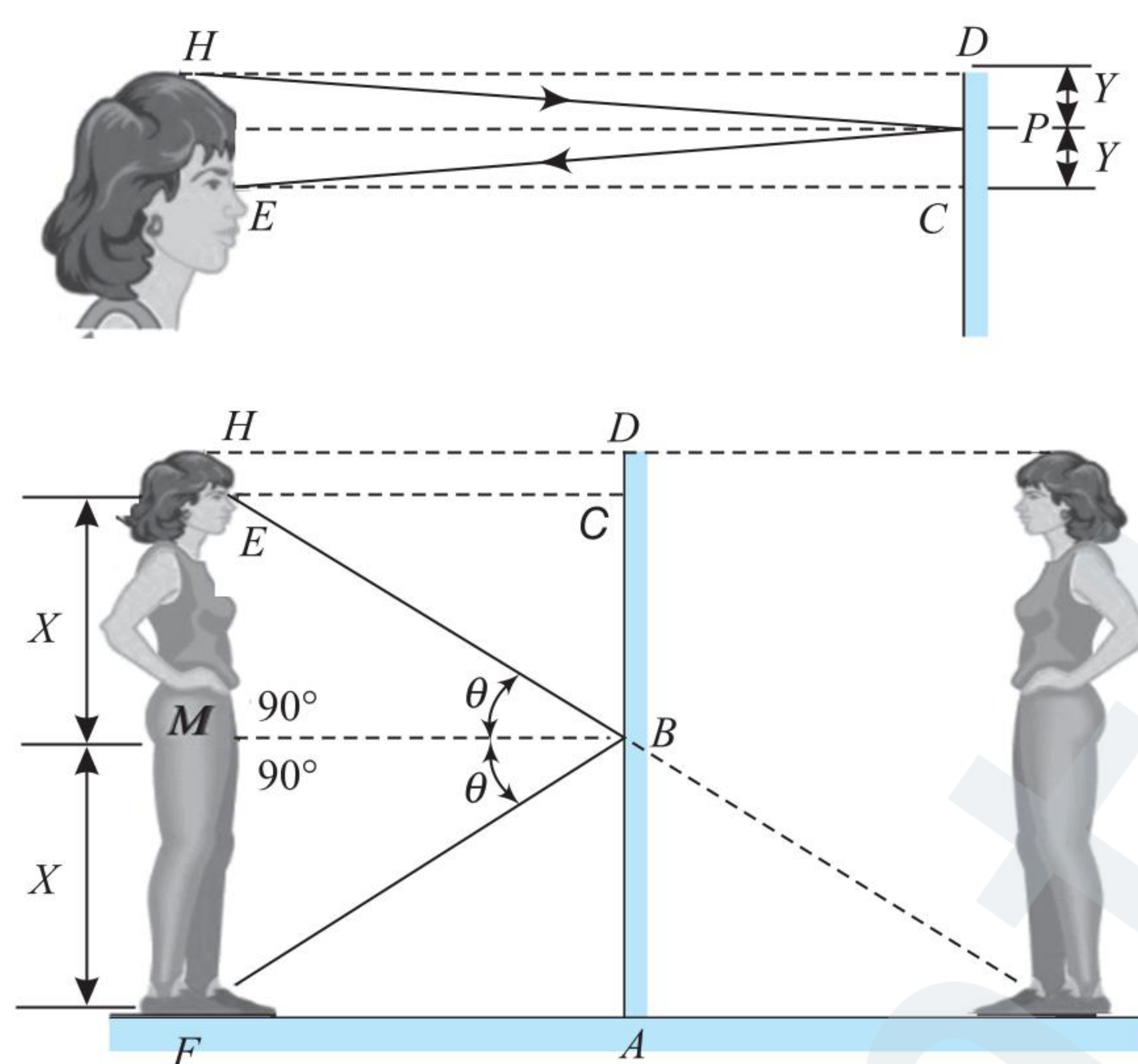
At point N_2 , the ray reflects again and finally it strikes again the mirror M_1 normally at point N_3 .

From simple geometry we can find the angle $ON_2N_3 = 90^\circ - \theta$. It means $\angle N_1N_2M_2$ should also be $90^\circ - \theta$, as glancing angles to the plane mirror should be equal. Hence the angle $N_1N_2N_3 = 2\theta$. In $\triangle N_1N_2N_3$: $\theta + 90^\circ + 2\theta = 180^\circ$ which gives $\theta = 30^\circ$.

ILLUSTRATION 1.4

A woman is standing in front of a plane mirror. What is the minimum mirror height that is necessary for her to see her full image.

Sol. The woman sees her image because light emanating from her body is reflected by the mirror (labeled $ABCD$ in figure) and enters her eyes. Consider, for example, a ray of light from her foot F . This ray strikes the mirror at B and enters her eyes at E . According to the law of reflection, the angles of incidence and reflection both are θ .



The section AB of the mirror is not necessary in order for the woman to see her full image. The section BC of the mirror that produces the image is one-half the woman's height between F and E . This follows because the right triangles FBM and EBM are identical. The blowup in figure illustrates a similar line of reasoning, starting with a ray from the woman's head at H . This ray is reflected from the mirror at P and enters her eyes. The top mirror section PD may be removed without disturbing this reflection. The necessary section CP is one-half the woman's height between her head at H and her eyes at E . We find, then, that only the sections BC and CP are needed for the woman to see her full length. The height of section BC plus section CP is exactly one-half the woman's height. The conclusions here are valid regardless of how far the person stands from the mirror. Thus, to view one's full length in a mirror, only a half-length mirror is needed.

FIELD OF VIEW

The field of view is said to be the area visible from the perspective of the observer through the mirror. It is the region where diverging rays from object or image are present. If our eyes are present in field of view and receive diverging rays, then only we can see

image. The field of view of image is decided by drawing extreme reflected or refracted rays.

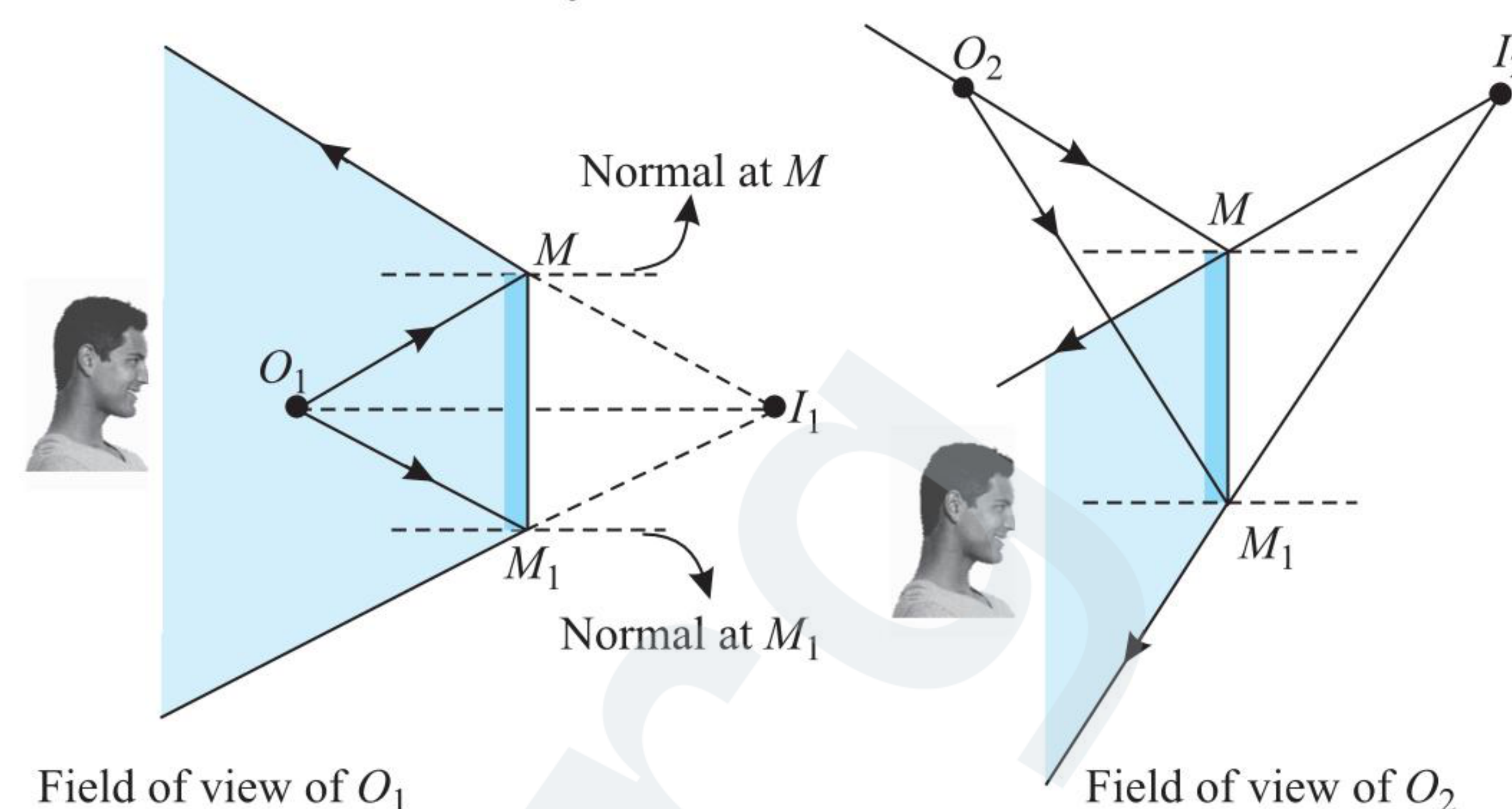
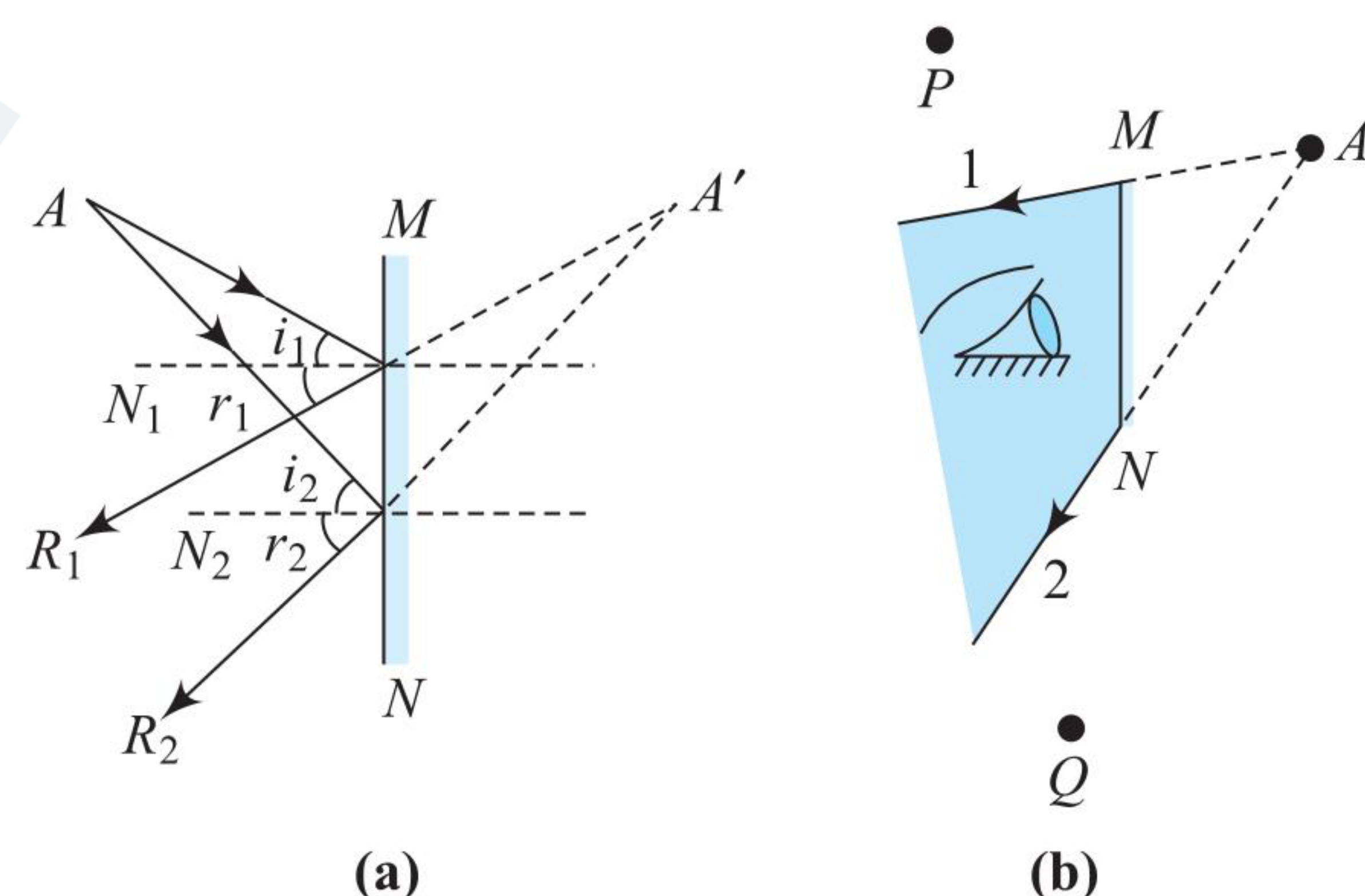


ILLUSTRATION 1.5

Figure shows a point object A and a plane mirror MN . Find the position of the image of object A , in mirror MN , by drawing ray diagram. Indicate the region in which the observer's eye must be present in order to view the image. (This region is called field of view.)

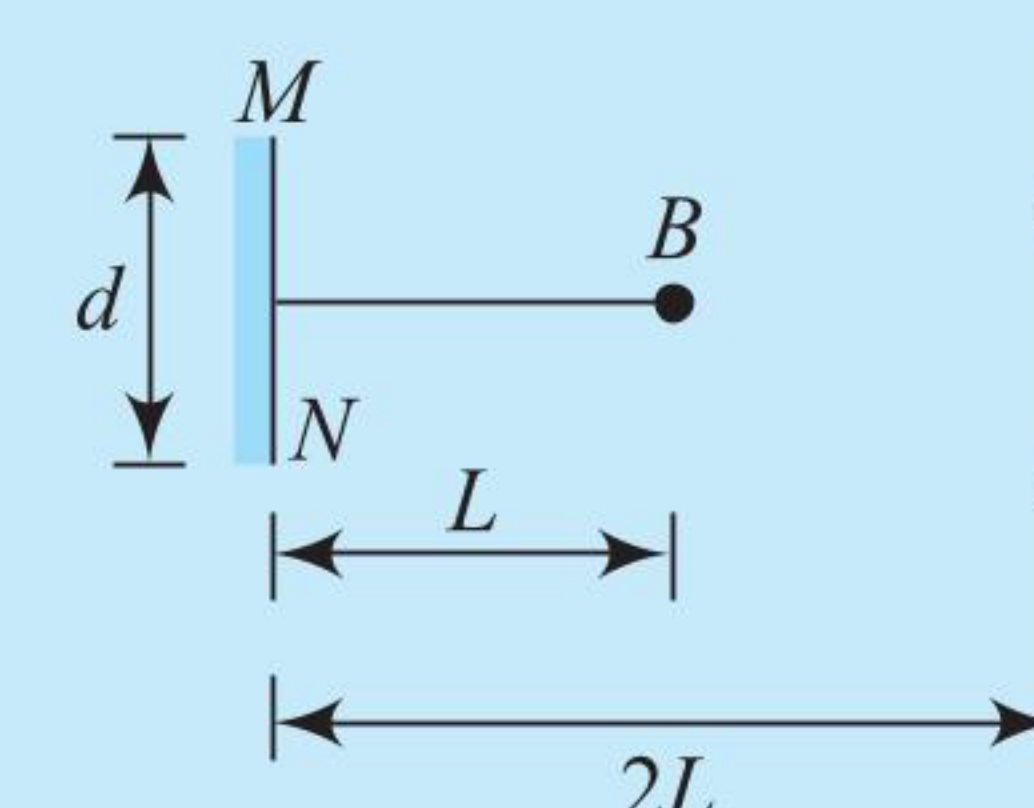
Sol. Consider any two rays emanating from the object [see Fig. (a)]. N_1 and N_2 are normals; $i_1 = r_1$ and $i_2 = r_2$.



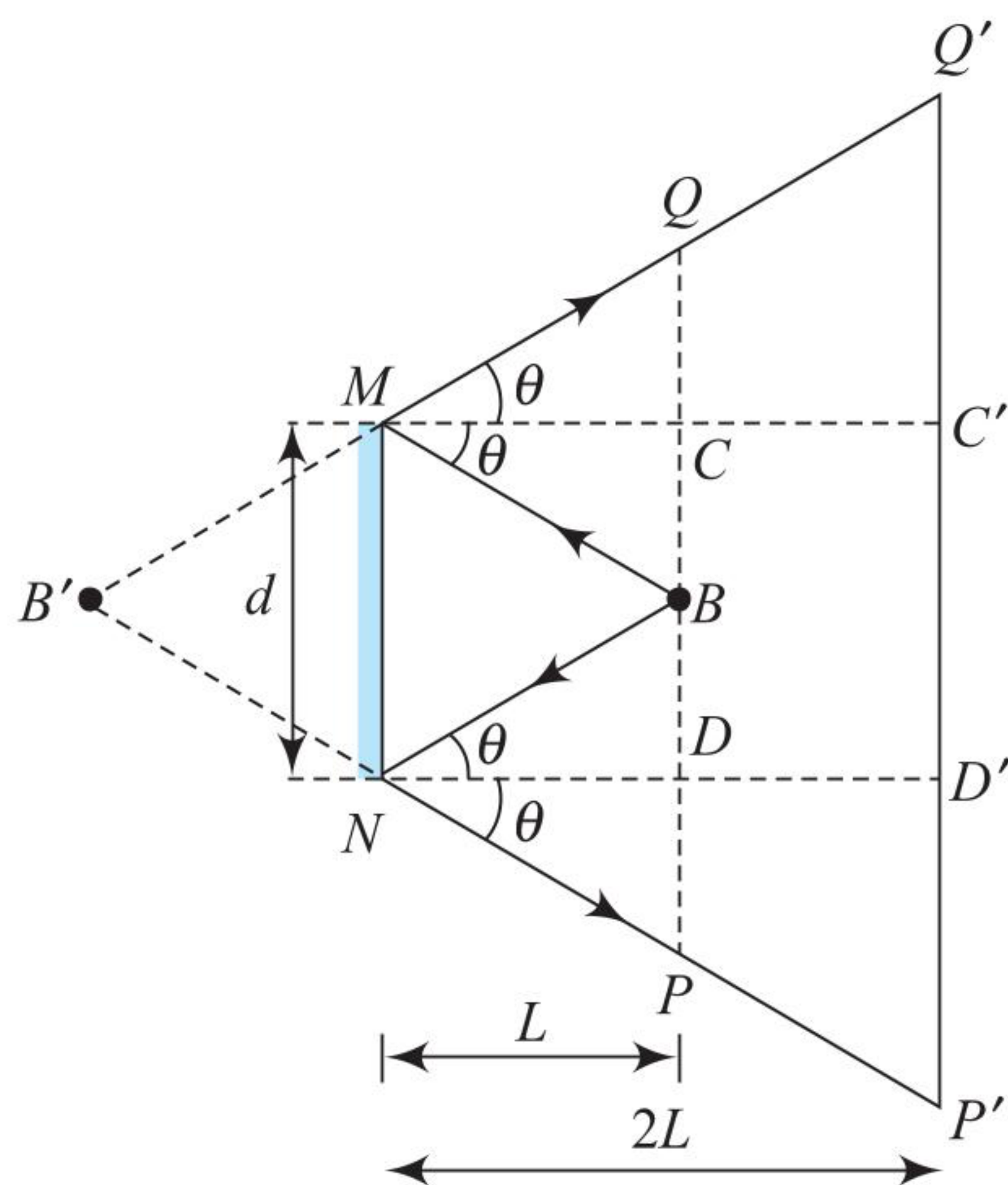
The meeting point of reflected rays R_1 and R_2 is image A' . Though only two rays are considered it must be understood that all rays from A reflect from mirror MN such that their meeting point is A' . To obtain the region in which reflected rays are present, join A' with the ends of mirror and extend. Figure (b) shows this region as shaded. In the figure, there are no reflected rays beyond the rays 1 and 2, therefore the observers P and Q will not be able to see the image because they do not receive any reflected ray.

ILLUSTRATION 1.6

A point source of light B is placed at a distance L in front of the centre of a mirror of width d hung vertically on a wall. A man walks in front of the mirror along a line parallel to the mirror at a distance $2L$ from it, as shown in figure. Find the greatest distance over which he can see the image of the light source from the mirror.



Sol. Draw the rays BM and BN incident on the mirror edges, and draw the reflected rays (see figure). From the figure, it is clear that the man can observe in the angular region $P'B'Q'$. The observer can see through the distance $P'Q'$, where reflected ray from B can meet the line.



From geometry, $CD = d = C'D'$

$$BD = DP = d/2; CB = QC = d/2$$

$$P'Q' = C'D' + P'D' + Q'C'$$

$$\Rightarrow \frac{ND}{DP} = \frac{ND'}{D'P'} \Rightarrow \frac{L}{d/2} = \frac{2L}{D'P'} \Rightarrow D'P' = d$$

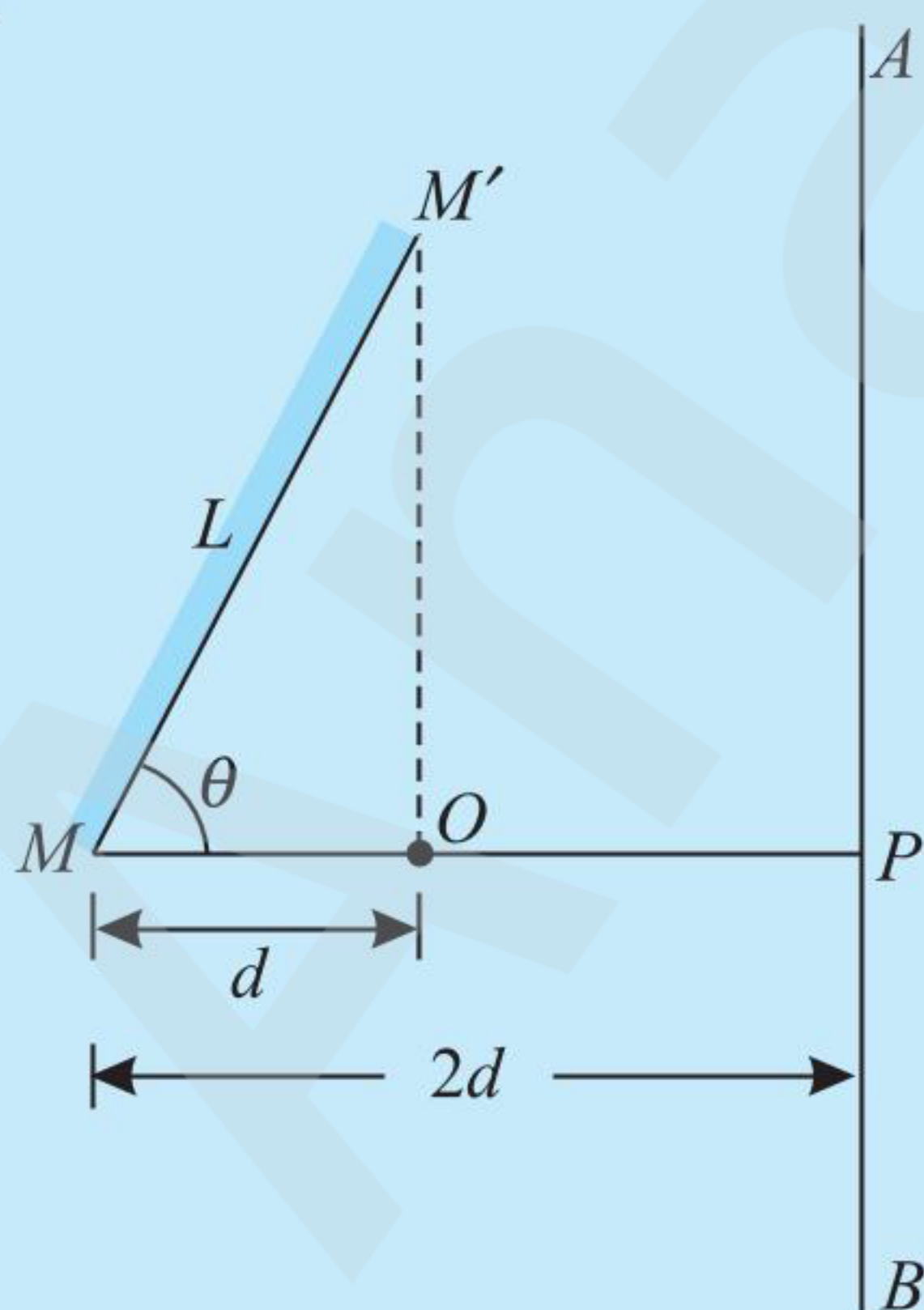
Similarly, $Q'C' = d$

Hence, distance through which the man can observe the image is

$$Q'C' + C'D' + D'P' = d + d + d = 3d$$

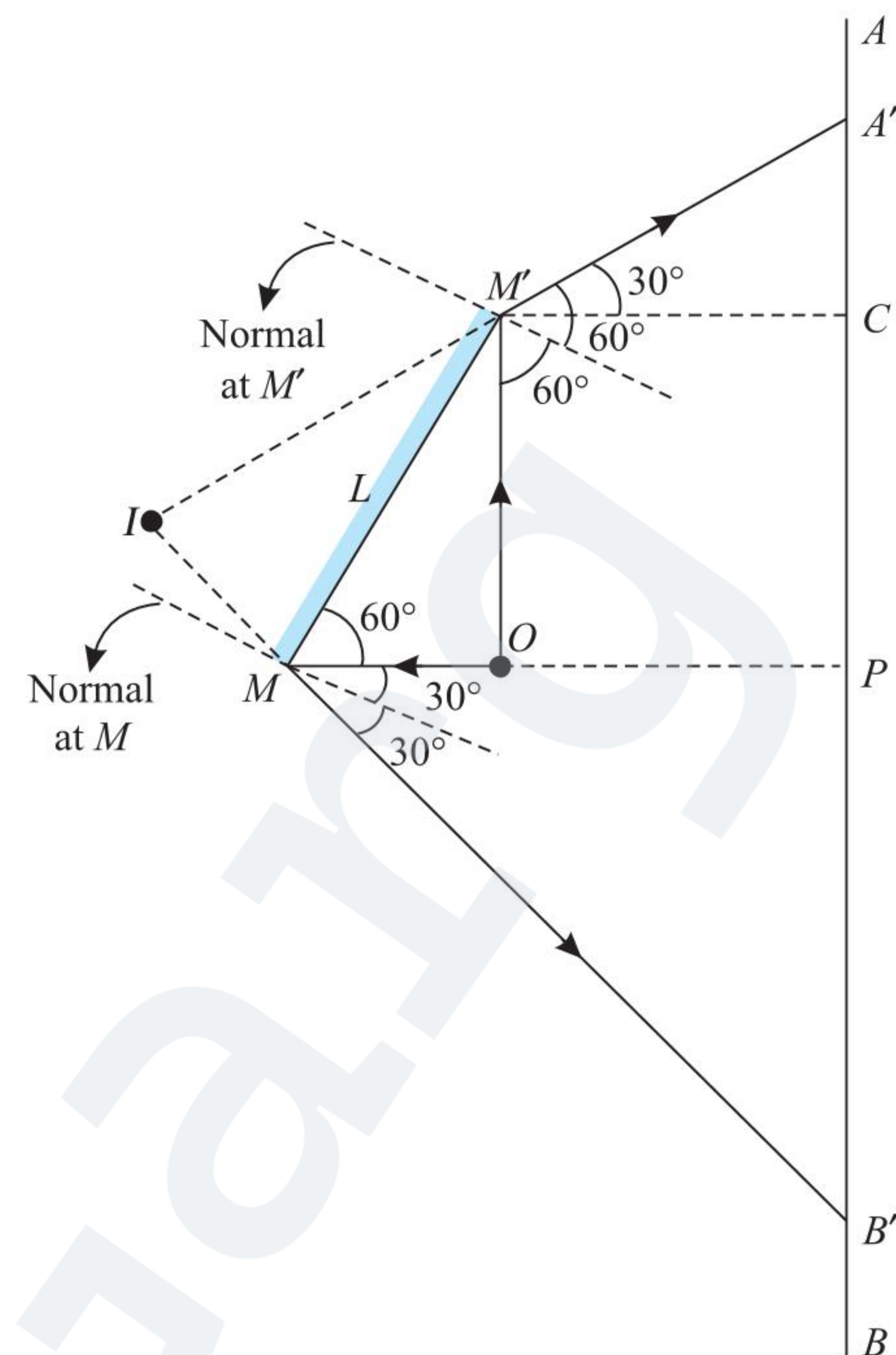
ILLUSTRATION 1.7

A point object O is kept in front of a plane mirror MM' . The line MOP makes an angle $\theta = 60^\circ$ with the mirror. An observer is walking along the line APB (perpendicular to MOP). Find the length of his path along APB in which he can see the image of the object. Given $MO = d$, $MP = 2d$ and length of the mirror $L = 2d$.



Sol. For find the length of path on the line AB , let us consider two rays from O , incident on the extreme ends of the mirror. Now plotting the ray diagram.

From ray diagram it is clear that one can see the image of point O , between points A' and B' on the line APB . Let us now calculate this length.



$$\text{In } \triangle MPB', PB' = MP \tan 60^\circ = 2d \times \sqrt{3} = 2\sqrt{3}d$$

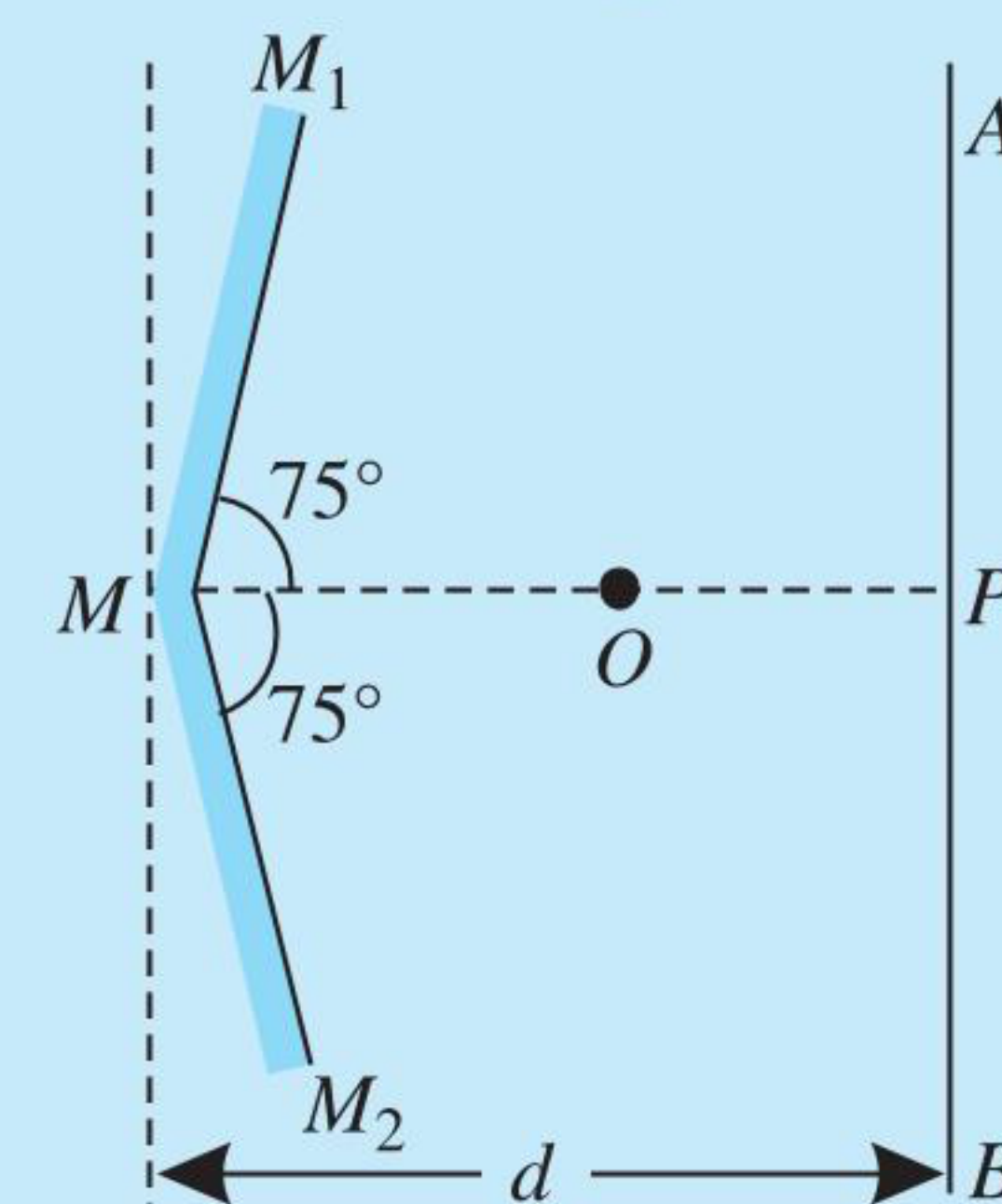
The length of section $A'P = A'C + CD$

$$A'P = d \tan 30^\circ + L \sin 60^\circ = d \times \frac{1}{\sqrt{3}} + 2d \times \frac{\sqrt{3}}{2} = \frac{4}{\sqrt{3}}d$$

$$\text{Hence } A'B' = A'P + PB' = \frac{4}{\sqrt{3}}d + 2\sqrt{3}d = \frac{10}{\sqrt{3}}d$$

ILLUSTRATION 1.8

Two large plane mirrors MM_1 and MM_2 are arranged at 150° as shown in the figure. O is a point object and AB is a long line perpendicular to the line OP . Find the length of the part of the line AB on which two images of the point object O can be seen.



Sol. The images of the point object O , formed by both the mirrors can be seen where the field of view of both the images overlap.

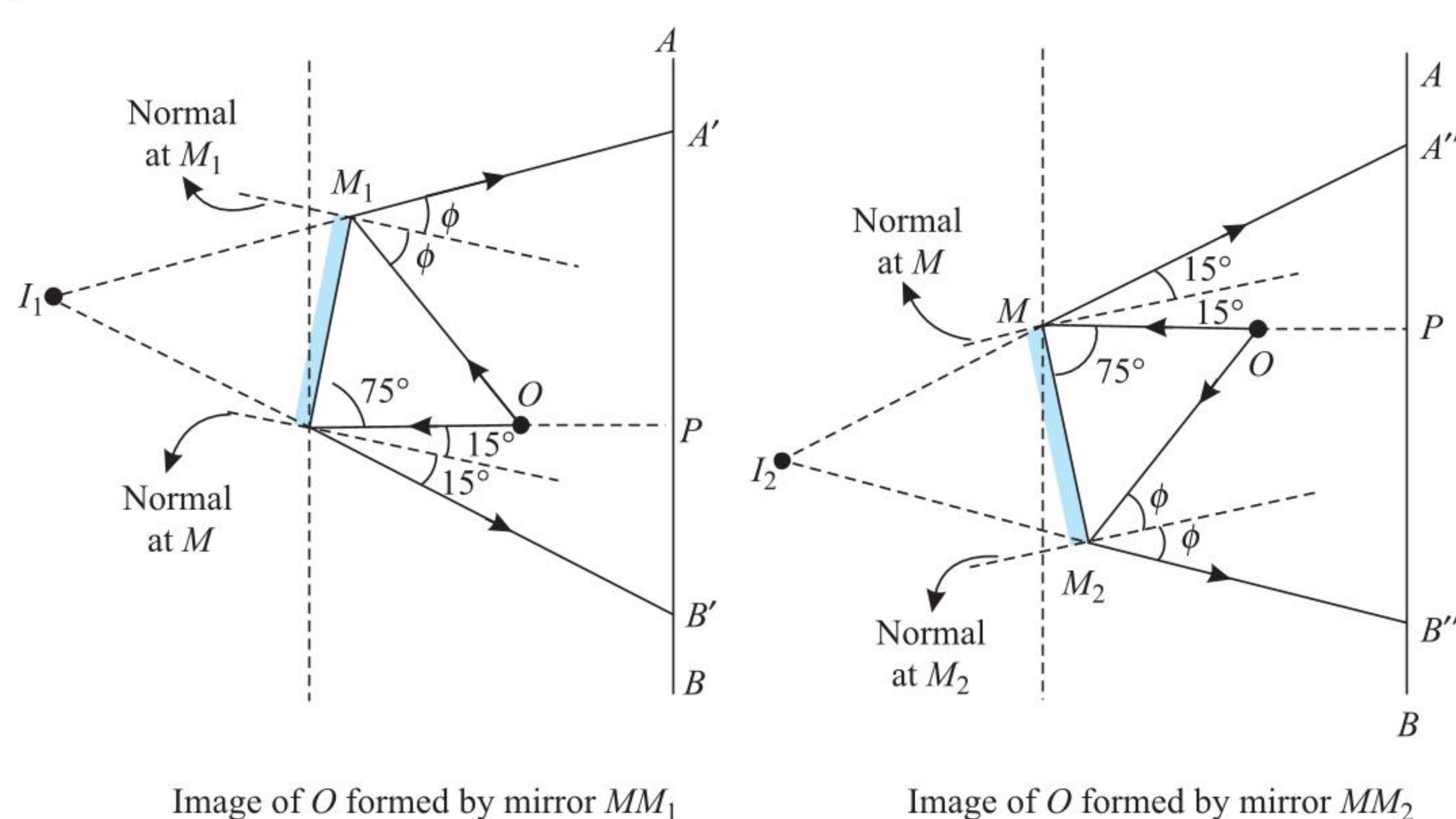
From ray diagram it is clear that overlapping region for field of view should be between $A''B'$.

Hence length of section $A''B' = 2PB'$

$$\text{In } \triangle MPB' : \frac{PB'}{MP} = \tan 30^\circ \Rightarrow PB' = MP \tan 30^\circ$$

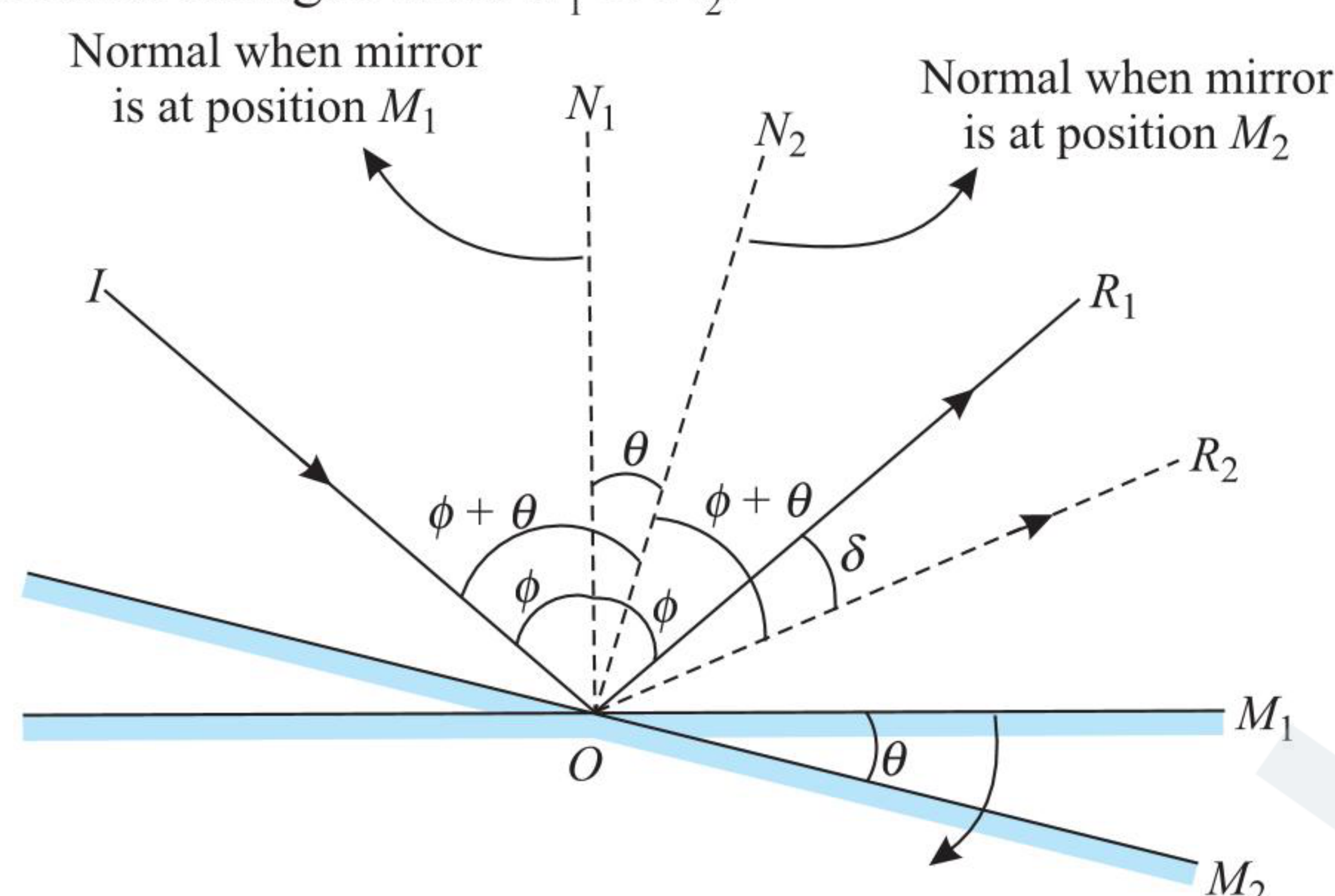
$$\text{or } PB' = d \times \frac{1}{\sqrt{3}} = \frac{d}{\sqrt{3}}$$

$$\text{Hence } A''B' = 2 \times \frac{d}{\sqrt{3}} = \frac{2d}{\sqrt{3}}$$



EFFECT OF ROTATION OF PLANE MIRROR

Direction of incident ray I is kept fixed while mirror M_1 is rotated by angle θ to position M_2 . Reflected ray R_1 changes to position R_2 and normal changes from N_1 to N_2 .



From figure it is clear that $\angle ABC = 2\phi + \delta = 2(\phi + \theta)$

or $\delta = 2\theta$

That means when incident ray is fixed, and mirror rotates through an angle θ :

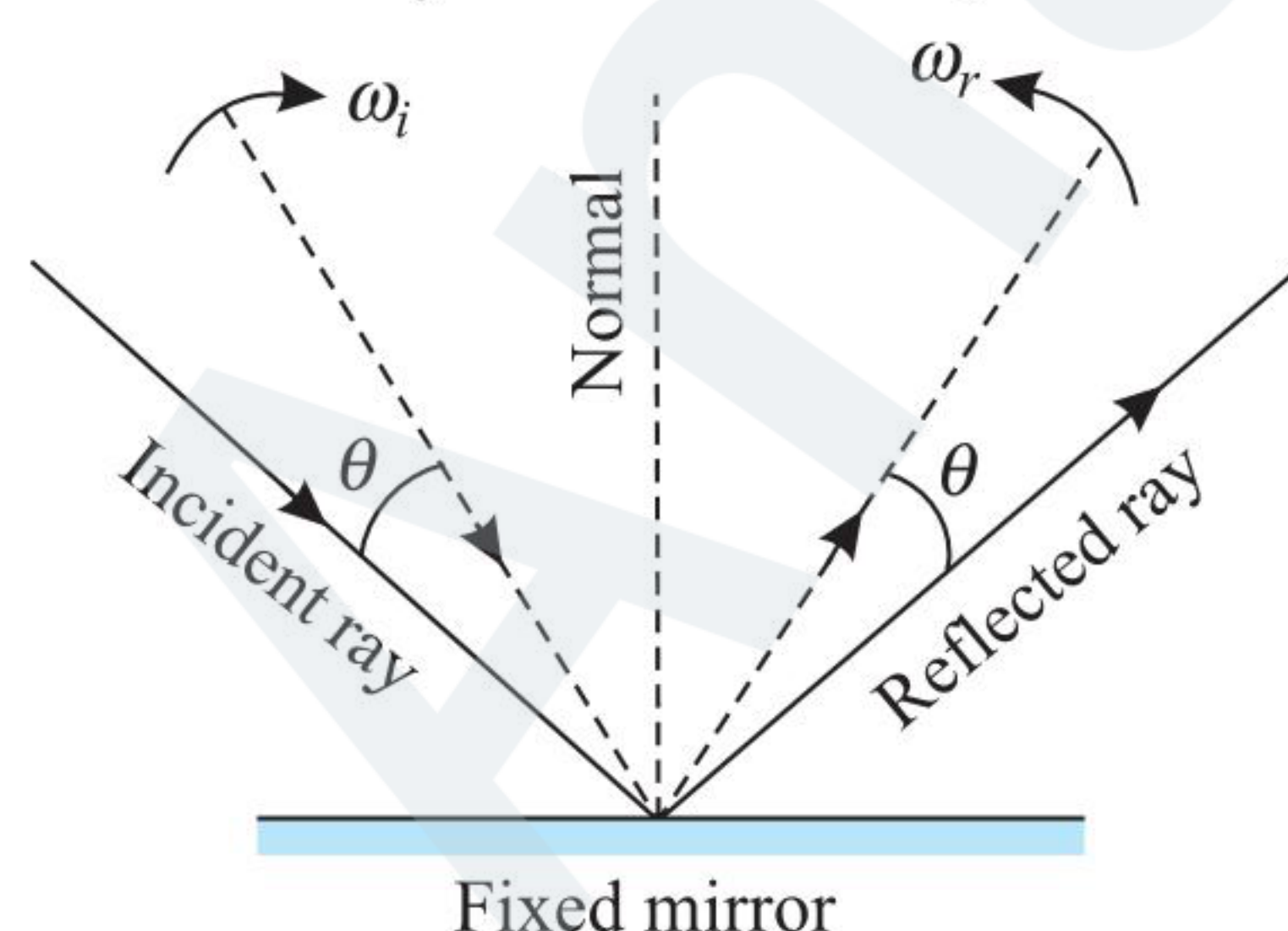
(a) Then reflected ray rotates through the angle 2θ in the same sense as the mirror rotates.

(b) The angular velocity and angular acceleration of new reflected ray becomes twice as that of mirror.

$$\omega_r = 2\omega_m \text{ and } \alpha_r = 2\alpha_m$$

(ω_r = angular velocity of reflected ray)

(ω_m = angular velocity of the mirror)



Note: If mirror is fixed and incident ray rotates:

ω_i = angular velocity of the incident ray

ω_r = angular velocity of reflected ray

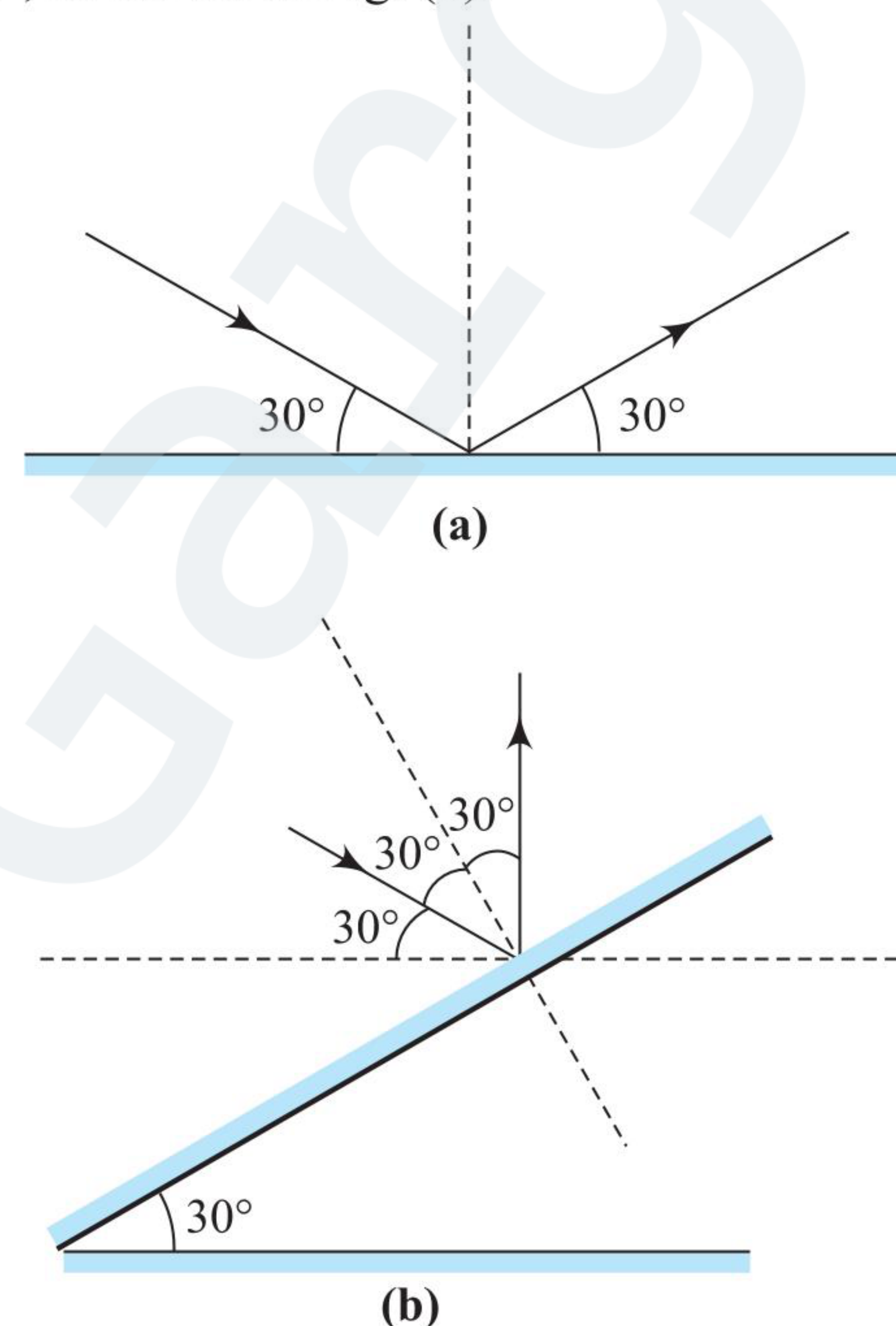
Then, $\omega_i = -\omega_r$

Negative sign represents that the direction of angular velocity is opposite to each other.

ILLUSTRATION 1.9

A light ray is incident on a plane mirror at an angle of 30° with the horizontal. At what angle with horizontal must a plane mirror be placed in its path so that it becomes vertically upwards after reflection?

Sol. To make the reflected ray vertical, it should be rotated through an angle of 60° . So, the mirror should be tilted by $60^\circ/2 = 30^\circ$, as shown in Fig. (b).



ANGLE OF DEVIATION

The angle of deviation is the angle made by the reflected ray with the direction of incident ray. Let us learn calculation of deviation angle through an illustration.

ILLUSTRATION 1.10

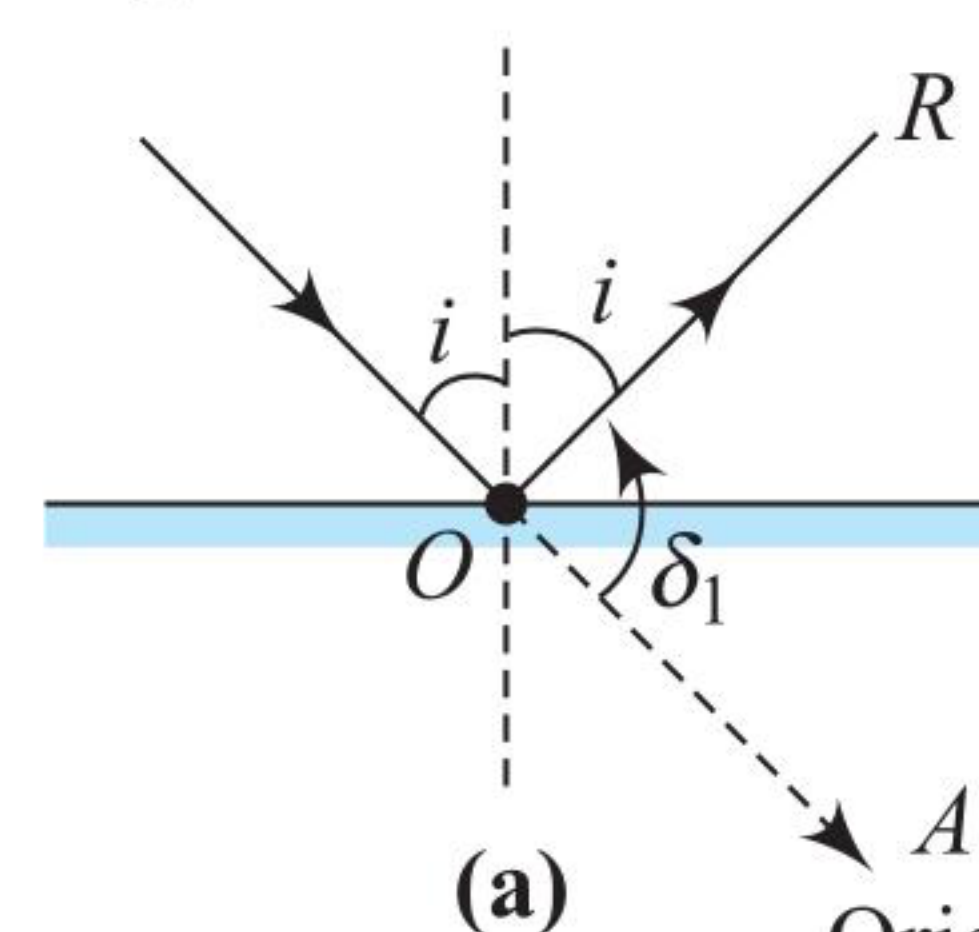
Show that for a light ray incident at an angle i on getting reflected the angle of deviation is $\delta = \pi - 2i$ or $\pi + 2i$.

Sol. From Fig. (b), it is clear that light ray bends either by δ_1 anticlockwise or by $\delta_2 (= 2\pi - \delta_1)$ clockwise.

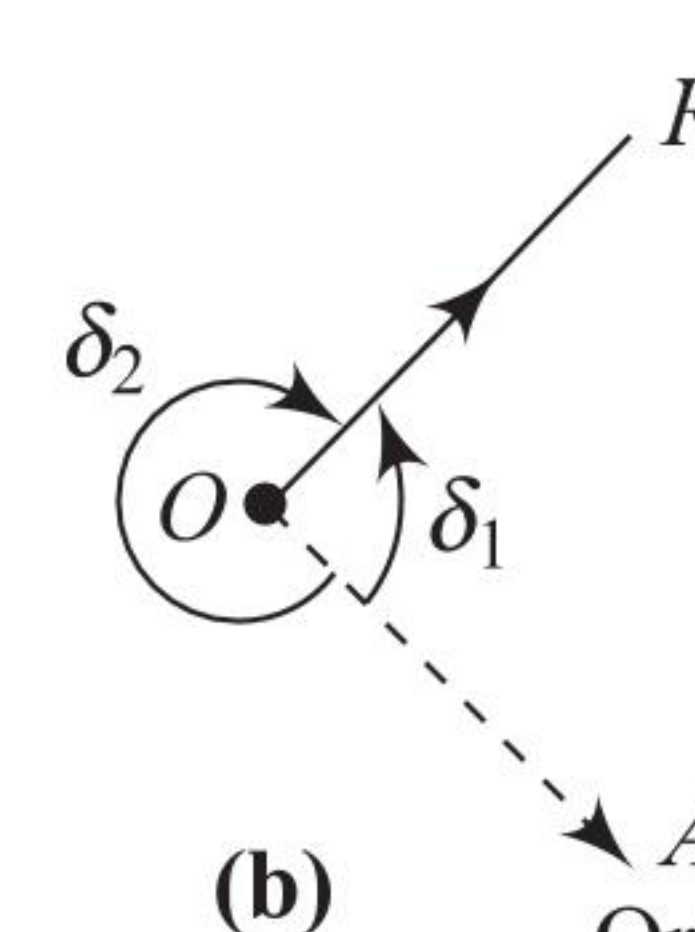
From Fig. (a),

$$\delta_1 = \pi - 2i$$

$$\therefore \delta_2 = 2\pi - (\pi - 2i) = \pi + 2i$$



Original direction of propagation



Original direction of propagation

FINDING ANGLE OF DEVIATION IN CASE OF SEVERAL MIRRORS

If several plane mirrors are placed on the path of a light ray, each mirror deviates the ray which can be clockwise or anticlockwise. Net deviation is the algebraic sum of the deviations due to the individual mirrors. In such cases, clockwise and anticlockwise deviations are given opposite signs. We take anticlockwise deviations as positive and clockwise deviations as negative.

ILLUSTRATION 1.11

Two plane mirrors are inclined to each other at an angle θ . A ray of light is reflected first at one mirror and then at the other. Find the total deviation of the ray.

Sol. Let,

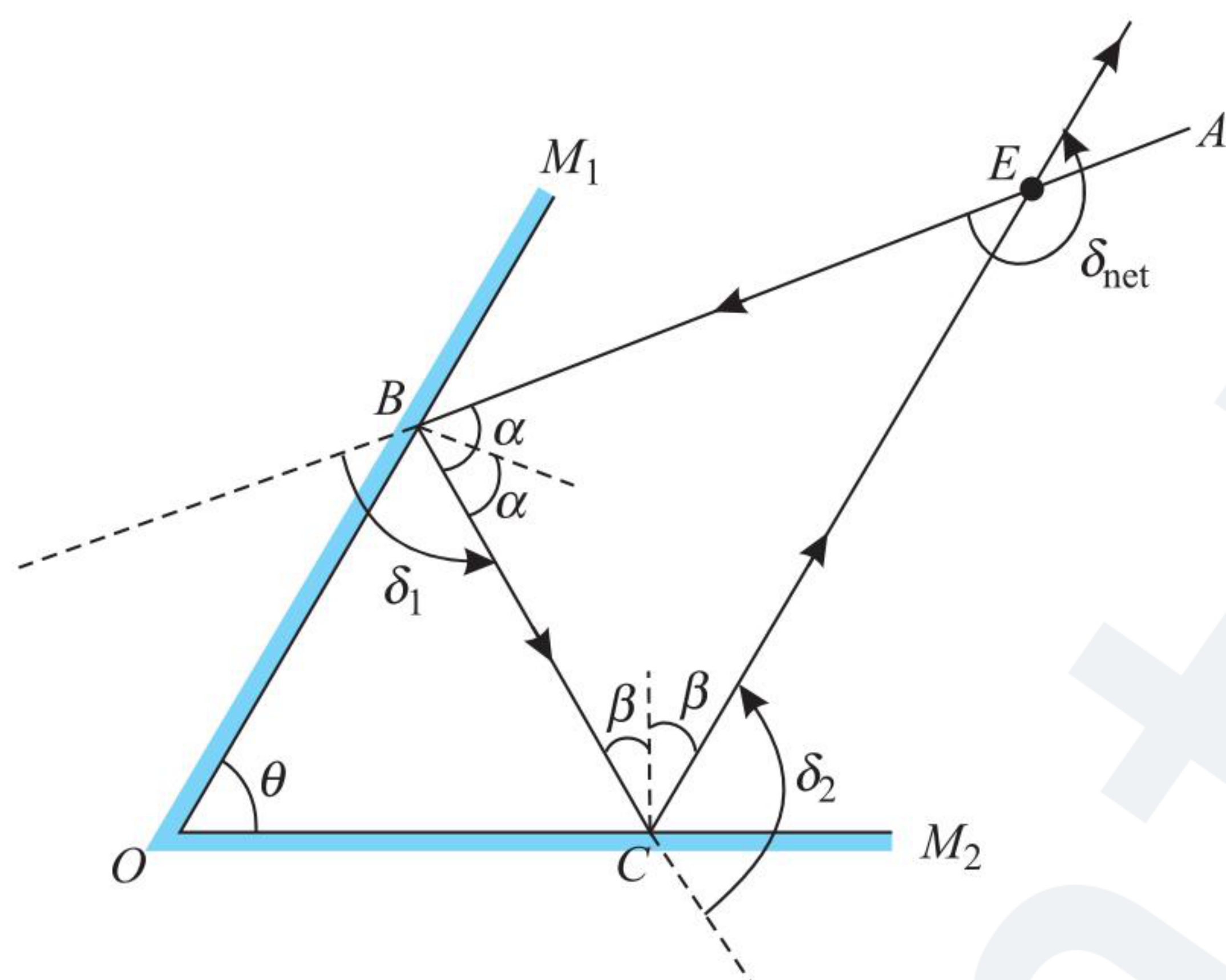
α = Angle of incidence for M_1

β = Angle of incidence for M_2

δ_1 = Deviation due to M_1

δ_2 = Deviation due to M_2

From figure, $\delta_1 = \pi - 2\alpha$, $\delta_2 = \pi - 2\beta$



Both deviations are in same sense i.e., anticlockwise

$$\delta_{\text{net}} = \text{Total deviation} = \delta_1 + \delta_2 = 2\pi - 2(\alpha + \beta)$$

Now in $\triangle OBC$,

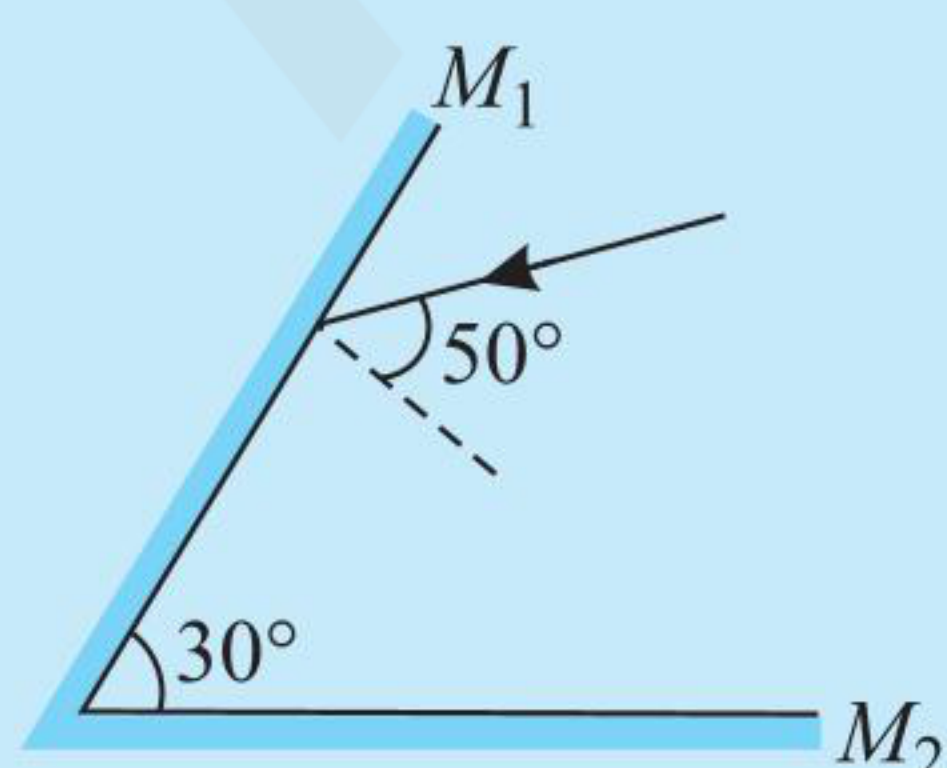
$$(90^\circ - \alpha) + (90^\circ - \beta) + \theta = 180^\circ$$

$$\text{or } \alpha + \beta = \theta$$

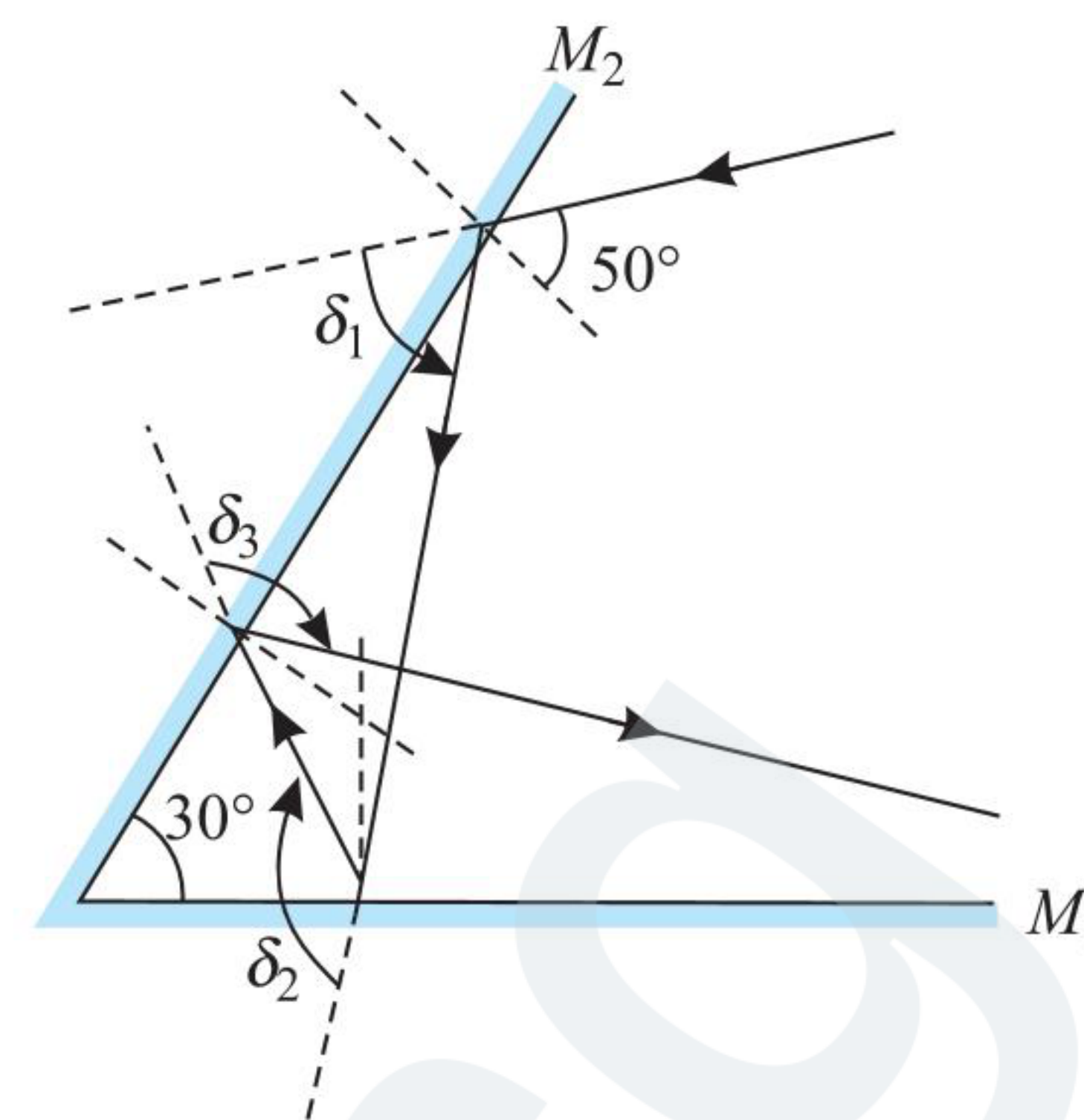
Hence, $\delta_{\text{net}} = 2\pi - 2\theta$ in anticlockwise sense and 2θ in clockwise sense, which evidently depends only on θ , the angle between the two mirrors.

ILLUSTRATION 1.12

Calculate deviation suffered by incident ray in situation as shown in figure after three successive reflections.



Sol. Net deviation is the algebraic sum of the deviations due to the individual mirrors. From figure,



$$\delta_1 = 180^\circ - 2 \times 50^\circ = 80^\circ \text{ anticlockwise}$$

$$\delta_2 = 180^\circ - 2 \times 20^\circ = 140^\circ \text{ clockwise}$$

$$\delta_3 = 180^\circ - 2 \times 10^\circ = 160^\circ \text{ clockwise}$$

Total deviation $\delta = \delta_1 + \delta_2 + \delta_3$. Taking anticlockwise sense of rotation positive and clockwise rotation as negative, we get

$$\delta = 80^\circ - 140^\circ - 160^\circ = -220^\circ$$

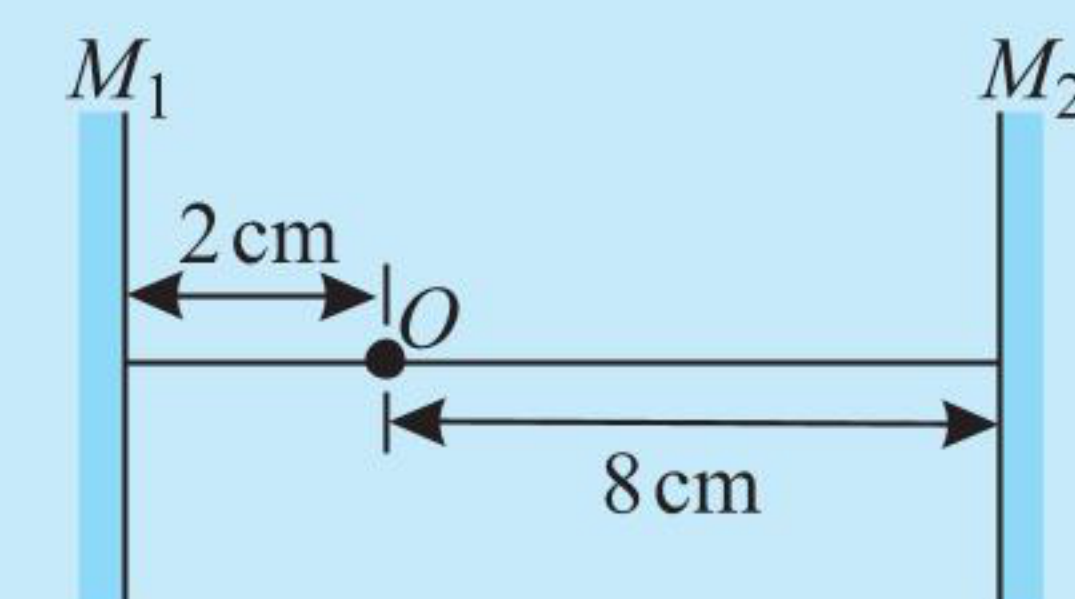
220° clockwise or 140° anticlockwise.

IMAGES FORMED BY TWO PLANE MIRRORS

If rays after getting reflected from one mirror strike second mirror, the image formed by first mirror will act as an object for second mirror, and this process will continue for every successive reflection. Let us understand this with the illustration discussed below.

ILLUSTRATION 1.13

Figure shows a point object placed between two parallel mirrors. Its distance from M_1 is 2 cm and that from M_2 is 8 cm. Find the distance of images from the two mirrors considering reflection on mirror M_1 first.



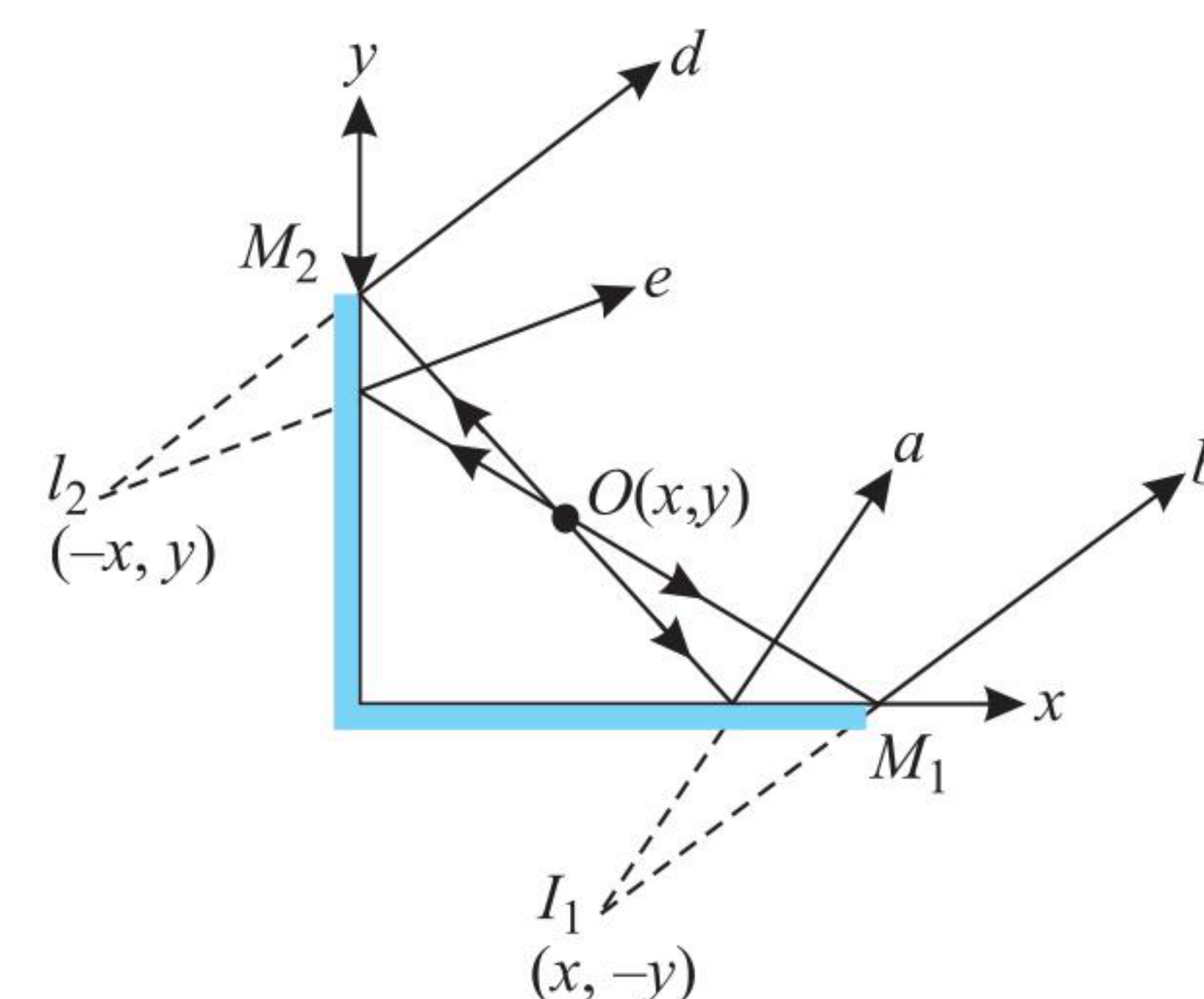
Sol. The image formed from the object will act as an object for next reflection. This process will continue again and again up to infinite reflection. Total number of images = ∞ .

ILLUSTRATION 1.14

Consider two perpendicular mirrors M_1 and M_2 and a point object O . Taking origin at the point of intersection of the mirrors and the coordinates of object as (x, y) , find the position and number of images.

Sol. We will first consider reflection from mirror-1 (M_1).

Let two rays a and b strike mirror M_1 and these rays will form image I_1 at $(x, -y)$, such that O and I_1 are equidistant from mirror M_1 . These rays do not form further image because they do not strike any mirror again. Now consider rays ' d ' and ' e ' strike mirror M_2 only and

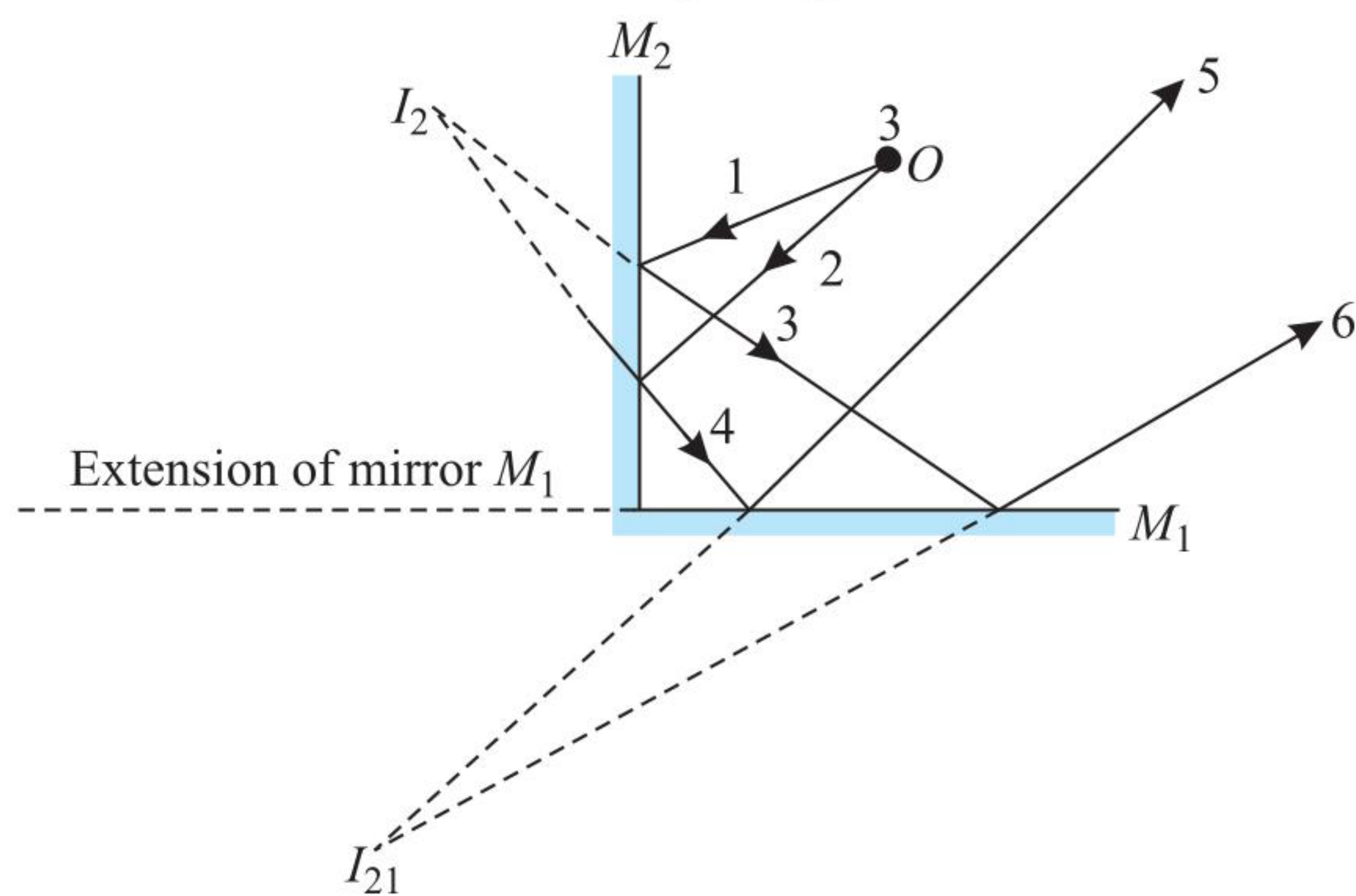


these rays will form image I_2 at $(-x, y)$, such that O and I_2 are equidistant from mirror M_2 .

Now, let us consider those rays which strike mirror M_2 first and then mirror M_1 . For incident rays 1 and 2 object is O , and the reflected rays 3 and 4 form image I_2 .

Now these reflected rays 3 and 4 incident on M_1 for which the object is I_2 , reflect as rays 5 and 6 and form image I_{21} . Rays 5 and 6 do not strike any mirror further, so image formation stops.

The images I_2 and I_{21} are equidistant from M_1 . Let us now summarize the situation in a single figure.



For rays reflecting first from M_1 and then from M_2 , first image I_1 (at $(x, -y)$) will be formed. I_1 will act as object for mirror M_2 and then its image I_{12} (at $(-x, -y)$) will be formed. I_{12} and I_{21} coincide. Hence, three images are formed.

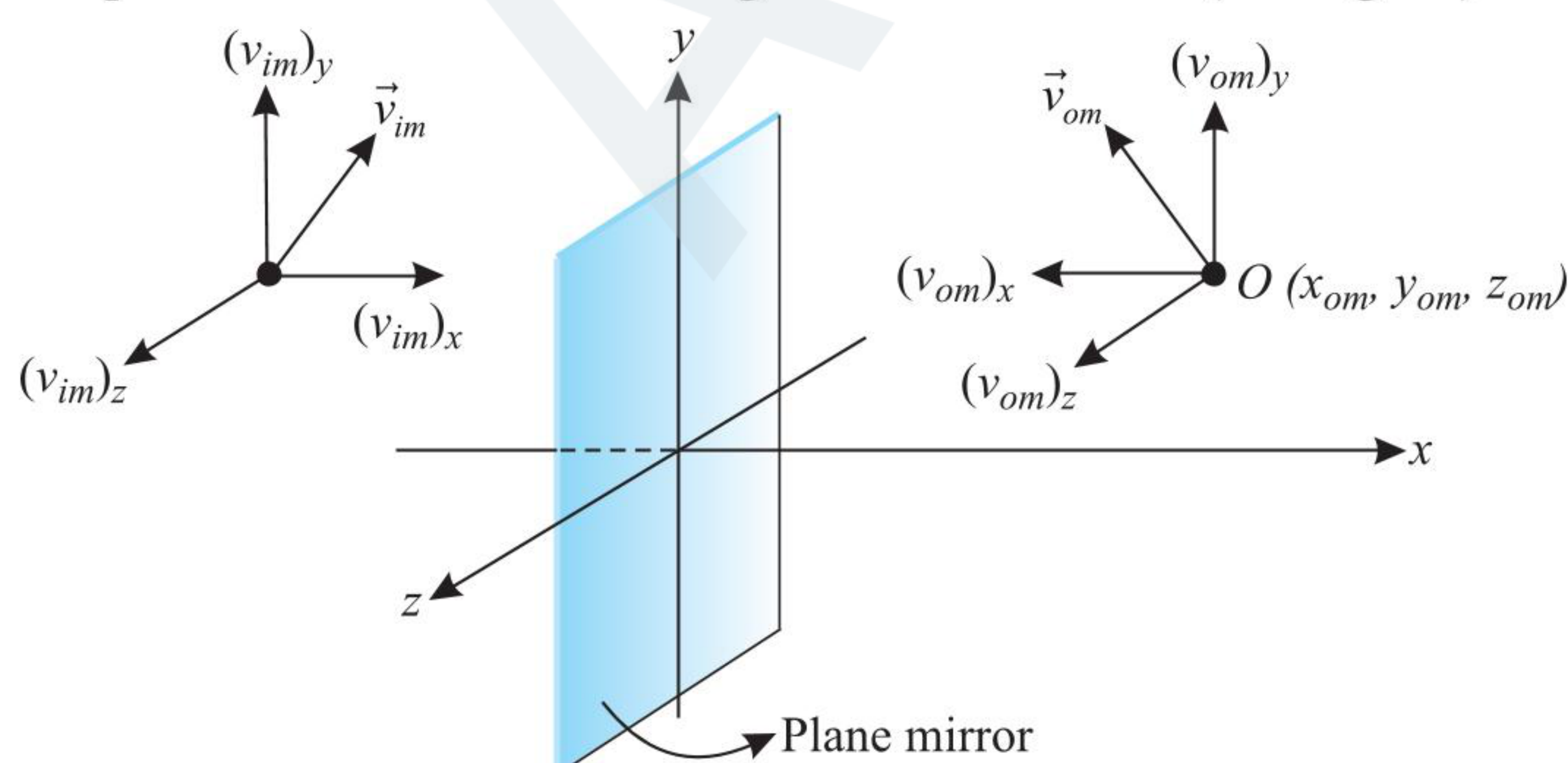
Important Points:

Number of images formed by two inclined mirrors:

1. If $\frac{360^\circ}{\theta} = \text{even number}$; number of images $= \frac{360^\circ}{\theta} - 1$
2. If $\frac{360^\circ}{\theta} = \text{odd number}$; number of images $= \frac{360^\circ}{\theta} - 1$, if the object is placed on the angle bisector.
3. If $\frac{360^\circ}{\theta} = \text{odd number}$; number of images $= \frac{360^\circ}{\theta}$, if the object is not placed on the angle bisector.
4. If $\frac{360^\circ}{\theta} \neq \text{integer}$, then count the number of images as explained above.

RELATION BETWEEN VELOCITY OF OBJECT AND IMAGE

In case of plane mirror, the distance of the object from the mirror is equal to the distance of image from the mirror (see figure).



Hence, from the mirror property:

$$x_{\text{im}} = -x_{\text{om}}, \quad y_{\text{im}} = y_{\text{om}} \quad \text{and} \quad z_{\text{im}} = z_{\text{om}}$$

Here, x_{im} means x -coordinate of image with respect to mirror. Similarly, others have also meaning.

Differentiating w.r.t. time, we get

$$\vec{v}_{(\text{im})x} = -\vec{v}_{(\text{om})x}; \quad \vec{v}_{(\text{im})y} = \vec{v}_{(\text{om})y}; \quad \vec{v}_{(\text{im})z} = \vec{v}_{(\text{om})z}$$

Here, $\vec{v}_{(\text{om})}$ = velocity of the object w.r.t. mirror

$\vec{v}_{(\text{im})}$ = velocity of the image w.r.t. mirror

$$\Rightarrow \vec{v}_i - \vec{v}_m = -(\vec{v}_o - \vec{v}_m)$$

Here, $\vec{v}_i$ = velocity of image with respect to ground

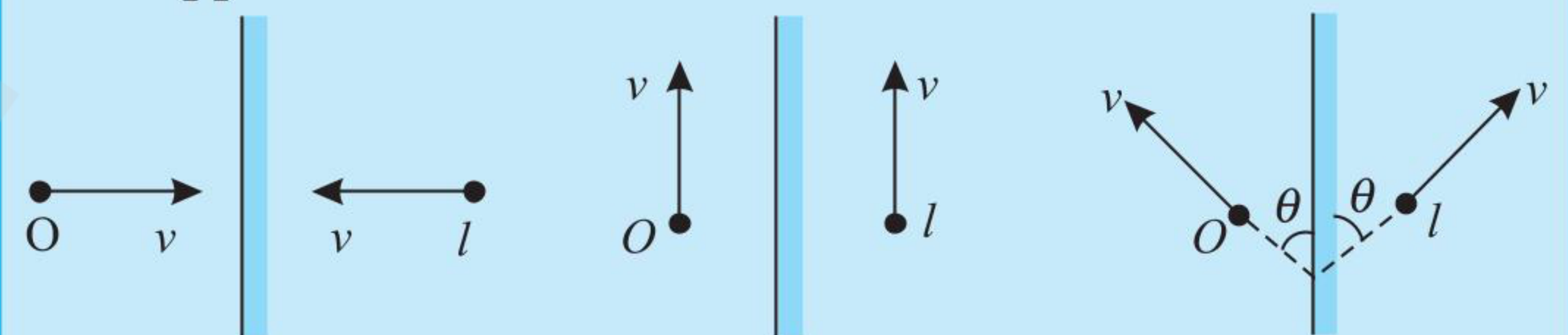
$\vec{v}_o$ = velocity of object w.r.t. ground

If mirror is at rest $\vec{v}_m = 0 \Rightarrow \vec{v}_i = -\vec{v}_o$

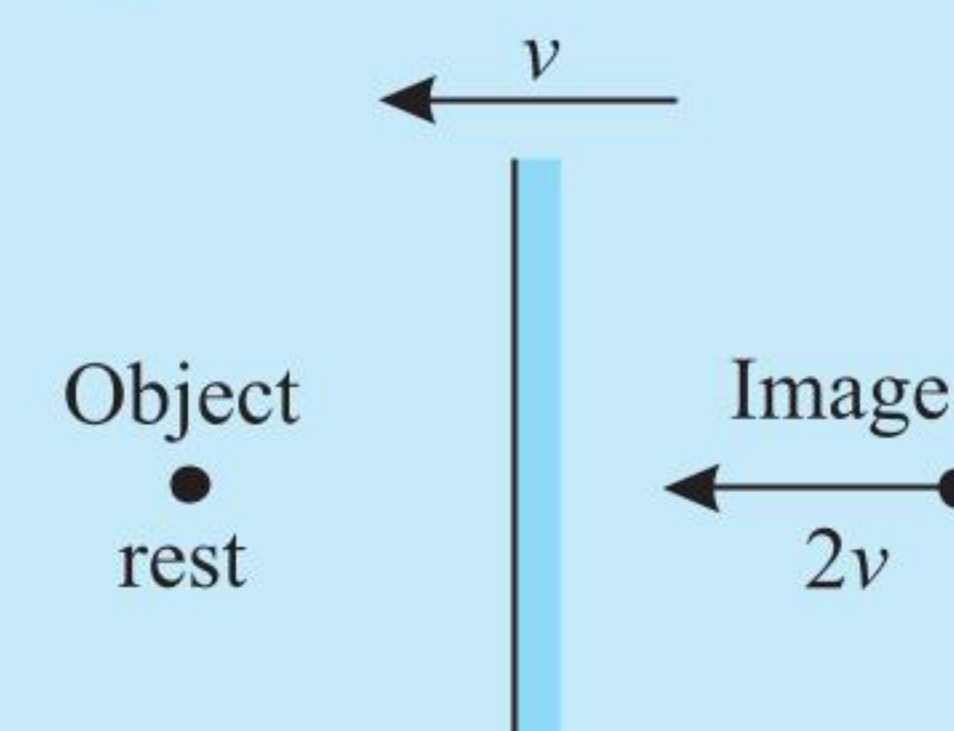
In the direction normal to the mirror = | Relative velocity of image w.r.t. mirror | = | Relative velocity of object w.r.t. mirror |
The velocity of object is equal to the velocity of image parallel to the mirror surface.

Important Points:

- With respect to plane mirror image speed is equal to the object speed.
- With respect to plane mirror image velocity and object velocity make equal angles from the plane mirror on two opposite sides of the mirror.



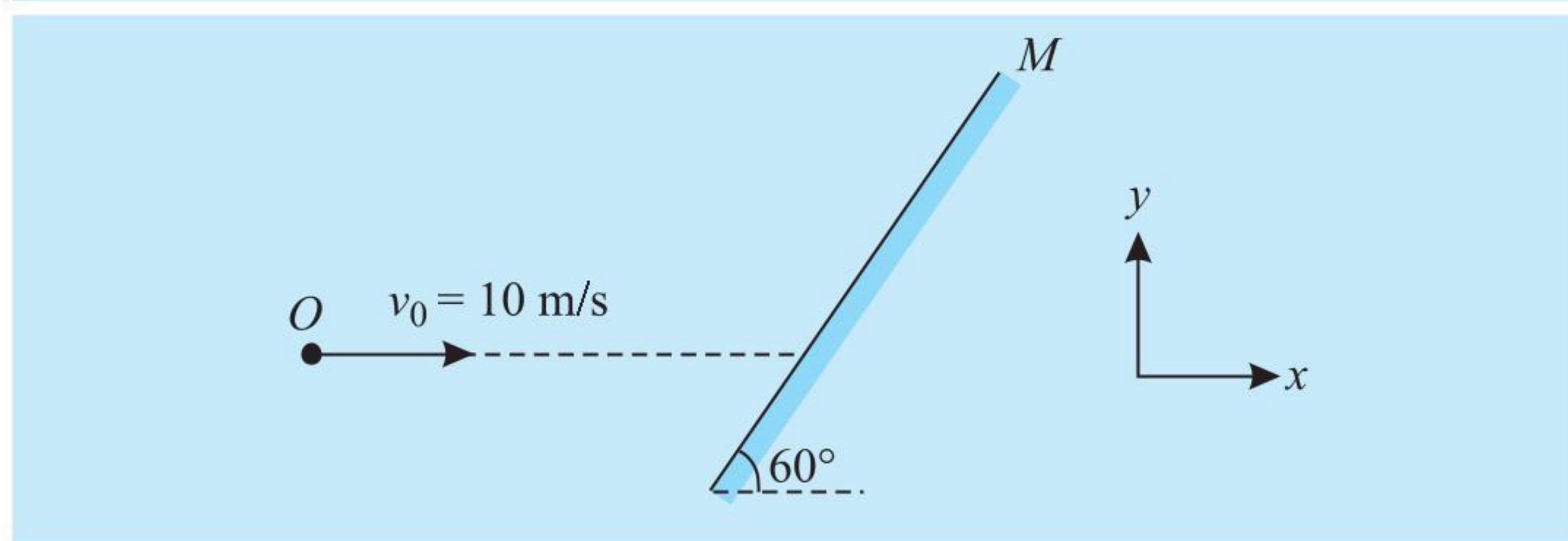
- Components of velocities of the object and image with respect to mirror which are along the mirror are equal.
- Components of velocities of the object and image with respect to mirror which are perpendicular to the mirror are equal and opposite.
- If a plane mirror moves towards a stationary object with speed v , the image moves towards the object with speed $2v$.



- If an angle moves towards (or moves away) from a stationary plane mirror with speed v , the image will also moves towards (or moves away) with same speed. The relative velocity of the image with respect to object will be $2v$.

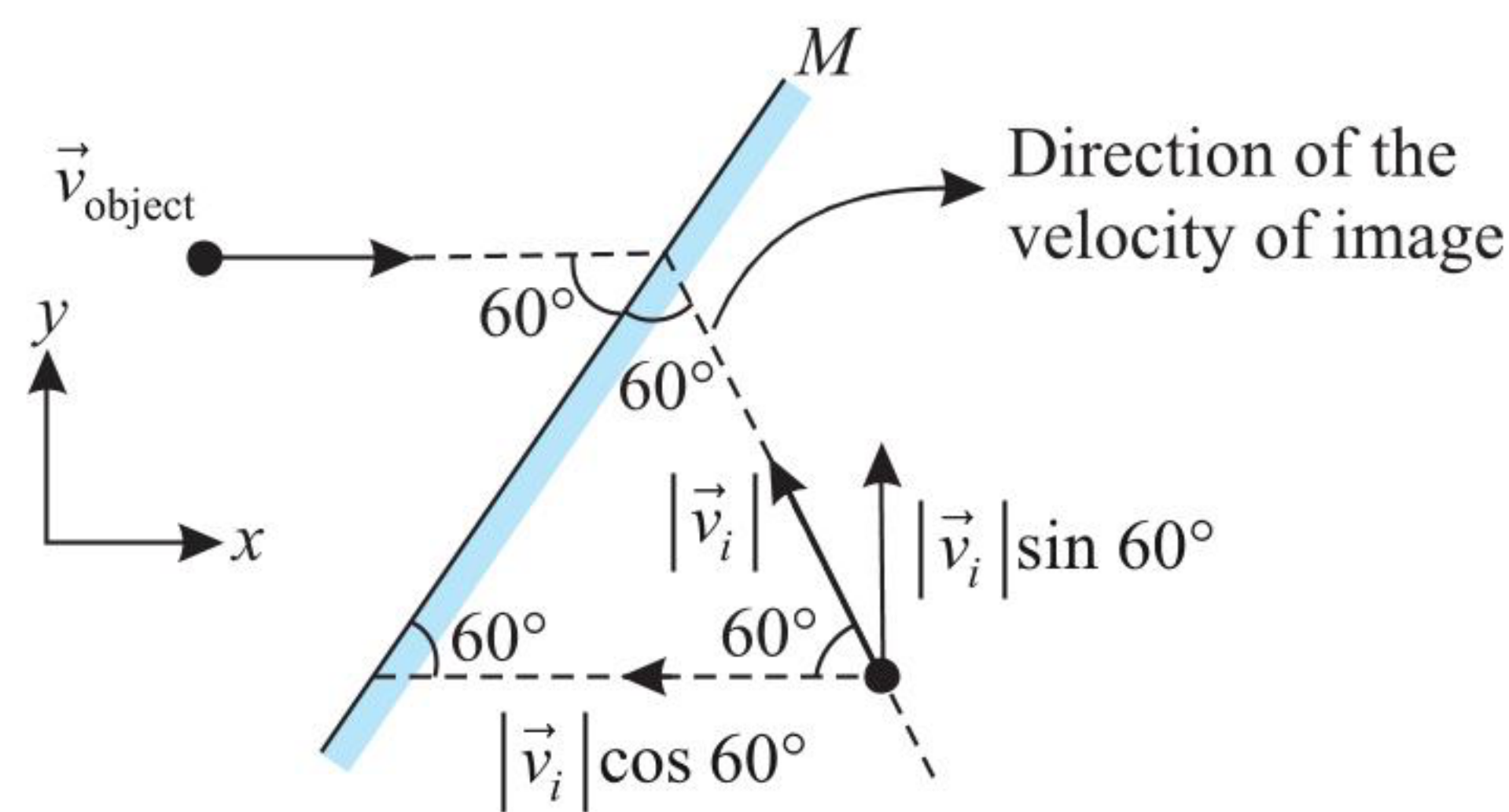
ILLUSTRATION 1.15

An object is moving horizontally with velocity 10 m/s towards a fixed plane mirror as shown in figure. Find the image velocity.



Sol. The image velocity and object velocity should make equal angle from the plane mirror on two opposite sides of the mirror, and speed of the object and image will be equal i.e.,

$$|\vec{v}_{i,m}| = |\vec{v}_{o,m}|$$

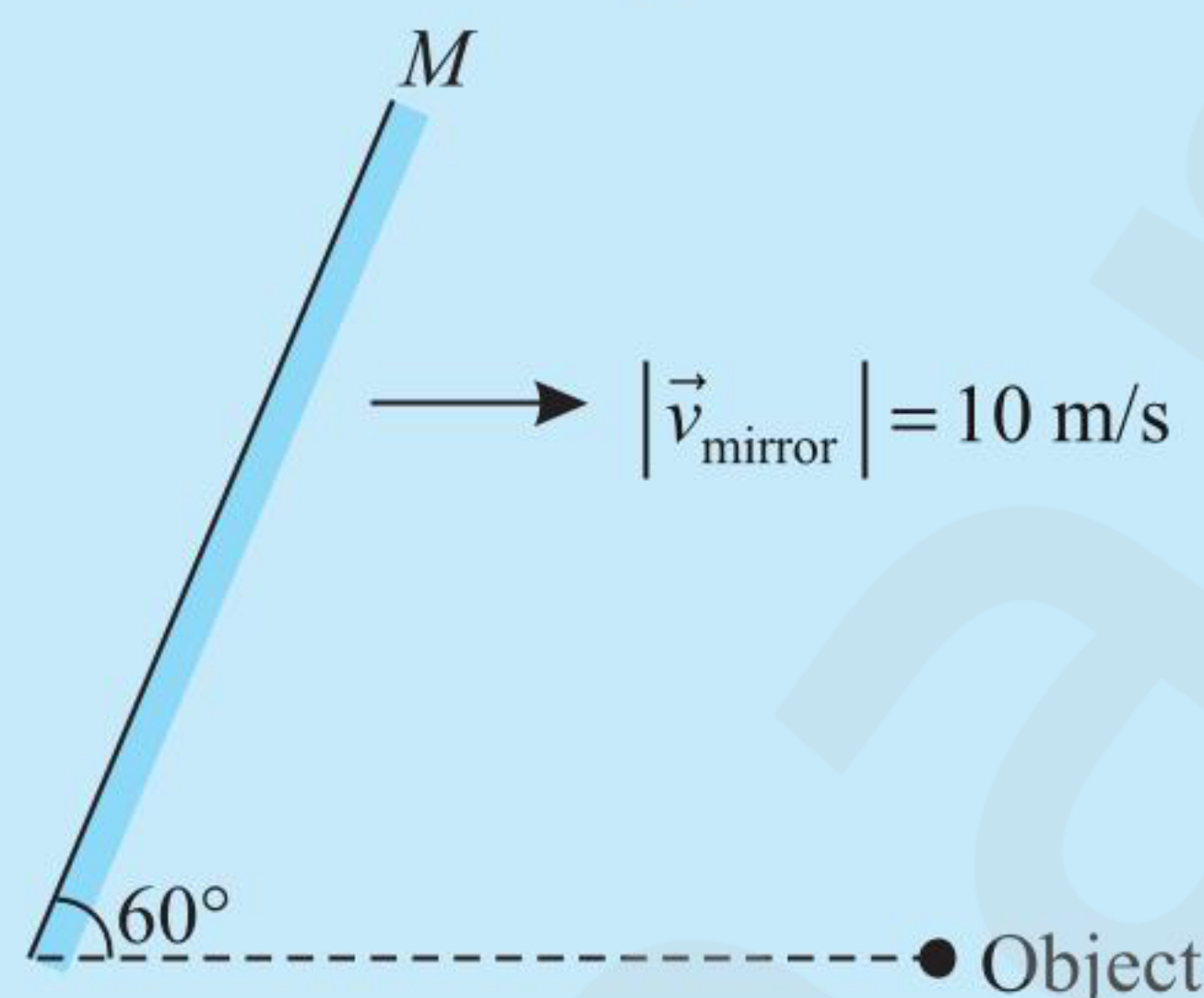


As mirror is at rest hence

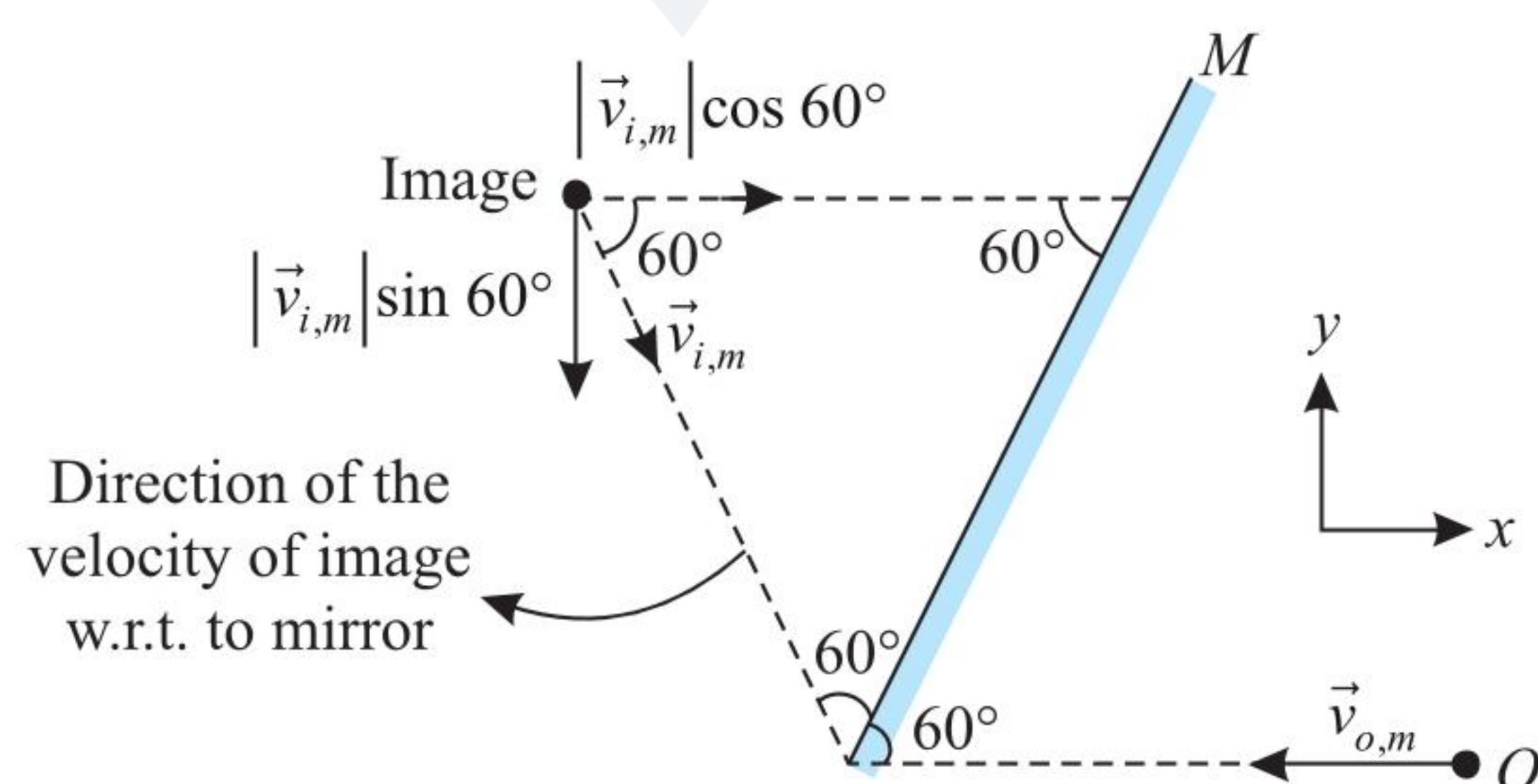
$$\begin{aligned} |\vec{v}_i| &= |\vec{v}_o| \\ \vec{v}_{\text{image}} &= |\vec{v}_i| \cos 60^\circ (-\hat{i}) + |\vec{v}_i| \sin 60^\circ \hat{j} \\ &= 10 \times \frac{1}{2} (-\hat{i}) + 10 \times \frac{\sqrt{3}}{2} (\hat{j}) \text{ (m/s)} \\ \vec{v}_{\text{image}} &= -5\hat{i} + 5\sqrt{3}\hat{j} \text{ (m/s)} \end{aligned}$$

ILLUSTRATION 1.16

A plane mirror is moving with speed 10 m/s towards an object which is at rest as shown in figure. Find the velocity of the image.



Sol. Let us analyse the motion of both the object and image w.r.t. mirror. In mirror frame of reference the object will move with velocity 10 m/s towards left. As we know, the image velocity and object velocity should make equal angle from the surface of the mirror on two opposite sides. The speed of the image with respect to mirror should be equal to the speed of the object with respect to mirror.



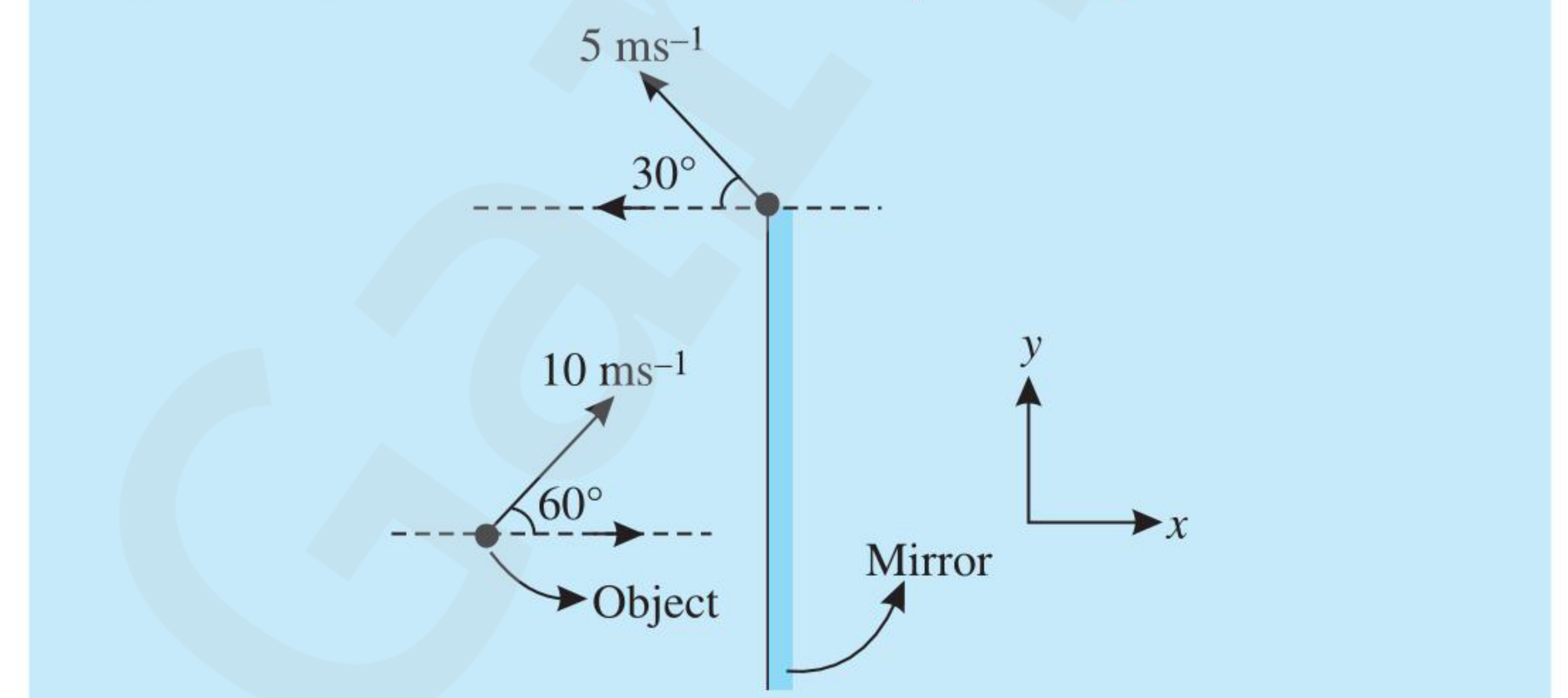
$$\begin{aligned} \text{Hence } \vec{v}_{i,m} &= |\vec{v}_{i,m}| \cos 60^\circ \hat{i} + |\vec{v}_{i,m}| \sin 60^\circ (-\hat{j}) \\ &= 10 \times \frac{1}{2} \hat{i} - 10 \times \frac{\sqrt{3}}{2} \hat{j} = (5\hat{i} - 5\sqrt{3}\hat{j}) \text{ m/s} \end{aligned}$$

$$\text{We can write, } \vec{v}_i = \vec{v}_{i,m} + \vec{v}_m = (5\hat{i} - 5\sqrt{3}\hat{j}) + 10\hat{i}$$

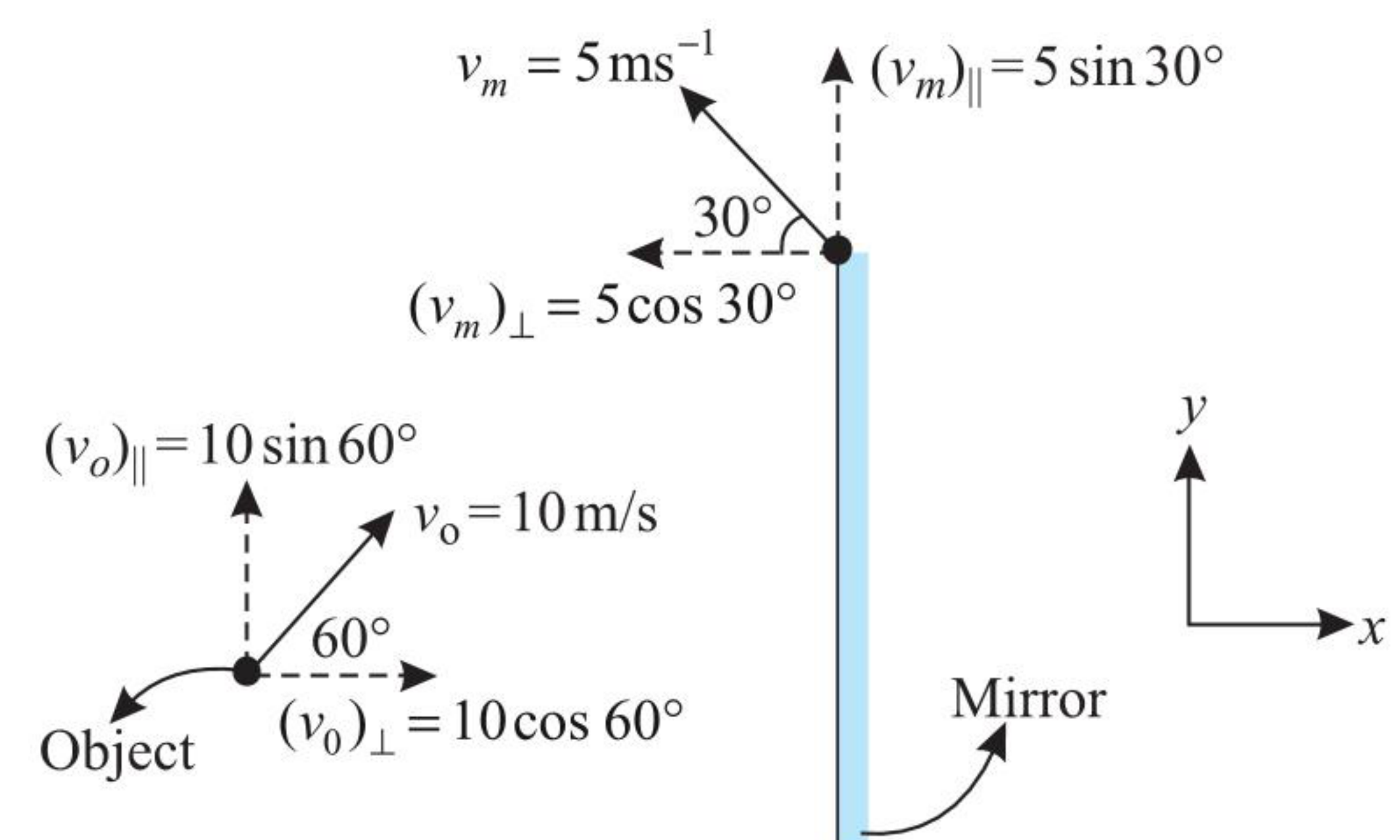
$$\Rightarrow \vec{v}_{\text{image}} = (15\hat{i} - 5\sqrt{3}\hat{j}) \text{ m/s}$$

ILLUSTRATION 1.17

Figure shows a plane mirror and an object that are moving towards each other. Find the velocity of image.



Sol. In the direction parallel to the surface of the mirror along y-axis.



$$(\vec{v}_{o,m})_{\parallel} = (\vec{v}_{i,m})_{\parallel}$$

$(\vec{v}_{o,m})_{\parallel}$ = Relative velocity of the object w.r.t. mirror parallel to its surface.

$$\text{Hence } (\vec{v}_o - \vec{v}_m)_{\parallel} = (\vec{v}_i - \vec{v}_m)_{\parallel} \text{ or } (\vec{v}_o)_{\parallel} = (\vec{v}_i)_{\parallel}$$

Hence velocity of the image parallel to the surface of the mirror should be

$$(\vec{v}_i)_{\parallel} = 10 \sin 60^\circ \hat{j} = 5\sqrt{3} \hat{j} \text{ (m/s)}$$

In the direction normal to the mirror or along x-axis.

$$(\vec{v}_{i,m})_{\perp} = -(\vec{v}_{o,m})_{\perp}$$

$(\vec{v}_{i,m})_{\perp}$ = Relative velocity of the image w.r.t. mirror perpendicular to the surface

$(\vec{v}_{o,m})_{\perp}$ = Relative velocity of the object w.r.t. mirror perpendicular to the surface

$$\text{Hence } (\vec{v}_i - \vec{v}_m)_{\perp} = -(\vec{v}_o - \vec{v}_m)_{\perp} \Rightarrow (\vec{v}_i)_{\perp} = 2(\vec{v}_m)_{\perp} - (\vec{v}_o)_{\perp}$$

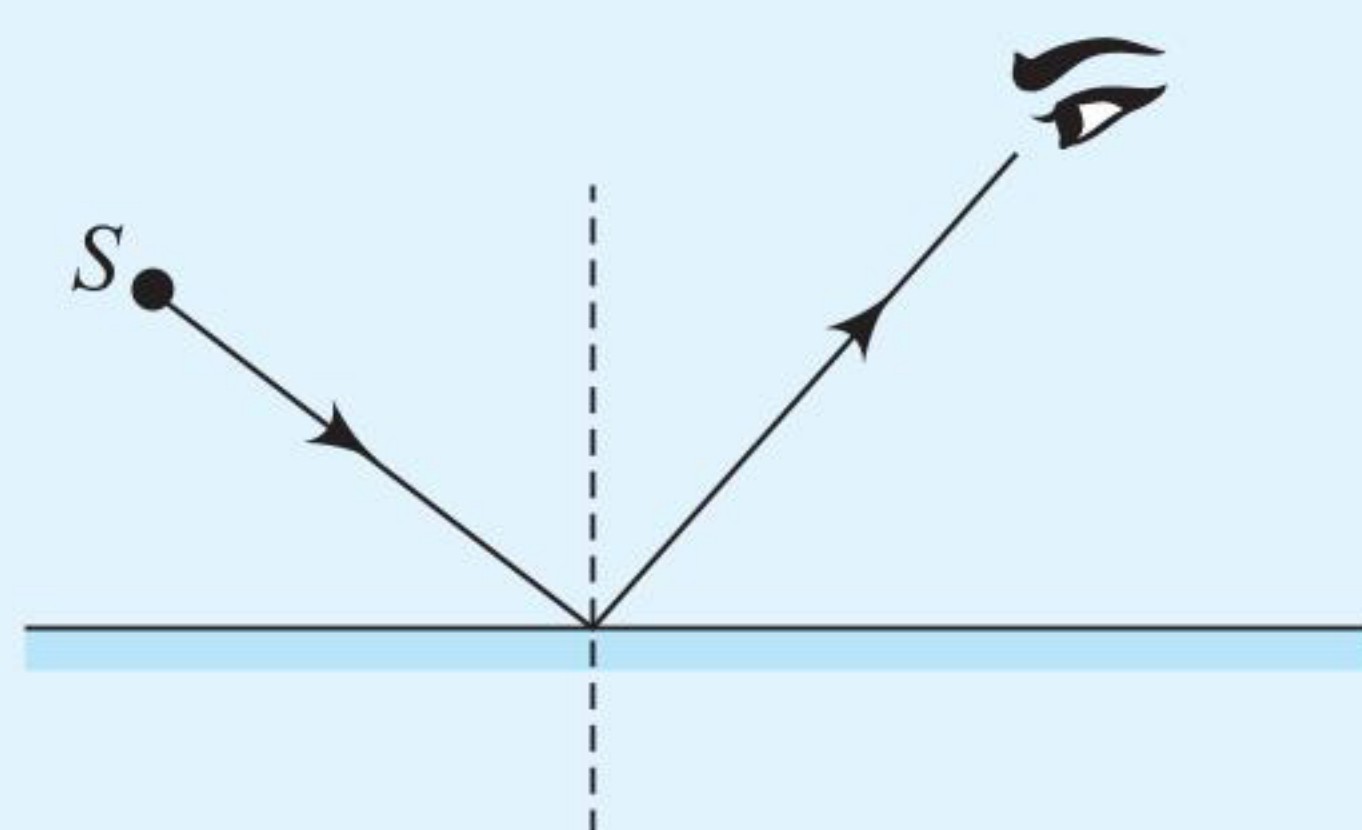
$$\begin{aligned}\vec{(v_i)}_{\perp} &= [2(-5 \cos 30^\circ \hat{i}) - (10 \cos 60^\circ \hat{i})] (\text{m/s}) \\ &= -5(1 + \sqrt{3}) \hat{i} (\text{m/s})\end{aligned}$$

Hence velocity of the image $\vec{v_i} = \vec{(v_i)}_{\parallel} + \vec{(v_i)}_{\perp}$

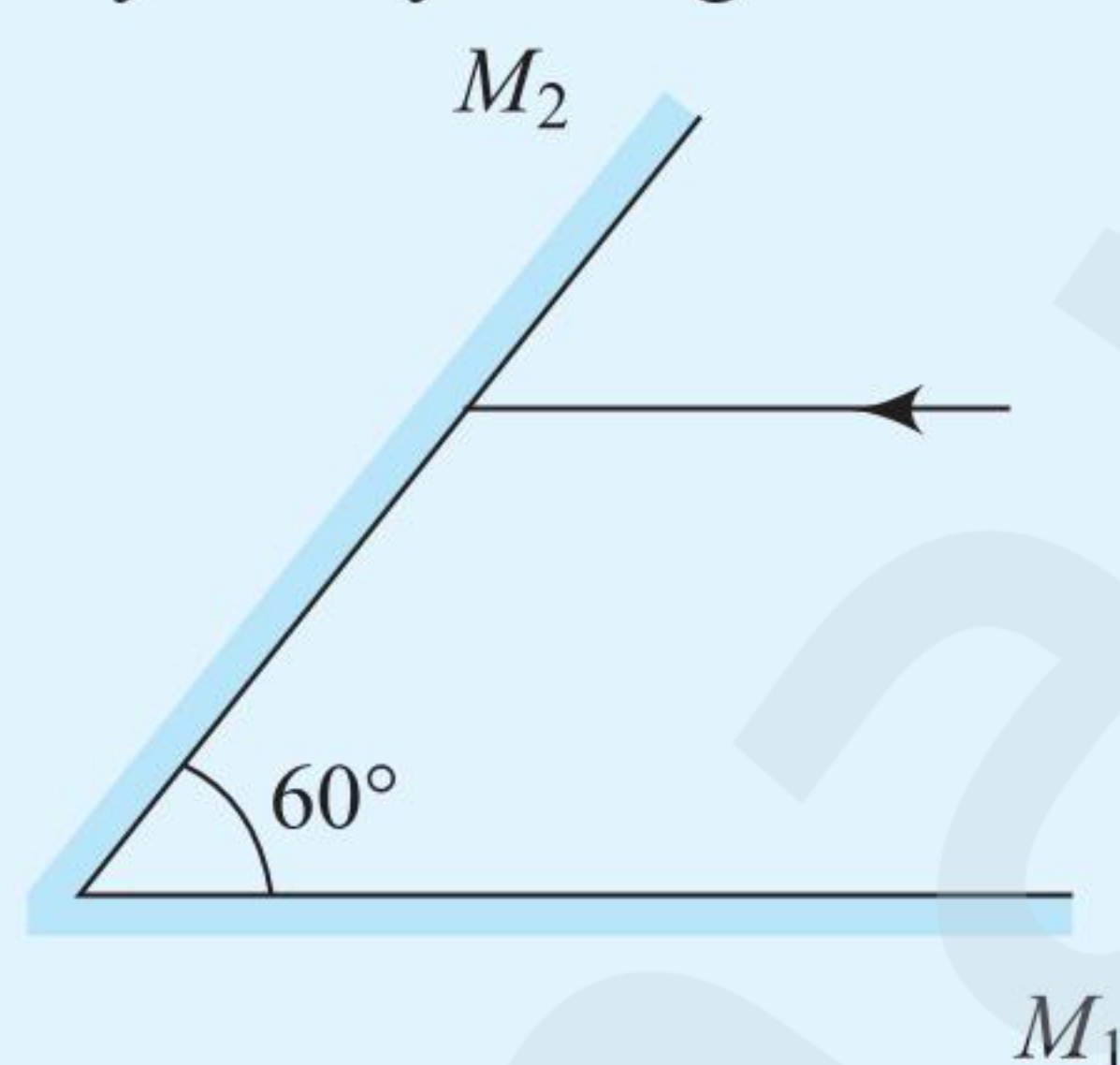
$$\text{or } \vec{v_i} = -5(1 + \sqrt{3}) \hat{i} + 5\sqrt{3} \hat{j} (\text{m/s})$$

CONCEPT APPLICATION EXERCISE 1.1

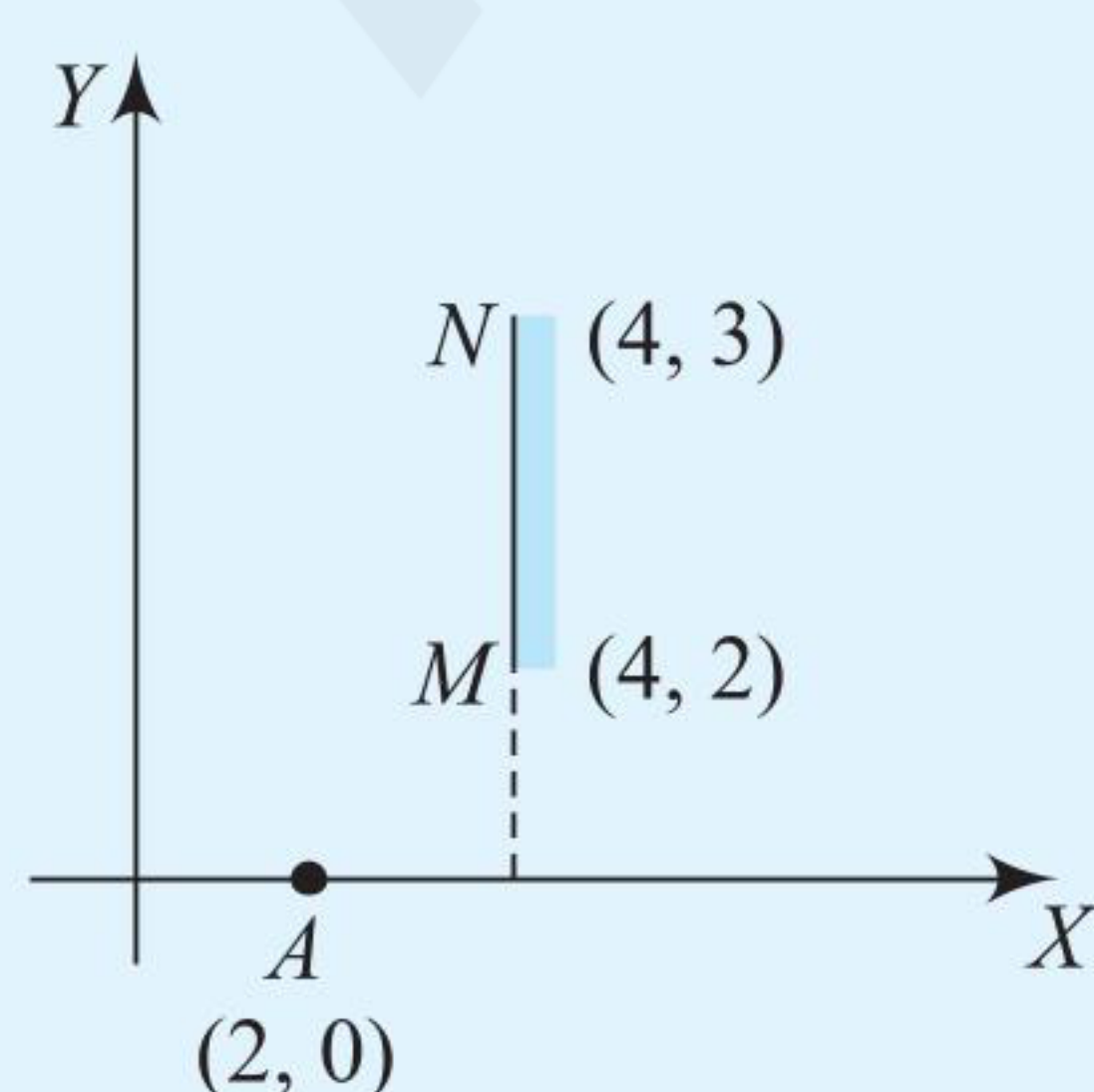
1. There is a point object and a plane mirror. If the mirror is moved by 10 cm away from the object, find the distance which the image will move.
2. Two plane mirrors are placed parallel to each other. The distance between the mirrors is 10 cm. An object is placed between the mirrors at a distance of 4 cm from one of them, say M_1 . What is the distance between the first image formed at M_1 and the second image formed at M_2 ?
3. A ray of light travels from a light source S to an observer after reflection from a plane mirror. If the source rotates in the clockwise direction by 10° , by what angle and in what direction must the mirror be rotated so that the light ray still strikes the observer?



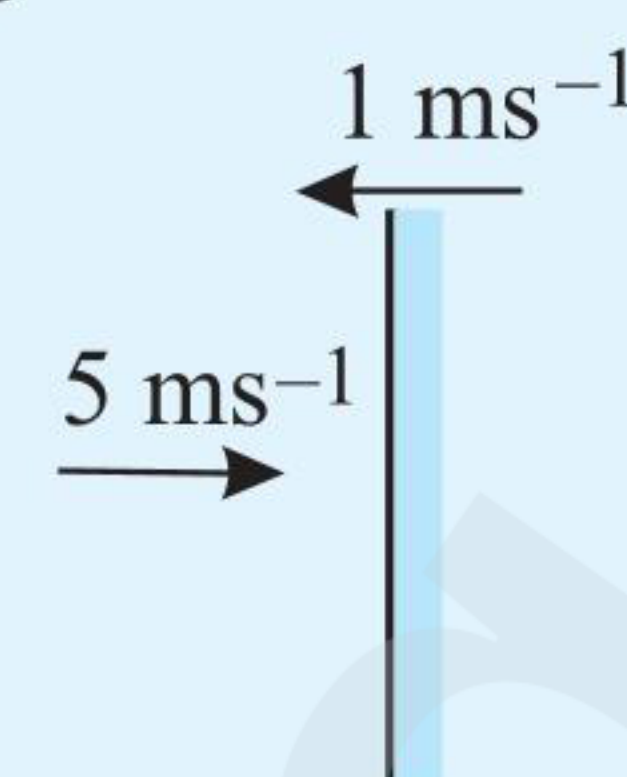
4. Two plane mirrors are inclined at an angle of 60° as shown in figure. A ray of light parallel to M_1 strikes M_2 . At what angle will the ray finally emerge?



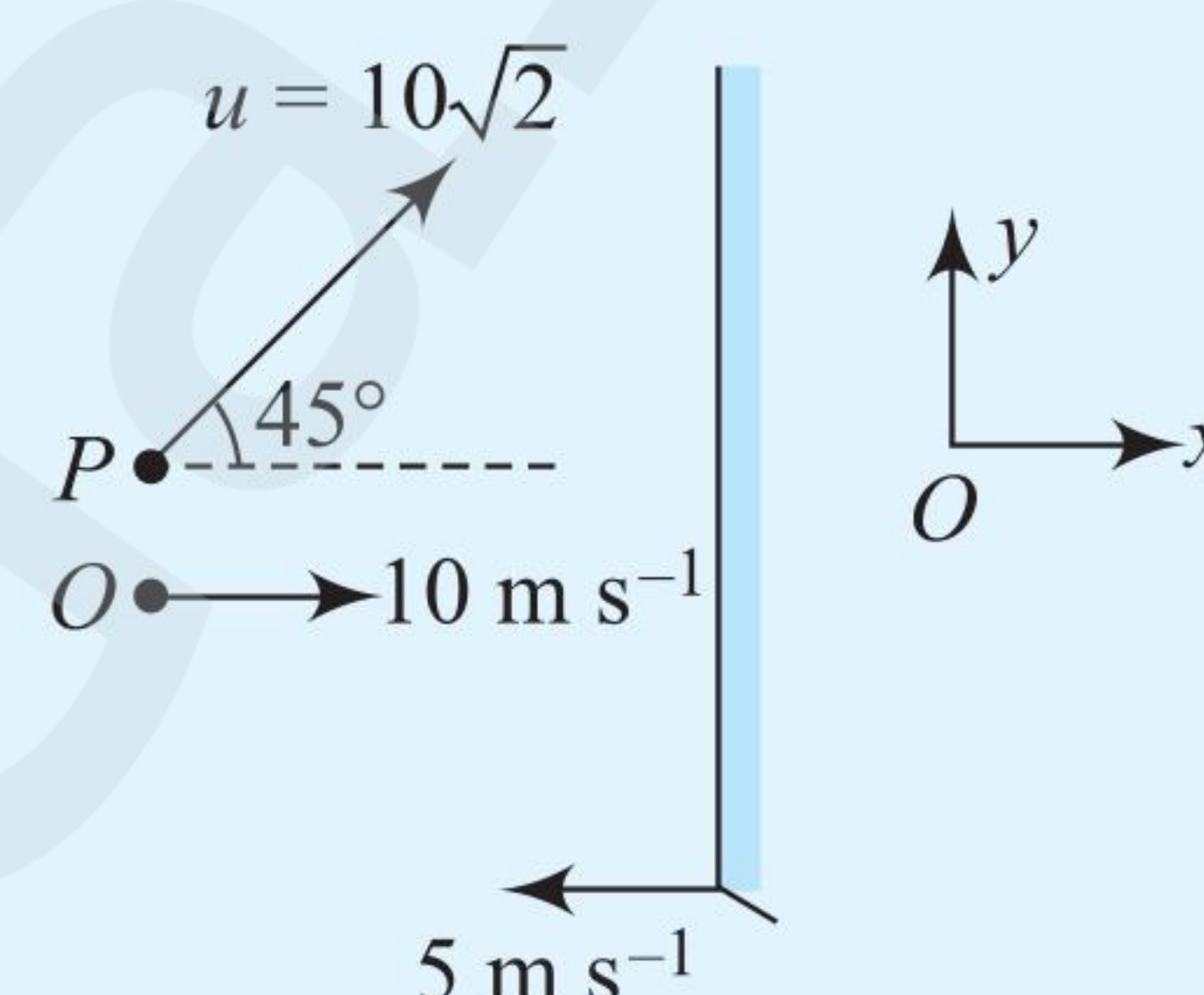
5. A man is standing at distance x from a plane mirror in front of him. He wants to see the entire wall in mirror which is at distance y behind the man. Find the minimum size of the mirror required.
6. Find the region on Y -axis in which reflected rays are present. Object is at $A(2, 0)$ and MN is a plane mirror, as shown in figure.



7. An object moves with 5 ms^{-1} toward right while the mirror moves with 1 ms^{-1} toward the left as shown in figure. Find the velocity of image.



8. In figure, a plane mirror is moving with a uniform speed of 5 ms^{-1} along negative x -direction and observer O is moving with a velocity of 10 ms^{-1} . What is the velocity of image of a particle P , moving with a velocity as shown in the figure, as observed by observer O ? Also find its direction.



ANSWERS

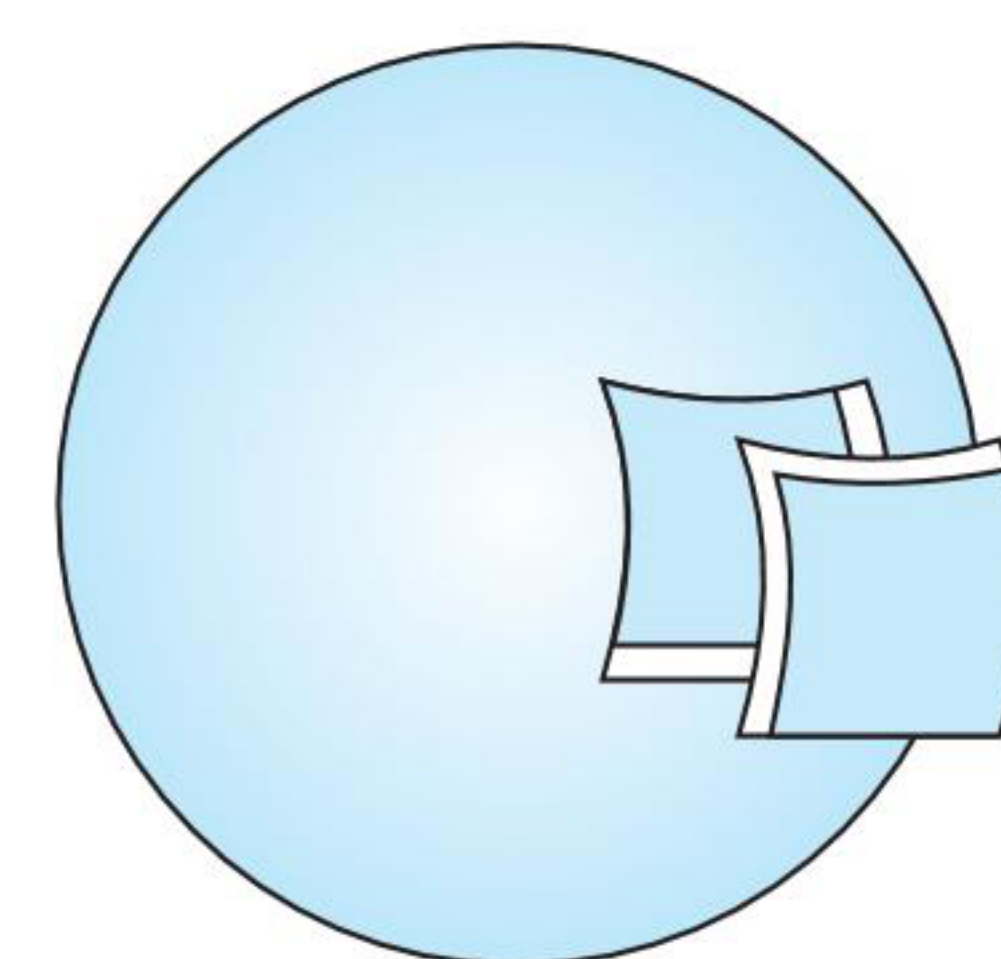
- | | | | |
|----------------------|----------|--|---------------|
| 1. 20 cm | 2. 28 cm | 3. 5° clockwise | 4. 60° |
| 5. $\frac{hx}{y+2x}$ | 7. 7 m/s | 8. $(-30\hat{i} + 10\hat{j}) \text{ ms}^{-1}; \tan^{-1}(-1/3)$ | |

REFLECTION FROM A CURVED SURFACE

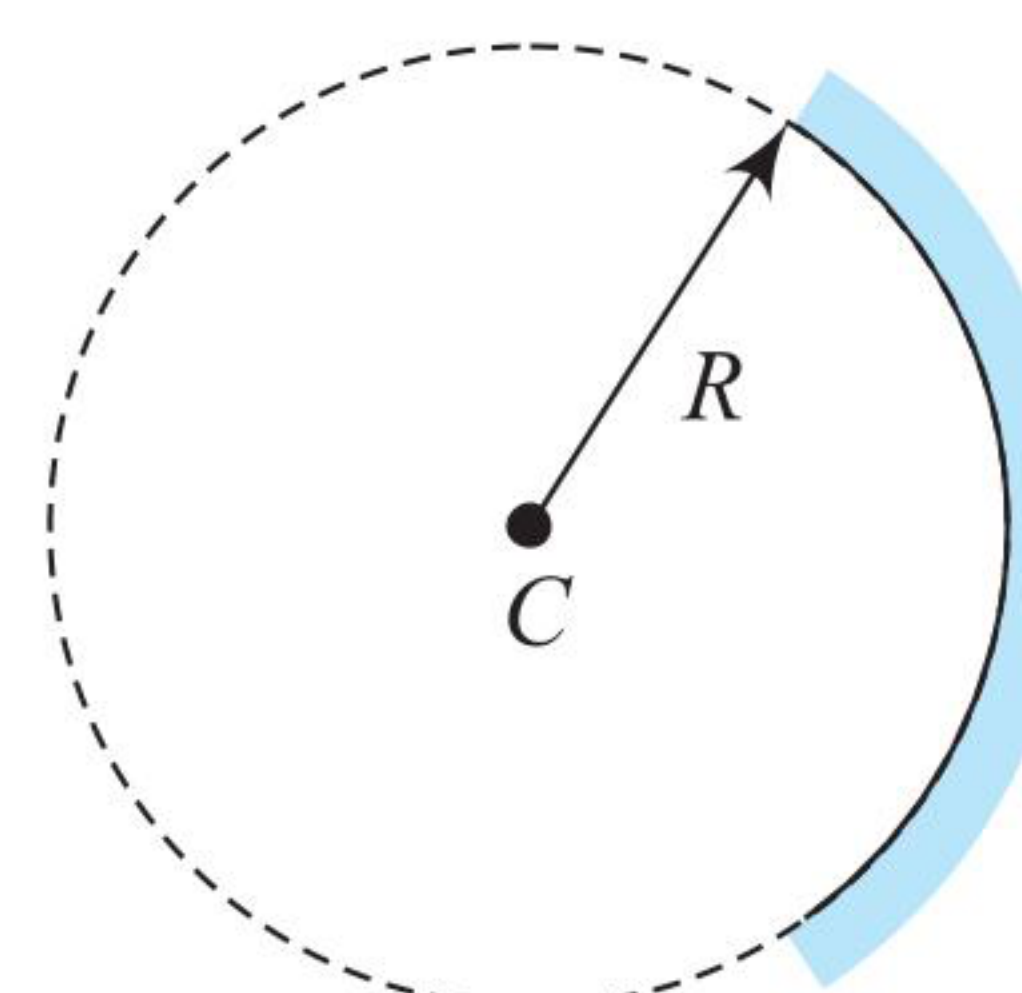
SPHERICAL MIRRORS

A **spherical mirror** is formed by polishing one surface of a part of sphere. A spherical mirror is a reflecting surface whose shape is a section of a spherical surface (see figure).

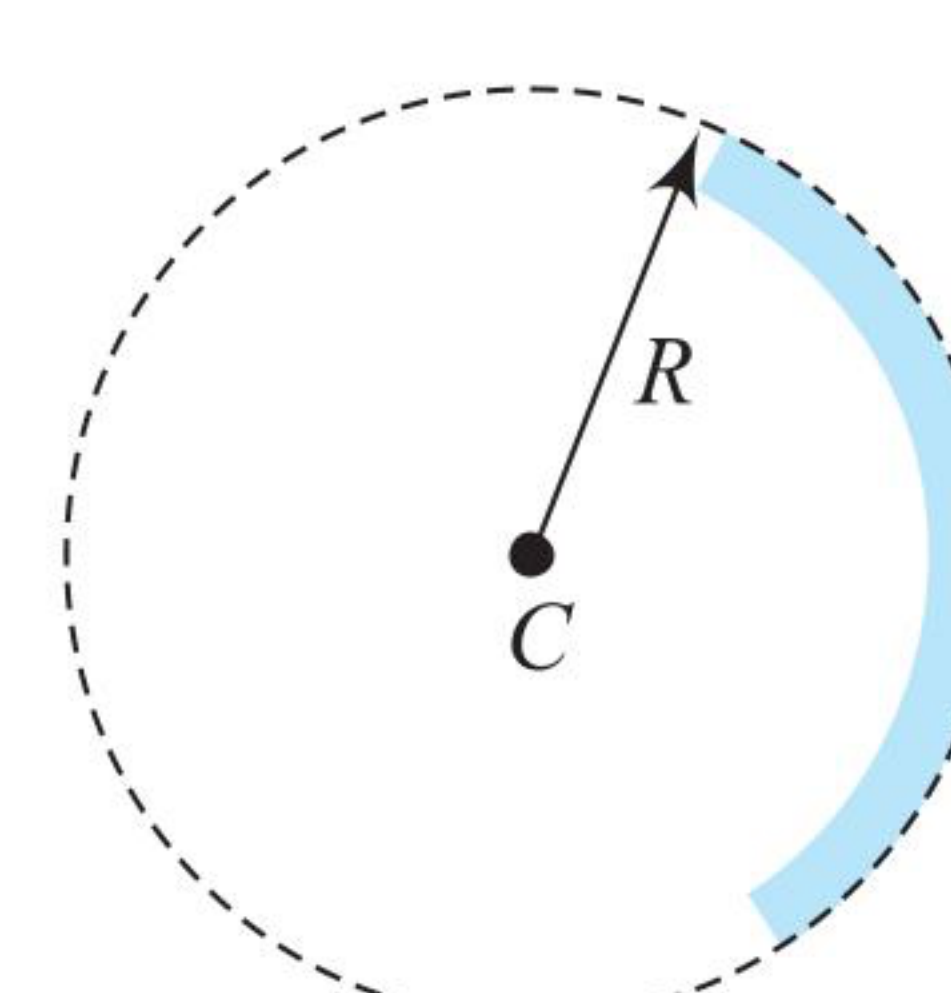
Depending upon which part is shining, the spherical mirror is classified as:



Spherical mirror



(a) Concave mirror



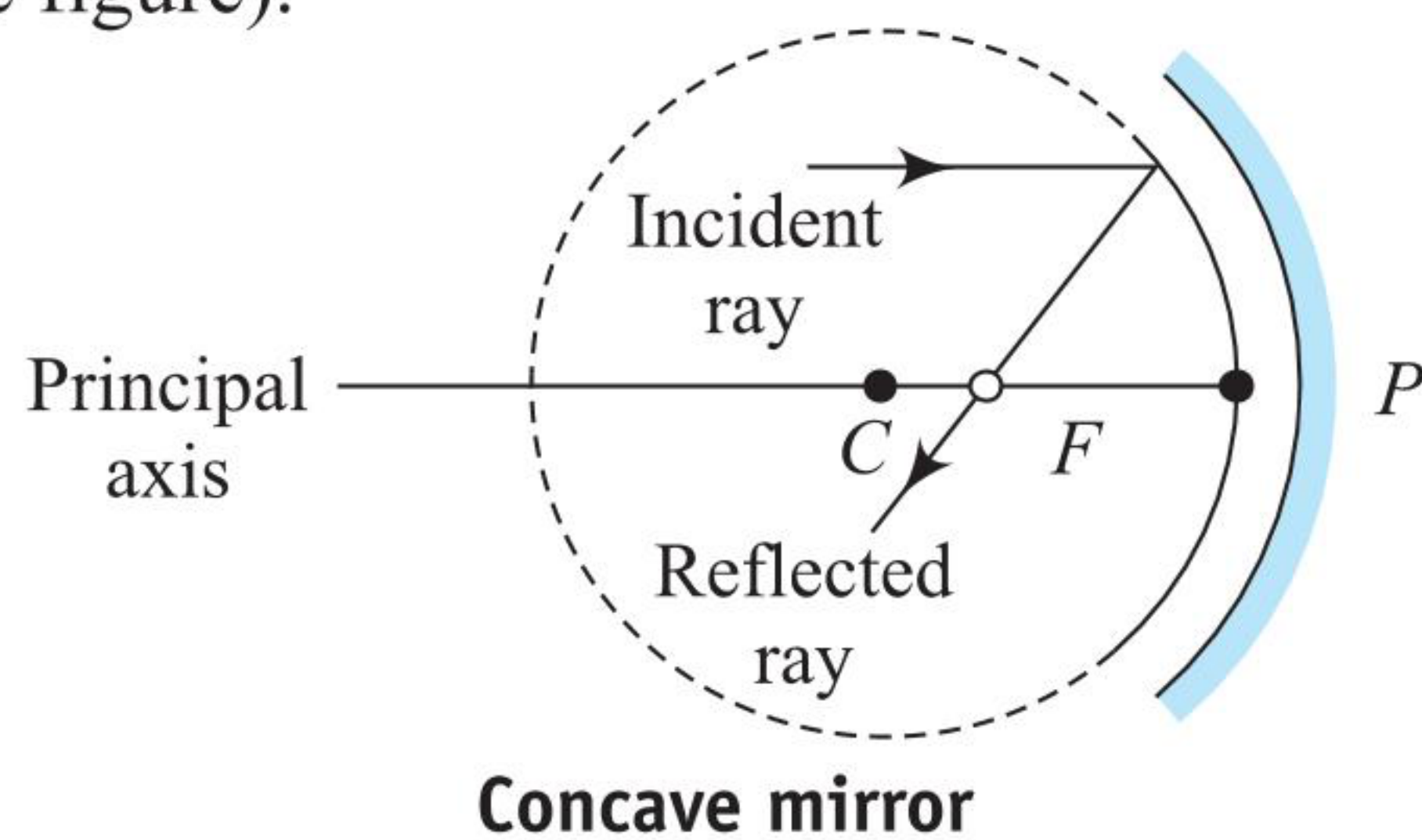
(b) Convex mirror

Concave mirror: If the inside surface of the mirror is polished, it is a concave mirror [see Fig. (a)].

Convex mirror: If the outside surface of the mirror is polished, it is a convex mirror [see Fig. (b)].

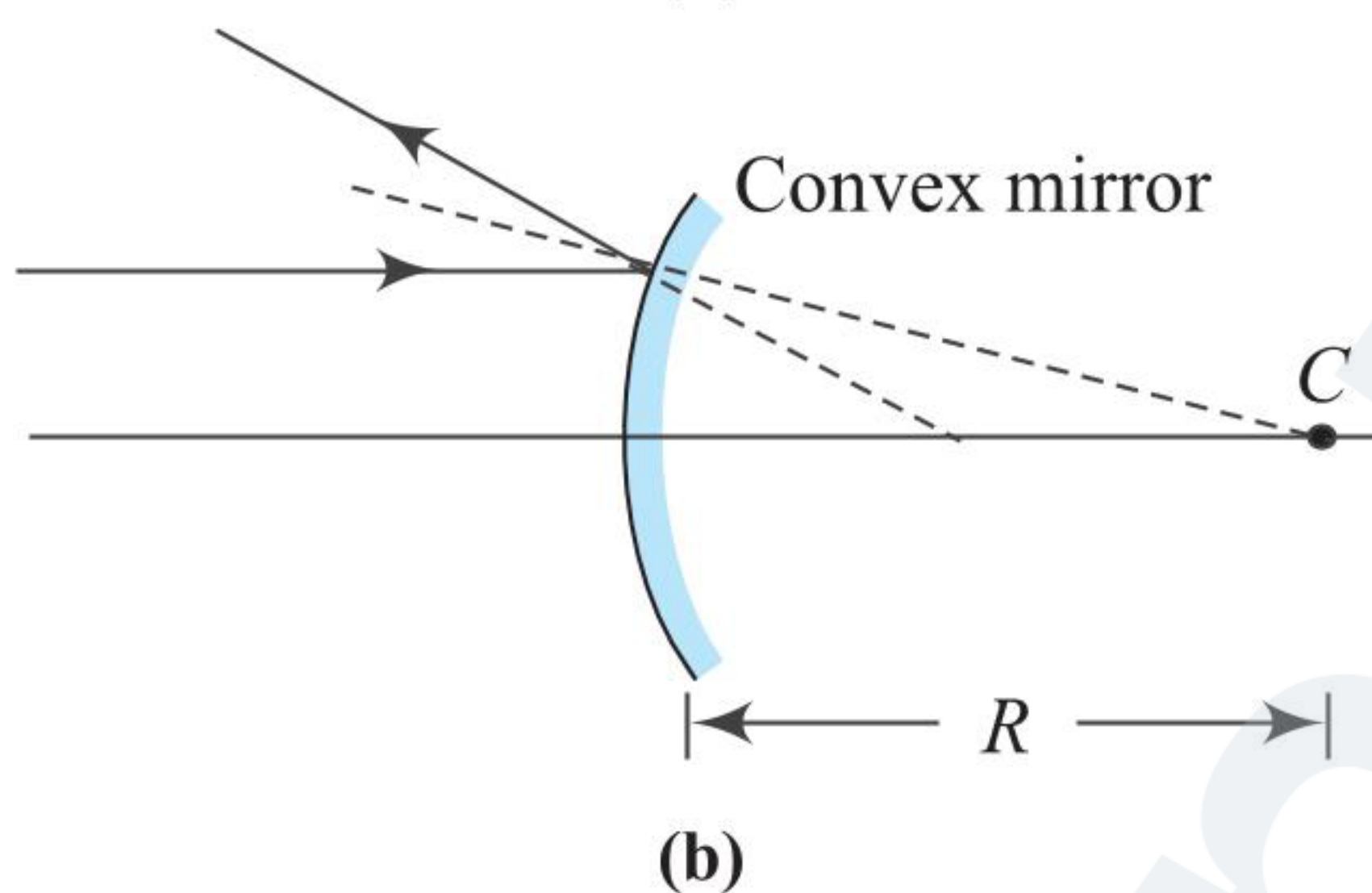
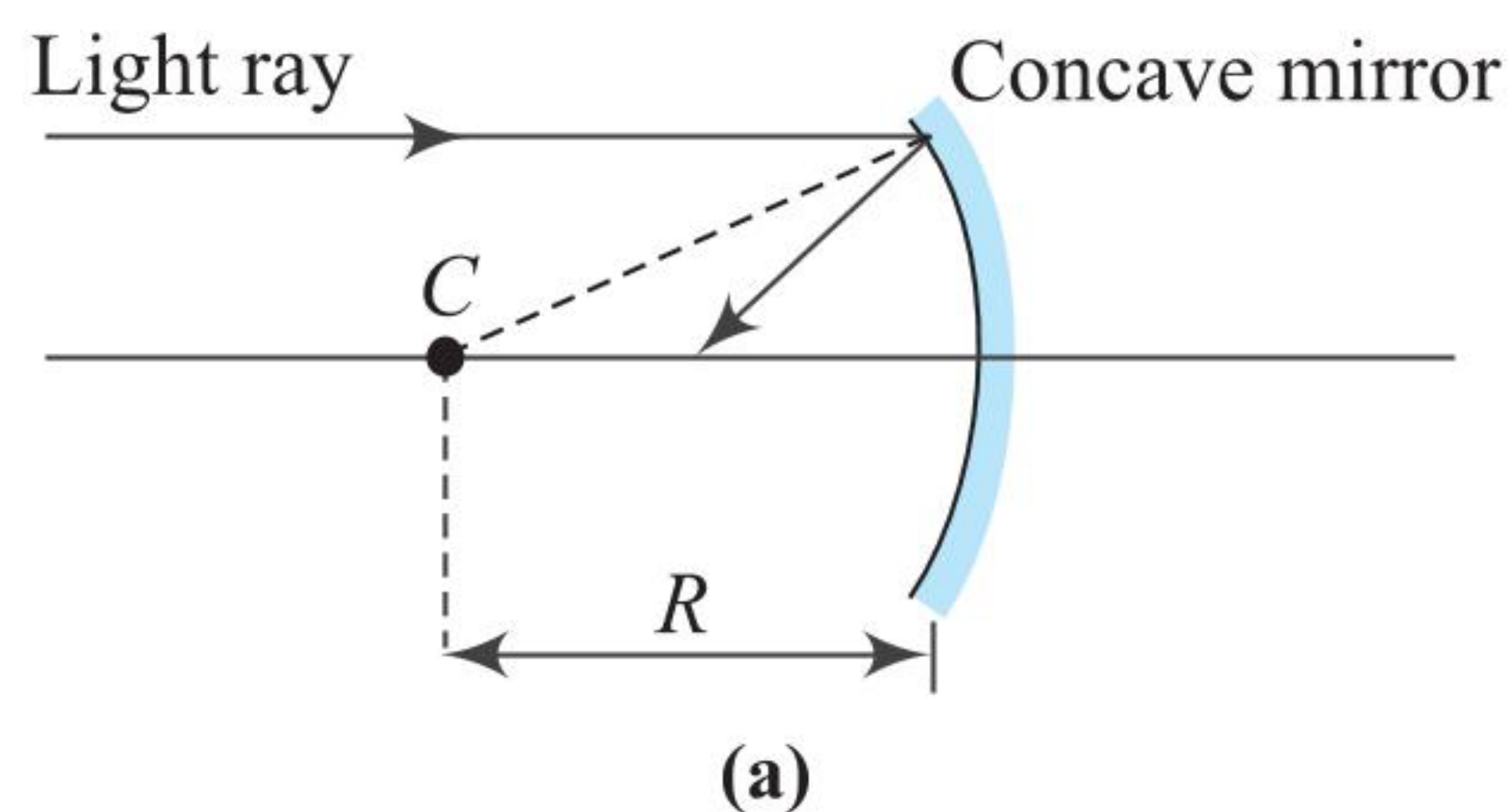
Important Terms

Pole (P): It is the geometrical centre of the spherical reflecting surface (see figure).



A point on the surface of the mirror from where the position of the object can be specified easily is called pole. The pole is generally taken at the mid point of reflecting surface.

Centre of curvature and radius of curvature: The centre of the sphere of which the mirror is a part, is called **centre of curvature (C)** [see Figs. (a) and (b)]. The radius of the sphere of which the mirror is a part is called **radius of curvature (R)**. A plane mirror can be treated as a special case of a spherical mirror: one which has an **infinite radius of curvature**.



Principal axis: It is the straight line joining the centre of curvature to the pole.

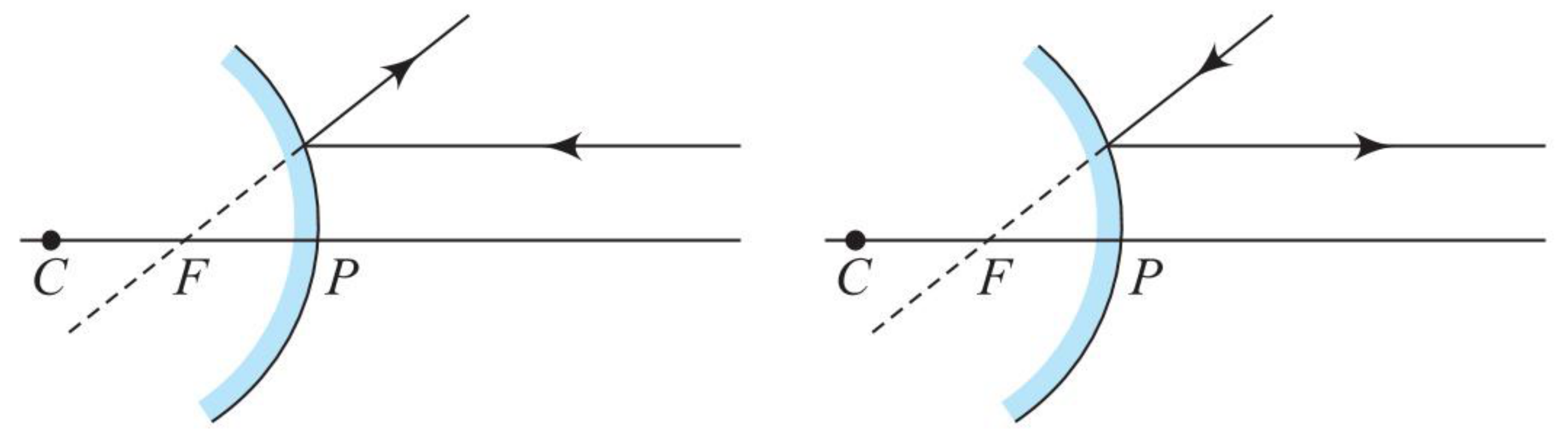
Principal focus (F): It is the point of intersection of all the reflected rays for which the incident rays strike the mirror (with small aperture) parallel to the principal axis. In a concave mirror it is real and in a convex mirror it is virtual. The distance from pole to focus is called **focal length**.

Aperture (related to the size of mirror): It is the diameter of the mirror.

Focus (F): When a narrow beam of rays of light, parallel to the principal axis and close to it, is incident on the surface of a mirror, the reflected beam is found to converge to or appears to diverge from a point on the principal axis. This point is called the focus (see figures).



(a) Concave mirror

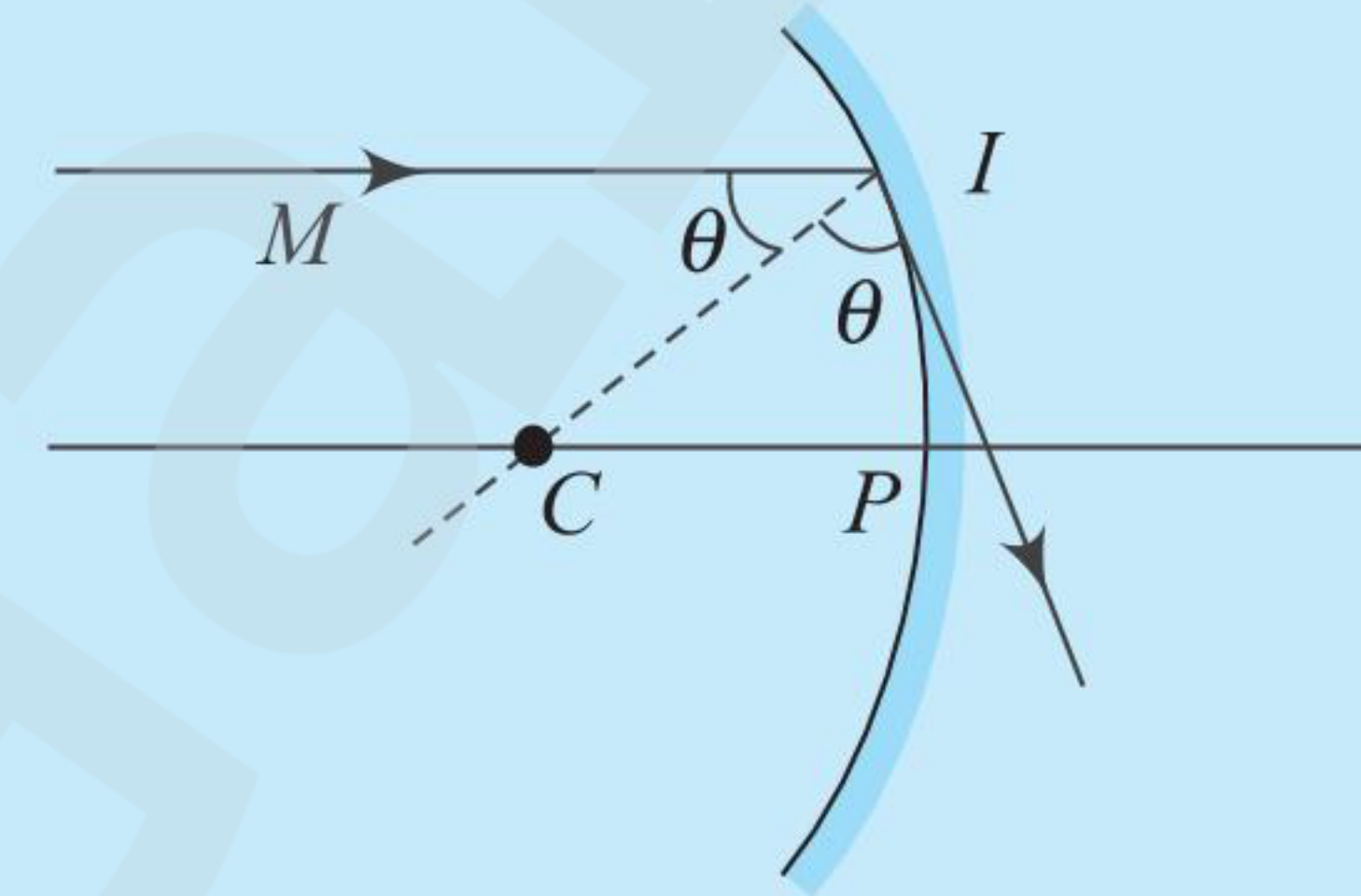


(b) Convex mirror

Focal length (f): It is the distance between the pole and the principal focus. For spherical mirrors, $f = R/2$.

ILLUSTRATION 1.18

Find the angle of incidence of the ray shown in figure for which it passes through the pole, given that $MI \parallel CP$.



Sol. From the figure, the ray MI is making an angle θ with normal IC . From law of reflection, $\angle MIC = \angle CIP = \theta$

As $MI \parallel CP \Rightarrow \angle MIC = \angle ICP = \theta$

Now, $CI = CP$

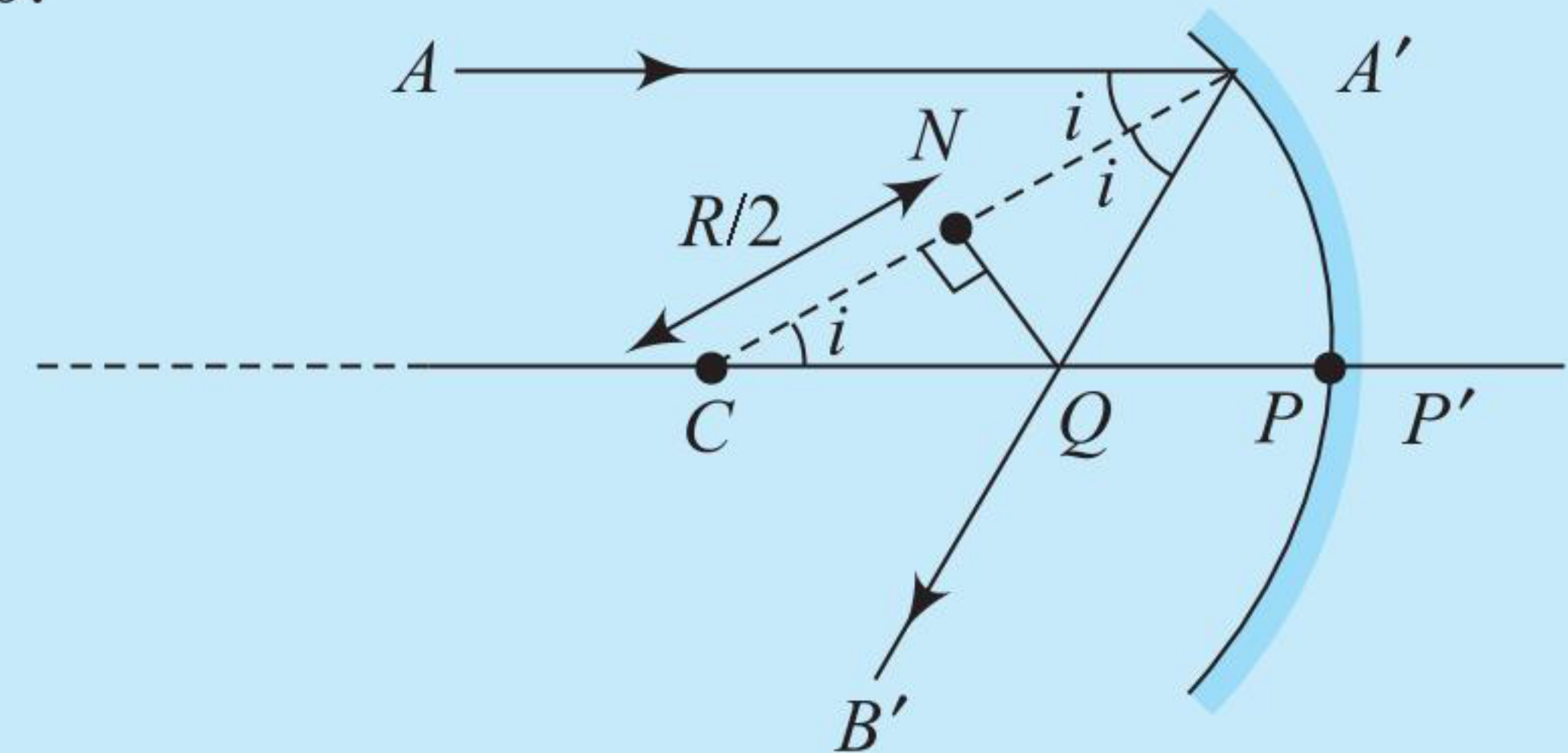
$\Rightarrow \angle CIP = \angle CPI = \theta$

$\therefore$ In $\triangle CIP$, all angles are equal, i.e., $3\theta = 180^\circ$

$\Rightarrow \theta = 60^\circ$

ILLUSTRATION 1.19

Find the distance CQ if incident light ray parallel to principal axis is incident at an angle i . Also, find the distance CQ if $i \rightarrow 0$.



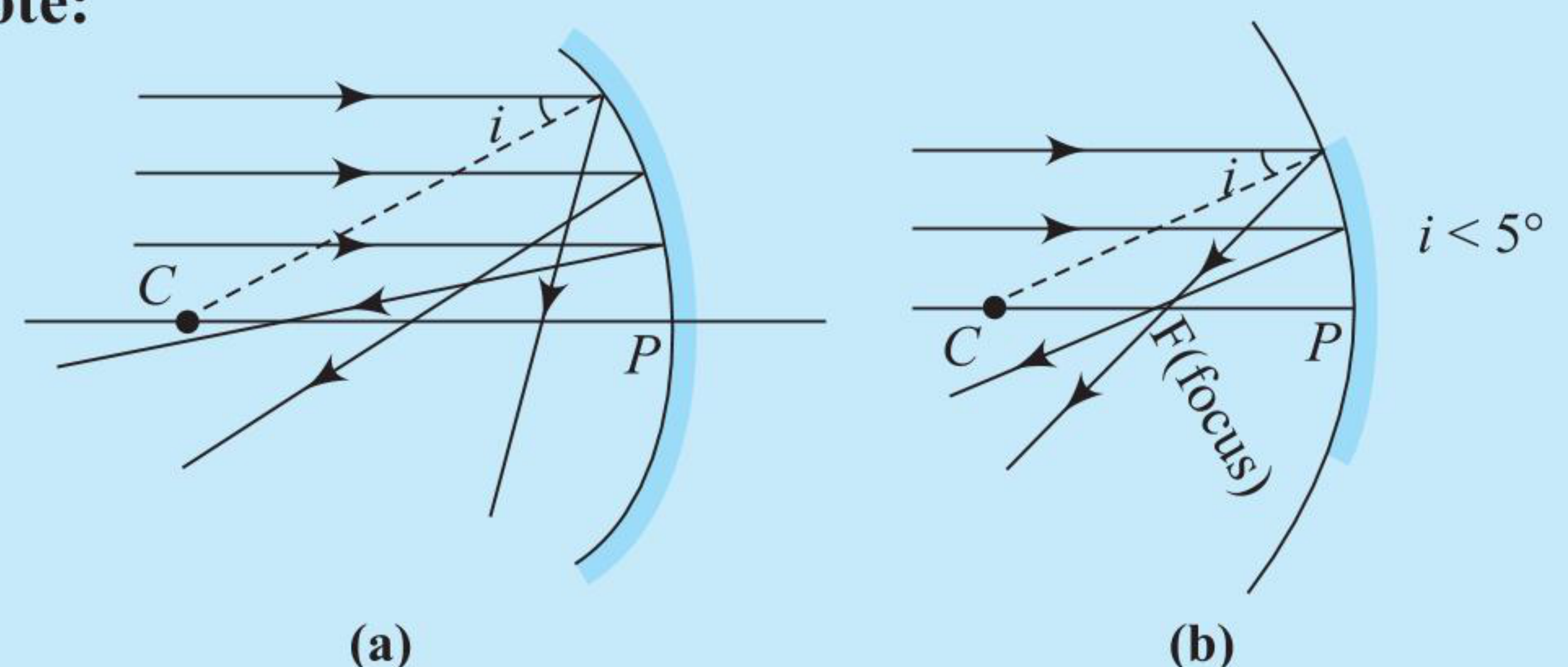
Sol. In triangle CNQ , $\cos i = R/2CQ \Rightarrow CQ = R/2 \cos i$
As i increases $\cos i$ decreases. Hence, CQ increases.

If i is a small angle, $\cos i \approx 1$ [see Fig. (b)].

$\therefore CQ = R/2$

So, paraxial rays meet at a distance equal to $R/2$ from centre of curvature, which is called focus.

Note:



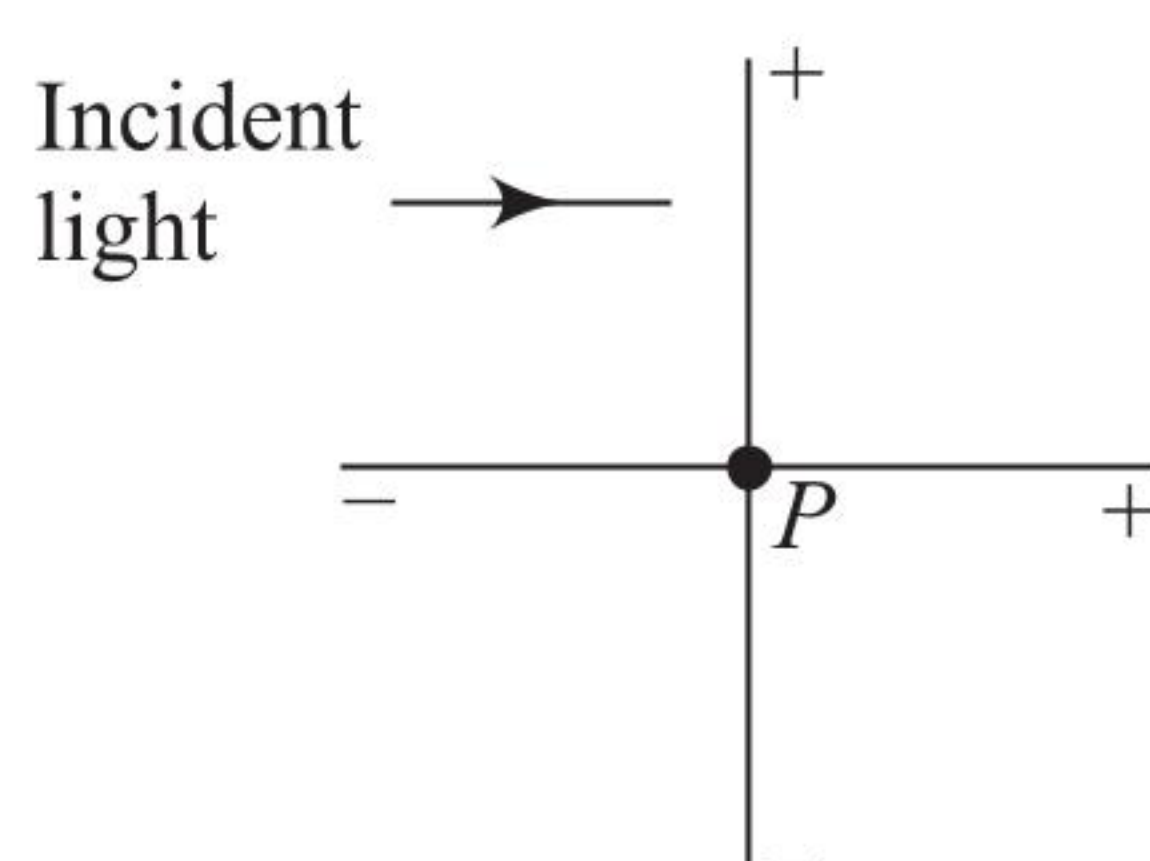
- (a) If angle of incident i is more, the rays will focus at different points on the principal axis.
- (b) If angle of incident i is small, the rays will focus at one point on principal axis. This the point is called focus.

SIGN CONVENTION: CARTESIAN CONVENTION

We will use following sign convention for problem solving in case of reflection as well as refraction.

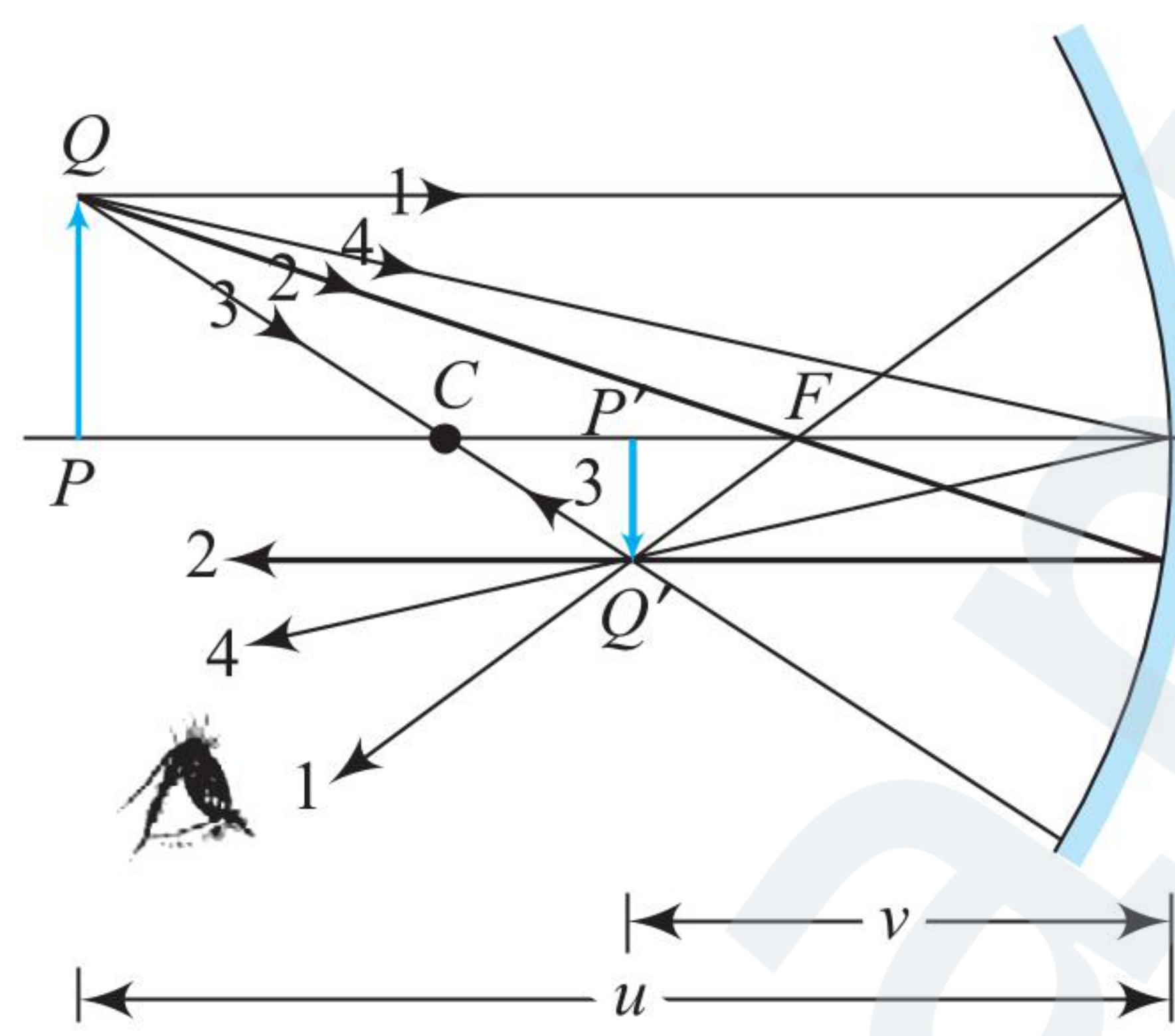
1. All distances are measured from the pole.
2. Distances measured in the direction of incident rays are taken as positive.
3. Distances measured in the direction opposite to that of the incident rays are taken as negative.
4. Distances above the principal axis are taken as positive.
5. Distances below the principal axis are taken as negative.

Angles measured from the normal in anticlockwise sense are positive, while that in clockwise sense are negative.

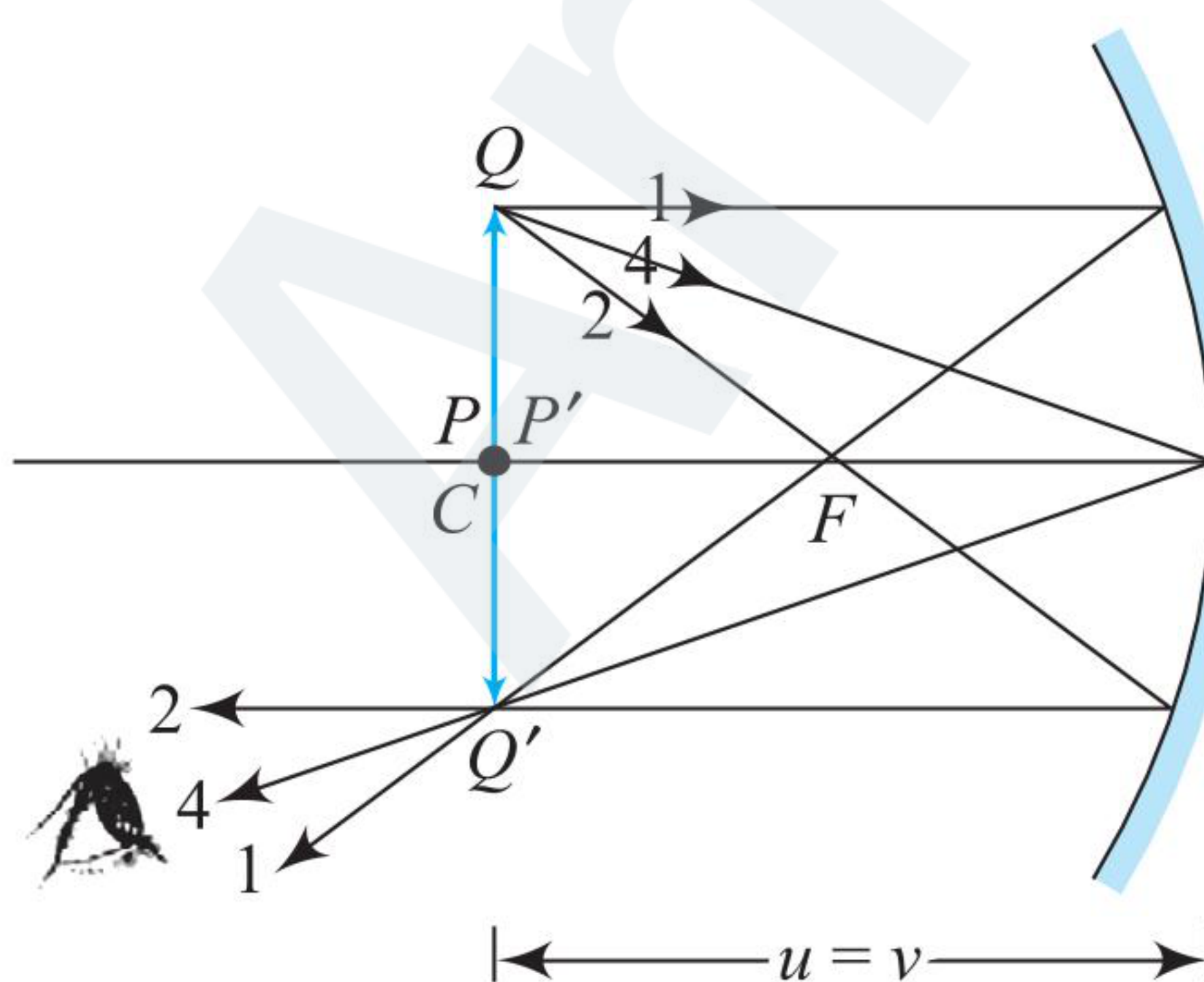


RULES FOR RAY DIAGRAMS

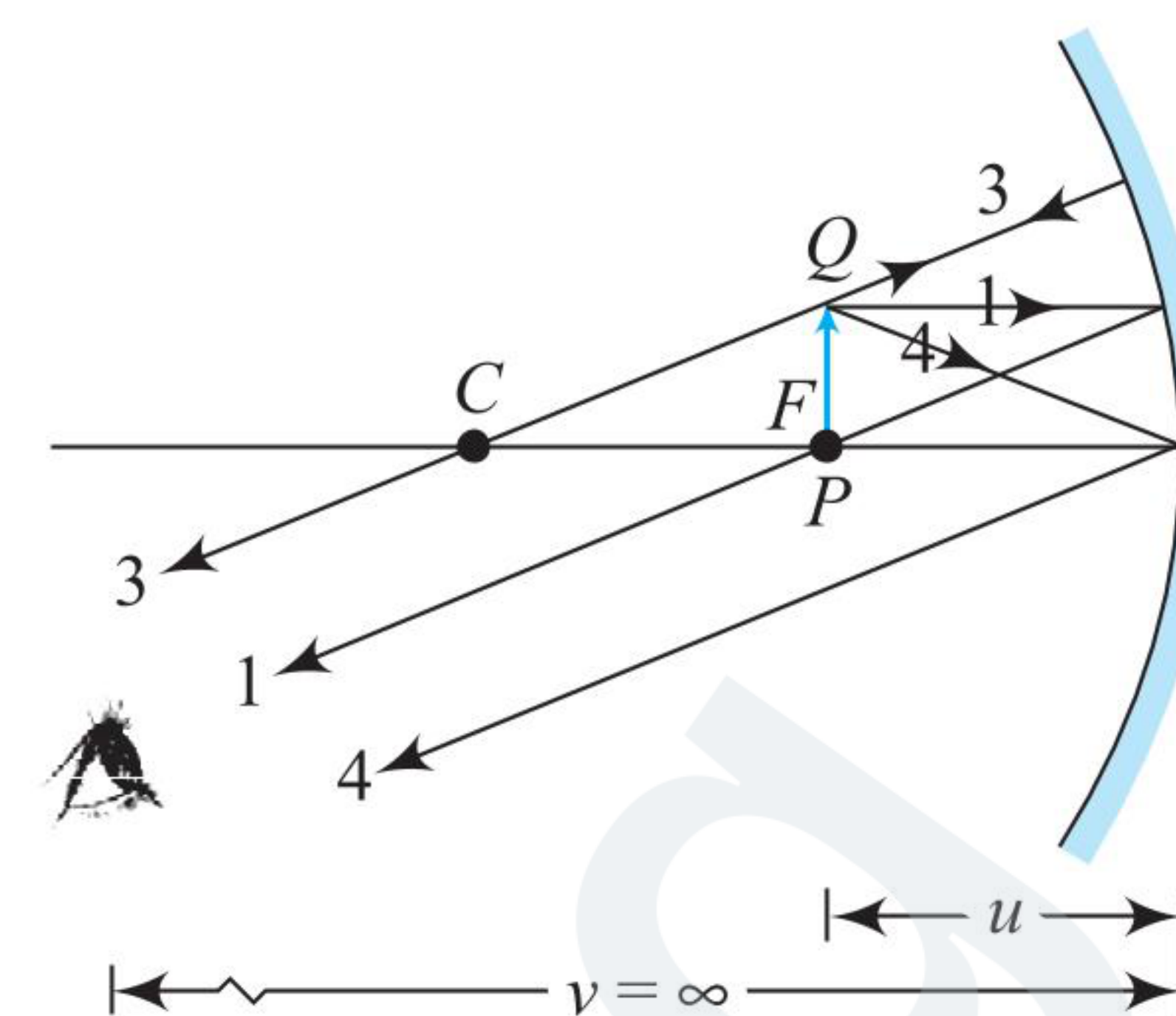
The position, size, and nature of the images formed by mirrors are conventionally expressed by ray diagrams (see figure).



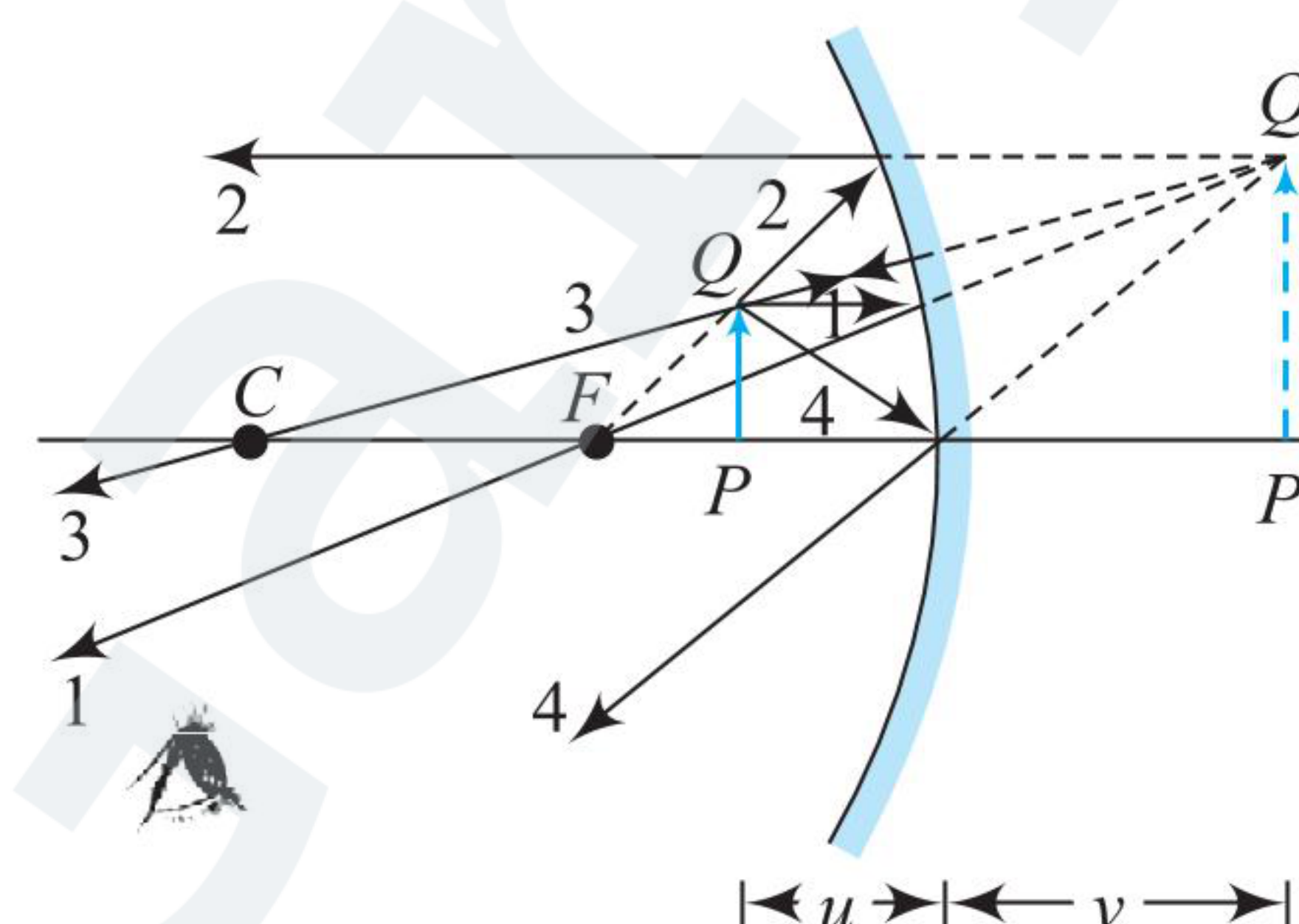
(a)



(b)



(c)



(d)

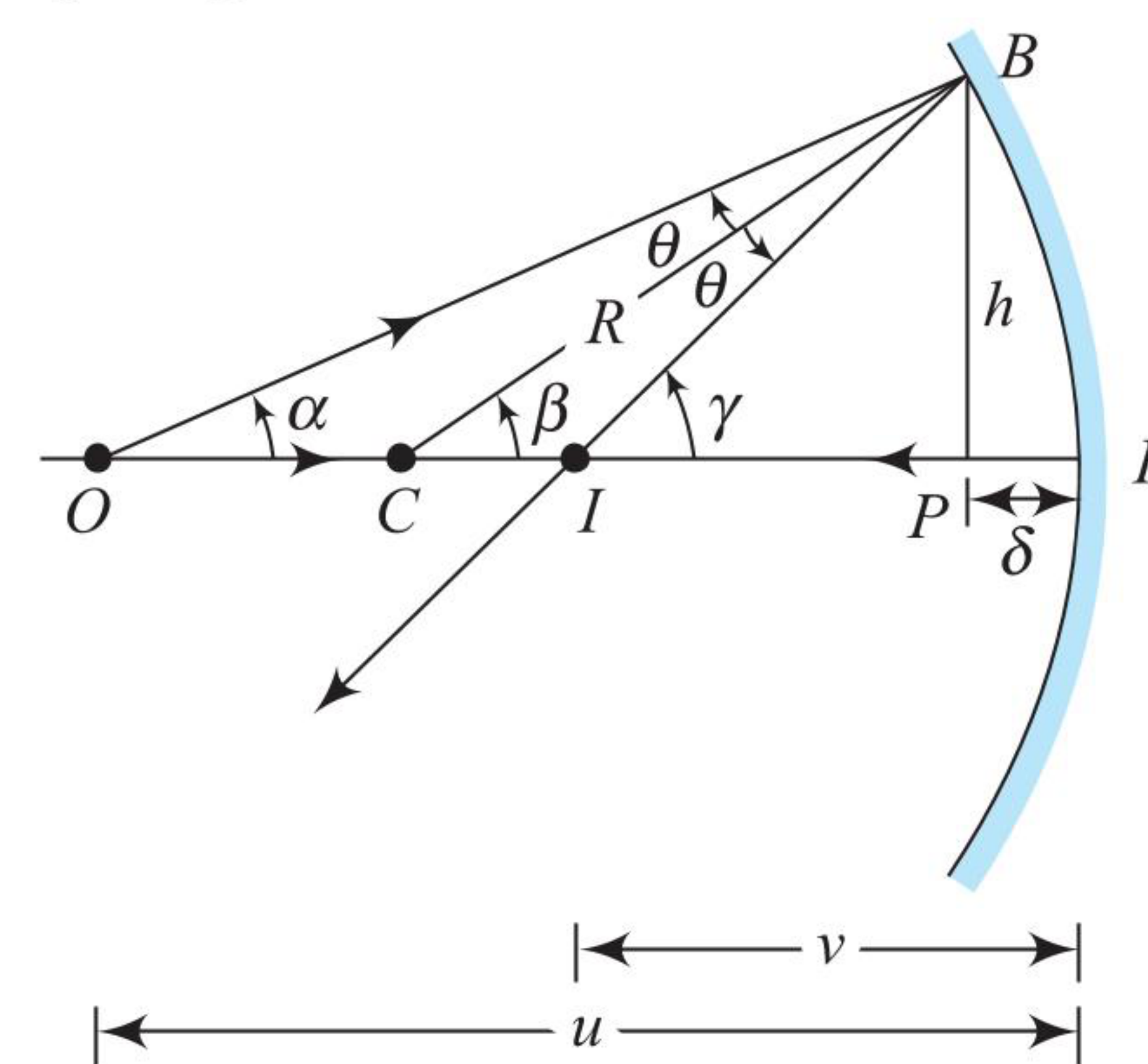
We can locate the image of any extended object graphically by drawing any two of the following four special rays:

1. A ray initially parallel to the principal axis is reflected through the focus of the mirror (1).
2. A ray passing through the centre of curvature is reflected back along itself (3).
3. A ray initially passing through the focus is reflected parallel to the principal axis (2).
4. A ray incident at the pole is reflected symmetrically (4).

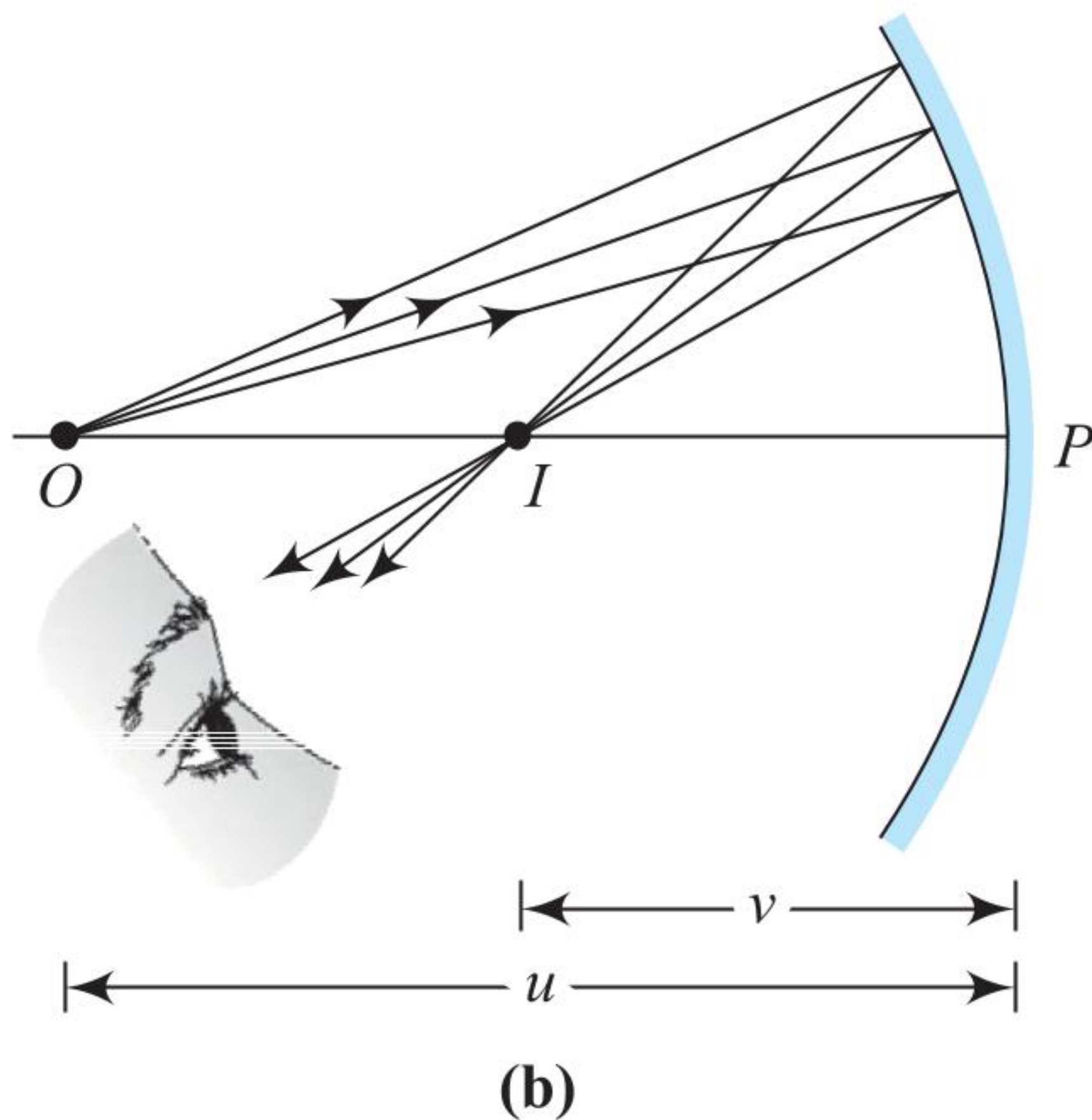
POSITION, SIZE AND NATURE OF IMAGE FORMED BY SPHERICAL MIRRORS

MIRROR FORMULA

Consider Fig. (a) where O is a point object and I is the corresponding image.



(a)



CB is normal to the mirror at B . By laws of reflection,

$$\angle OBC = \angle CBI = \theta$$

$$\alpha + \theta = \beta, \quad \beta + \theta = \gamma, \quad \alpha + \gamma = 2\beta$$

For small aperture of the mirror, α, β, γ

$$\alpha \approx \tan \alpha, \quad \beta \approx \tan \beta, \quad \gamma \approx \tan \gamma, \quad P' \rightarrow P$$

$$\Rightarrow \tan \alpha + \tan \gamma = 2 \tan \beta$$

$$\Rightarrow \frac{BP'}{OP'} + \frac{BP'}{IP'} = 2 \frac{BP'}{CP'}$$

Applying sign convention,

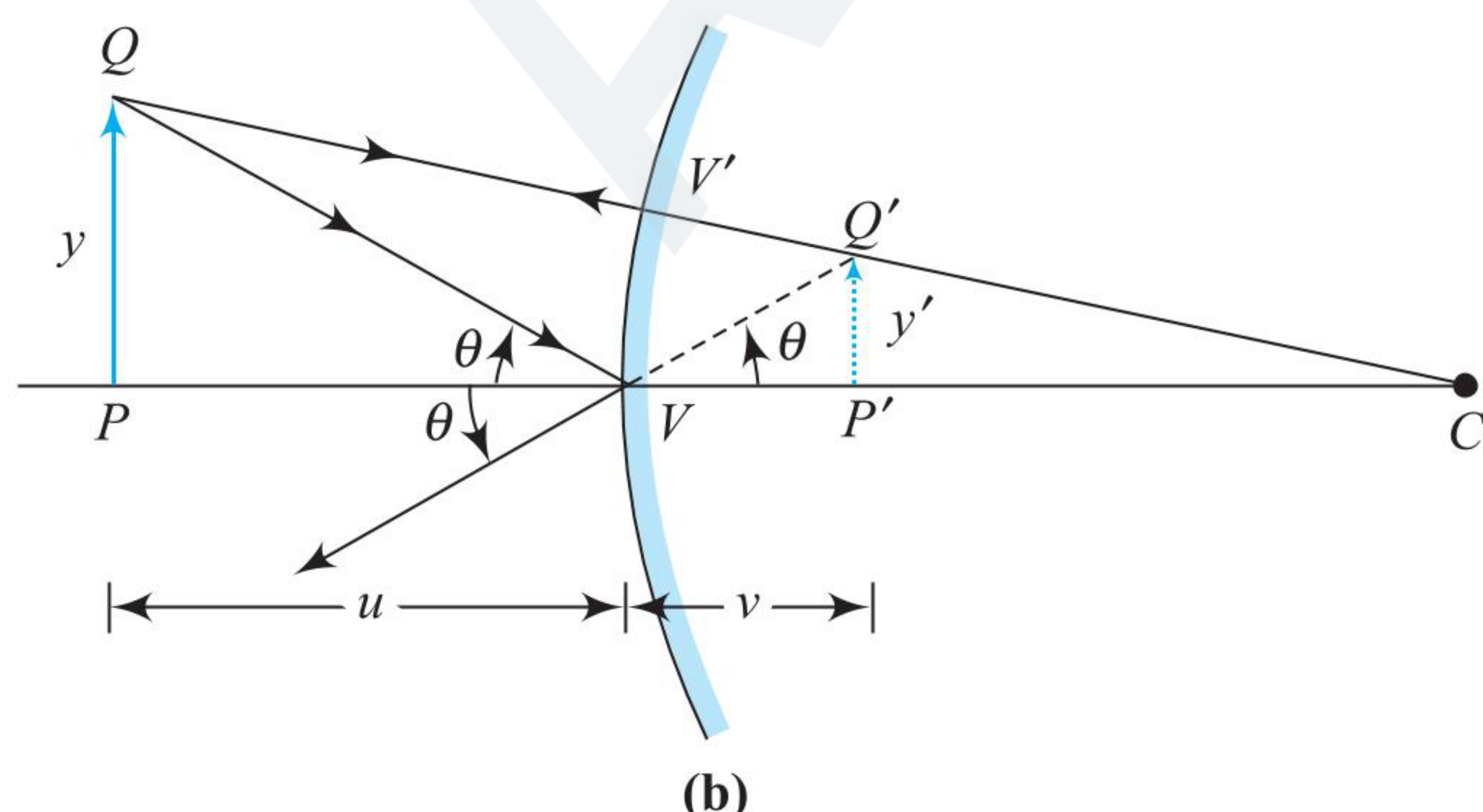
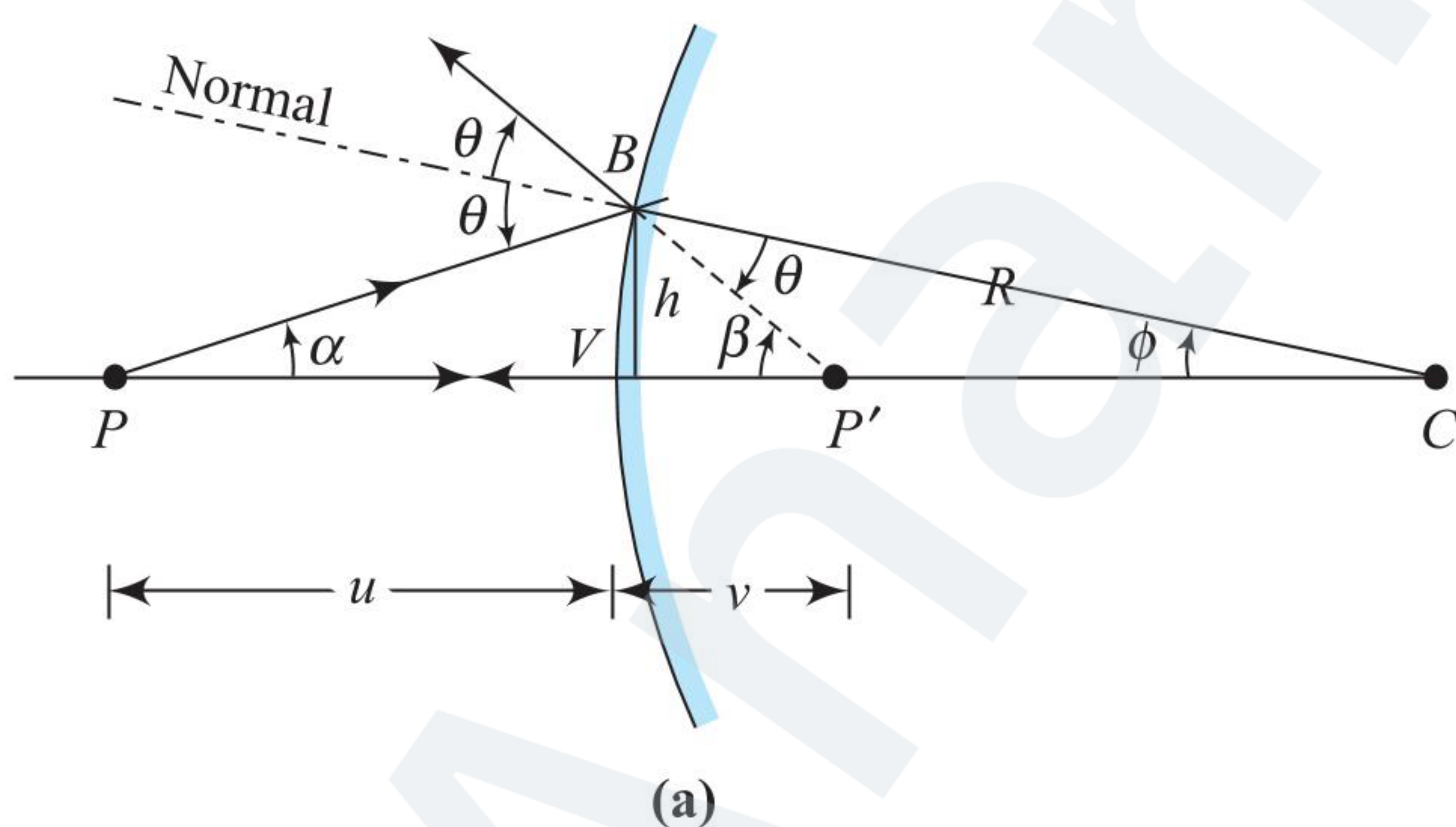
$$u = -OP, \quad v = -IP, \quad R = -CP$$

$$\Rightarrow -\frac{1}{u} + \left(-\frac{1}{v}\right) = -\frac{2}{R}$$

If $u = \infty, \frac{1}{v} = \frac{2}{R}$, but by definition, if $u = \infty, v = f$

$$\text{Hence, } f = \frac{R}{2} \text{ and } \frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

For convex mirrors, an exactly similar formula emerges (see figure).



MAGNIFICATION

The lateral magnification is defined as the ratio

$$m_v = \frac{\text{height of image}}{\text{height of object}} = \frac{h_i}{h_0}$$

To compute the vertical magnification, consider the extended object OA shown in figure. The base of the object, O , will map on to a point I on the principal axis which can be determined from the equation $(1/u) + (1/v) = (1/f)$. The image of the top of the object, A , will map on to a point A' that will lie on the perpendicular through I . The exact location can be determined by drawing a ray from A passing through the pole and intercepting the line through I at A' .

Consider the triangles APO and $A'PI$ in the figure. As the two triangles are similar, we get

$$\tan \alpha = \frac{AO}{PO} = \frac{A'I}{PI} \quad \text{or} \quad \frac{A'I}{AO} = \frac{PI}{PO}$$

Applying the sign convention, we get, $u = -PO$

$$v = -PI \Rightarrow h_0 = +AO \Rightarrow h_i = -A'I$$

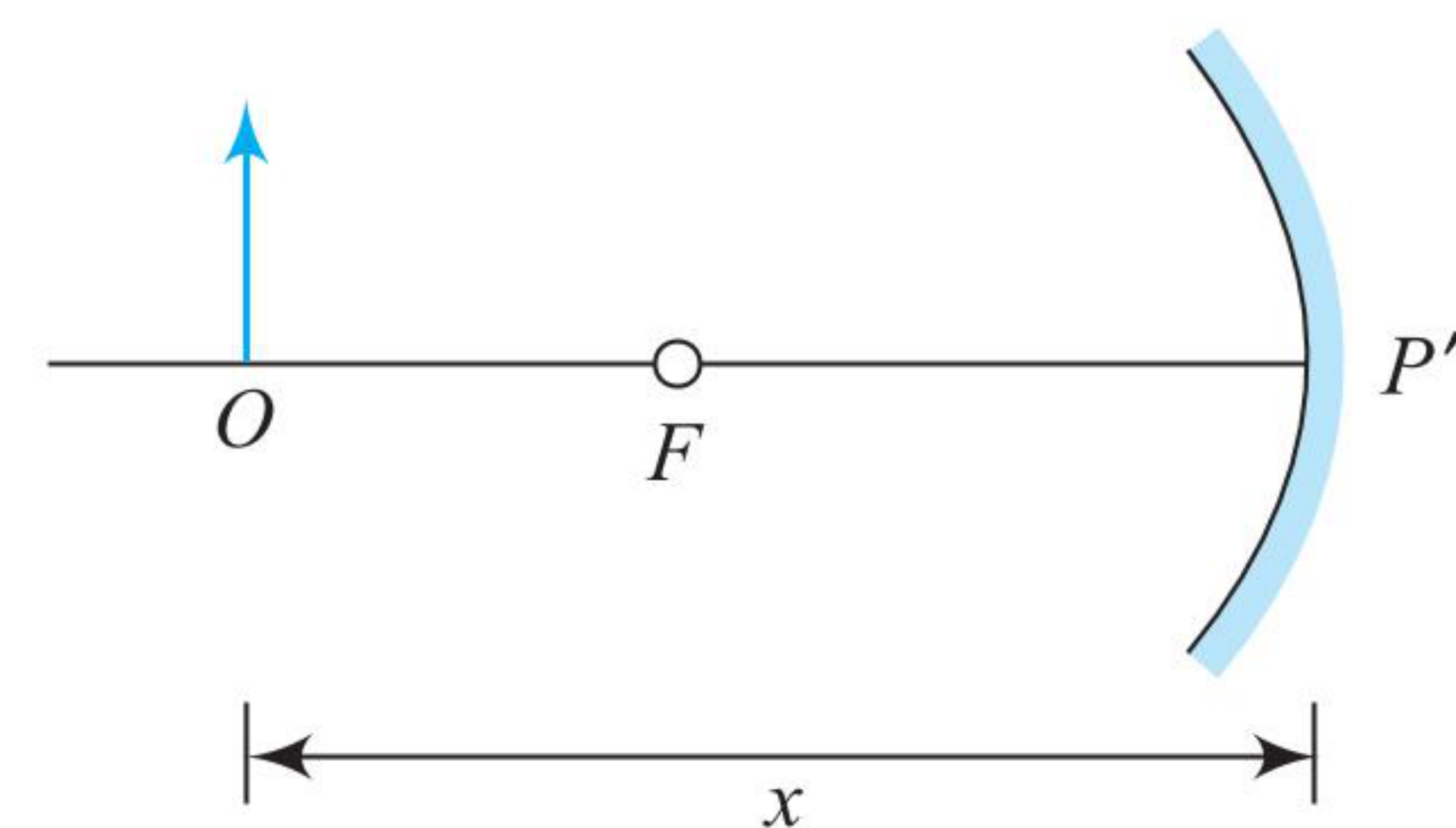
$$\text{Therefore, } -\frac{h_i}{h_0} = \frac{v}{u} \quad \text{or} \quad m_v = \frac{h_i}{h_0} = -\frac{v}{u}$$

A real image of an extended object in front of a concave mirror

MAGNIFICATION FROM A CONCAVE MIRROR

Figure shows a concave mirror of focal length f_0 in front of which an object O is placed at a distance x from the pole P . According to Cartesian sign convention, the mirror formula may be modified as $u = -x; f = -f_0$. Thus,

$$\frac{1}{v} + \frac{1}{-x} = \frac{1}{-f_0} \quad \text{or} \quad v = \frac{xf_0}{f_0 - x}$$

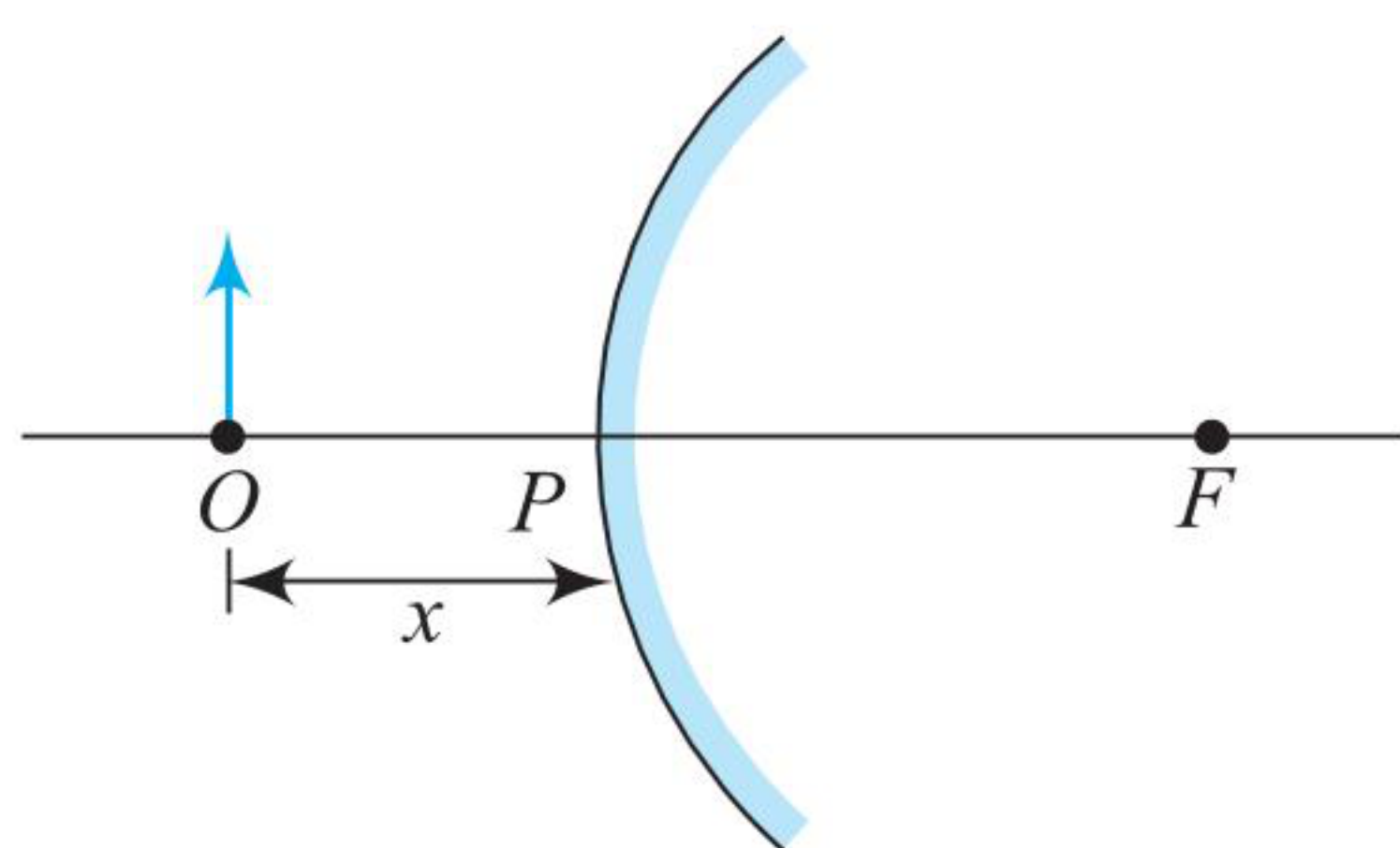


And the magnification formula may be modified as

$$m = \frac{-v}{u} = \frac{y}{x} = \frac{f_0}{f_0 - x}$$

MAGNIFICATION FROM A CONVEX MIRROR

Figure shows a convex mirror of focal length f_0 in front of which an object O is placed at a distance x from the pole P . According to Cartesian sign convention, the formulae may be modified as $u = -x$ and $f = +f_0$.



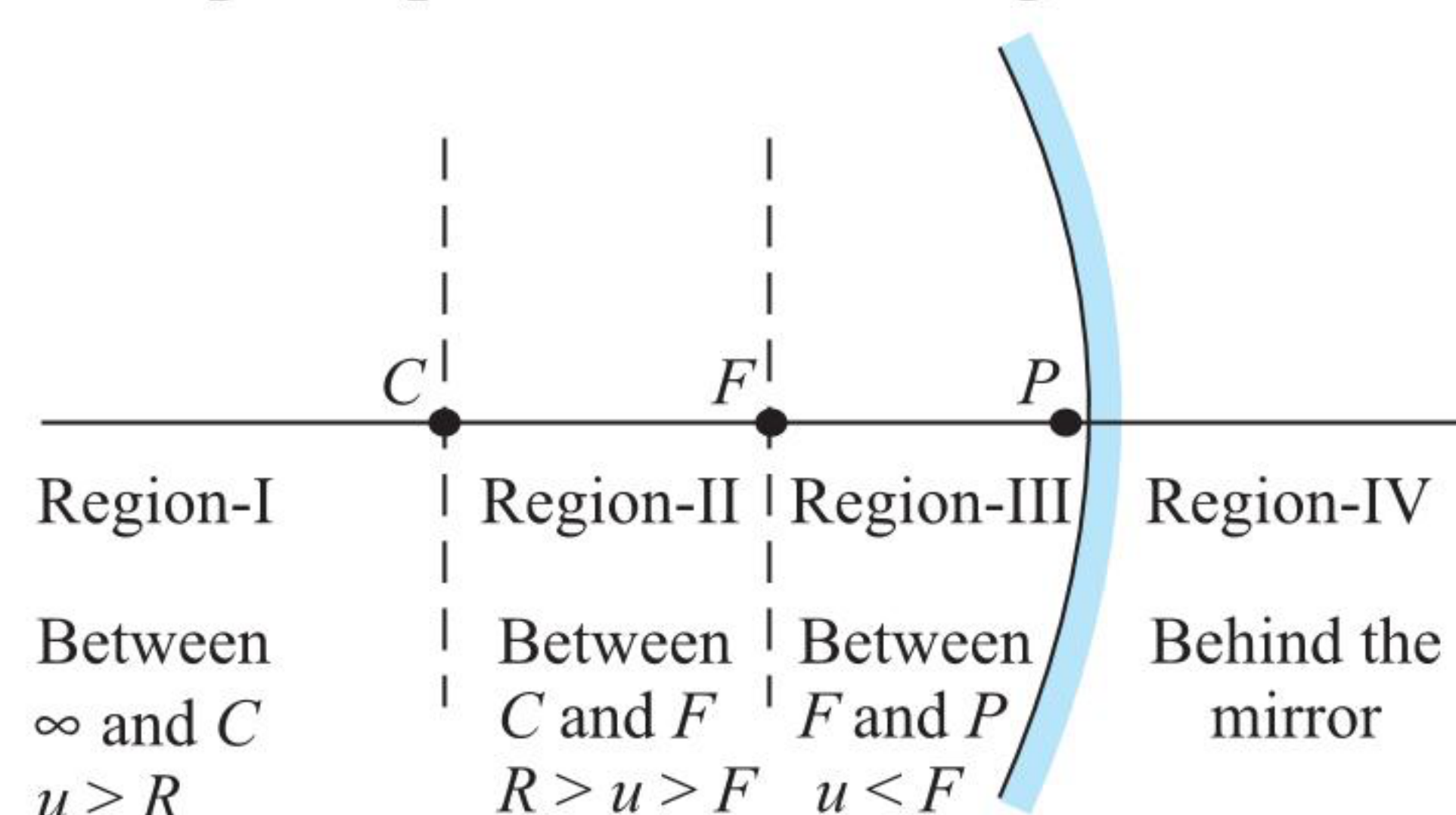
Thus,
$$v = \frac{x f_0}{f_0 + x}.$$

The above expression shows that whatever may be the value of x ($|x| > f_0$), v is always positive and its value is always less than or equal to f_0 . The magnification formula may be modified as $m = f_0 / (f_0 + x)$. It explains that m always lies between 0 and +1.

IMAGE FORMATION

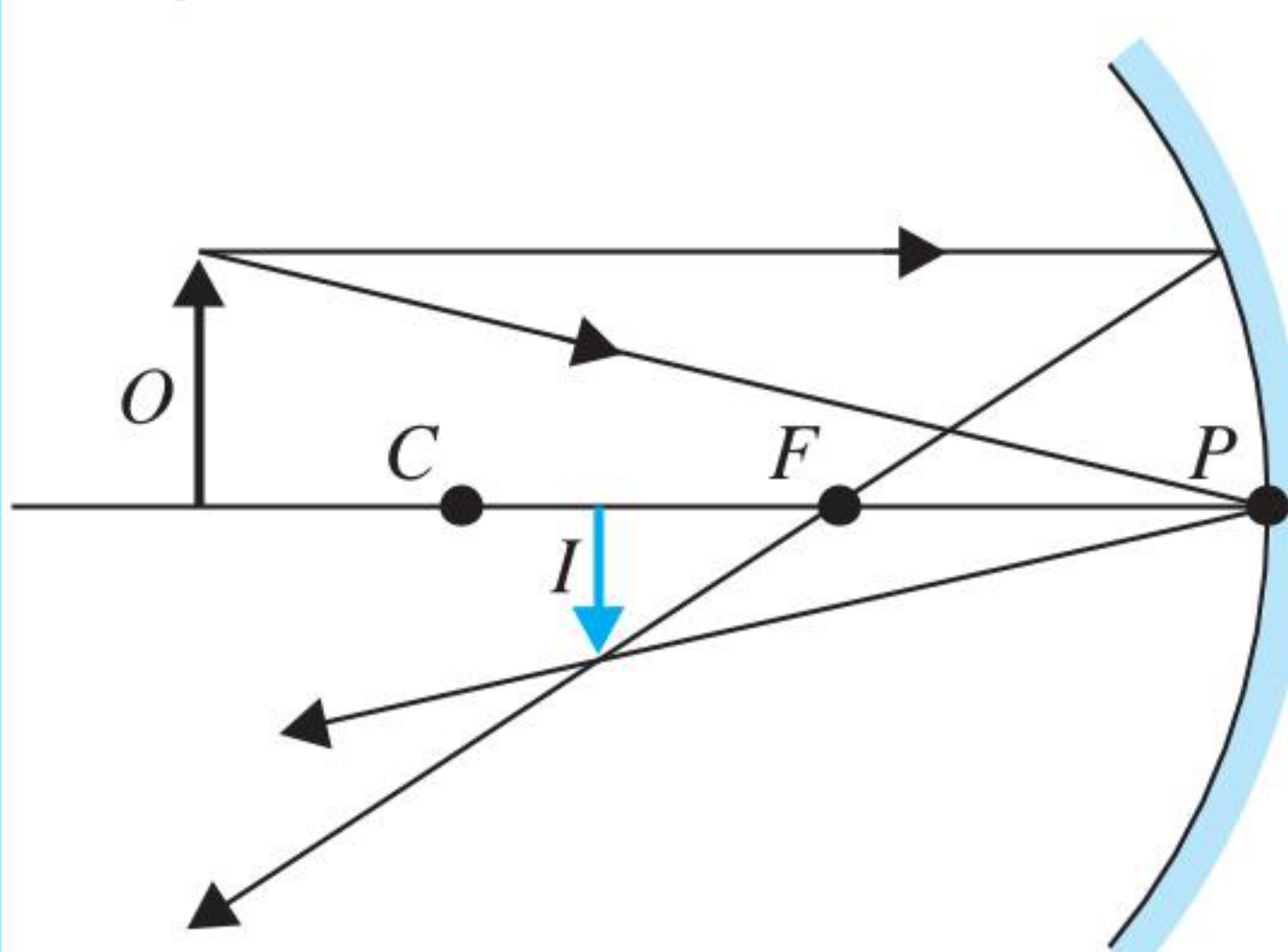
NATURE OF IMAGE FORMED BY CONCAVE MIRROR

We can divide principal axis in four regions.



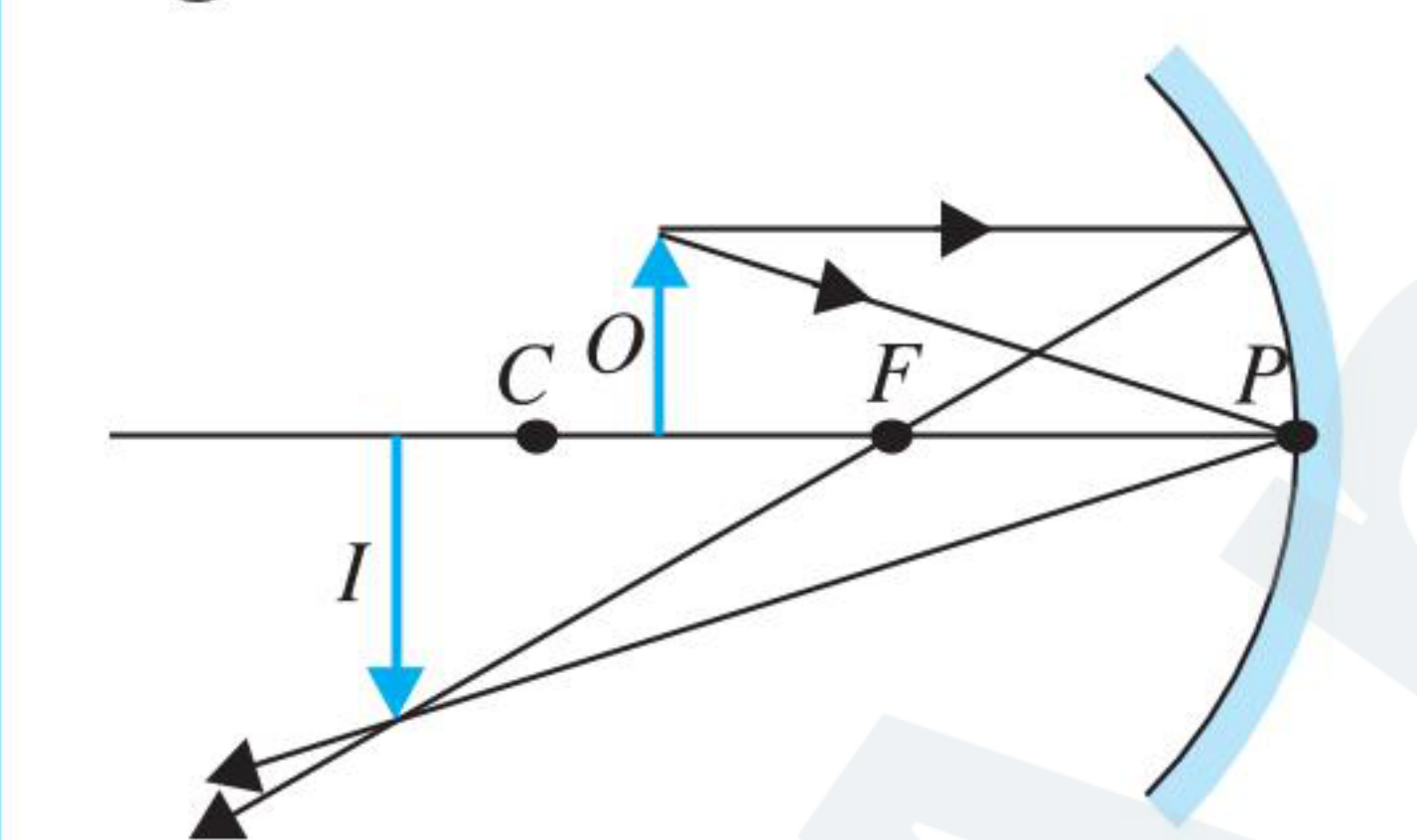
Now we analyze the corresponding image formed.

Region-I



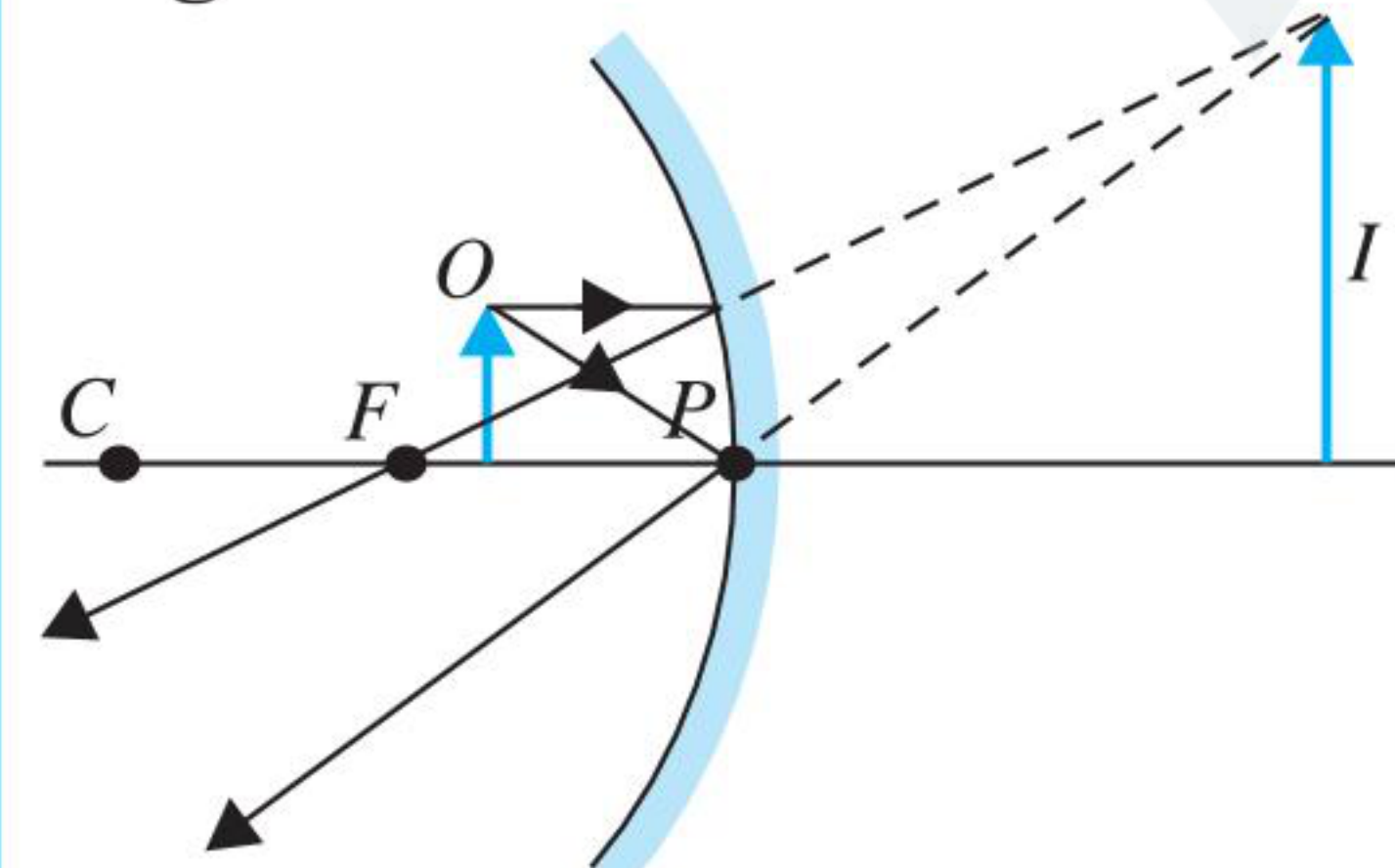
- Object placed between centre of curvature and infinity. Image real, inverted, smaller (diminished)
- If object is moved away from centre of curvature towards infinity, magnification goes on decreasing.

Region-II



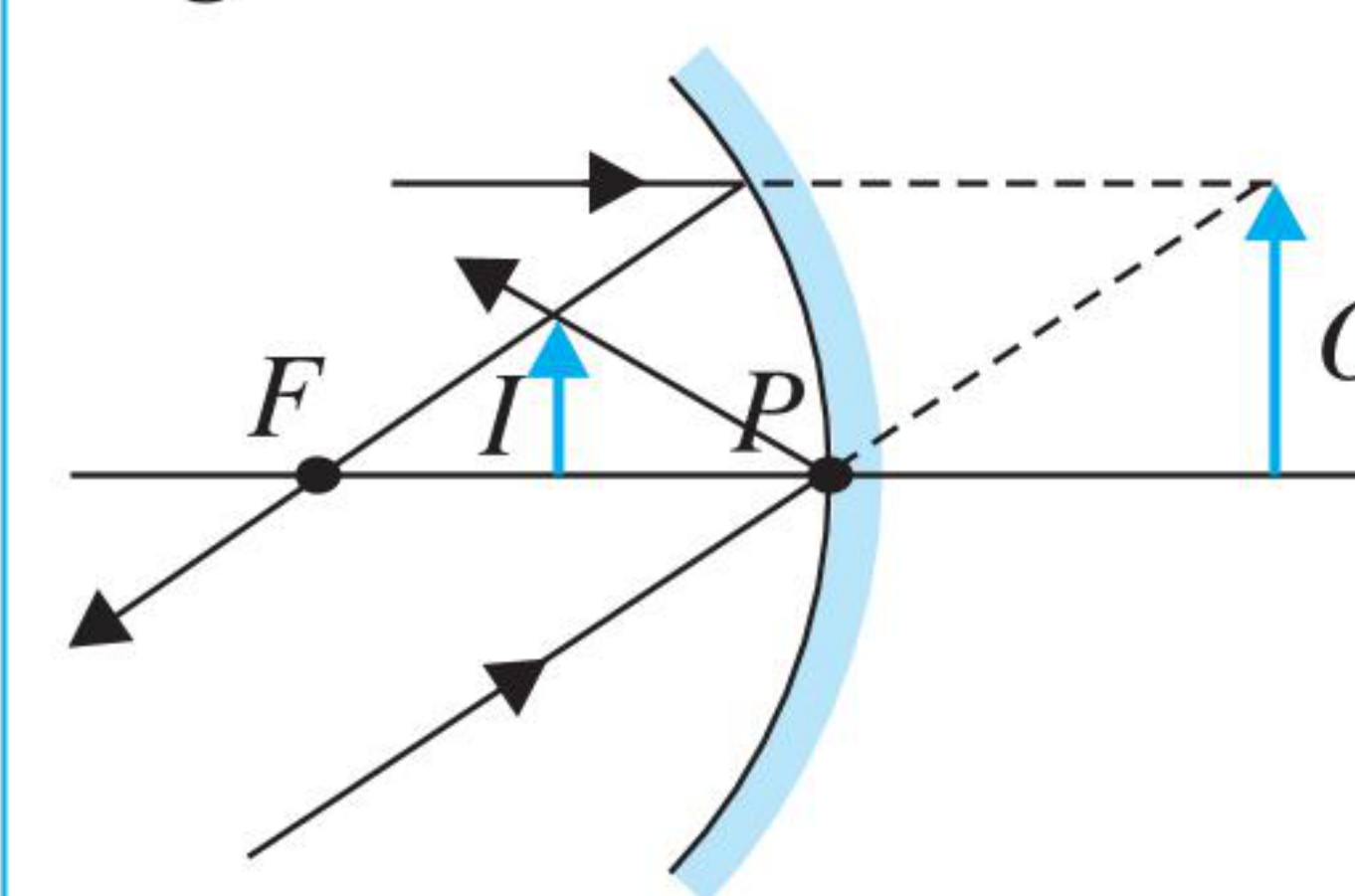
- Object placed between f and centre of curvature. Image is real, inverted, magnified.
- If object is moved from focus towards centre of curvature, magnification goes on decreasing at center of curvature it becomes unity.

Region-III



- Object placed between pole and focus. Image is virtual, erect, magnified.
- If object is moved towards pole magnification goes on decreasing at pole magnification m tends to unity.

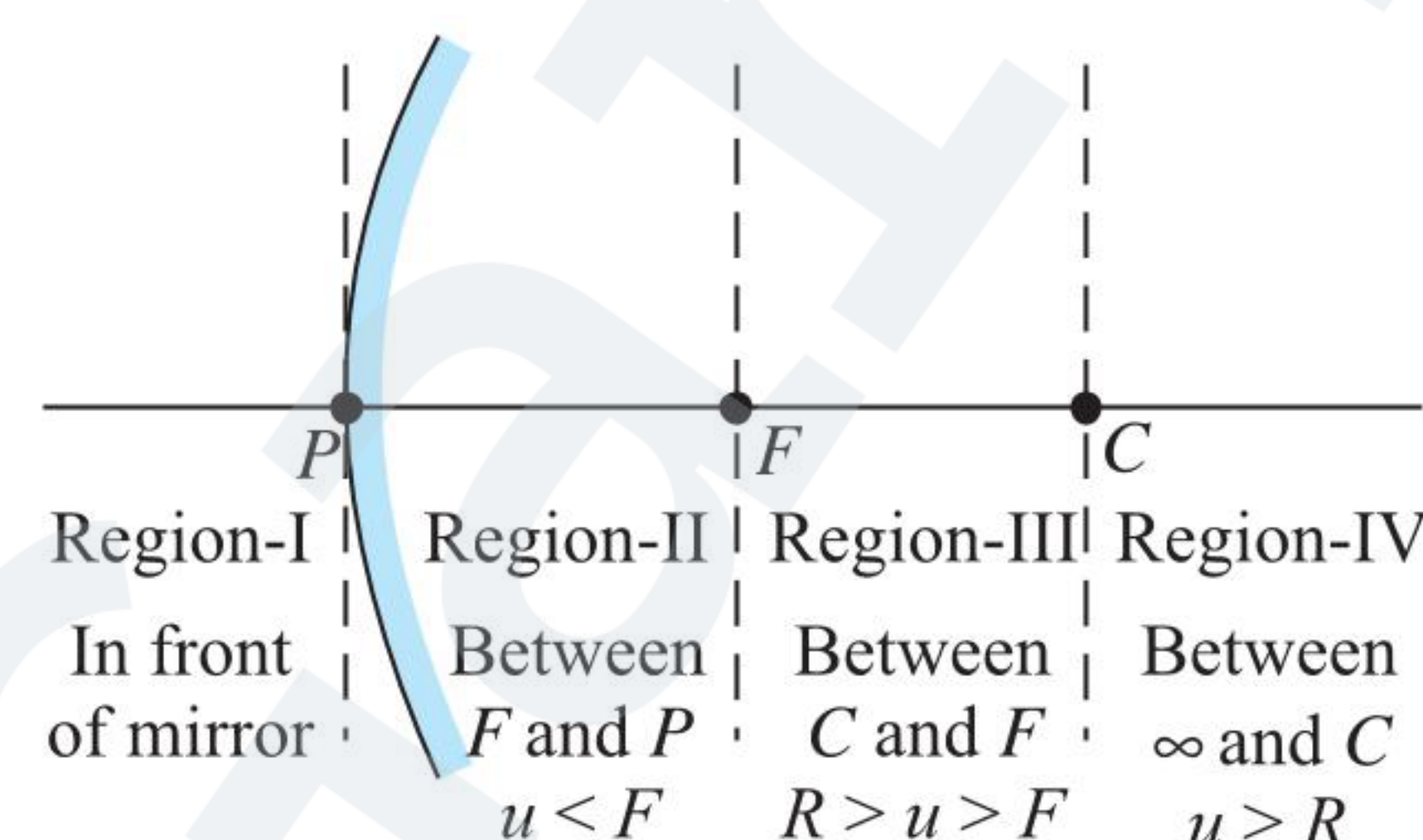
Region-IV



- Object is virtual point object. Image real, erect, smaller (diminished)
- As object moves away from pole, magnification goes on decreasing.

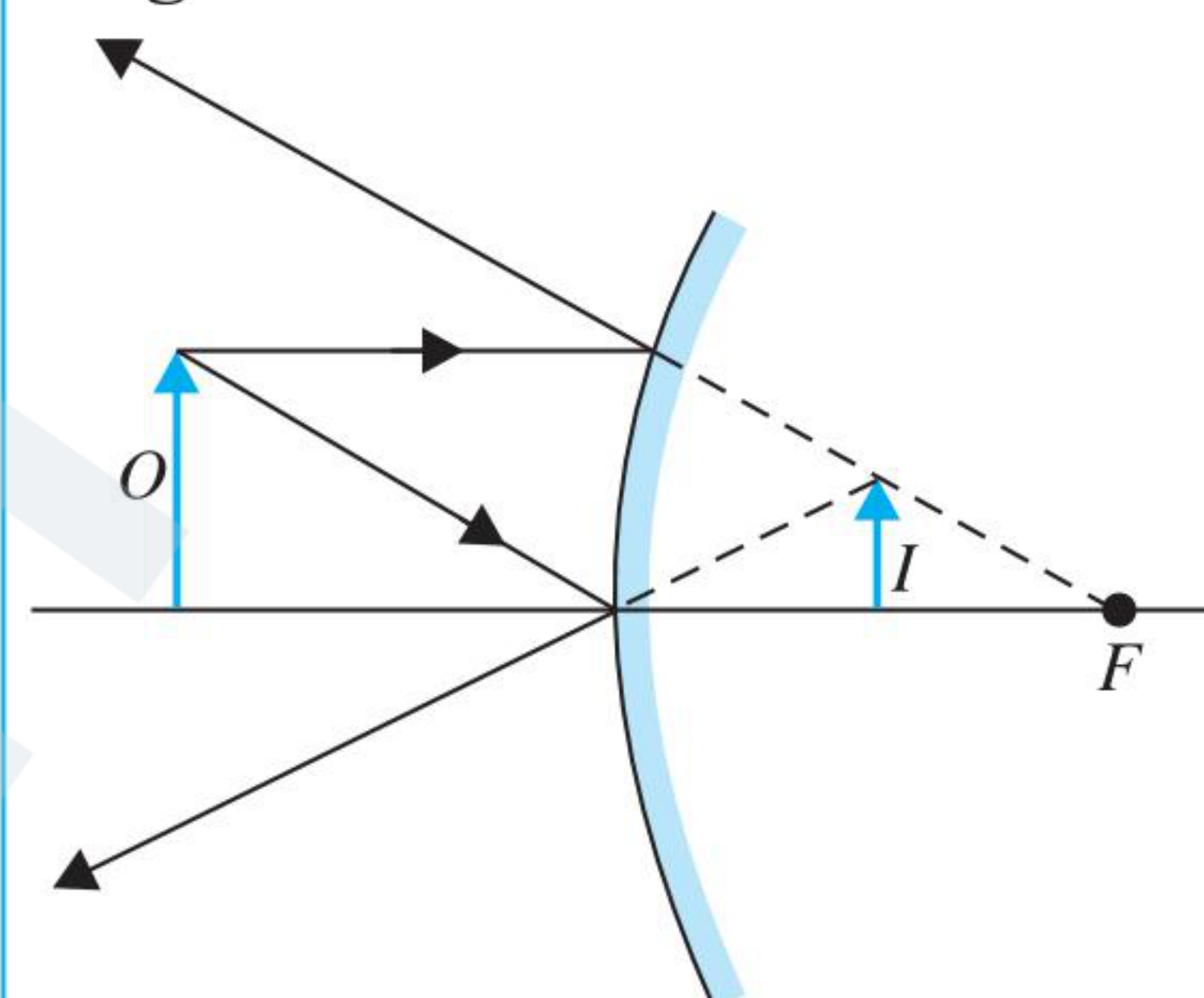
NATURE OF IMAGE FORMED BY CONVEX MIRROR

Figure shows convex mirror with four different object positions as shown in figure.



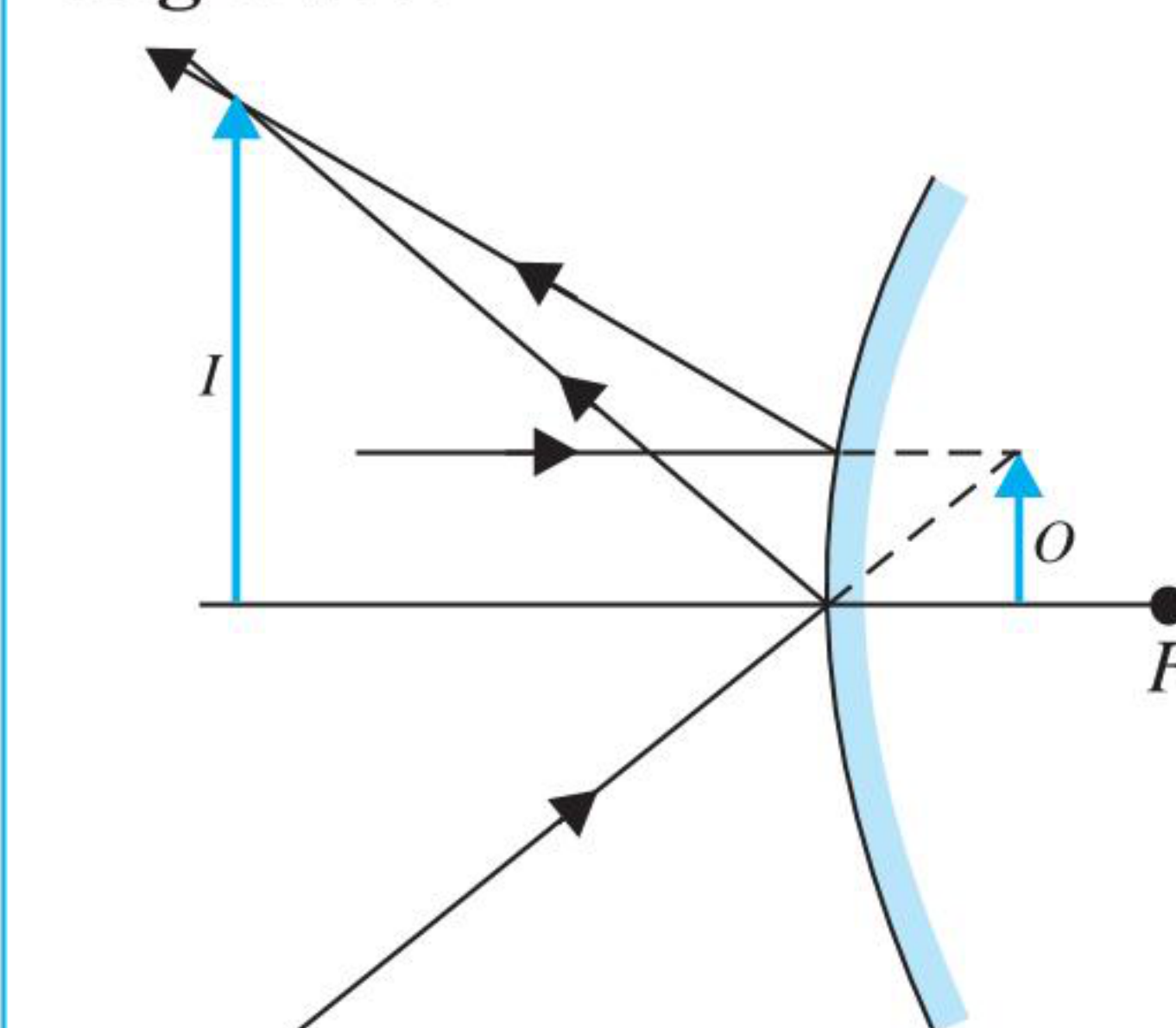
Now we analyze the corresponding image formed.

Region-I



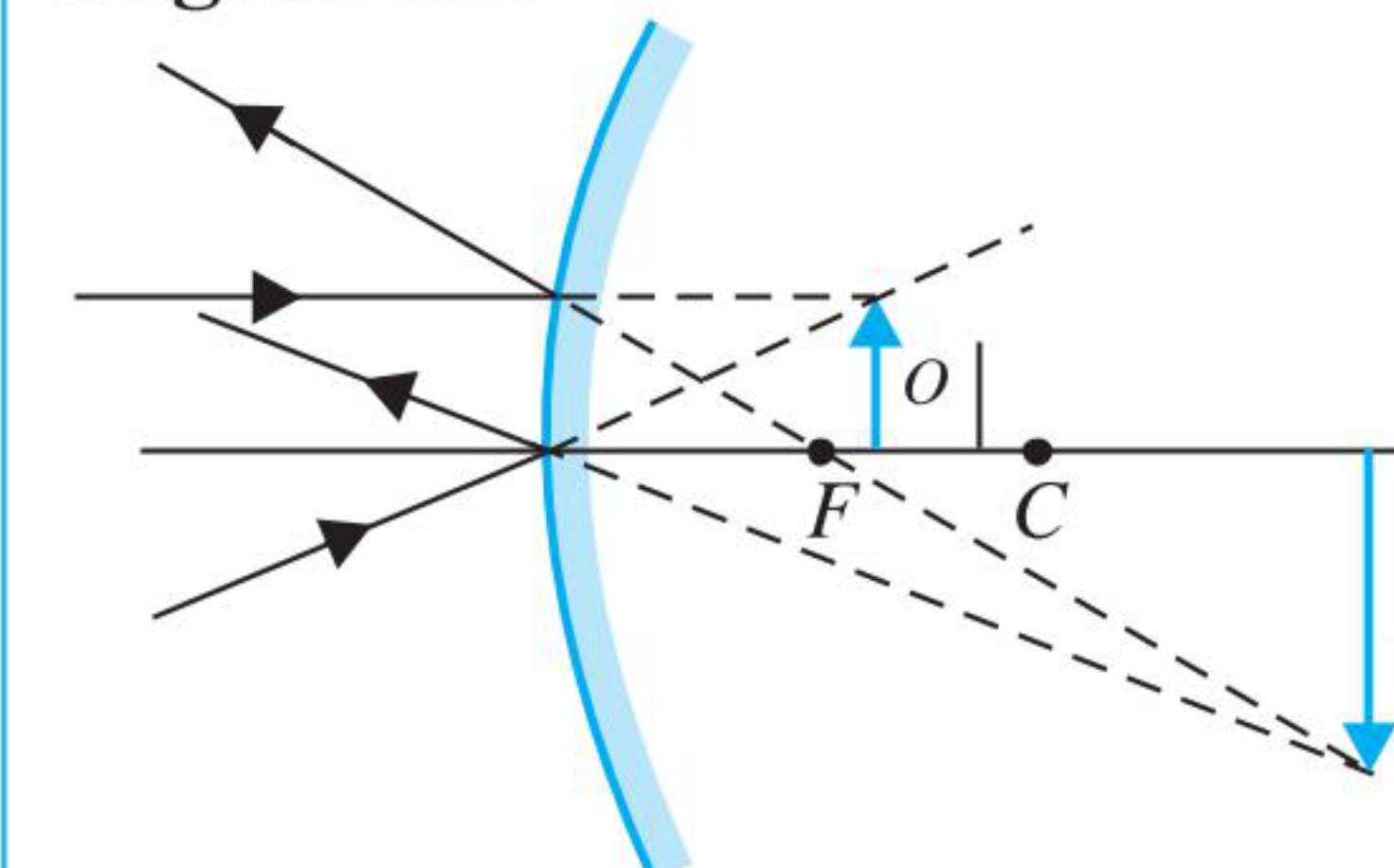
- Object placed in front of mirror. For all positions of object in front of mirror. Image is virtual, erect, smaller in size.
- As object moved towards pole, magnification increases and tends to unity at pole.

Region-II

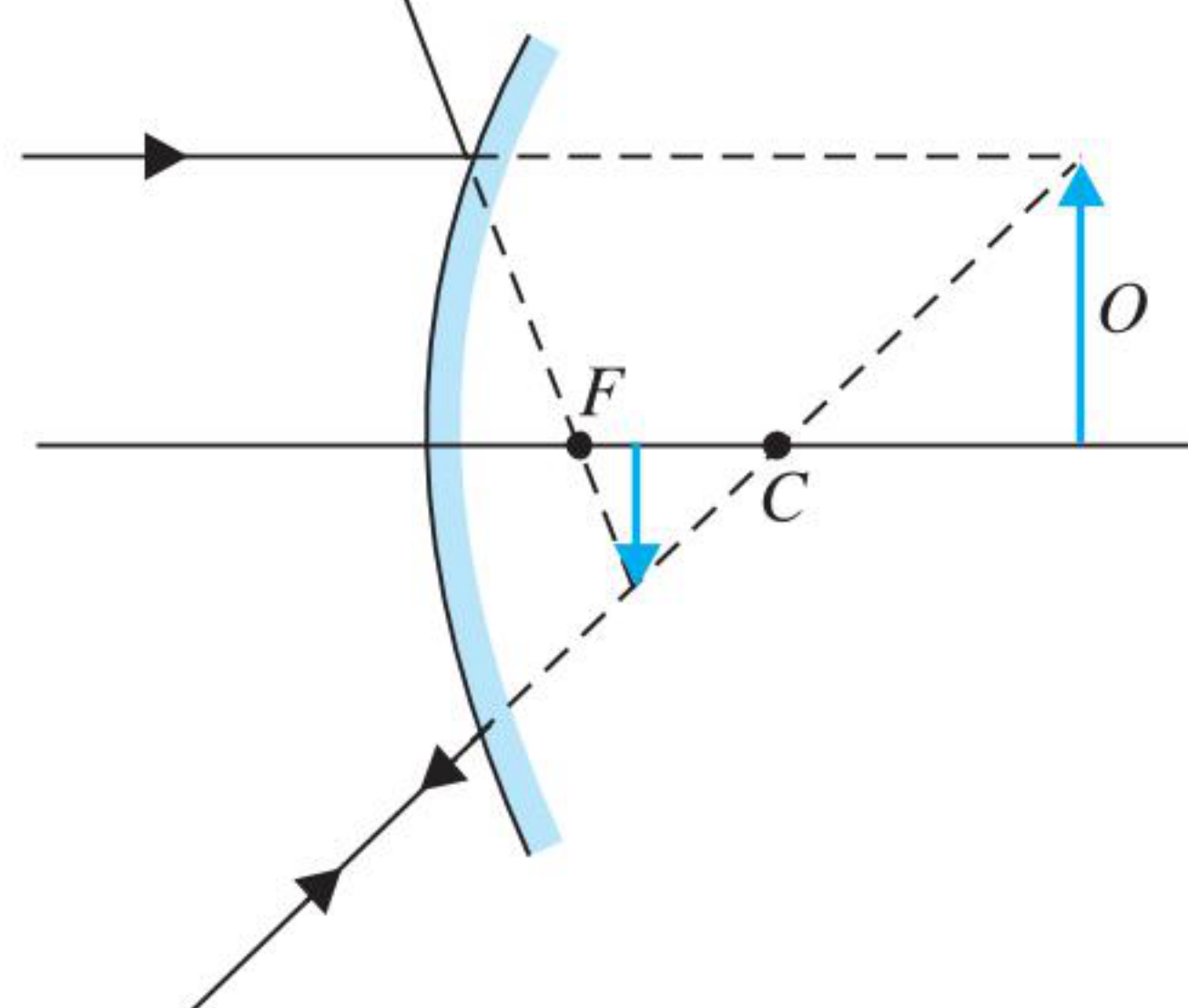


- Virtual object placed between pole and focus. Image formed is real, erect, enlarged.
- As object is moved away from pole, magnification increases.

Region-III



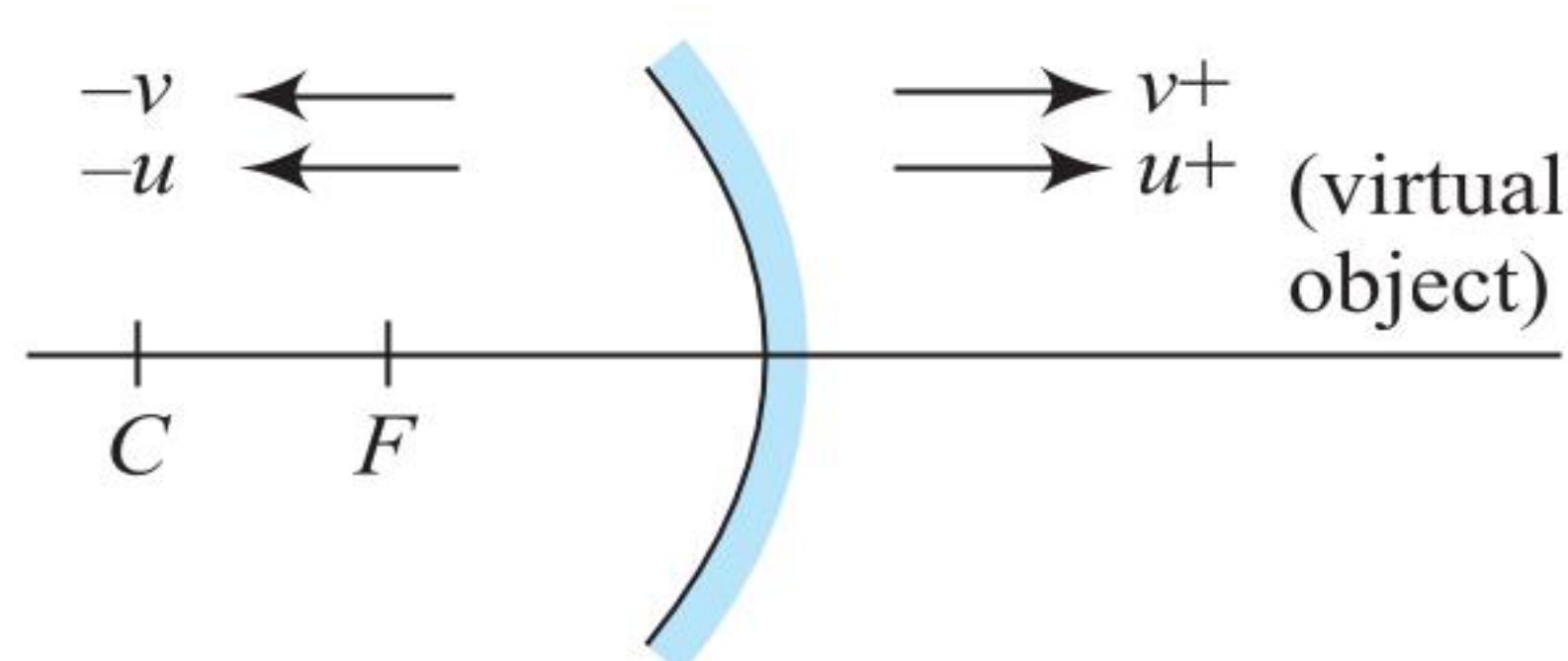
- Virtual object placed between F and C . Image formed virtual, inverted, enlarged.
- As object is moved toward C magnification decreases.

Region-IV

- Virtual object is placed beyond C . Image is virtual inverted smaller.
- As object is moved away from C , image size further decrease magnification decreases.

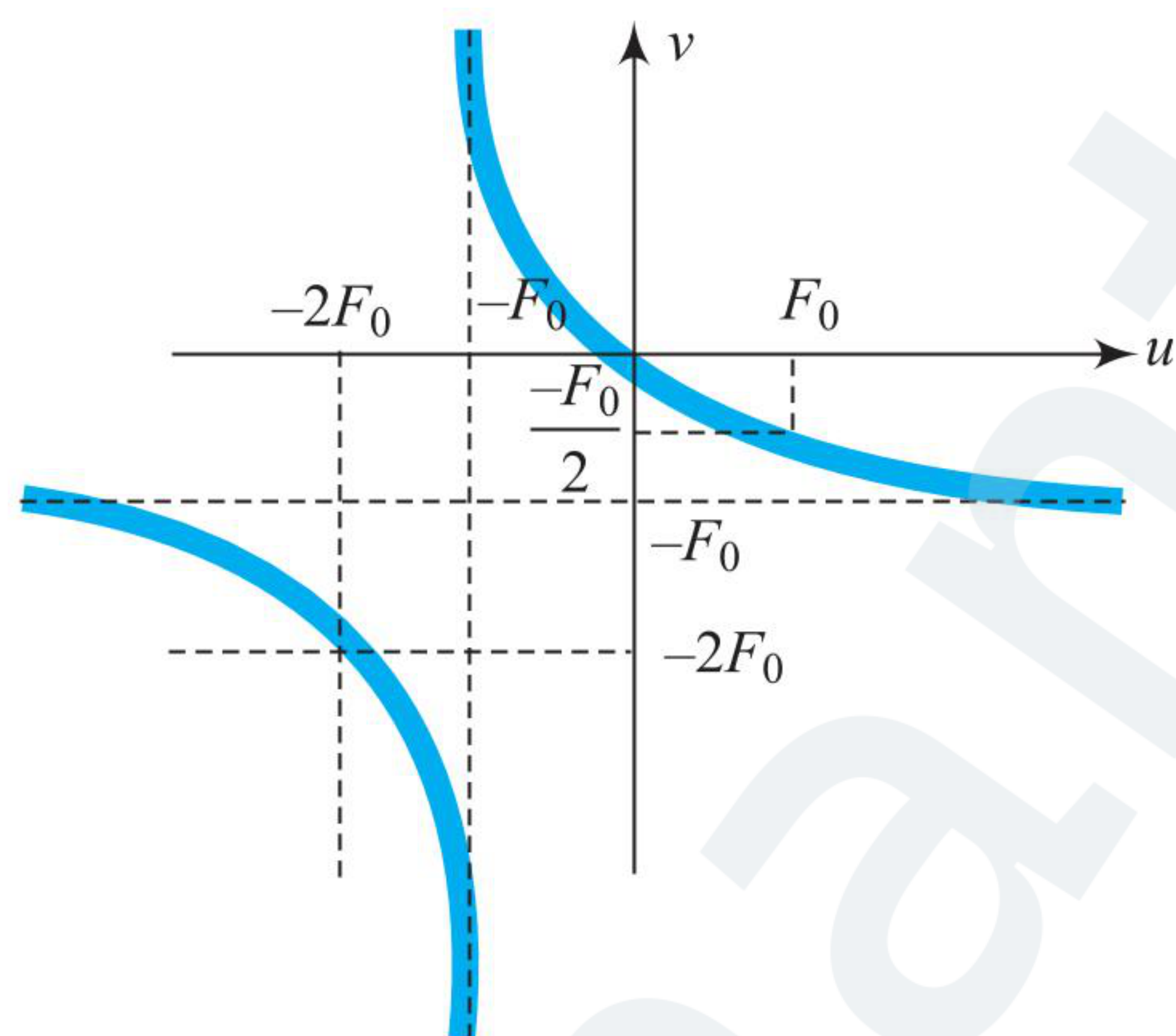
OBJECT DISTANCE (u) VS IMAGE DISTANCE (v) GRAPH

Concave mirror: $v = \frac{uf}{u-f}$, f is negative. Let $f = -F_0$.



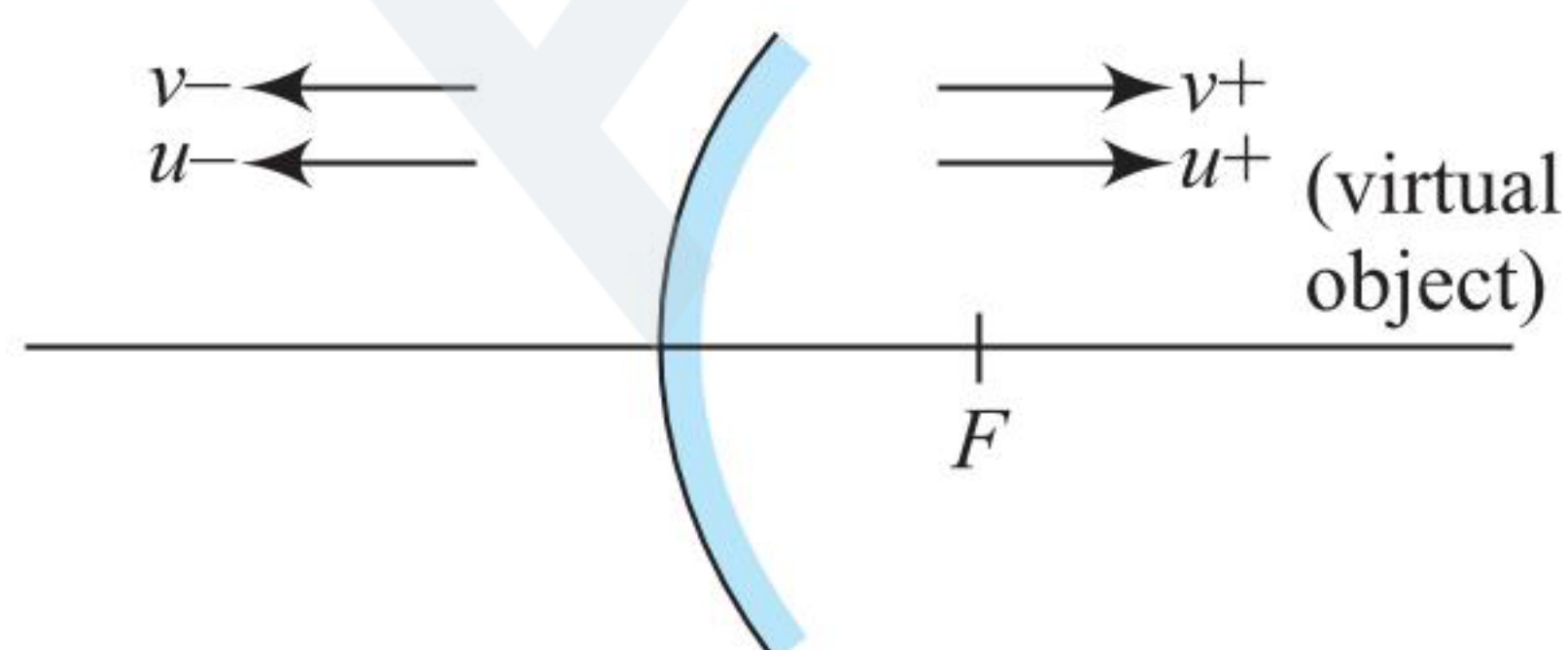
u	$-\infty$	$-2F_0$	$-F_0$	0	F_0	∞
v	$-F_0$	$-2F_0$	$\pm\infty$	0	$-\frac{F_0}{2}$	F_0

$$v = \frac{-uF_0}{u + F_0}$$

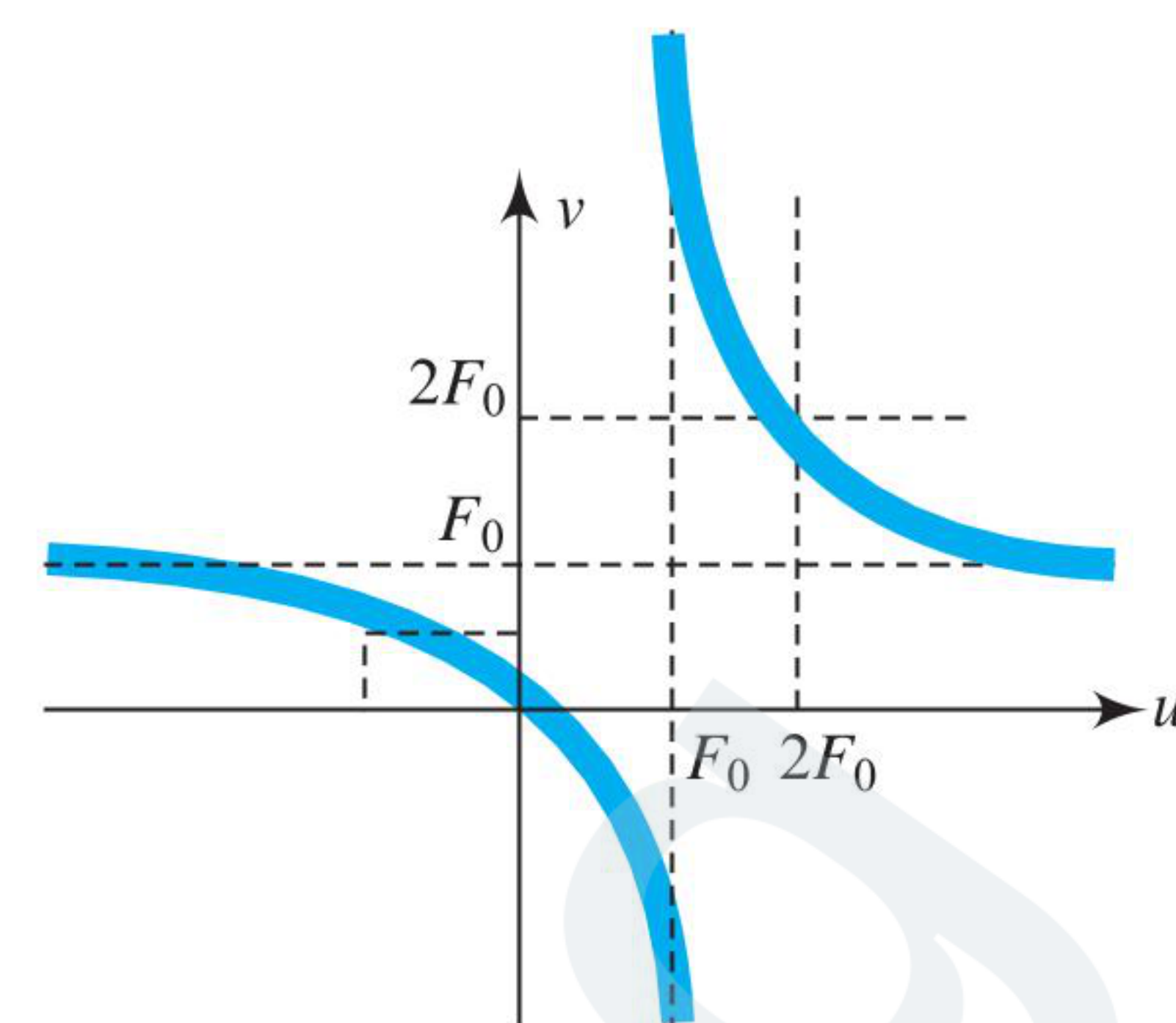


Convex mirror: $v = \frac{uf}{u-f}$

f is positive. Let $f = F_0 \Rightarrow v = \frac{uF_0}{u - F_0}$



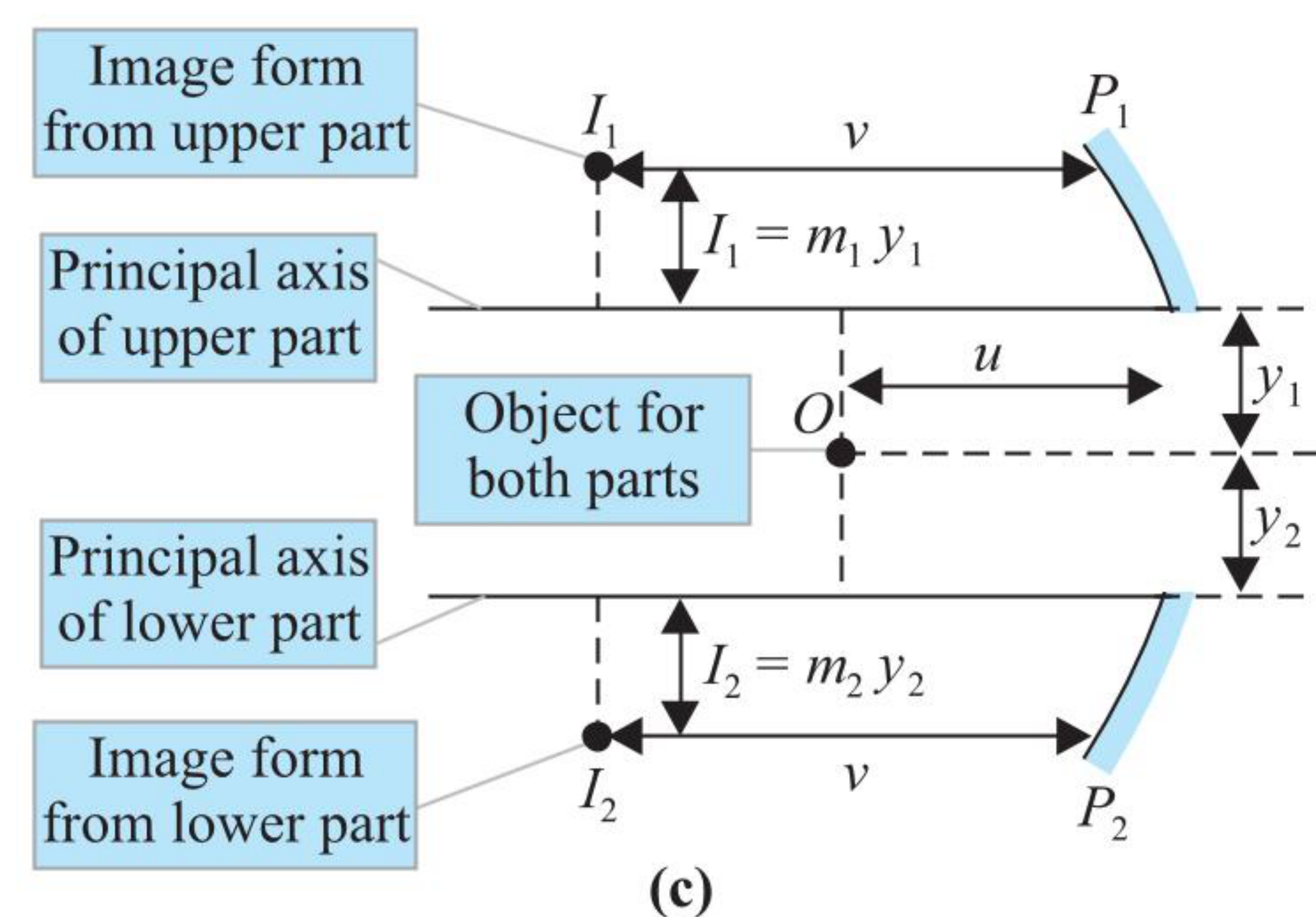
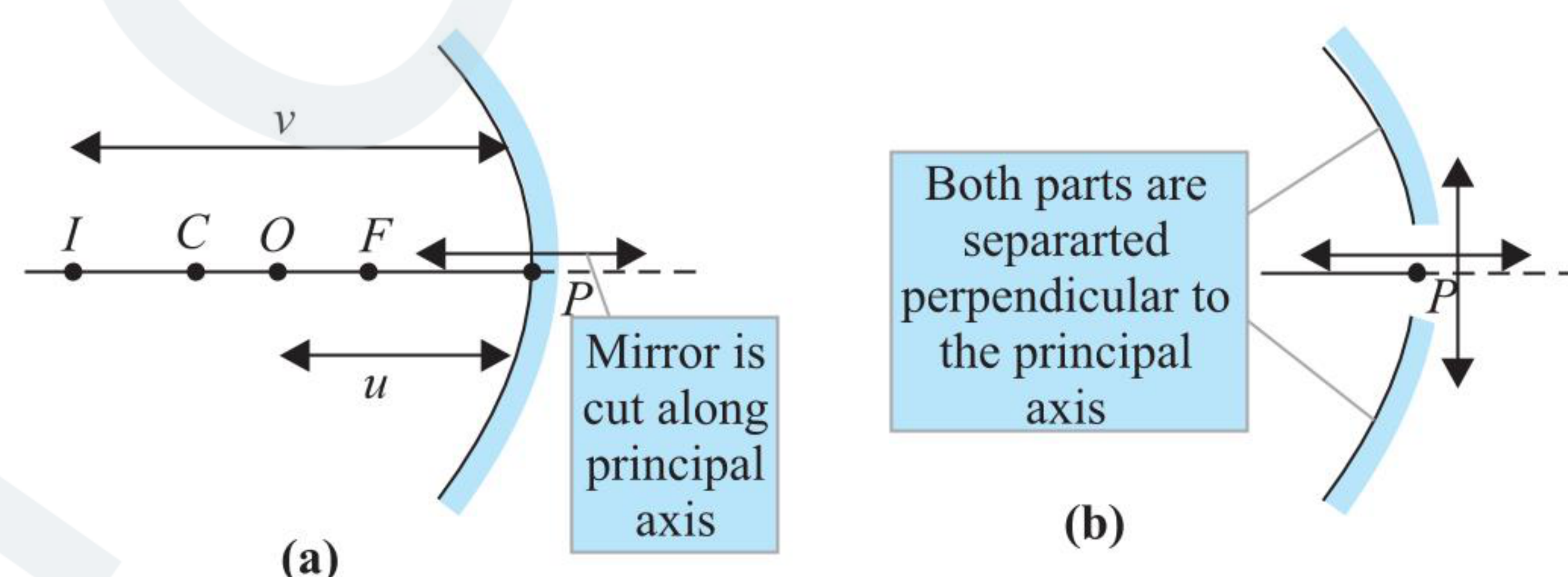
u	$-\infty$	$-F_0$	0	F_0	$2F_0$	∞
v	F_0	$\frac{F_0}{2}$	0	$\pm\infty$	$2F_0$	F_0



CUTTING OF A MIRROR

Mirror is cut along its principal axis and two parts are separated perpendicular to the principal axis.

1. For first part the object O will be below principal axis and for second part the object O will be above principal axis.
2. For first part image will form above its principal axis, and for second part the image will form below principal axis at a distance $m_1 y_1$ and $m_2 y_2$, respectively.



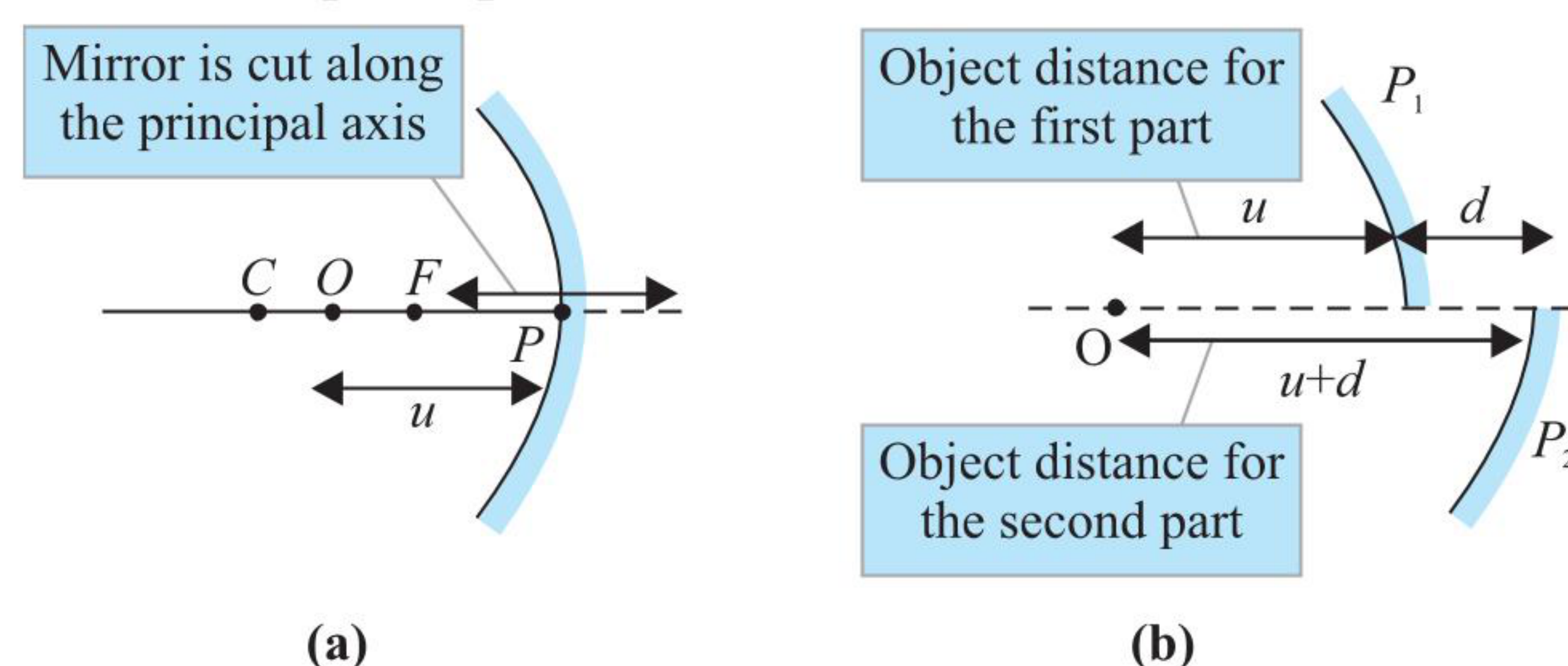
For image partition from principal axis,

$$m = -\frac{v}{u} = \frac{I_1}{O} \Rightarrow |I_1| = my_1$$

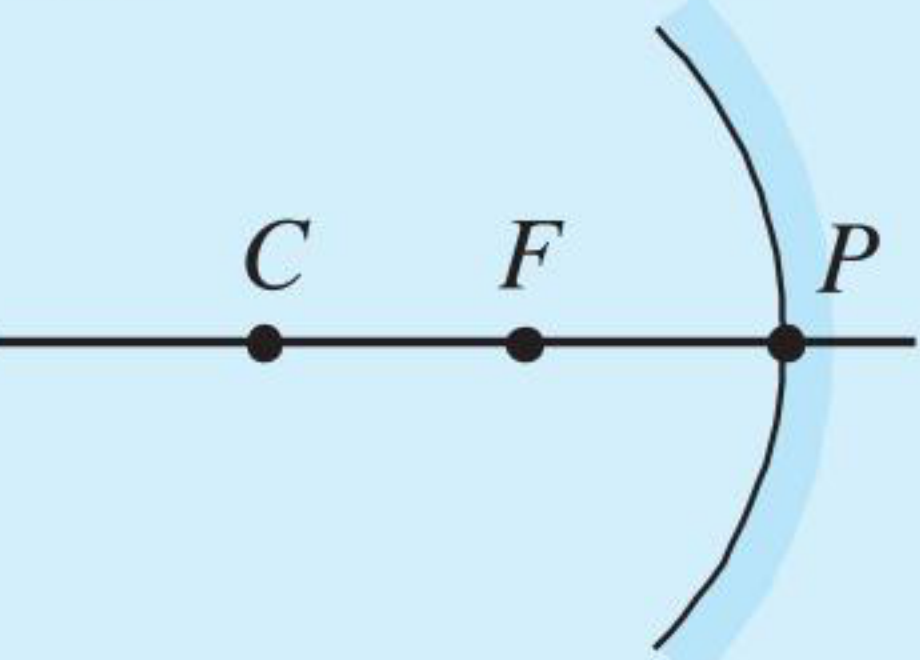
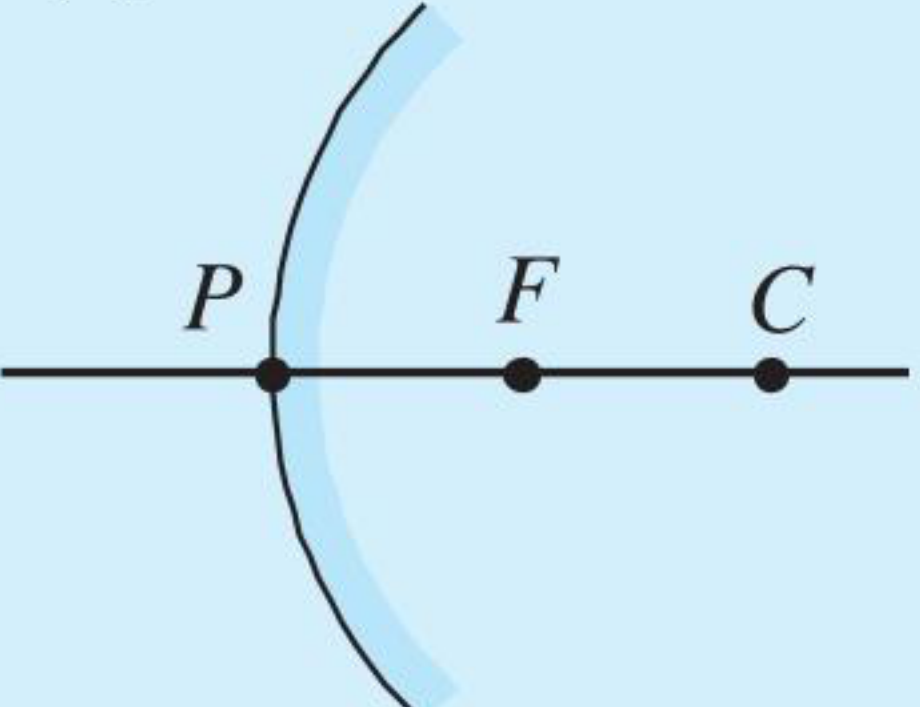
Thus, $s = my_1 + y_1 + my_2 + y_2$; $s = (m + 1)(y_1 + y_2)$

If s is the separation distance between two images I_1 and I_2 of the same object O formed due to two halves of the mirror.

Mirror is cut along its principal axis and two parts are separated parallel to the principal axis.

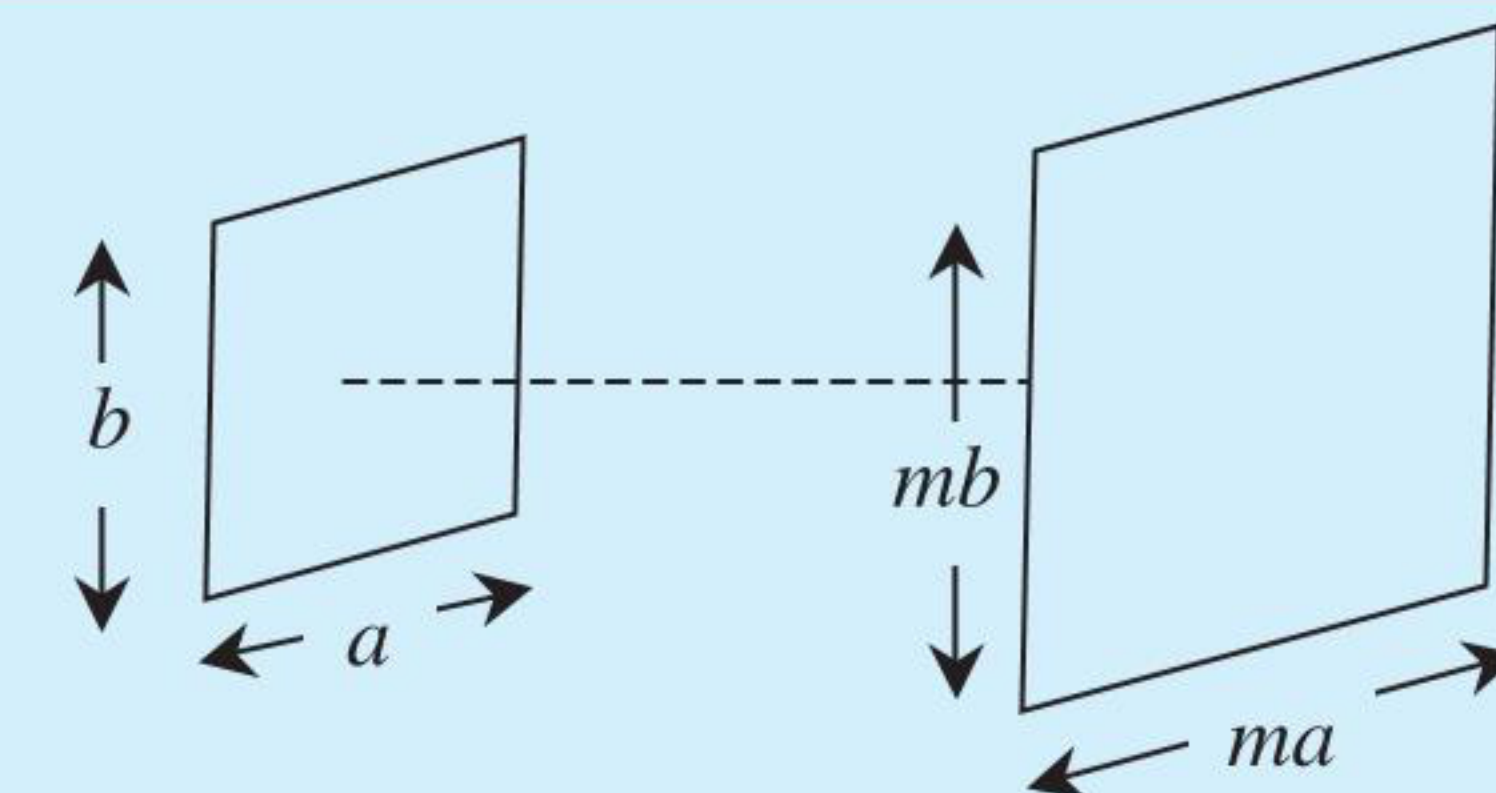


Position, size and nature of image formed by the spherical mirror

Mirror	Location of the object	Location of the image	Magnification, size of the image	Nature	
				Real virtual	Erect inverted
(a) Concave 	At infinity, i.e., $u = \infty$	At focus, i.e., $v = f$	$m \ll 1$, diminished	Real	inverted
	Away from centre of curvature ($u > 2f$)	Between f and $2f$, i.e., $f < v < 2f$	$m < 1$, diminished	Real	inverted
	At centre of curvature $u = 2f$	At centre of curvature, i.e., $v = 2f$	$m = 1$, same size as that of the object	Real	inverted
	Between centre of curvature and focus: $F < u < 2f$	Away from the centre of curvature $v > 2f$	$m > 1$, magnified	Real	inverted
	At focus, i.e., $u = f$	At infinity, i.e., $v = \infty$	$m = \infty$, magnified	Real	inverted
	Between pole and focus $u < f$	$v > u$	$m > 1$, magnified	Virtual	erect
(b) Convex 	At infinity, i.e., $u = \infty$	At focus, i.e., $v = f$	$m < 1$, diminished	Virtual	erect
	Anywhere between infinity and pole	Between pole and focus	$m < 1$, diminished	Virtual	erect

Note:

- In case of convex mirrors, as the object moves away from the mirror, the image becomes smaller and moves closer to the focus.
- For convex mirror maximum image distance is its focal length.
- In concave mirror, minimum distance between a real object and its real image is zero (i.e., when $u = v = 2f$).

Linear magnification		Areal magnification
Transverse	Longitudinal	
When an object is placed perpendicular to the principal axis, then linear magnification is called lateral or transverse magnification. It is given by $m = \frac{I}{O} = -\frac{v}{u} = \frac{f}{f-u} = \frac{f-v}{f}$ (*Always use sign convention while solving the problems)	When object lies along the principal axis then its longitudinal magnification $m = \frac{I}{O} = \frac{-(v_2 - v_1)}{(u_2 - u_1)}$ If object is small; $m = -\frac{dv}{du} = \left(\frac{v}{u}\right)^2$ Also length of image $= \left(\frac{v}{u}\right)^2 \times \text{Length of object } (L_o)$ $(L_i) = \left(\frac{f}{u-f}\right)^2 \cdot L_o$	 If a 2D-object is placed with its plane perpendicular to principal axis Its areal magnification $M_s = \frac{\text{Area of image } (A_i)}{\text{Area of object } (A_o)} = \frac{ma \times mb}{ab} = m^2$ $\Rightarrow m_s = m^2 = \frac{A_i}{A_o}$

Note:

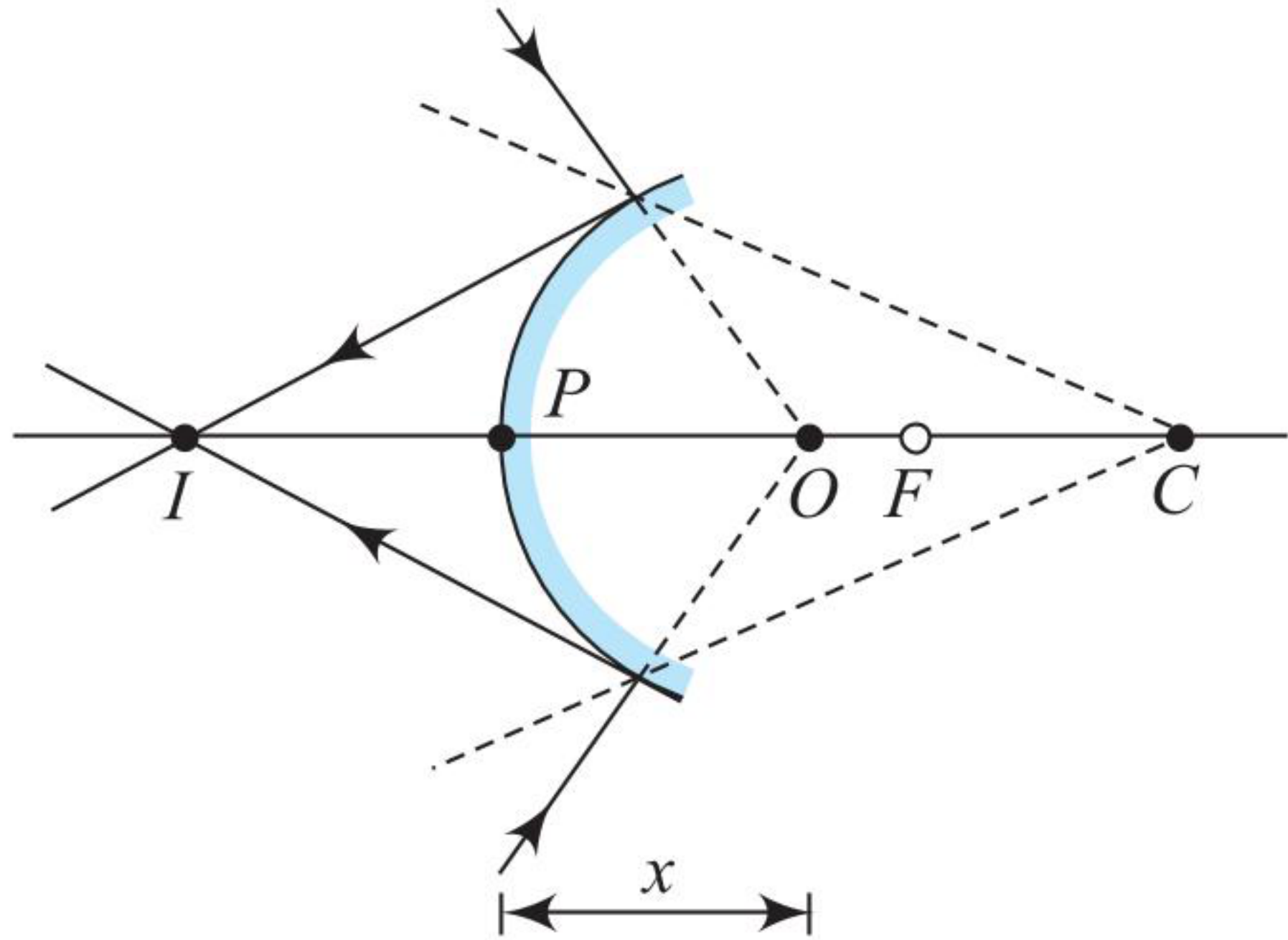
- Do not put the sign of quantity which is to be determined.
- If a spherical mirror produces an image m times the size of the object ($m = \text{magnification}$) then u , v and f are given by the following:

$$u = \left(\frac{m-1}{m}\right)f, \quad v = -(m-1)f \quad \text{and} \quad f = \left(\frac{m}{m-1}\right)u \quad (\text{use sign convention})$$

ILLUSTRATION 1.20

Can a convex mirror form a real image? Explain.

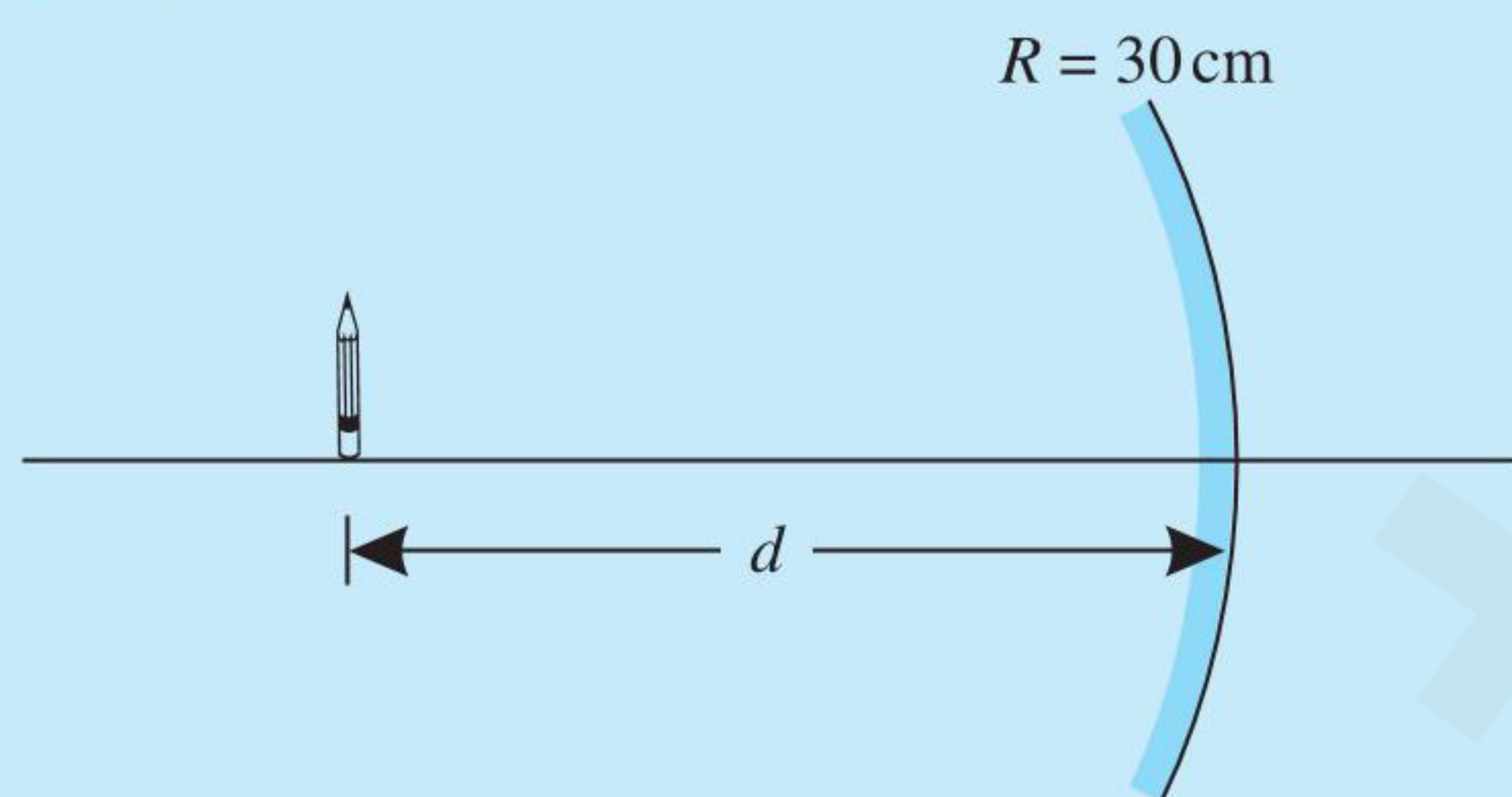
Sol. Yes, only when the object is virtual and is placed between F and P . Figure shows a convex mirror exposed to a converging beam which converges to a point that lies between F and P .



$$v = \frac{-xf}{f_0 - x}; v \text{ becomes negative (real image) only when } x < f_0.$$

ILLUSTRATION 1.21

A concave mirror has radius of curvature $R = 30$ cm. A pencil lies perpendicular to the axis of the mirror at each of the following object distances:



- (a) $d = 40$ cm (b) $d = 30$ cm (c) $d = 15$ cm (d) $d = 5$ cm
Find the image of the pencil, magnification of each image.

Sol. The incident ray is travelling towards right, all the distance measured in rightward direction should be positive,

$$\text{The focal length of mirror is } f = \frac{R}{2} = -15 \text{ cm.}$$

$$\text{From mirror equation, } \frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

- (a) In this case $d = 40$ cm, the object is placed on left side of centre of curvature and $u = -40$ cm

$$\frac{1}{v} + \frac{1}{(-40)} = \frac{1}{(-15)} \Rightarrow v = -24 \text{ cm}$$

The minus sign shows that the image is in front of the mirror, real.

$$\text{Magnification, } m = -\frac{v}{u} = -\left(\frac{-24}{-40}\right) = -0.6$$

As $m < 1$, the image is diminished in size, minus sign shows that the image is inverted.

- (b) In this case $d = 30$ cm, the object is placed at the centre of curvature. In this case the image will form at object position. The image is real, formed in front of the mirror.

$$\text{Magnification, } m = -\left(\frac{v}{u}\right) = -\left(\frac{-30}{-30}\right) = -1$$

As $m = -1$, the size of the object and the image are equal, minus sign shows that the image is inverted.

- (c) In this case $d = 15$ cm, the object is placed at the focus. In this case the image will form at infinity.

$$\text{Magnification, } m = -\frac{v}{u} = -\frac{-\infty}{(-15)} = -\infty$$

The image is enlarged real, formed in front of the mirror and will form at infinity.

- (d) In this case $d = 5$ cm, the object is placed on right side of the focus and $u = -5$ cm

$$\frac{1}{v} + \frac{1}{(-5)} = \frac{1}{(-15)} \Rightarrow v = +7.5 \text{ cm}$$

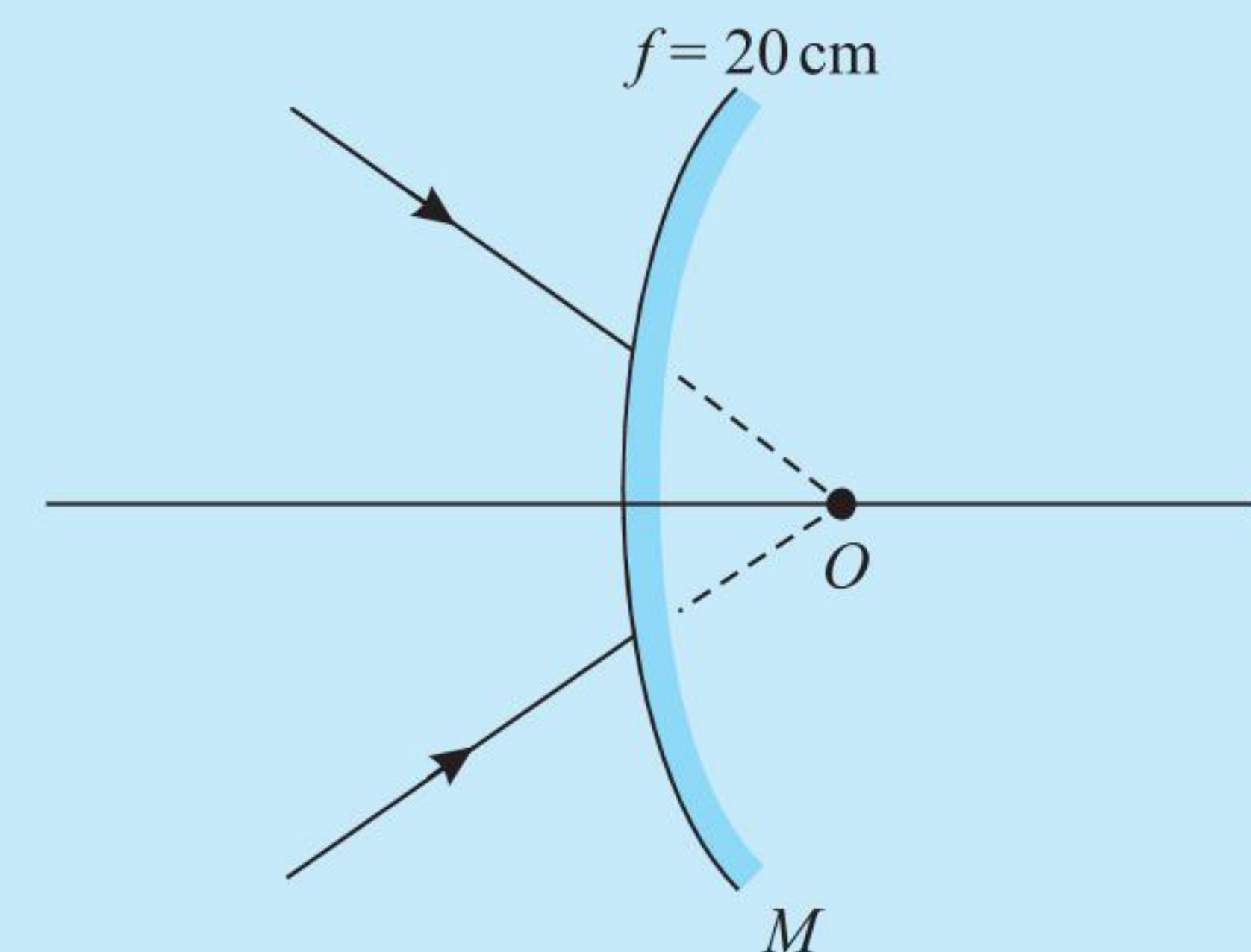
The plus sign shows that the image is behind the mirror, virtual. It is a general observation that an object placed inside the focal point of a concave mirror produces a virtual image.

$$m = -\left(\frac{v}{u}\right) = -\left(\frac{+7.5}{-5}\right) = +1.5$$

As $m > 1$, the image is magnified; plus, sign shows that it is erect.

ILLUSTRATION 1.22

A beam of light converges towards a point O , behind a convex mirror of focal length 20 cm.

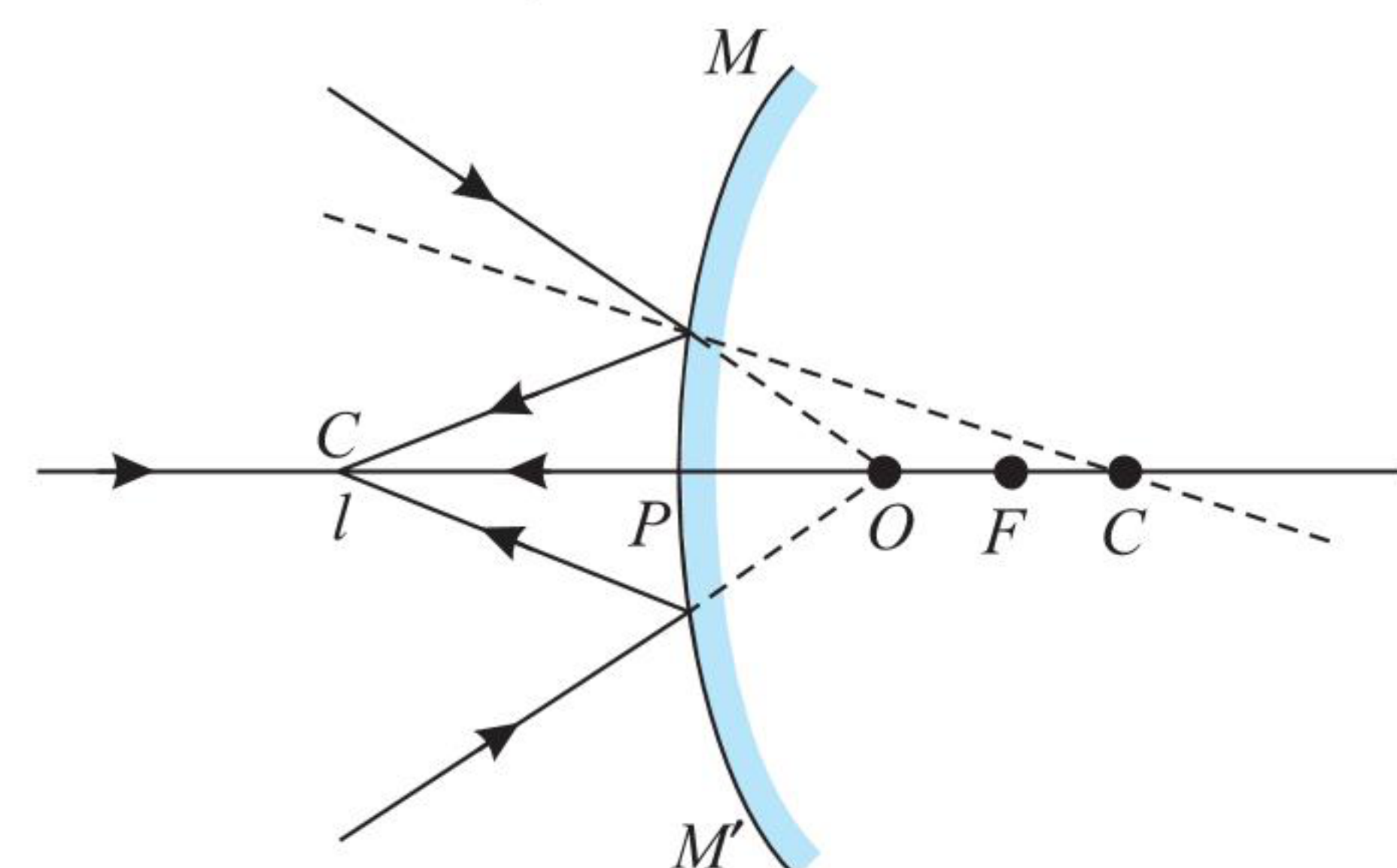


Find the nature and position of image if the point O is

- (a) 10 cm behind the mirror
(b) 30 cm behind the mirror

Sol. The incident ray is travelling towards right, all the distance measured in rightward direction should be positive,

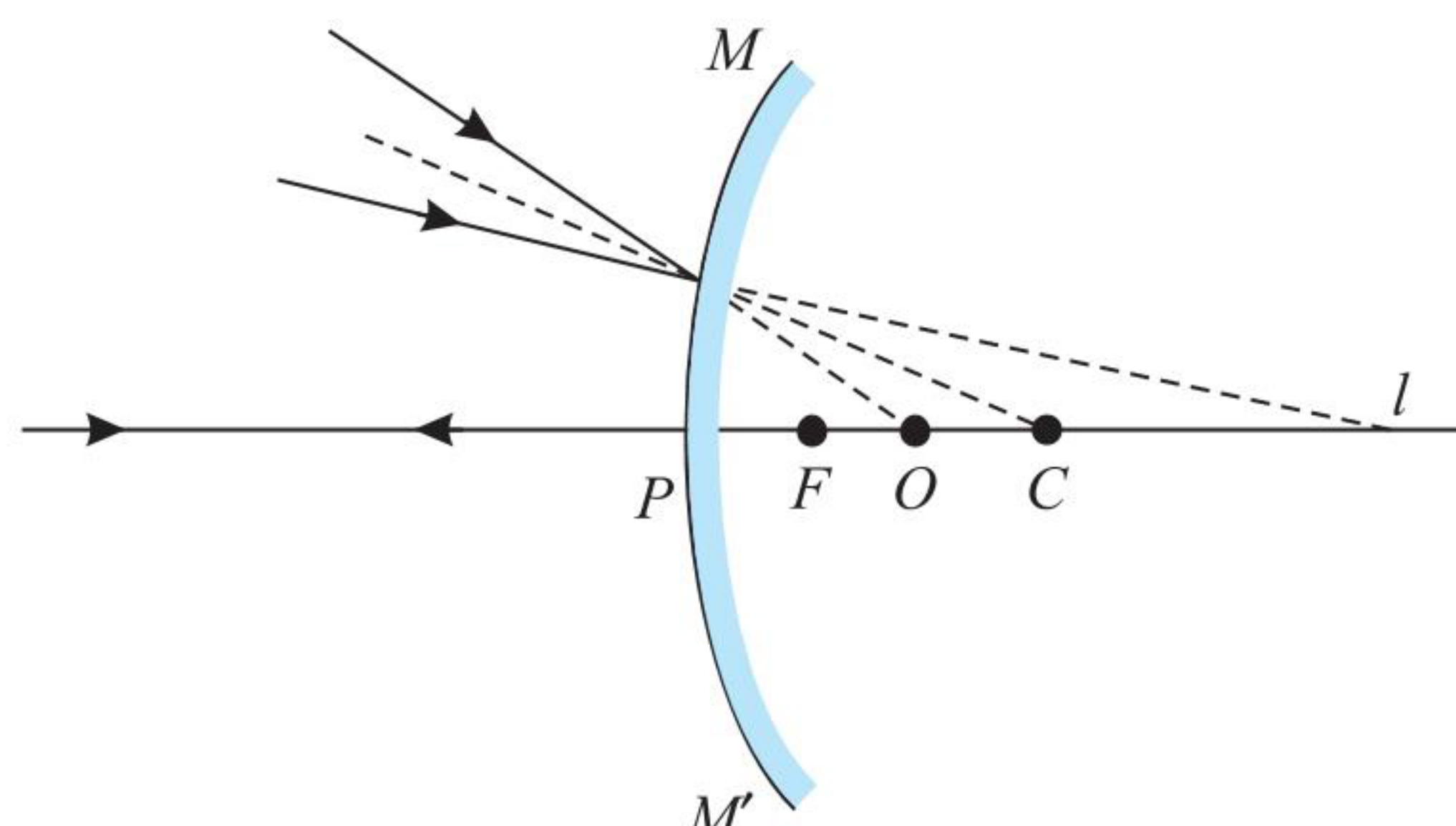
- (a) The beam of light converges towards O , behind the mirror. For this situation object will be virtual as shown in figure. Here $u = +10$ cm and $f = +20$ cm.



$$\text{So, } \frac{1}{v} + \frac{1}{+10} = \frac{1}{+20} \text{ i.e., } v = -20 \text{ cm}$$

i.e., the image will be at a distance of 20 cm in front of the mirror and will be real, erect and enlarged with $m = -(-20/10) = +2$.

- (b) For this situation also object will be virtual as shown in figure.



Here, $u = +30 \text{ cm}$ and $f = +20 \text{ cm}$

$$\text{So, } \frac{1}{v} + \frac{1}{+30} = \frac{1}{+20} \text{ i.e., } v = +60 \text{ cm}$$

i.e., the image will be at a distance of 60 cm behind the mirror and will be virtual, inverted and enlarged with $m = -(+60/30) = -2$.

ILLUSTRATION 1.23

The focal length of a concave mirror is 30 cm. Find the position of the object in front of the mirror, so that the image is three times the size of the object.

Sol. As the object is in front of the mirror it is real and for real object the magnified image formed by concave mirror can be inverted (i.e., real) or erect (i.e., virtual), so there are two possibilities:

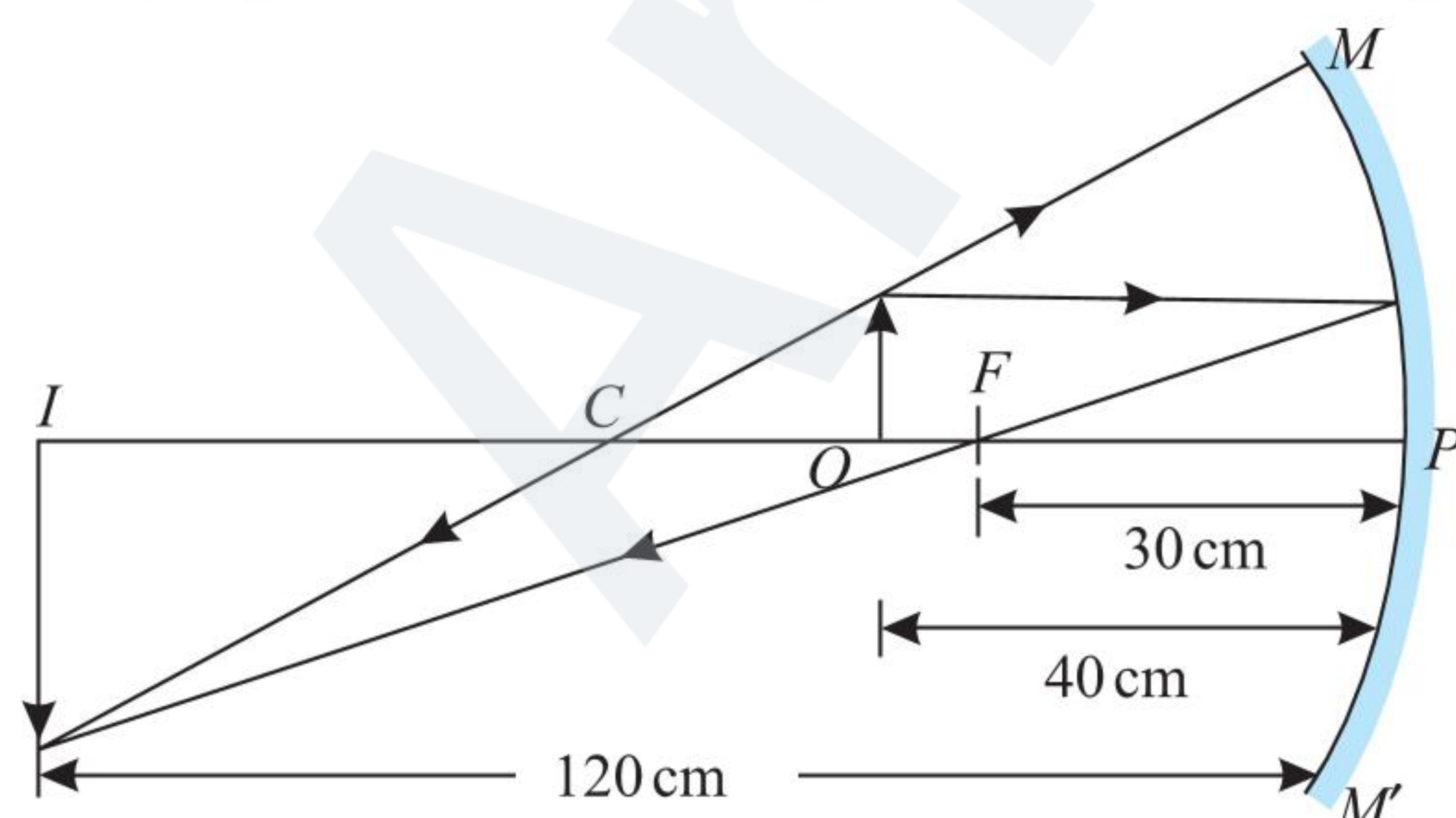
- (a) If the image is inverted (i.e., real)

$$m = -\left(\frac{v}{u}\right) = -3 \text{ i.e., } v = 3u$$

$$\text{So from mirror formula, } \frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\text{We get, } \frac{1}{3u} + \frac{1}{u} = \frac{1}{(-30)} \text{ i.e., } u = -40 \text{ cm}$$

i.e., object must be at a distance of 40 cm in front of the mirror (i.e., between C and F) as shown in the figure.



- (b) If the image is erect (i.e., virtual)

$$m = -(v/u) = 3 \text{ i.e., } v = -3u$$

So from mirror formula,

$$\frac{1}{(-3u)} + \frac{1}{u} = \frac{1}{(-30)} \text{ i.e., } u = -20 \text{ cm}$$

i.e., object must be at a distance of 20 cm in front of the mirror (i.e., between F and P) as shown in the figure).

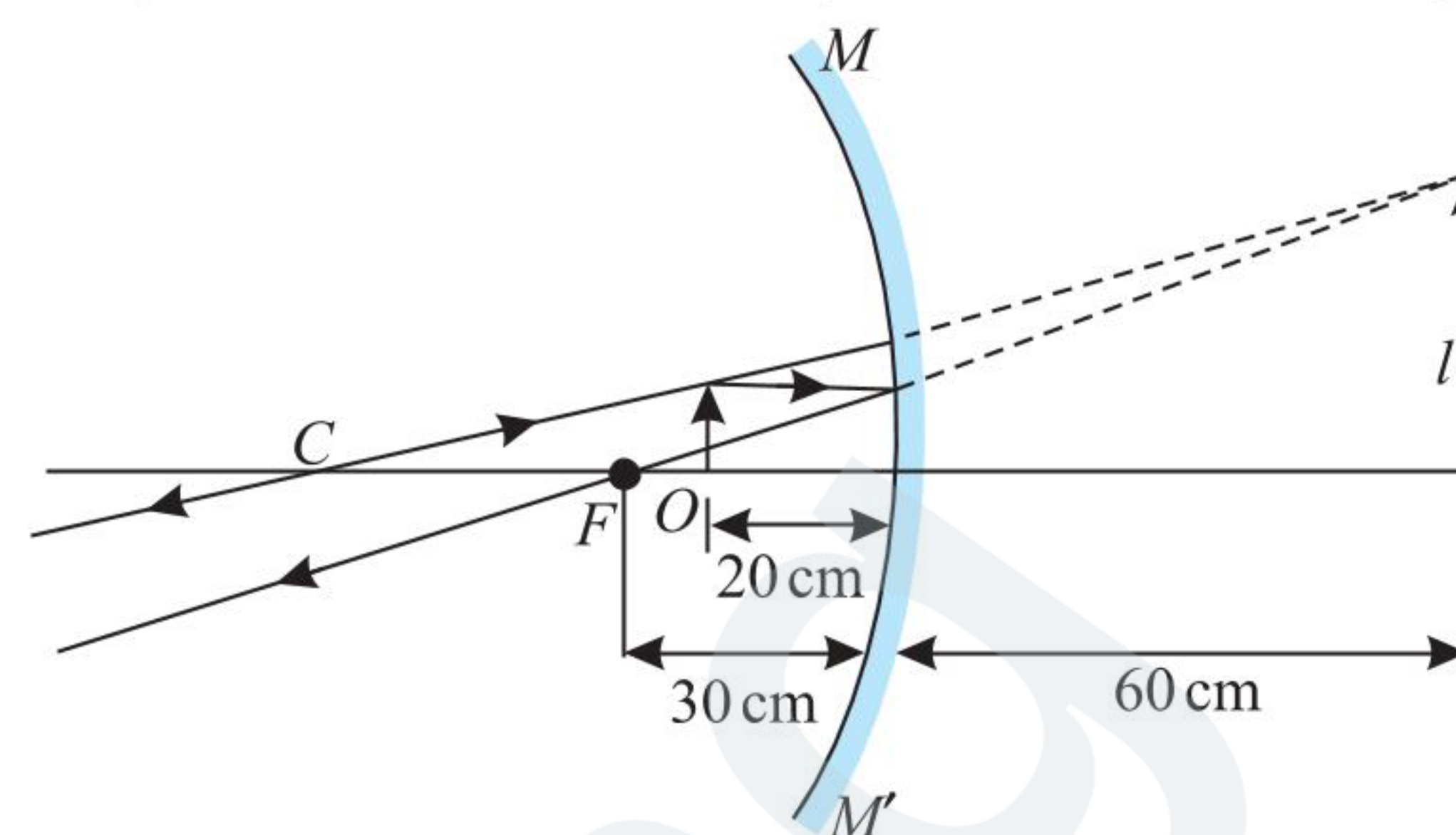


ILLUSTRATION 1.24

An image of a candle on a screen is found to be double its size. When the candle is shifted by a distance of 5 cm, then the image becomes triple its size. Find the nature and radius of curvature of the mirror.

Sol. Since the image is formed on the screen, it is real. Real object and real image imply concave mirror.

$$\text{Applying } m = \frac{f}{f - u} \text{ or } -2 = \frac{f}{f - u} \quad \dots(i)$$

$$\text{After shifting, } -3 = \frac{f}{f - (u + 5)} \quad \dots(ii)$$

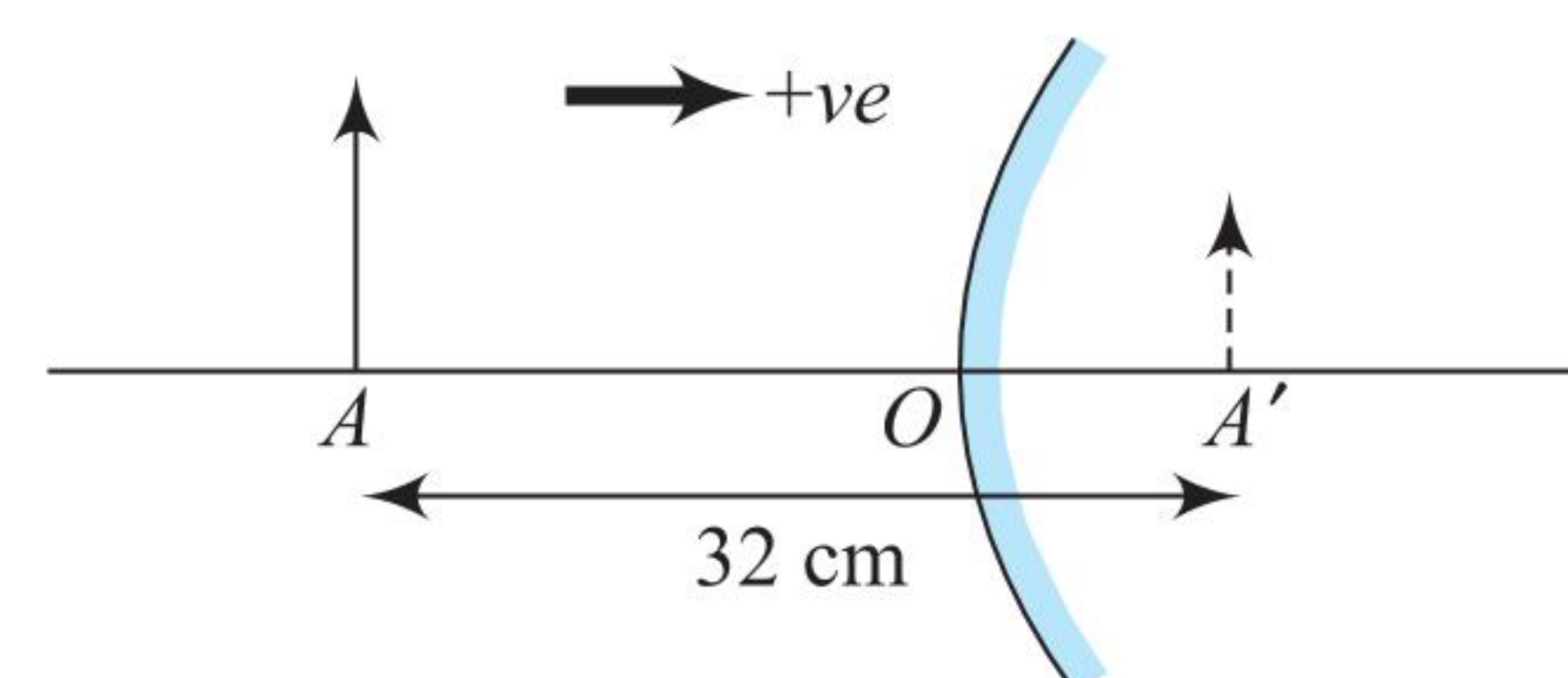
Here, care should be taken that distance of the object becomes $u + 5$ not $u - 5$. In a concave mirror the size of real image will increase only when the real object is brought closer to the mirror. In doing so, its x -coordinate will increase. From Eqs. (i) and (ii), we get

$$f = -30 \text{ cm or } R = 60 \text{ cm}$$

ILLUSTRATION 1.25

The distance between a real object and its image in a convex mirror of focal length 12 cm is 32 cm. Find the size of image if the object size is 1 cm.

Sol. Let x and y be the magnitudes of object and image distances.



We have $AA' = 32 \text{ cm}$

$$\Rightarrow AO + A'O = 32 \text{ cm}$$

$$\Rightarrow (x + y) = 32 \quad \dots(i)$$

And also, $u = -x, v = +y$

$$\frac{1}{-x} + \frac{1}{y} = \frac{1}{+12} \quad \dots(ii)$$

Solving Eqs. (i) and (ii) simultaneously, we can get u and v .

The relevant answers are $u = -24 \text{ cm}, v = +8 \text{ cm}$

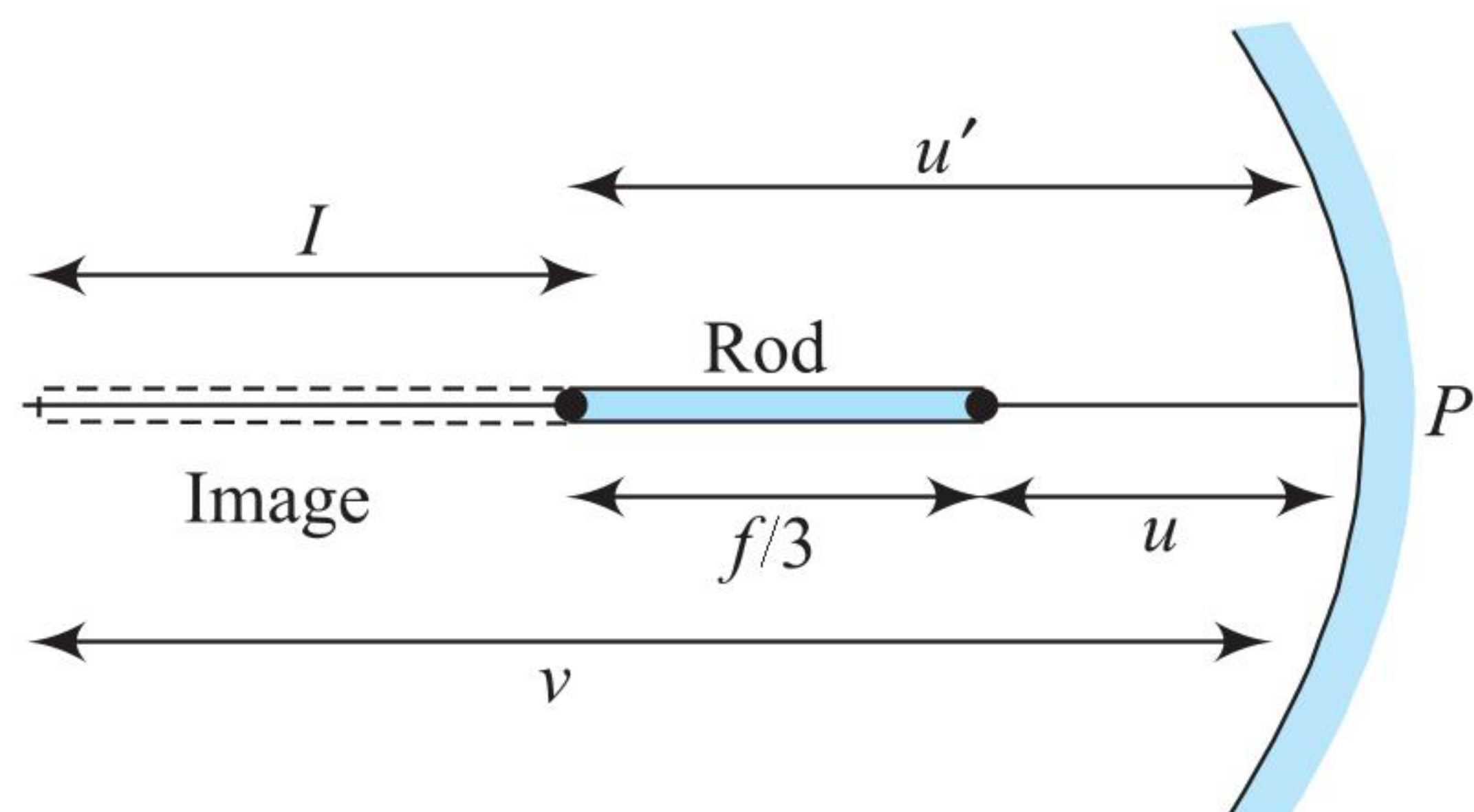
$$\text{Using } I = -1\left(\frac{+8}{-24}\right) = +\frac{1}{3}$$

So, the image size is $1/3 \text{ cm}$.

ILLUSTRATION 1.26

A thin rod of length $f/3$ is placed along the optical axis of a concave mirror of focal length f such that its image which is real and elongated just touches the rod. Calculate the magnification.

Sol. As in question, image touches the rod, i.e., image and object coincide, hence one end of the rod should be at the centre of curvature. It is also written that image is enlarged, it indicates that the orientation of rod should be toward focus then only we can get enlarged image along the principal axis. Let l be the length of the image.



$$\text{Then, } m = \frac{l}{f/3} \Rightarrow l = \frac{mf}{3}$$

Also, one end of the image coincides with the object, $u' = 2f$.

$$\text{Now, } u' = u + \frac{f}{3} \Rightarrow u = 2f - \frac{f}{3} = \frac{5f}{3}$$

$$v = -\left(u + \frac{f}{3} + \frac{mf}{3}\right)$$

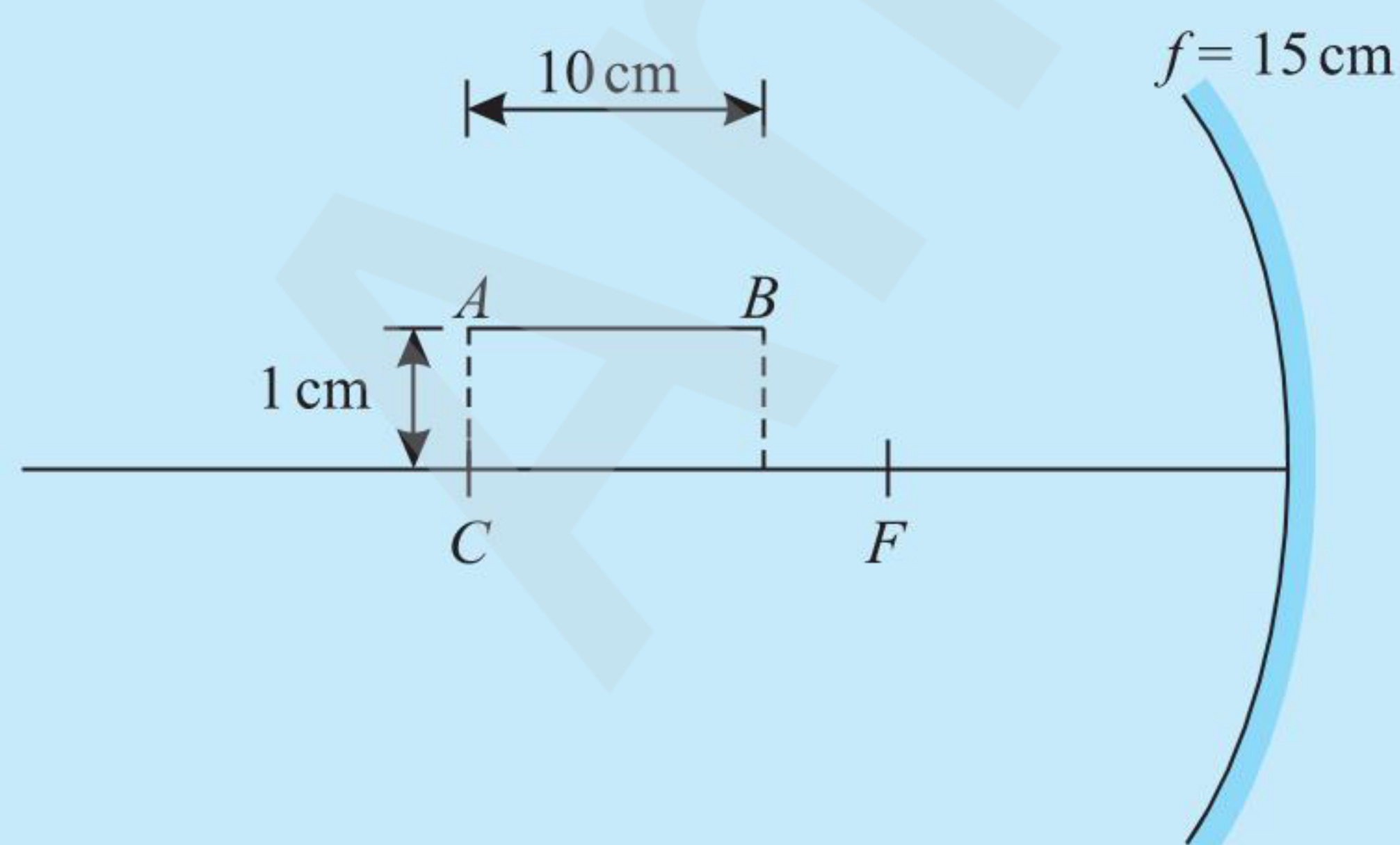
Putting in mirror formula, we get

$$\frac{1}{u + f/3 + mf/3} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{3}{5f + f + mf} + \frac{3}{5f} = \frac{1}{f}$$

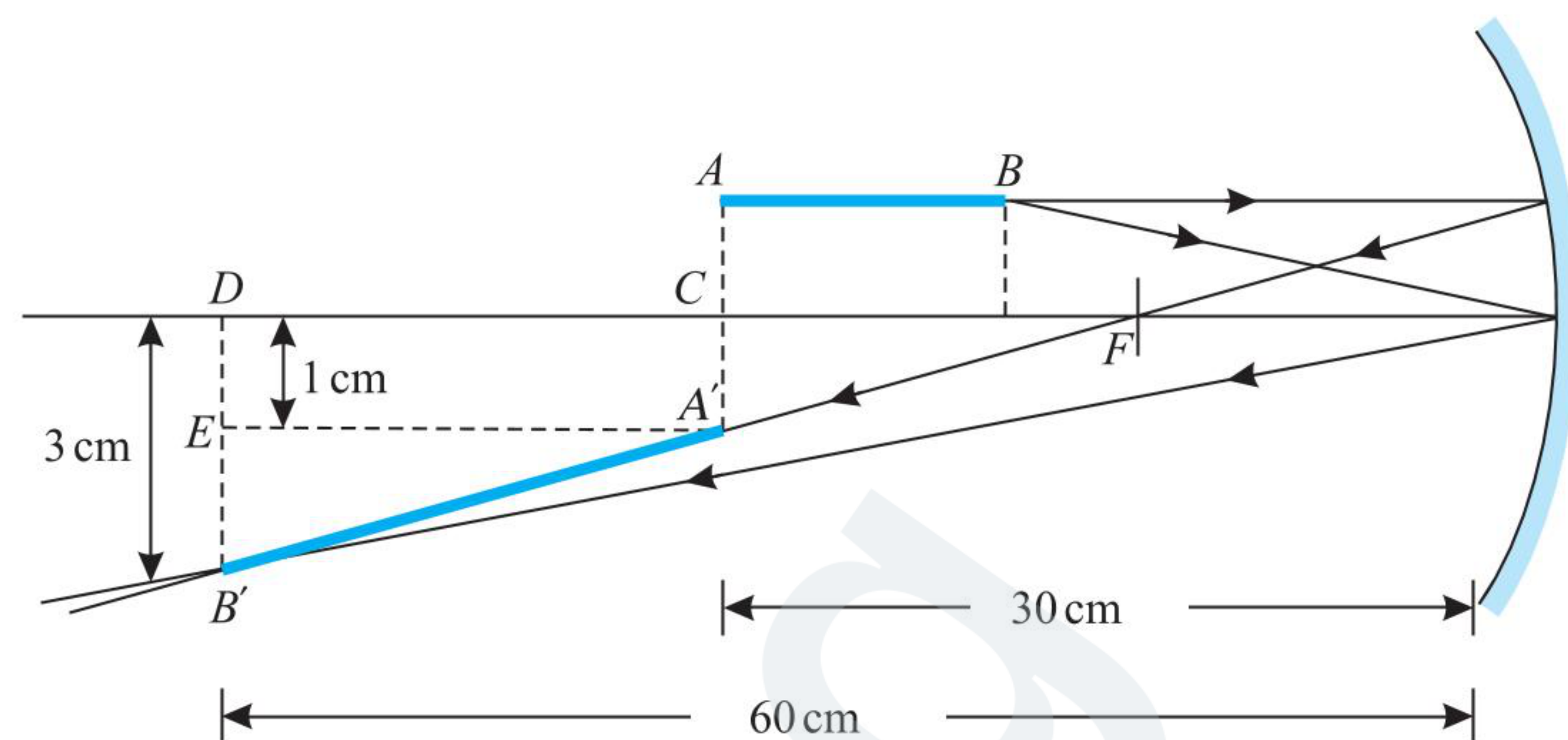
$$\Rightarrow \frac{1}{m+6} = \frac{2}{15} \Rightarrow m = \frac{3}{2}$$

ILLUSTRATION 1.27

A metal wire AB of length 10 cm is placed in front of a concave mirror of focal length 15 cm as shown in figure. Find the length of the wire.



Sol. The image of end A of the wire is placed at a distance $R = 2f = 30$ cm, hence image of A will form at a distance 30 cm in front of the mirror but below the principal axis at a same distance as it is above the principal axis i.e., 1 cm.



Now consider the image of end B ,

For B ; $u = -(30 - 10) = -20$ cm, $f = -15$ cm

Using mirror formula: $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$$\frac{1}{v} + \frac{1}{-20} = \frac{1}{-15} \Rightarrow v = -60 \text{ cm}$$

Now using, $m = \frac{I}{O} = -\frac{v}{u}$

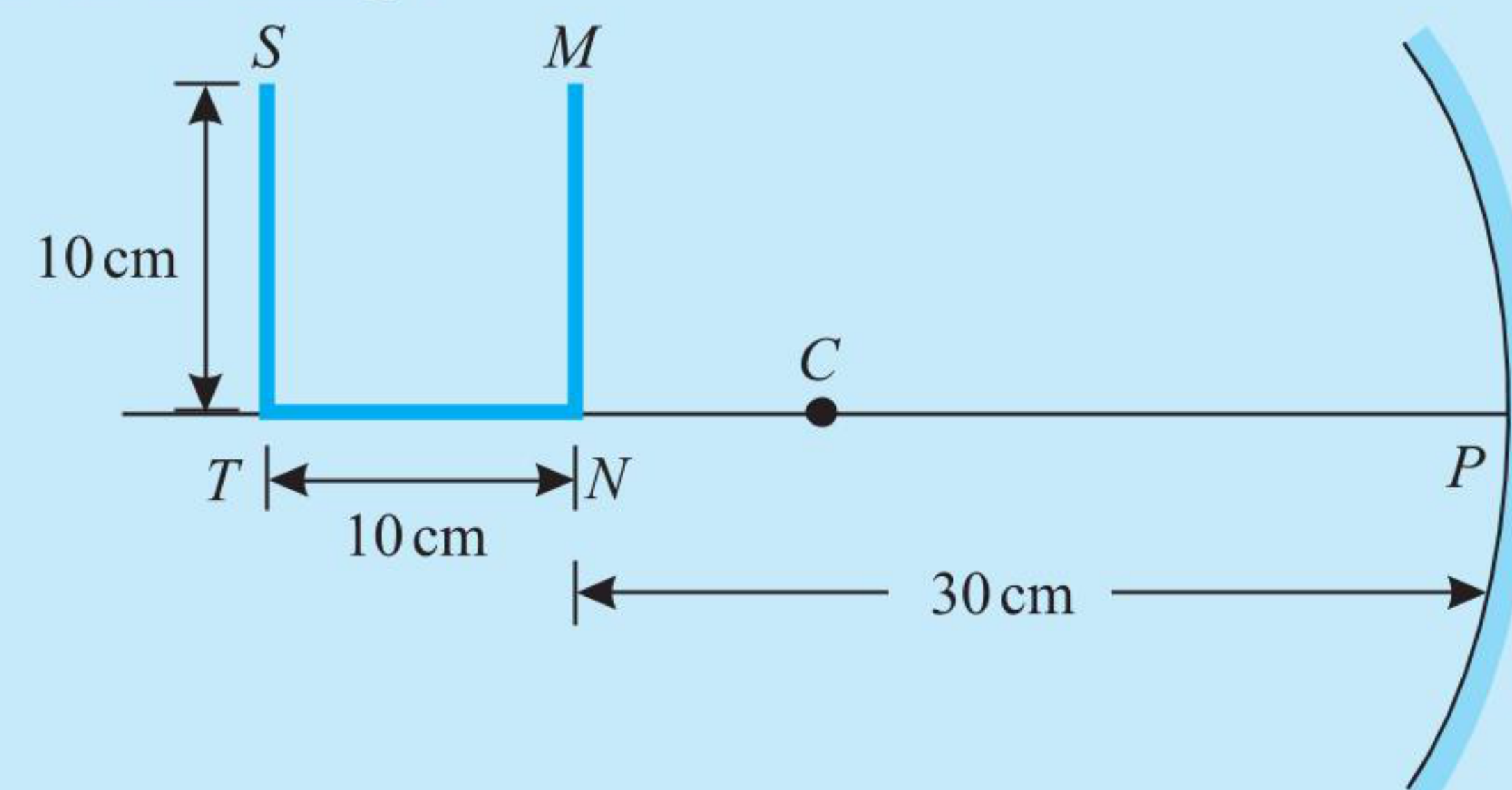
$$\frac{I}{1} = -\frac{(-60)}{(-20)} \Rightarrow I = -3 \text{ cm}$$

Length of the image $A'B' = \sqrt{(A'E)^2 + (B'E)^2}$

$$\Rightarrow A'B' = \sqrt{(30)^2 + (2)^2} = \sqrt{904} \text{ cm or } 2\sqrt{226} \text{ cm}$$

ILLUSTRATION 1.28

A U-shaped wire is placed before a concave mirror having radius of curvature 20 cm as shown in figure. Find the total length of the image.



Sol. The incident ray is travelling towards right, all the distance measured in rightward direction should be positive, For part MN ,

$$u = -30 \text{ cm, } f = -10 \text{ cm}$$

Using, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

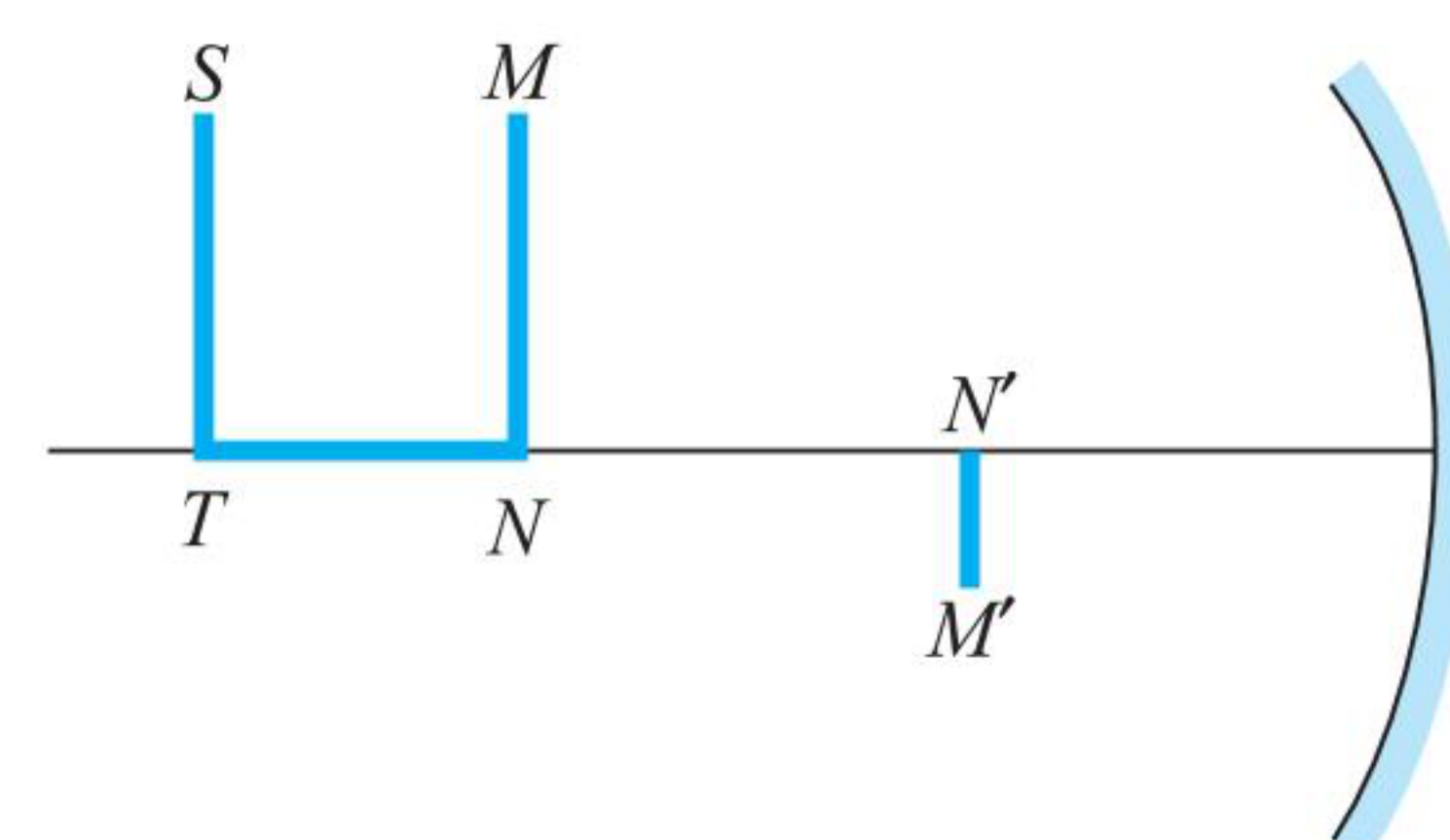
$$\frac{1}{v} + \frac{1}{-30} = \frac{1}{-10} \Rightarrow v = -15 \text{ cm}$$

The minus sign shows that the image is in front of the mirror, real.

Magnification, $m = \frac{I}{O} = -\frac{v}{u}$

$$\Rightarrow \frac{I}{(+10)} = -\frac{(-15)}{(-30)} = -\frac{1}{2}$$

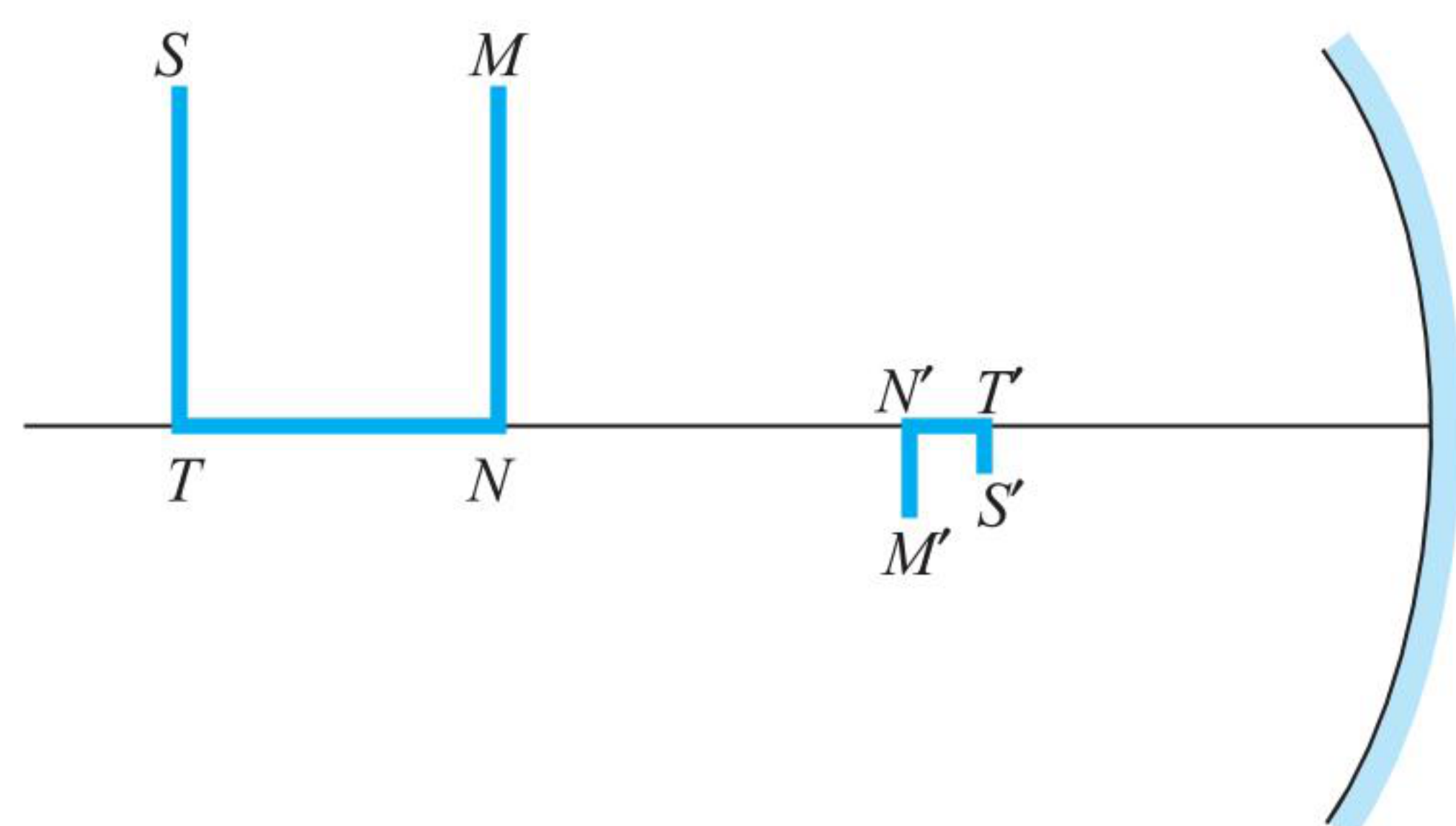
$$\Rightarrow I = M'N' = -5 \text{ cm}$$



For image of ST , $u = -40$ cm and $f = -10$ cm

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} + \frac{1}{(-40)} = \frac{1}{(-10)} \Rightarrow v = -\frac{40}{3} \text{ cm}$$



For length of image of ST , $m = \frac{I}{O} = -\frac{v}{u}$

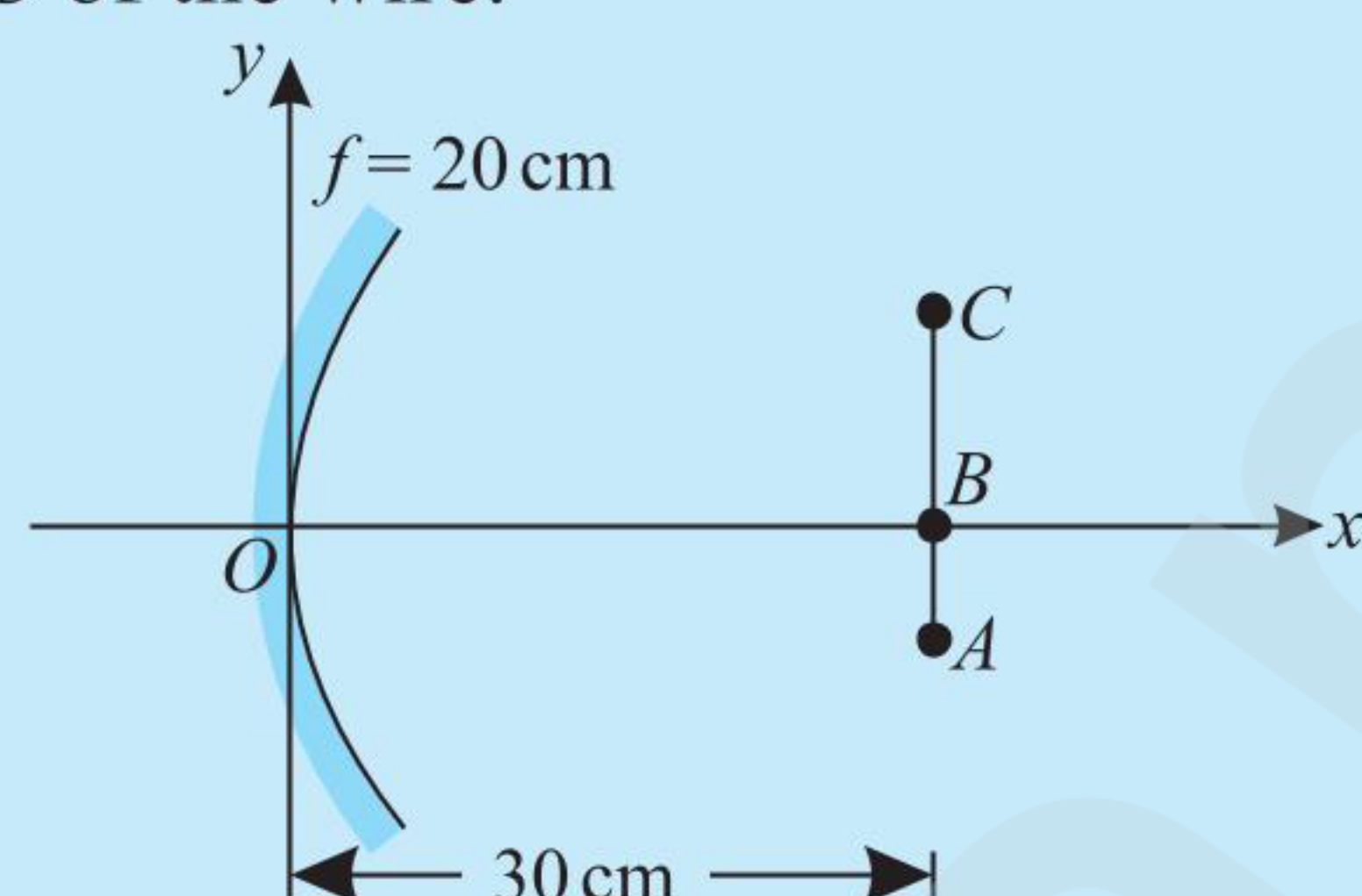
$$\frac{I}{10} = -\left(\frac{-40/3}{-40}\right) \Rightarrow I = S'T' = -\frac{10}{3} \text{ cm}$$

Length of image of NT , $T'N' = 15 - \frac{40}{3} = \frac{5}{3}$ cm

Length of wire in image = $5 + \frac{10}{3} + \frac{5}{3} = 10$ cm

ILLUSTRATION 1.29

A metal wire ABC of length 3 cm is placed in front of a concave mirror of focal length 20 cm as shown in figure. If $AB = 1$ cm and $BC = 2$ cm, then find the coordinates image of point A and C of the wire.



Focal length of the concave mirror shown in figure is 20 cm.

$ab = 1$ mm and $bc = 2$ mm

For the given situation, make its image.

Sol. $u = -30$ cm and $f = -20$ cm

Using the mirror formula, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$, we have $\frac{1}{v} + \frac{1}{-30} = \frac{1}{-20}$

Solving this equation, we get $v = -60$ cm

Now, $m = -\frac{v}{u} = -\frac{(-60)}{(-30)} = -2$

The point C is 2 cm above the principal axis and magnification is -2 . Hence, image C will be formed 4 cm below the principal axis. Similarly, point A is 1 cm below the principal axis and value of m is -2 . Hence, image of A will be formed 2 cm above the principal axis. Image with ray diagram of C is as shown below.

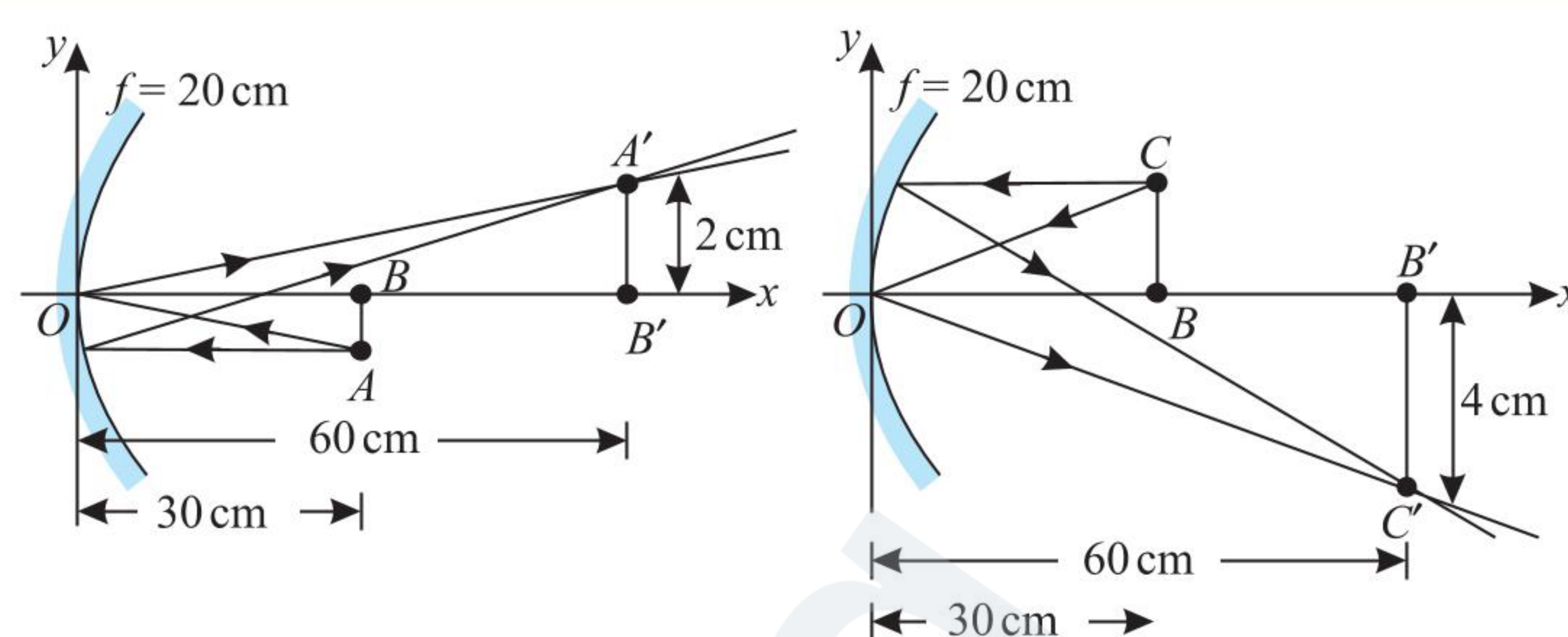


ILLUSTRATION 1.30

Point I is the image of a point source of light O in a spherical mirror whose optical axis is X_1X_2 (shown in figure). Find by construction the position of the centre of the mirror and its focus.



Sol. As the image and object both are on the opposite side of the principal axis, hence the mirror should be concave. For finding the position of the mirror and its focus, let us go step by step.

Step I: The line joining object and image position intersect at principal axis at the optical centre C of the mirror. Hence the point of intersection of ray OI with axis X_1X_2 is the position of centre of curvature of the mirror (C).

Step II: We know the ray incident on the mirror at its pole is reflected symmetrically w.r.t. the principal axis, let us plot point I_1 symmetrical to I and draw ray OI_1 until it intersects the axis at point P (see in given figure). This point should be the pole of the mirror.

Step III: The focus can be found by the usual construction of ray OM parallel to the axis. The reflected ray must pass through focus F (lying on the principal axis of the mirror) and through I .

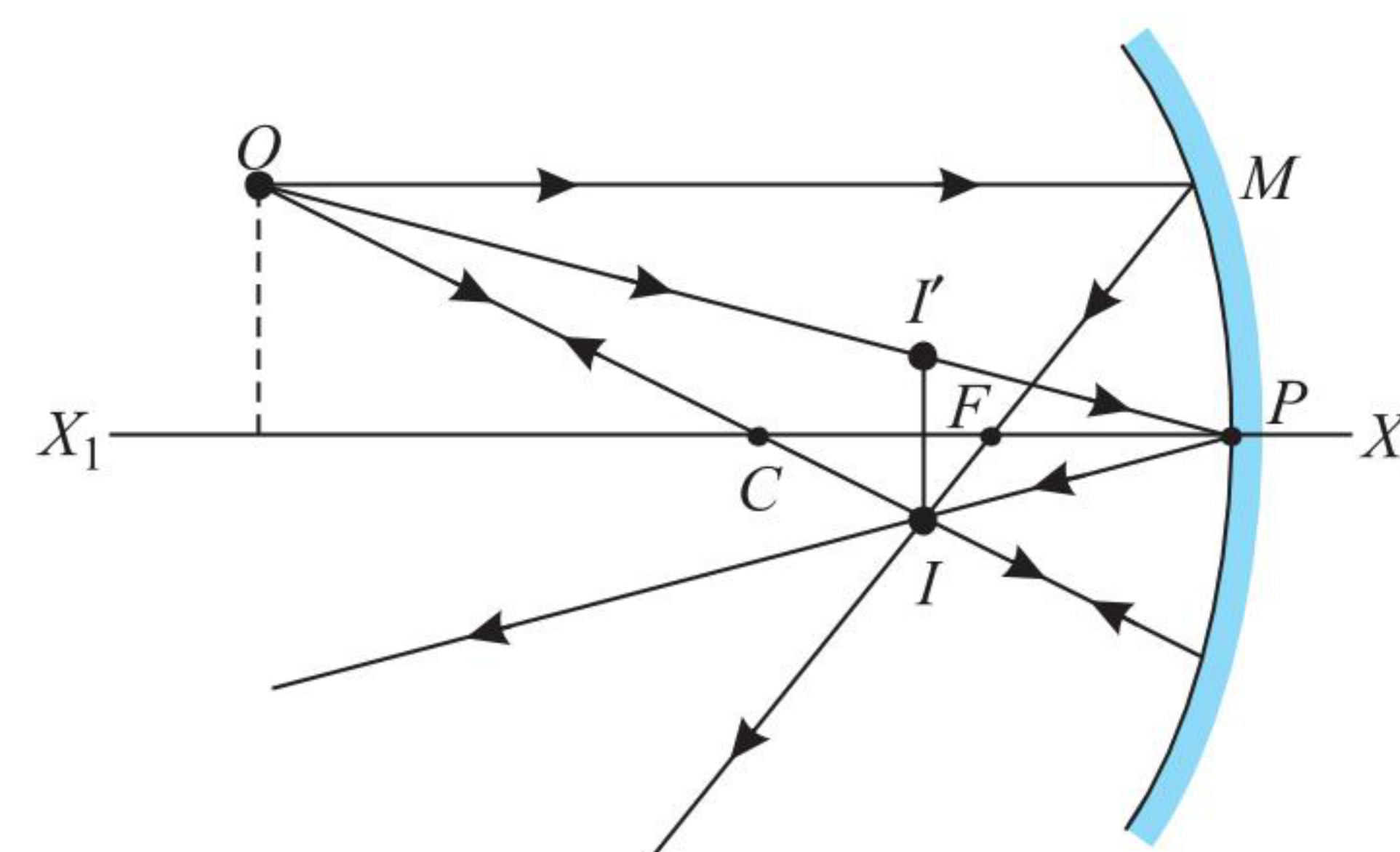
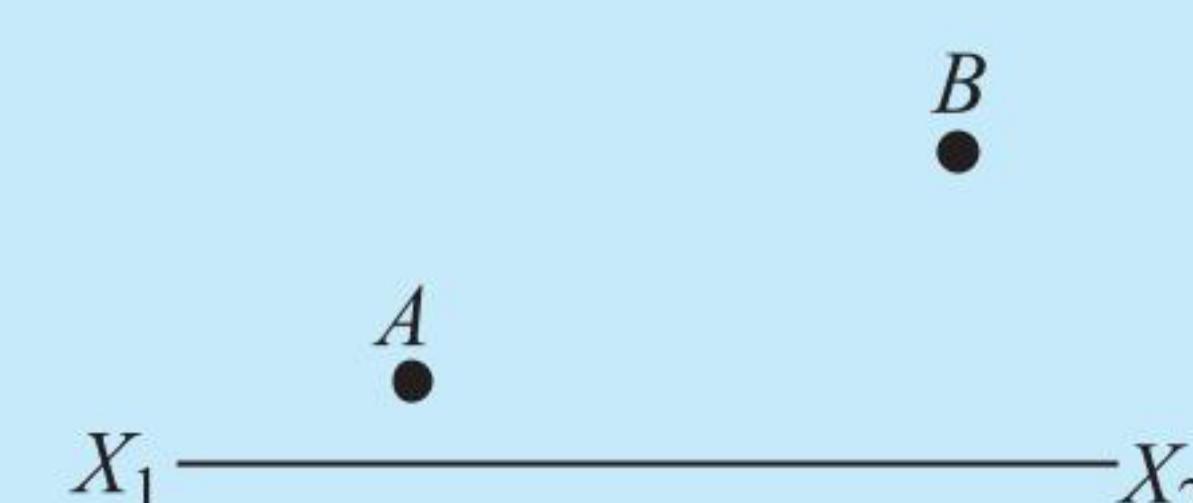


ILLUSTRATION 1.31

The positions of optical axis X_1X_2 of a spherical mirror, the source and the image are known (as shown in figure). Find by construction the positions of the centre of the mirror, its focus, and the pole for the cases

- (a) A — object, B — image;
(b) B — object, A — image.



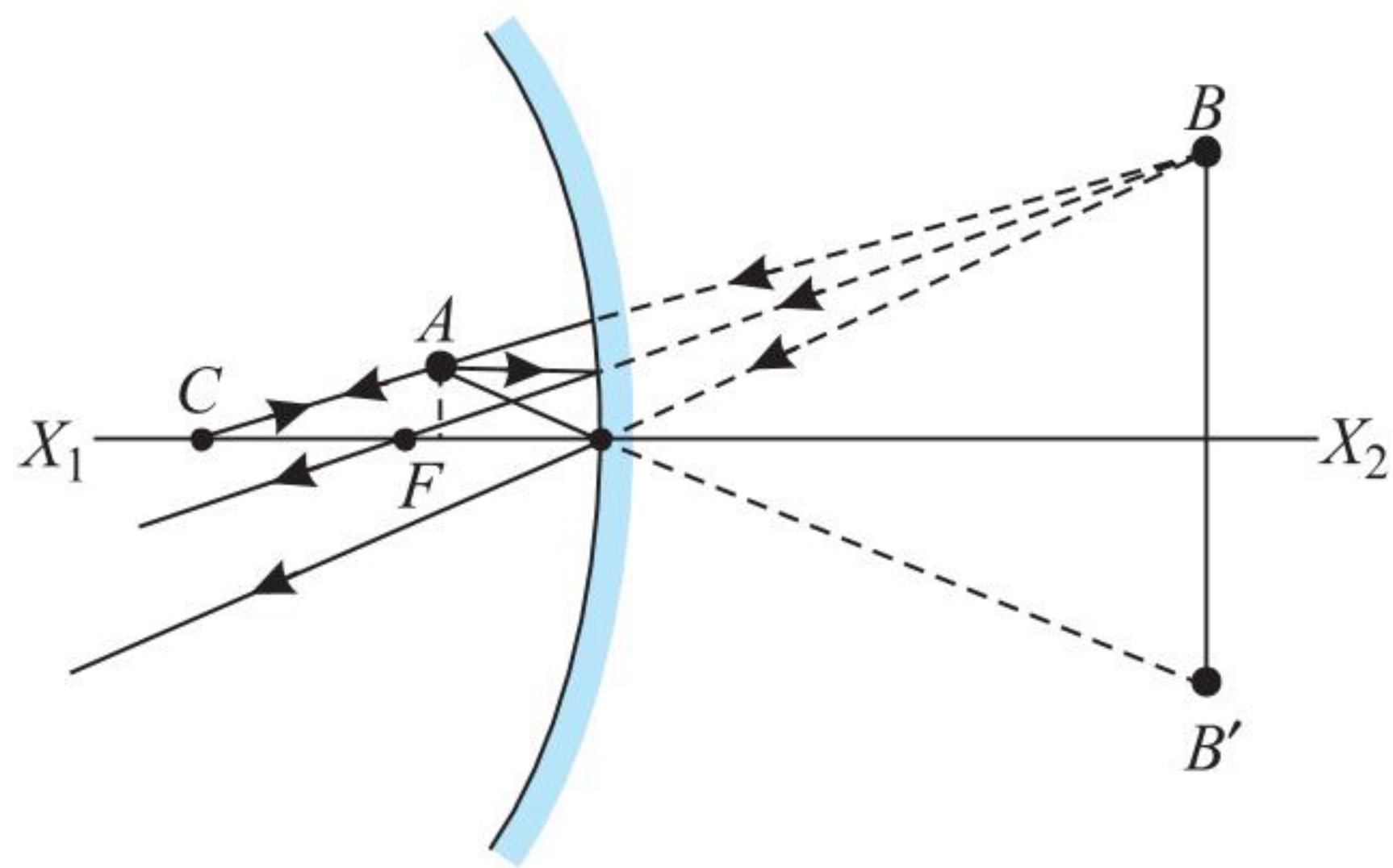
Sol. As the image and object both are on the same side of the principal axis, hence the mirror can be concave or convex.

(a) For finding the position of the mirror and its focus, let us go step by step.

Step I: The line joining object position (A) and image position (B) intersect at principal axis at the optical centre C of the mirror

Step II: The position of the pole P can be found by constructing the path of the ray APA' reflected in the pole with the aid of symmetrical point A' .

Step III: The position of the mirror focus F is determined by means of the usual construction of ray AMF parallel to the axis.



(b) This construction can also be used to find the centre C of the mirror and pole P as we have done in case (a) (see figure). The reflected ray BM will pass parallel to the optical axis of the mirror. For this reason, to find the focus.

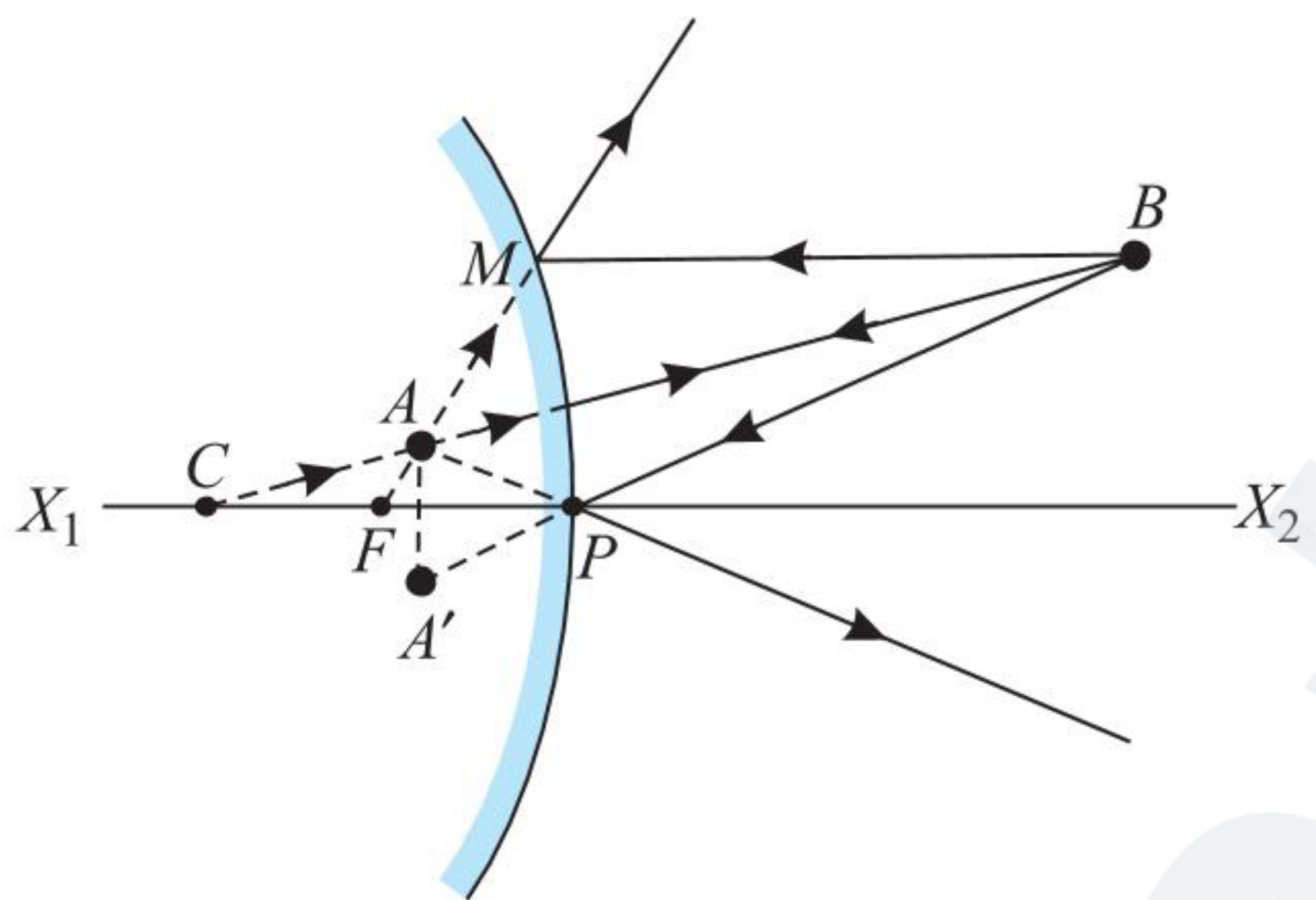
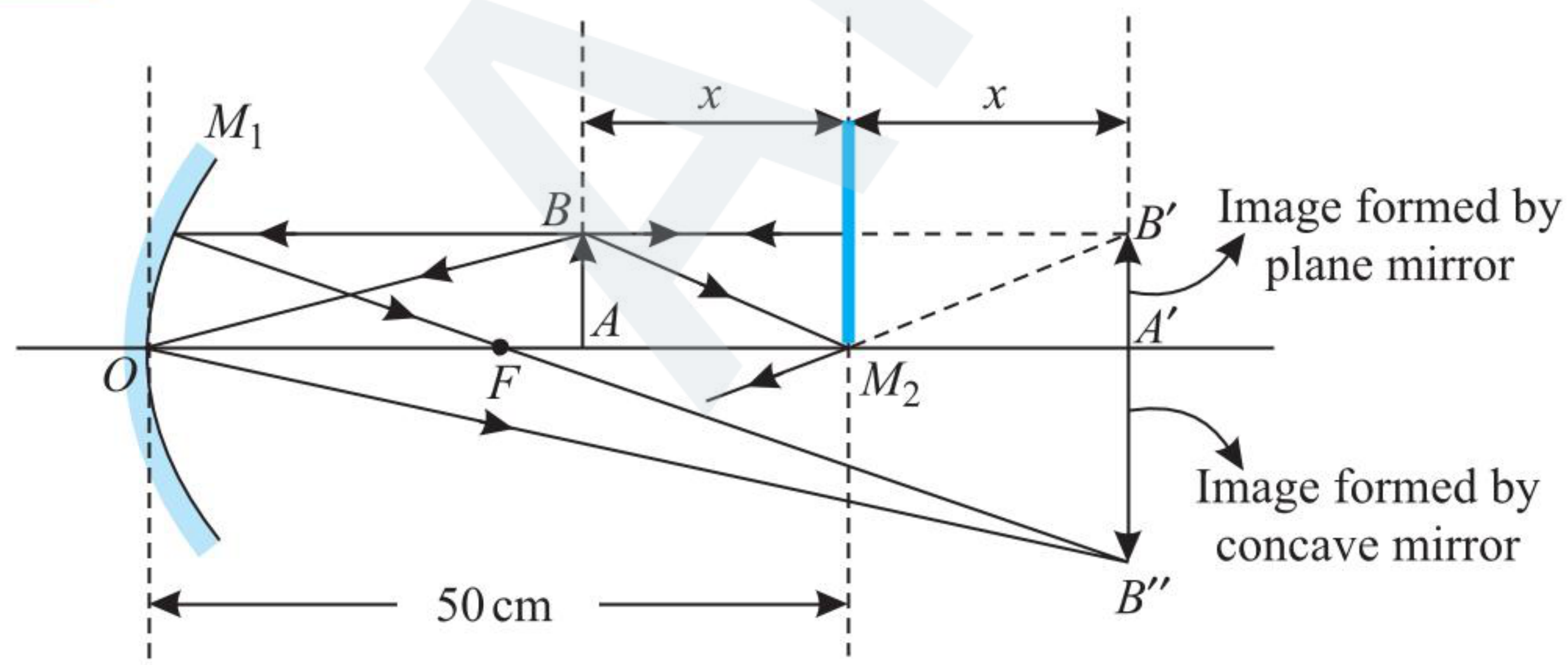


ILLUSTRATION 1.32

A plane mirror is placed at a distance of 50 cm from a concave mirror of focal length 16 cm. Where should a short object be placed between the mirrors and facing both the mirrors so that its virtual image in the plane mirror coincides with the real image in concave mirror? What is the ratio of the sizes of the two images?

Sol. Let the object be at a distance x from the plane mirror.



The distance of object from concave mirror is $(50 - x)$. For the plane mirror, object and image distances are equal.

$$A'M_2 = AM_2 = x$$

Distance of the image from concave mirror,

$$OA' = OM_2 + A'M_2 = (50 + x)$$

The incident ray direction on concave mirror is from right to left. Taking leftward direction as positive.

For the concave mirror, $u = -(50 - x)$, $v = -(50 + x)$, $f = -16$ cm. Using the mirror formula,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f},$$

we have,

$$\frac{1}{-(50 + x)} + \frac{1}{-(50 - x)} = \frac{1}{-16} \Rightarrow x = 30 \text{ cm}$$

The object must be placed at a distance of 30 cm from the plane mirror.

The ratio of image sizes = $\frac{A'B'}{A'B''} = \frac{AB}{A'B''}$

But AB = Size of the object

and $A'B''$ = Size of the image formed by concave mirror

$$|m| = \frac{I}{O} = \frac{v}{u} = \frac{50 + x}{50 - x} = \frac{50 + 30}{50 - 30} = 4$$

The real image formed by the concave mirror is four times the size of virtual image formed by the plane mirror.

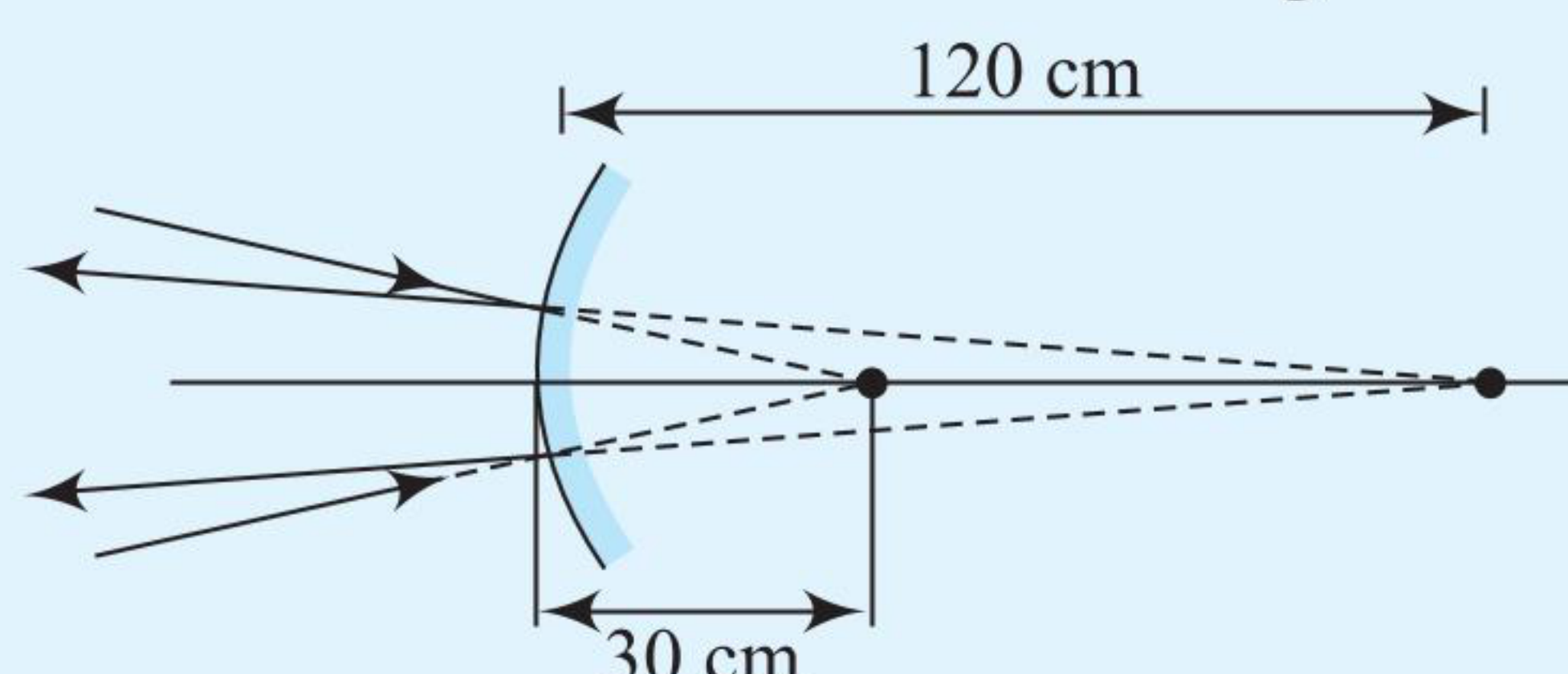
CONCEPT APPLICATION EXERCISE 1.2

1. State the following statements as True or False.

- A convex mirror cannot form a real image for a real object.
- The image formed by a convex mirror is always diminished and erect.
- Virtual image formed by a concave mirror is always enlarged.
- Only in the case of a convex mirror, it may happen that the object and its image move in same direction.
- In the case of a concave mirror, the image always move faster than the object.
- If an object is placed in front of a diverging mirror at a distance equal to its focal length, then the height of image formed is half of the height of object.
- For two positions of an object, a concave mirror can form enlarged image.
- Concave mirror is used as a rear view mirror in motor vehicles.
- If some portion of the mirror is covered, then complete image will be formed but of reduced brightness.
- A plane mirror always forms an erect image of same size as that of the object.
- The image formed by a plane mirror has left-right reversal.
- A virtual object means a converging beam.

- A particle moves in a circular path of radius 5 cm in a plane perpendicular to the principal axis of a convex mirror with radius of curvature 20 cm. The object is 15 cm in front of the mirror. Calculate the radius of the circular path of the image.
- A body of length 6 cm is placed 10 cm from a concave mirror of focal length 20 cm. Find the position, size, and nature of the image.
- An object is placed 15 cm from a mirror and an image is captured on the screen with magnification 2. Calculate the focal length of the mirror and determine if it is concave or convex.

5. An object is placed 15 cm from a mirror and an erect image of size 5 cm is seen. Determine the focal length and the nature of the mirror. Assume the object size is 15 cm.
6. A beam of light converges to a point on a screen S . A mirror is placed in front of the screen at a distance of 10 cm from the screen. It is found that the beam now converges at a point 20 cm in front of the mirror. Find the focal length of the mirror.
7. Converging rays are incident on a convex spherical mirror so that their extensions intersect 30 cm behind the mirror on the optical axis. The reflected rays form a diverging beam so that their extensions intersect the optical axis 1.2 m from the mirror. Determine the focal length of the mirror.



8. A concave mirror gives a real image magnified 4 times. When the object is moved 3 cm the magnification of the real image is 3 times. Find the focal length of mirror.
9. The image of a real object in a convex mirror is 4 cm from the mirror. If the mirror has a radius of curvature of 24 cm, find the position of object and magnification.
10. When an object is placed at a distance of 25 cm from a mirror, the magnification is m_1 . The object is moved 15 cm farther away with respect to the earlier position, and the magnification becomes m_2 . If $m_1/m_2 = 4$, then calculate the focal length of the mirror.
11. A concave mirror forms a real image three times larger than the object on a screen. The object and screen are moved until the image becomes twice the size of the object. If the shift of the object is 6 cm, find the shift of screen.

ANSWERS

1. (a) True; (b) False; (c) True; (d) False; (e) False; (f) True; (g) True; (h) False; (i) True; (j) True; (k) True; (l) True
2. 2 cm 4. 10 cm (concave) 5. 7.5 cm (convex)
6. 20 cm 7. 24 cm 8. 36 cm 9. 6 cm, $\frac{2}{3}$
10. 20 cm 11. 36 cm

IMAGES FORMED BY TWO MIRRORS

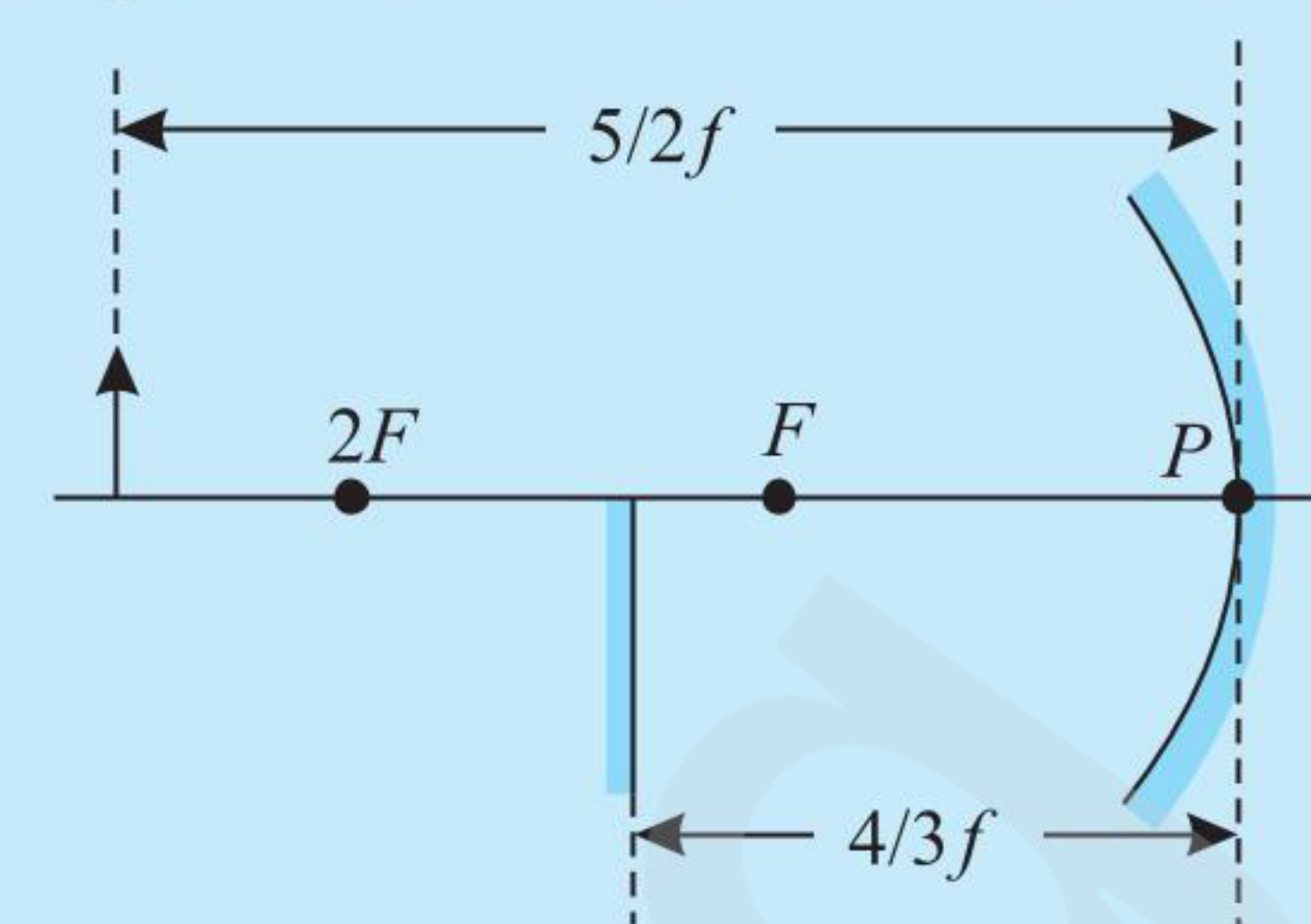
If rays after getting reflected from one mirror strike second mirror, the image formed by first mirror will act as an object for second mirror, and this process will continue for every successive reflection. And the overall lateral magnification is given by $m_1 \times m_2 \times m_3 \times \dots$, where $m_1, m_2, \dots$ etc. are lateral magnifications produced by individual mirrors.

Let us understand this with the illustration discussed below.

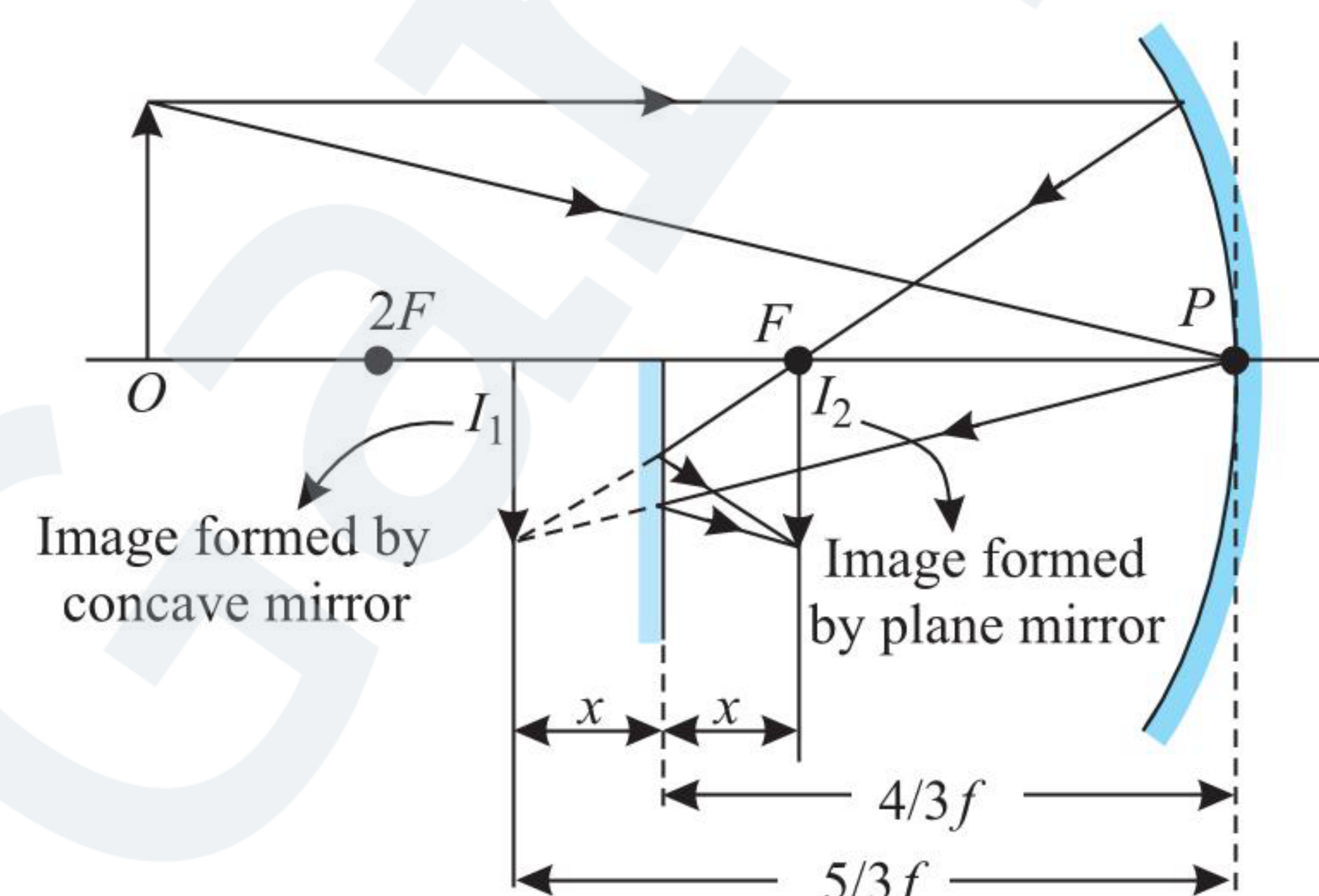
ILLUSTRATION 1.33

A vertically erect object placed on the optic axis at a distance $(5/2)f$ from a concave mirror of focal length f as shown in the figure. If a plane mirror is placed perpendicular to the

optic axis at a distance $(4/3)f$ from the pole, facing concave mirror, find the position and nature of the final image formed.



Sol. Two rays come from the tip of the object. After reflection from concave mirror, they converge to form a real image at I_1 .



As the plane mirror is introduced in between, the image I_1 acts as virtual object for plane mirror. Hence the reflected rays from the concave mirror get reflected from the plane mirror to form an image I_2 . In fact I_2 is the real image of the image I_1 of the object O . The incident ray is travelling towards right, the distance measured from rightward direction should be positive,

For the concave mirror, $u = -\frac{5}{2}f$ and $f = -f$

Using the mirror formula, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$,

$$\frac{1}{v} + \frac{1}{(-5/2 f)} = \frac{1}{(-f)} \Rightarrow v = -\frac{5}{3}f$$

(Negative sign indicates the image is left the mirror.)

The image I_1 is formed which behaves as a virtual object for the plane mirror and therefore finally a real erect image I_2 of same size, will be formed at equal distance x in front of the plane mirror as shown in the figure.

Hence, I_2 will be formed at a distance $\left(\frac{5}{3}\right)f - 2x$, from concave mirror, where

$$x = \frac{5}{3}f - \frac{4}{3}f = \frac{1}{3}f$$

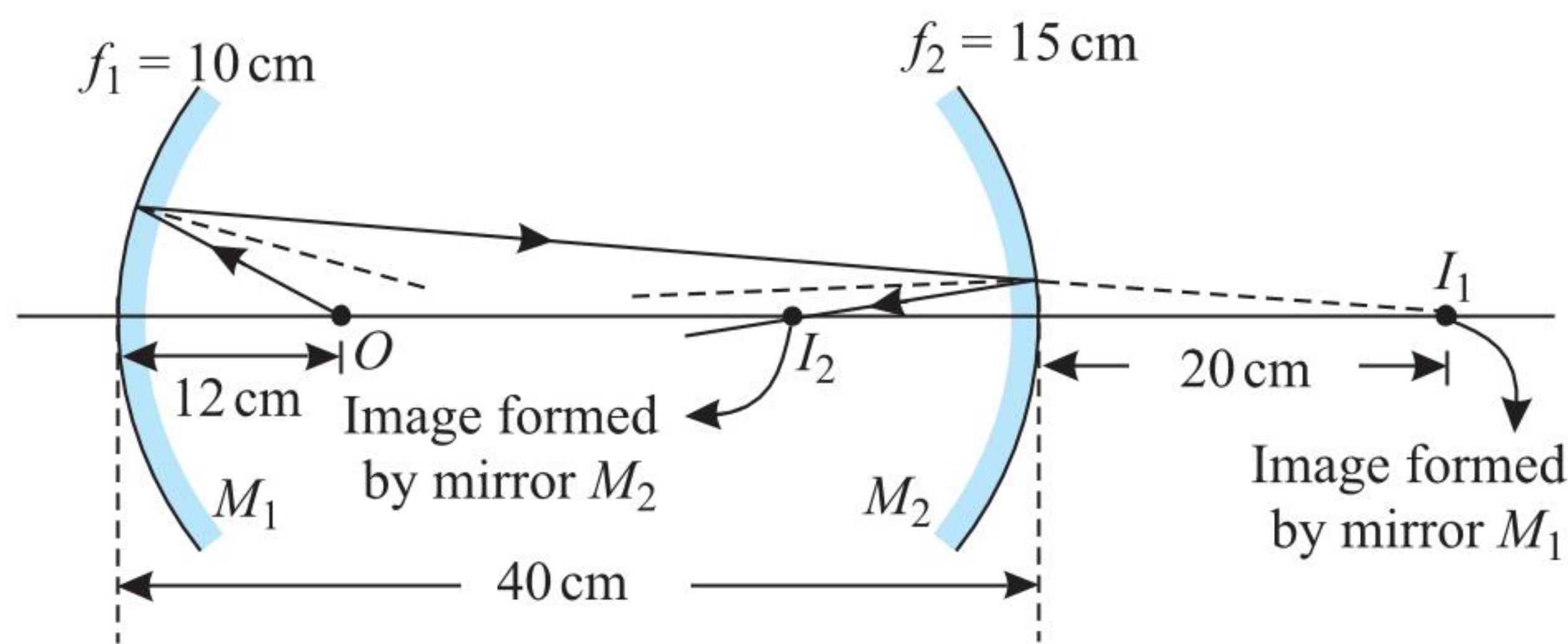
The final image distance $v = \left(\frac{5}{3}\right)f - 2\left(\frac{1}{3}f\right) = f$

Hence the final image is formed at the focus of the concave mirror.

ILLUSTRATION 1.34

Two concave mirrors are placed 40 cm apart and are facing each other. A point object lies between them at a distance of 12 cm from the mirror of focal length 10 cm. The other mirror has a focal length of 15 cm. Find the location of final image formed after two reflections—first at the mirror nearer to the object and second at the other mirror.

Sol. The initial incident ray is travelling towards left, all the distance measured from leftward direction should be positive,



First reflection from M_1 : $u = -12$ cm, $f = -10$ cm

Using the mirror formula, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$,

we have, $\frac{1}{v} + \frac{1}{(-12)} = \frac{1}{(-10)} \Rightarrow v = -60$ cm

The rays after reflection are going to converge at a point $v = 60$ cm from the first mirror, i.e., $60 - 40 = 20$ cm behind the other mirror.

Second reflection from M_2 : As initial incident rays are directed toward left, all quantities towards right will be taken negative.

The converging rays falling at the second mirror create a virtual object for this mirror at I_2 with object distance

$$u = -20 \text{ cm and } f = +15 \text{ cm,}$$

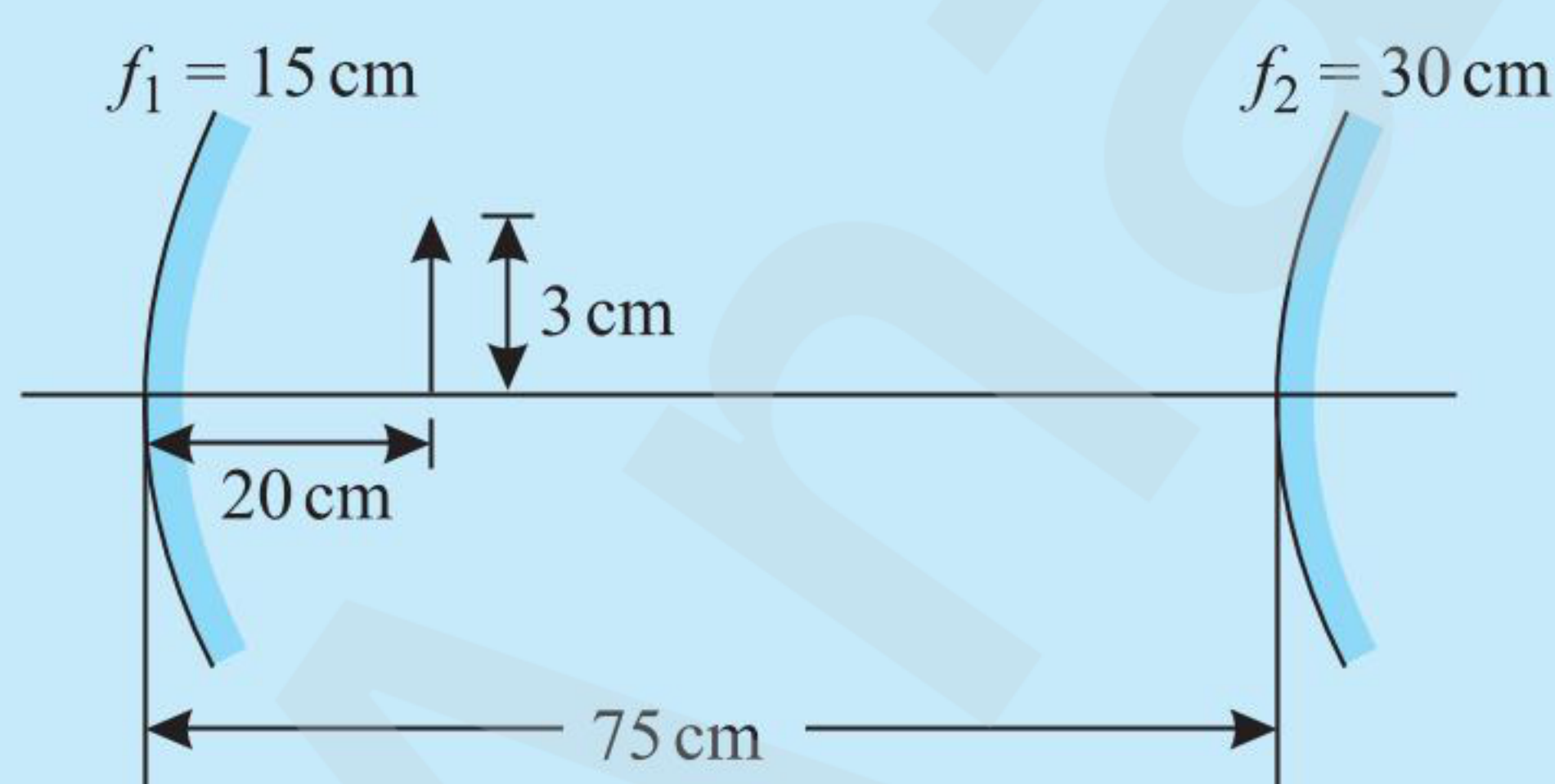
we have, $\frac{1}{v} + \frac{1}{(-20)} = \frac{1}{(+15)} \Rightarrow v = +\frac{60}{7}$ cm

As leftward direction is positive, image will be formed to the left of mirror.

Hence, the final image is formed at a distance of $\frac{60}{7}$ cm from the second mirror and is real.

ILLUSTRATION 1.35

A concave mirror and a convex mirror of focal lengths 15 cm and 30 cm are placed at a distance of 75 cm. An object of height 3 cm is placed at a distance of 20 cm from the concave mirror. First ray is incident on the concave mirror then on the convex mirror. Find size, position, and nature of the image.



Sol. First ray is incident on the concave mirror, it means initial ray travelling towards left. It means all the distance measured from leftward direction should be positive.

Hence for concave mirror, $u = -20$ cm, $f = -15$ cm

Using $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$; $\frac{1}{v} + \frac{1}{(-20)} = \frac{1}{(-15)} \Rightarrow v = -60$ cm

Magnification by concave mirror, $m_1 = -\frac{v}{u} = -\frac{(-60)}{(-20)} = -3$

The image formed by concave mirror will act as object for convex mirror.

For convex mirror, $u = +(75 - 60) = 15$ cm, $f = -30$ cm

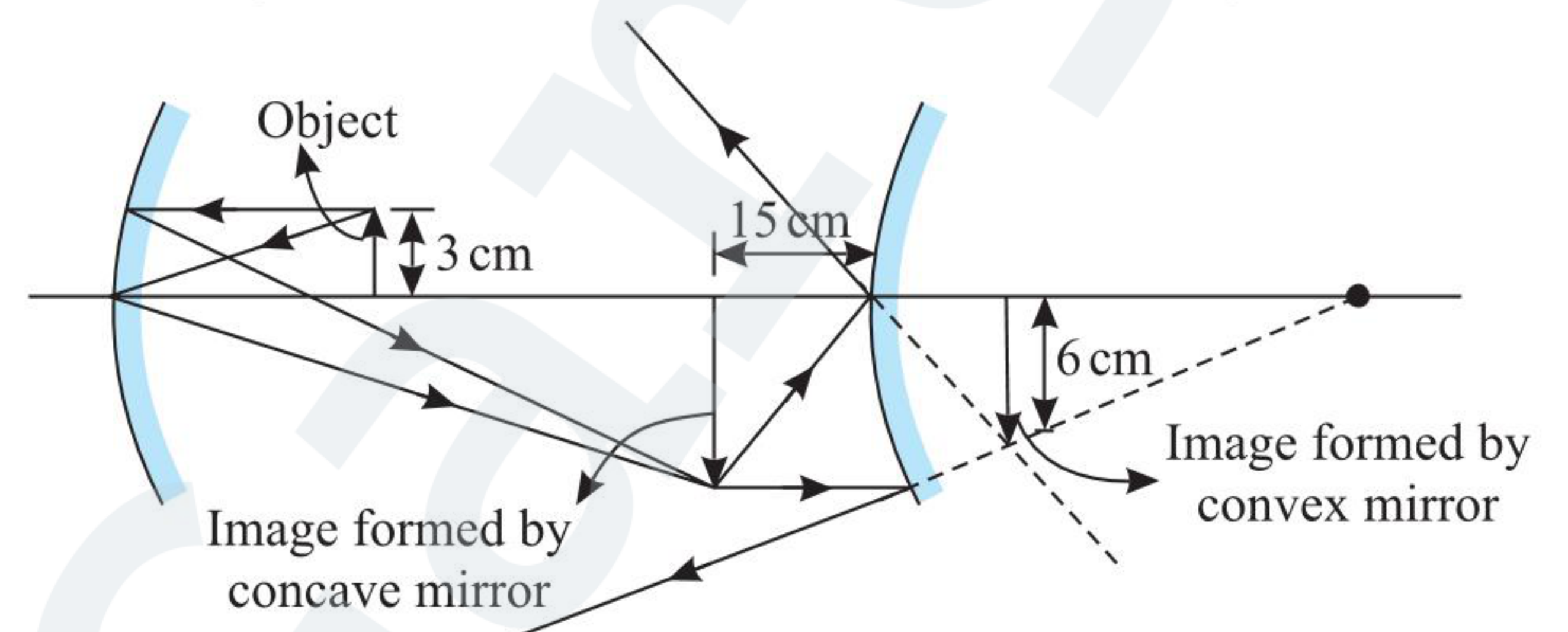
Using $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$; $\frac{1}{v} + \frac{1}{(+15)} = \frac{1}{(-30)} \Rightarrow v = -10$ cm

Magnification by convex mirror, $m_2 = -\frac{v}{u} = -\frac{(-10)}{(+15)} = \frac{2}{3}$

Overall lateral magnification, $m = m_1 m_2 = -3 \times \frac{2}{3} = -2$

Size of image = $-2 \times$ size of object = $(-2) \times 3 = -6$ cm

Final image will be virtual, inverted, and enlarged.



RELATION BETWEEN OBJECT AND IMAGE VELOCITY

Case (i). When object is moving along the principal axis: Let a point object is moving along the principal axis of a concave mirror of focal length f . We can relate the object and image positions by the mirror formula

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \quad \dots(i)$$

Differentiate Eq. (i) with respect to time.

$$\Rightarrow -\frac{1}{v^2} \frac{dv}{dt} - \frac{1}{u^2} \frac{du}{dt} = 0$$

$$\text{or } \frac{dv}{dt} = -\left(\frac{v^2}{u^2}\right) \frac{du}{dt} \quad \dots(ii)$$

Here, $\frac{du}{dt}$ is the rate by which u is changing. It is the object speed with respect to or object velocity when mirror is stationary.

Similarly, $\frac{dv}{dt}$ is the rate by which v (distance between image and mirror) is changing. It is image speed if with respect to or object velocity when mirror is stationary.

Let $\frac{dv}{dt} = V_{i,m}$ = velocity of image w.r.t. mirror

and, $\frac{du}{dt} = V_{o,m}$ = velocity of object w.r.t. mirror

$$\text{Also, } \frac{v^2}{u^2} = m^2$$

Then Eq. (ii) becomes

$$V_{i,m} = -m^2 V_{o,m} \quad \dots(iii)$$

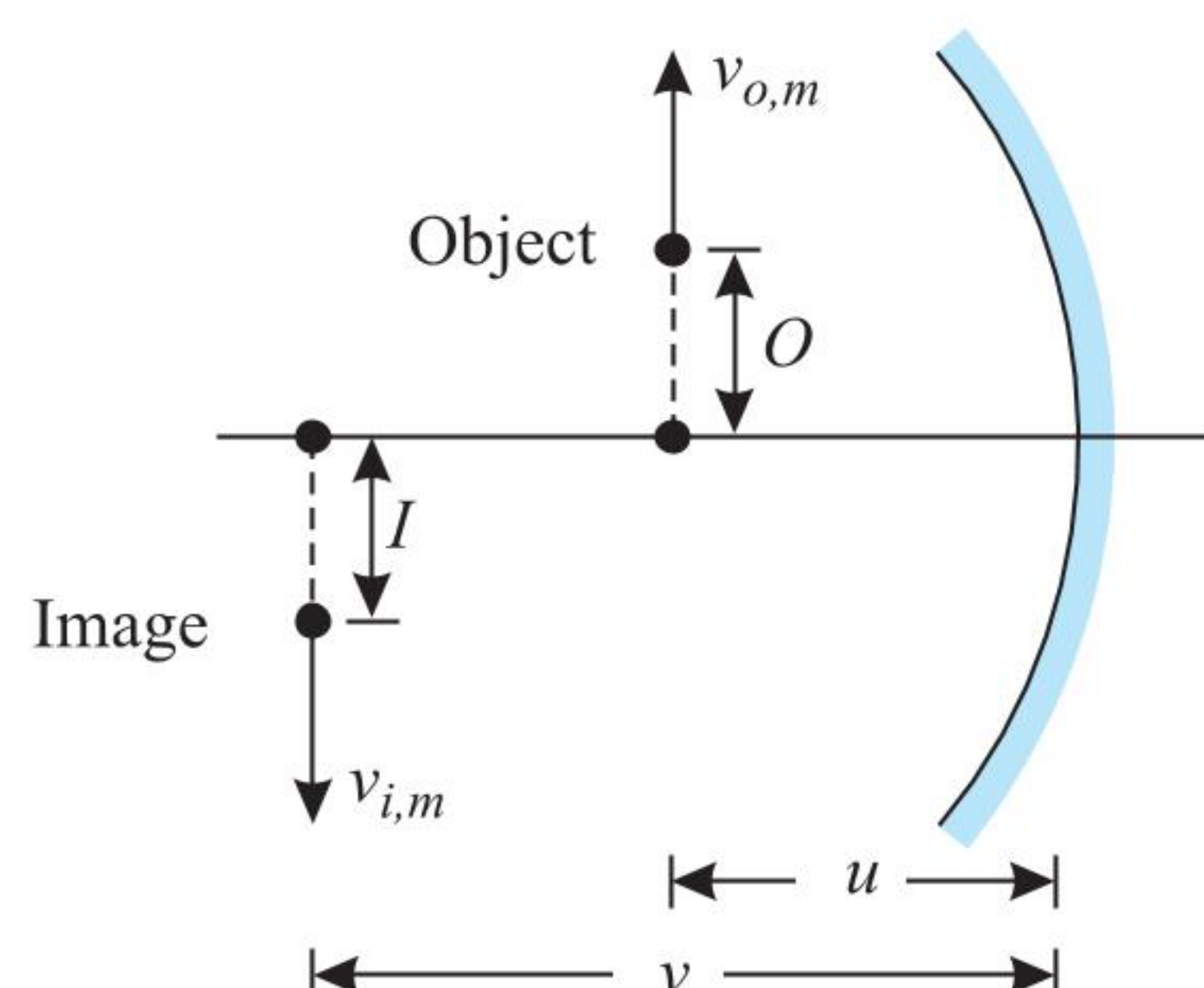
The negative sign shows that if u is decreasing, v will increase, i.e., if real object approaches the mirror, its real image will recede from the mirror.

Important Points:

- Image speed = $m^2 \times$ Object speed
- In vector notation, Eq. (iii) can be written as

$$\vec{V}_{i,m} = -m^2 \vec{V}_{o,m} \Rightarrow (\vec{V}_i - \vec{V}_m) = -m^2 (\vec{V}_o - \vec{V}_m)$$
- Object moves between infinite and centre of curvature
 In this case, $|m| < 1$, hence $V_{i,m} < V_{o,m}$
 The speed of image is less than speed of object
 When the object is at centre of curvature $|m| = 1$,
 hence $V_{i,m} = V_{o,m}$
- Object moves between centre of curvature and focus.
 In this case, $|m| > 1$, hence $V_{i,m} > V_{o,m}$
 Speed of image is more than speed of object.

Case (ii). When object is moving normal to the principal axis: In this case a point object is moving normal to the principal axis of a concave mirror as shown in figure. In this situation the object distance will not change, it makes image distance also constant.



For constant values of v and u the value of transverse magnification also constant or $m = \frac{I}{O} = -\frac{v}{u}$ is a constant. We can write,

$$I = mO \quad \dots (i)$$

Differentiate Eq. (i) with respect to time, we get

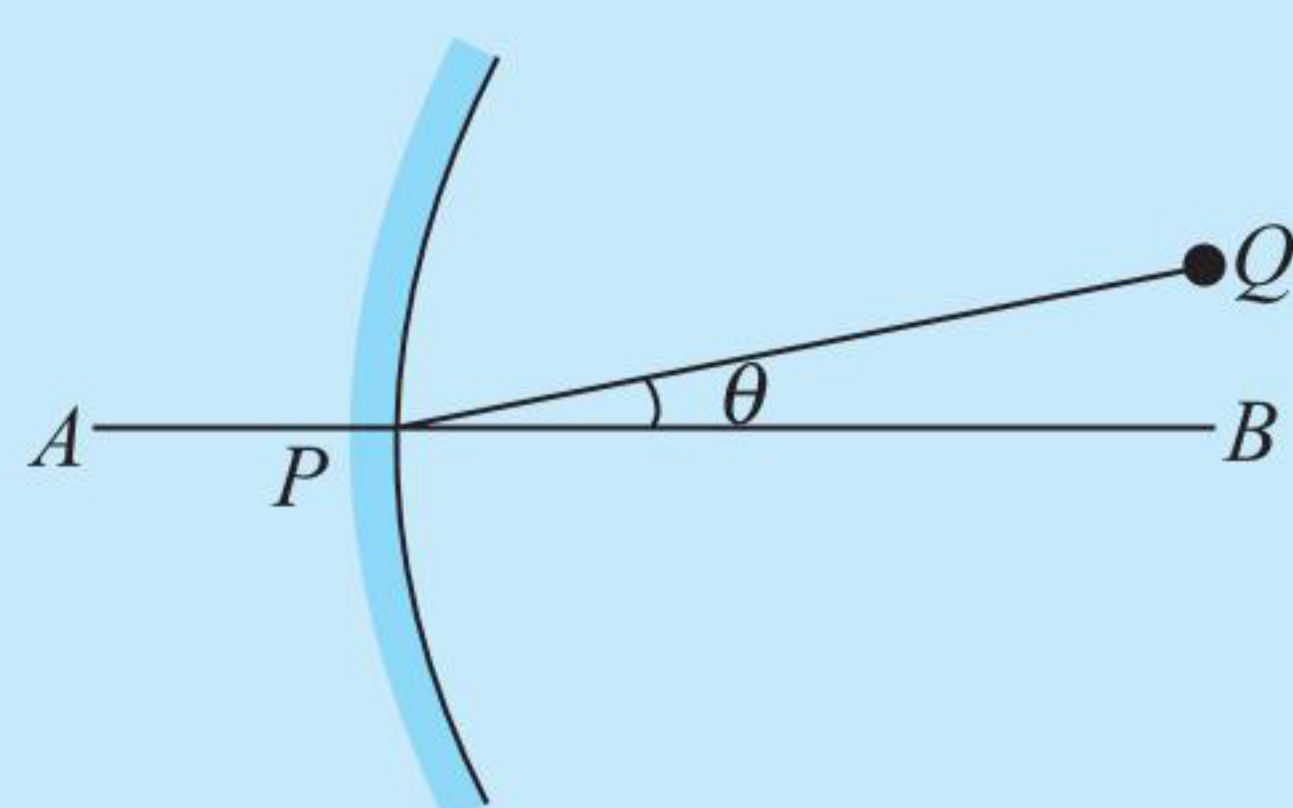
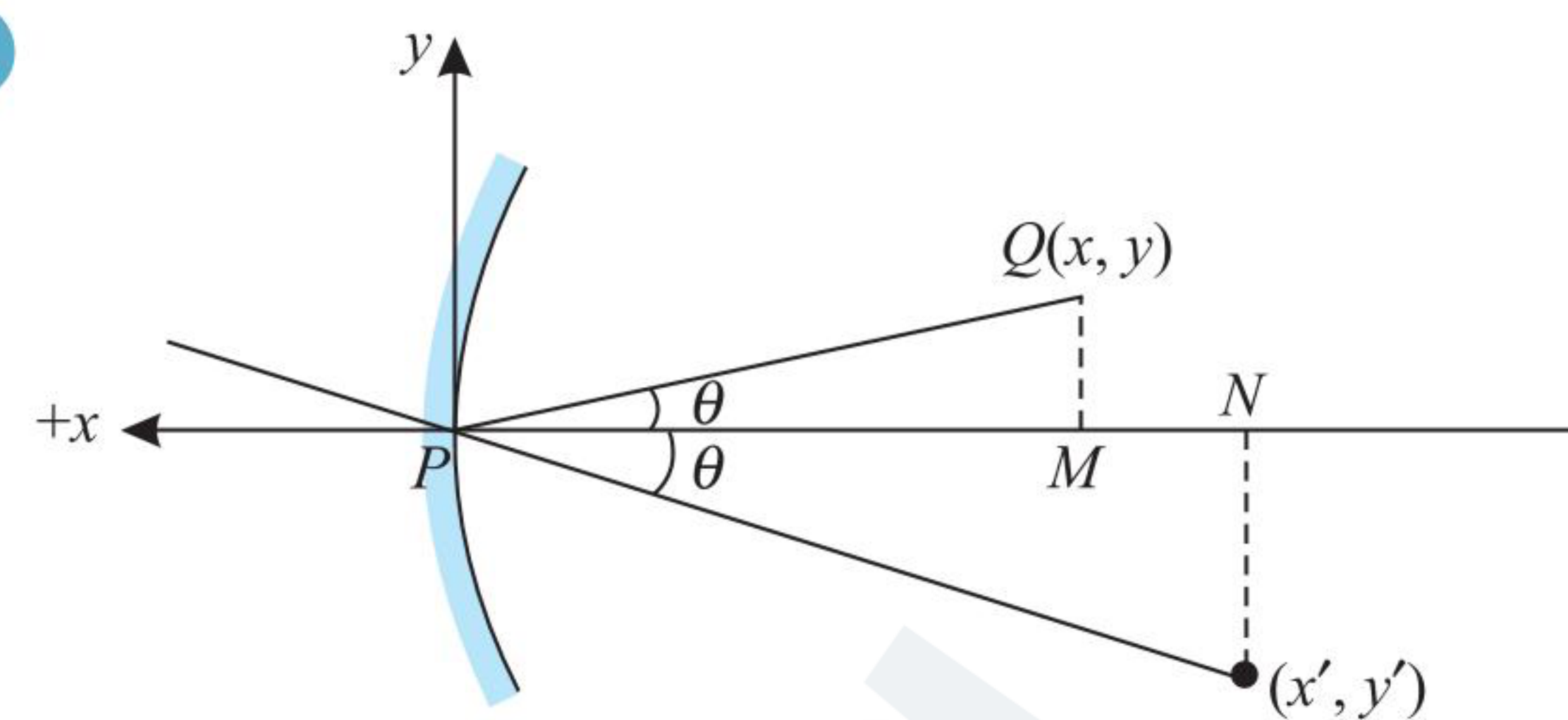
$$\frac{dI}{dt} = m \frac{dO}{dt} \Rightarrow (V_{i,m})_y = m (V_{o,m})_y$$

Here $\frac{dI}{dt} = (V_{i,m})_y$ = velocity of image w.r.t. mirror in the direction perpendicular to principal axis

And, $\frac{dO}{dt} = (V_{o,m})_y$ = velocity of object w.r.t. mirror in the direction perpendicular to principal axis

ILLUSTRATION 1.36

In figure, AB is the principal axis of the concave mirror. A point object moves on the line PQ which makes small angle θ with the principal axis. Show that image also moves in straight line making same angle θ with principal axis.

**Sol.**

Let $\tan \theta = a$ then equation of the line PQ is

$$y = ax \quad \dots (i)$$

Let $y' = y$ -coordinate of the image and

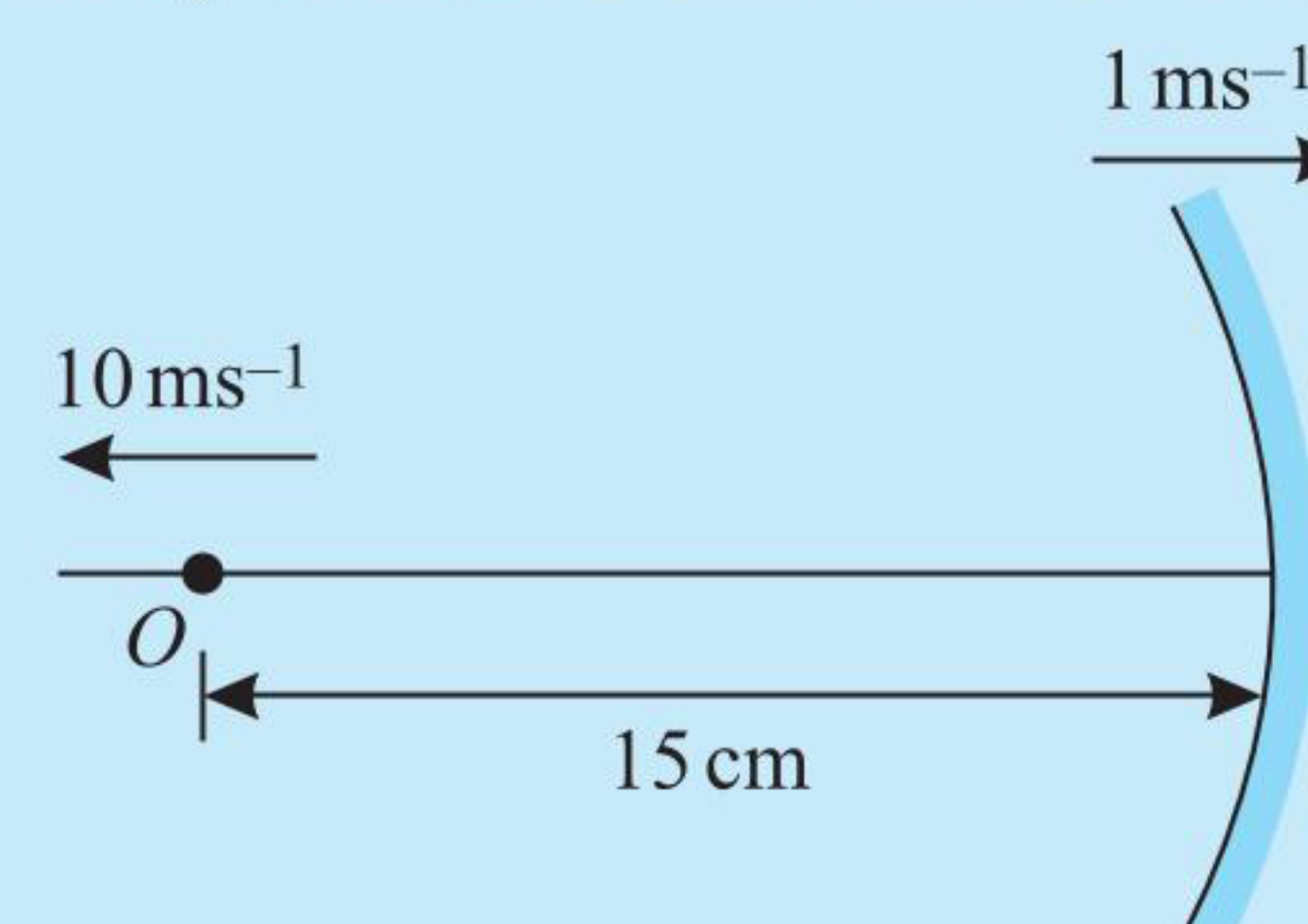
$x' = x$ -coordinate of image. By using the formula of magnification,

$$\frac{y'}{y} = -\frac{x'}{x} \Rightarrow y' = -\frac{y}{x} x' = -(a) x' = -ax' \Rightarrow y' = -ax'$$

This is an equation of a straight line, making angle θ with the principal axis. (Hence, proved.)

ILLUSTRATION 1.37

A concave mirror of focal length 10 cm and a point object both are moving away from each other with velocities 1 ms^{-1} and 10 ms^{-1} respectively as shown in figure. Find the velocity of image when the object is at a distance 15 cm from the mirror.



Sol. Using $\frac{1}{v} + \frac{1}{u} = \frac{2}{R}$, we get

$$\frac{1}{v} - \frac{1}{15} = -\frac{1}{10} \Rightarrow v = -30 \text{ cm}$$

Now, using $\vec{V}_{i,m} = -m^2 \vec{V}_{o,m}$

$$\text{Or } (\vec{V}_i - \vec{V}_m) = -m^2 (\vec{V}_o - \vec{V}_m)$$

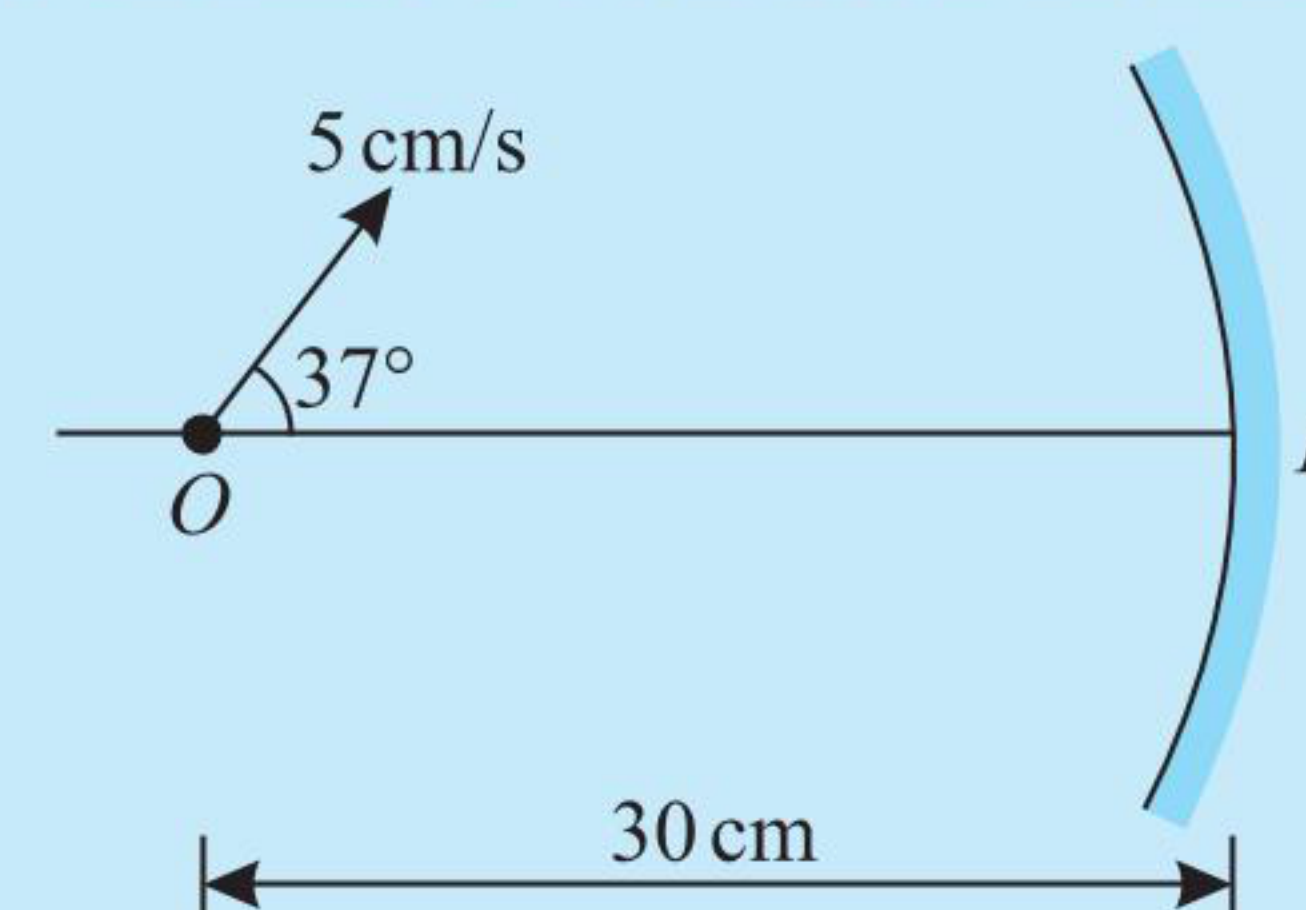
$$\Rightarrow [\vec{V}_i - (1)] = -\left(\frac{(-30)}{(-15)}\right)^2 [(-10) - (1)]$$

$$\Rightarrow V_i = 45 \text{ cm s}^{-1} \text{ towards right}$$

So, the image will move with velocity 45 cm s^{-1} towards right.

ILLUSTRATION 1.38

A point object is moving with velocity 5 cm/s at angle 37° towards a stationary concave mirror of focal length 20 cm as shown in figure. Find the image position and its velocity when the object is at a distance 30 cm from the mirror.



Sol. We have mirror formula

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \quad \dots(i)$$

Substituting the values, $u = -30$ cm and $f = -20$ cm in Eq. (i), we have

$$\frac{1}{v} + \frac{1}{-30} = \frac{1}{-20}$$

Solving this equation, we get $v = -60$ cm

Further, $m = -\frac{v}{u} = -\frac{(-60)}{(-30)} = -2$ and $m^2 = 4$

Let us make the component of the object velocity along the principal axis and along normal to principal axis.

$$\vec{V}_{o,m} = 5 \cos 37^\circ \hat{i} + 5 \sin 37^\circ \hat{j} \text{ (cm/s)}$$

$$\Rightarrow \vec{V}_{o,m} = 4\hat{i} + 3\hat{j} \text{ (cm/s)}$$

Now, using $\vec{V}_{i,m} = -m^2 \vec{V}_{o,m}$ for finding the component of the image velocity along principal axis

$$\text{Or } (\vec{V}_i - \vec{V}_m) = -m^2 (\vec{V}_o - \vec{V}_m)$$

$$(\vec{V}_i - 0) = -4(4\hat{i} - 0) \Rightarrow (\vec{V}_i)_x = -16\hat{i} \text{ cm/s}$$

The velocity of image in the direction perpendicular to principal axis,

$$(\vec{V}_{i,m})_y = -m(\vec{V}_{o,m})_y = -2 \times 3\hat{j} = -6\hat{j} \text{ cm/s}$$

Hence the velocity of image, $\vec{V}_{\text{image}} = -16\hat{i} - 6\hat{j} \text{ (cm/s)}$

The position and velocity of image is shown below.

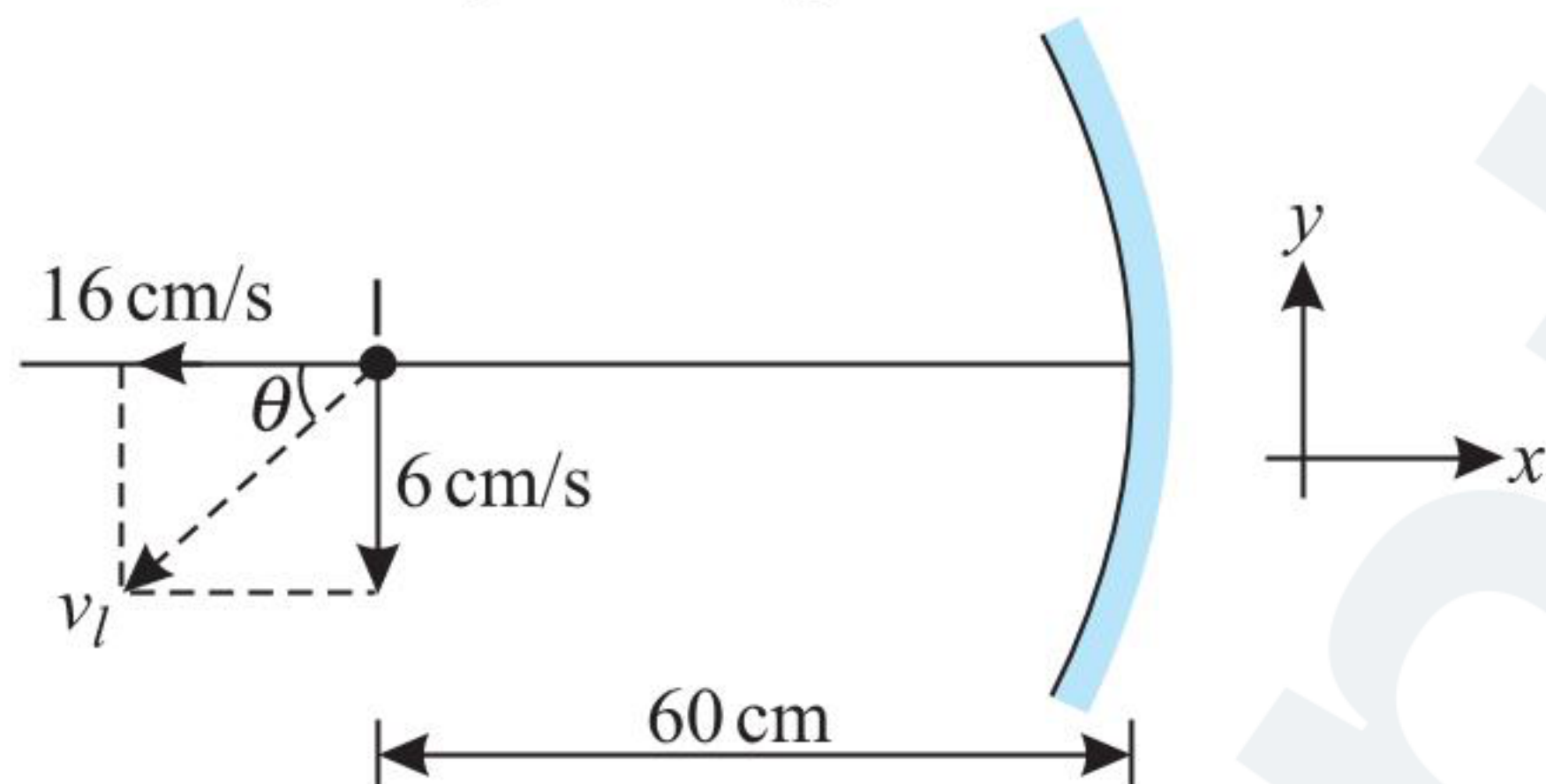
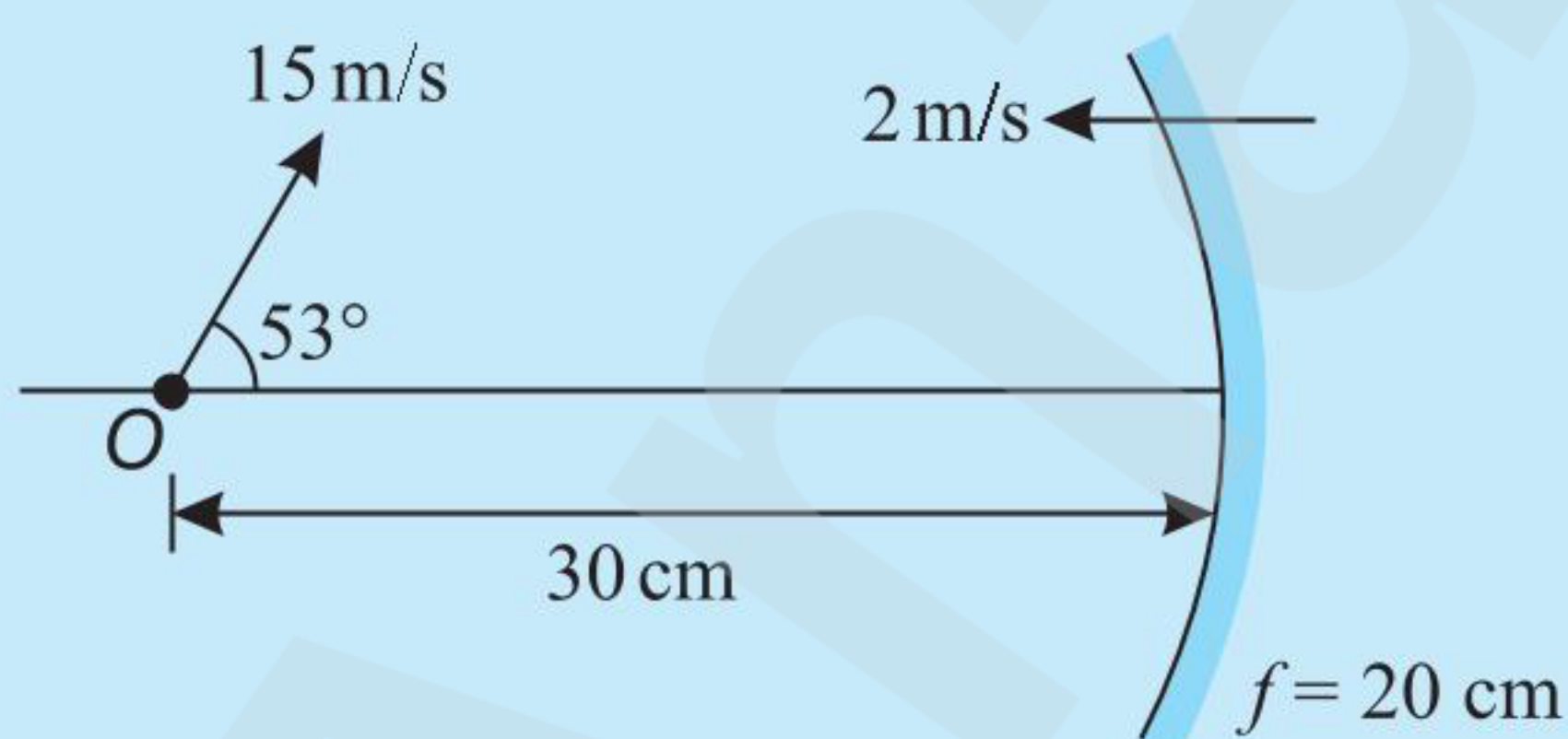


ILLUSTRATION 1.39

Find the velocity of image in a situation as shown in the figure.



Sol. Given: Object distance $u = -30$ cm, $f = -20$ cm

Velocity of the object $\vec{v}_o = (9\hat{i} + 12\hat{j})$ m/s and velocity of the mirror $\vec{v}_m = -2\hat{i}$ m/s

Using mirror formula: $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

Magnification $m = -\frac{v}{u} = -\frac{(-60)}{(-30)} = -2$

Velocity of the image with respect to mirror in the direction along principal axis (x-direction)

$$(\vec{V}_{i,m})_x = -m^2 (\vec{V}_{o,m})_x \Rightarrow (\vec{v}_i - \vec{v}_m)_x = -m^2 (\vec{v}_o - \vec{v}_m)_x$$

$$\left[\vec{v}_i - (-2\hat{i}) \right] = -(-2)^2 \left[(9\hat{i}) - (-2\hat{i}) \right]$$

$$(\vec{v}_i)_x = -46\hat{i} \text{ m/s}$$

Velocity of image w.r.t. mirror in y-direction

$$(\vec{V}_{i,m})_y = m(\vec{V}_{o,m})_y \Rightarrow (\vec{v}_i - \vec{v}_m)_y = m(\vec{v}_o - \vec{v}_m)_y$$

$$\left[\vec{v}_i - 0 \right] = (-2) \left[(12\hat{j}) - 0 \right]$$

$$\Rightarrow (\vec{v}_i)_y = -24\hat{j} \text{ (m/s)}$$

Hence velocity of the image $\vec{v}_i = (-46\hat{i} - 24\hat{j})$ m/s

GRAPHICAL METHOD OF DETERMINING THE FOCAL LENGTH OF A CONCAVE MIRROR

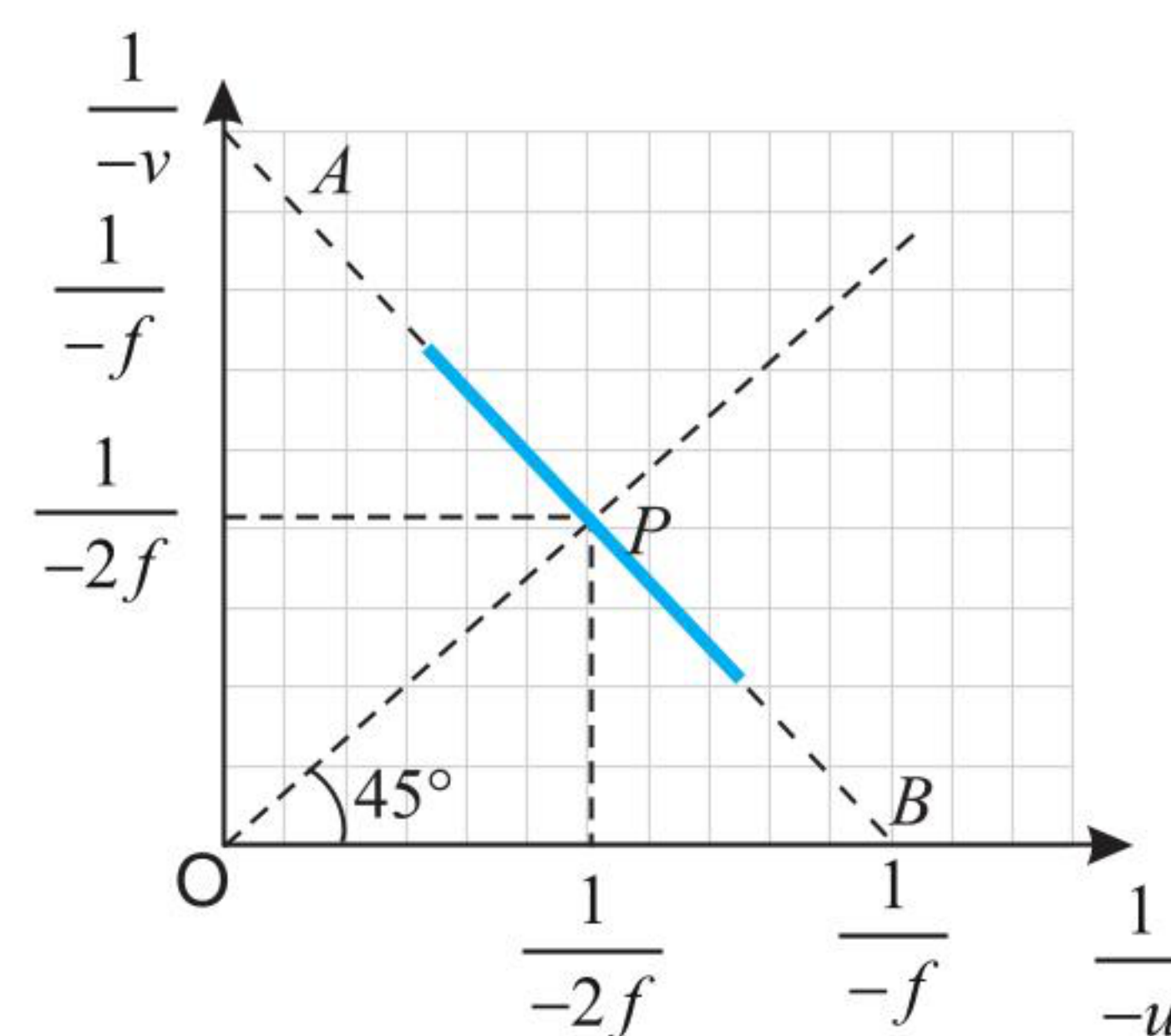
It forms real and inverted image of an object placed beyond its focus. From mirror equation,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

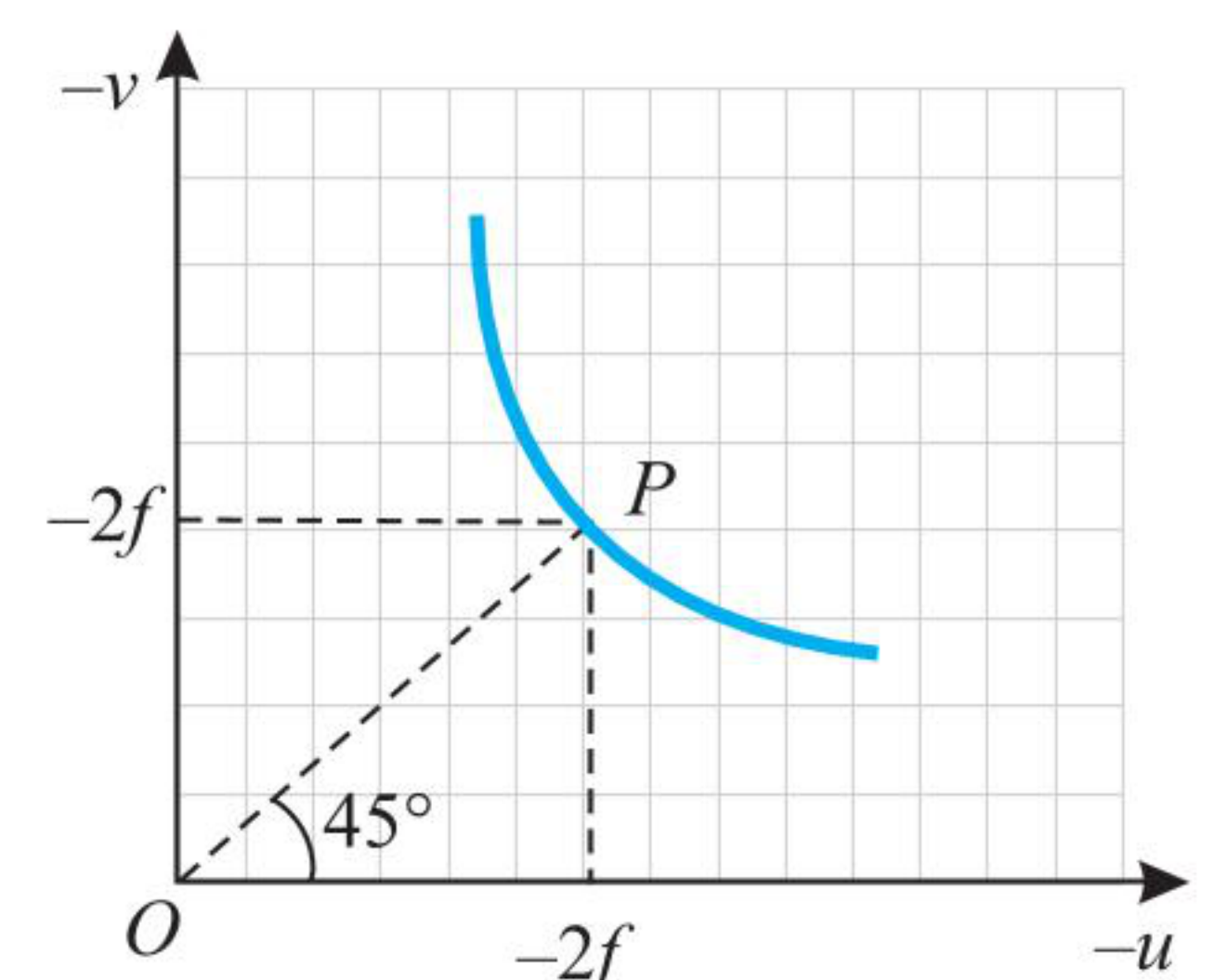
Using Cartesian sign convention, we have $u = -x$, $v = -y$ and $f = -f$

or $\frac{1}{-y} + \frac{1}{-x} = \frac{1}{-f}$

A graph between $\frac{1}{-v}$ and $\frac{1}{-u}$ is a straight line, as shown in Fig. (a).



(a)



(b)

Note that the slope of the straight line is -1 and the intercepts on the horizontal and vertical axes are equal. It is equal to $\frac{1}{-f}$.

A straight line OP at an angle of 45° with the horizontal axis is drawn which intersects the line AB at P . The coordinates of the

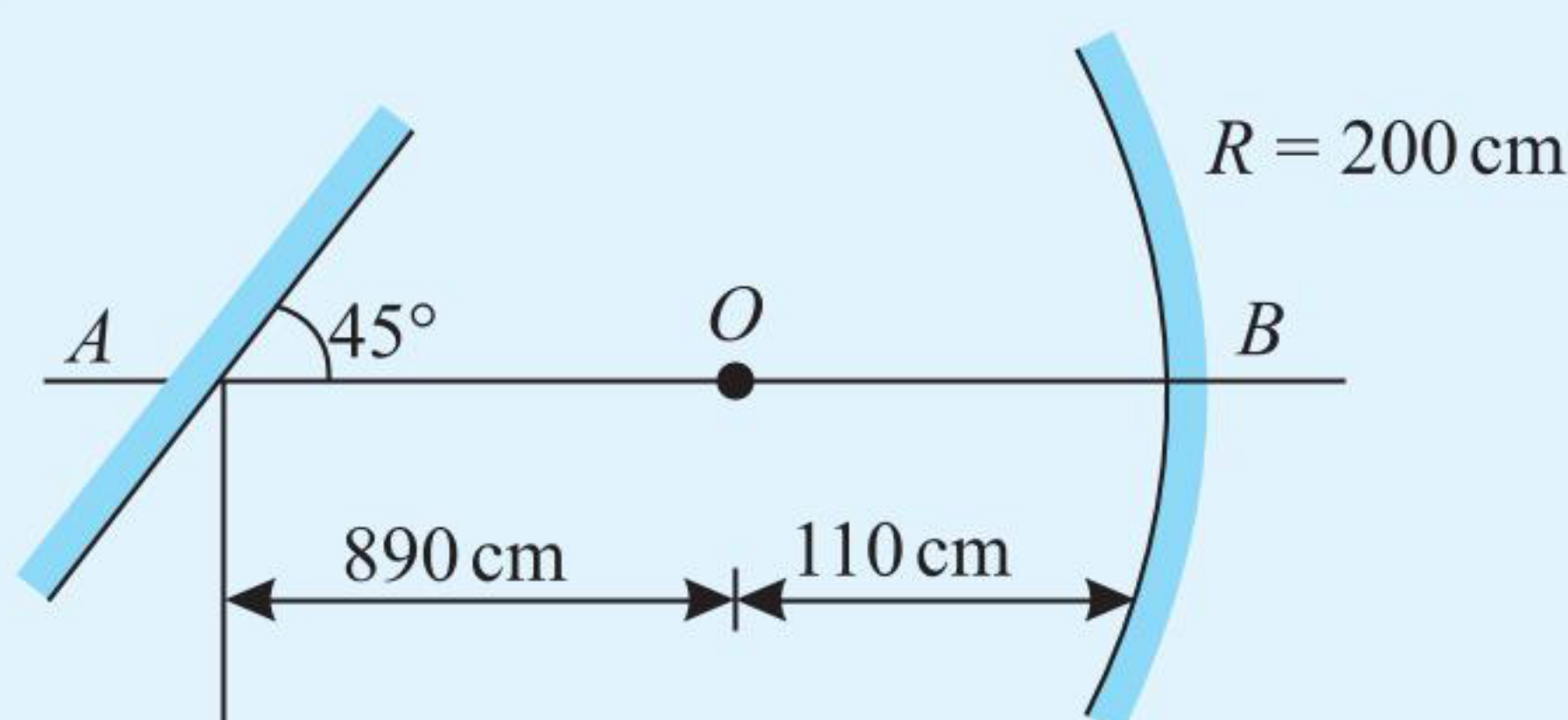
point P are $\left(\frac{1}{-2f}, \frac{1}{-2f} \right)$.

The focal length of the mirror can be calculated by measuring the coordinates of either of the points A , B or P .

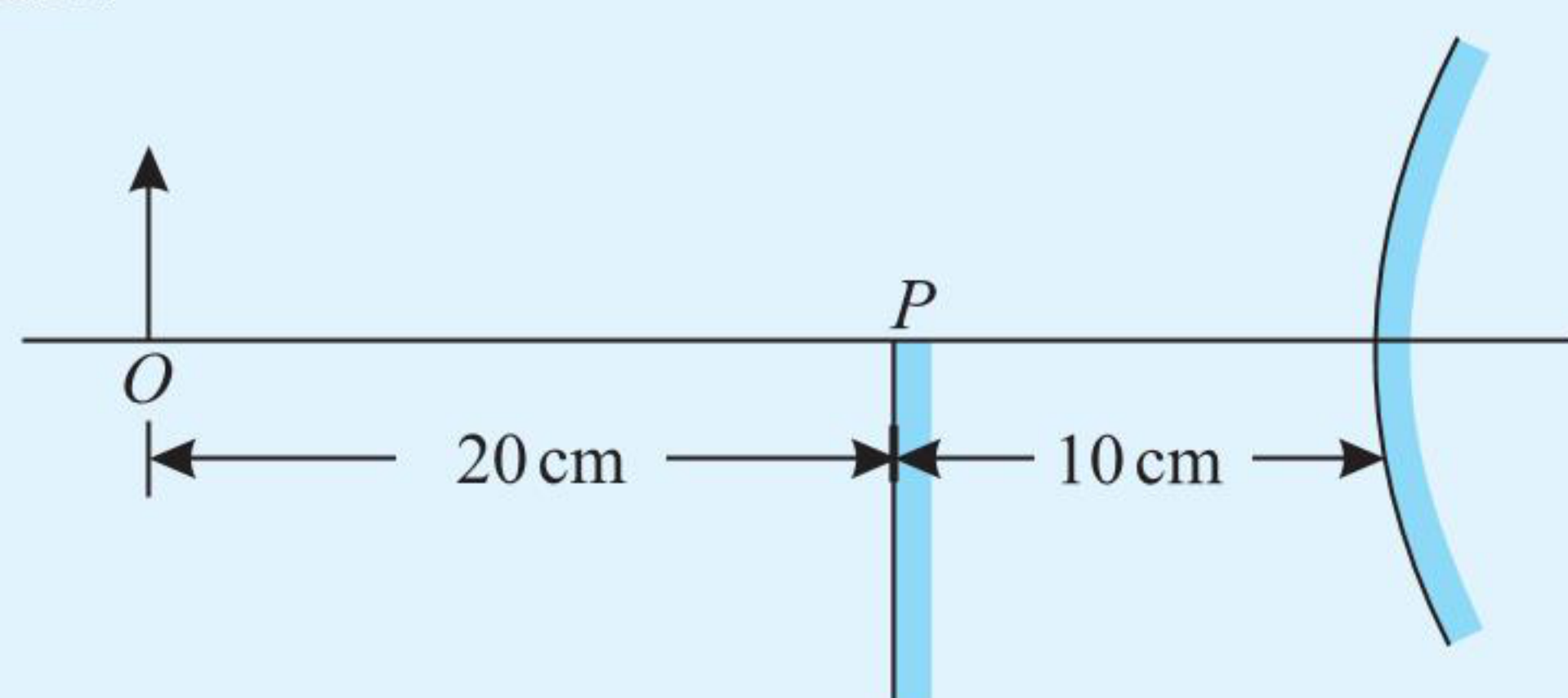
Alternatively, a graph between $-v$ and $-u$ can also be plotted, which is a curve as shown in Fig. (b). A line drawn at an angle of 45° from the origin intersects it at the point P whose coordinates are $(-2f, -2f)$. By measuring the coordinates of this point, the focal length of the mirror can also be measured.

CONCEPT APPLICATION EXERCISE 1.3

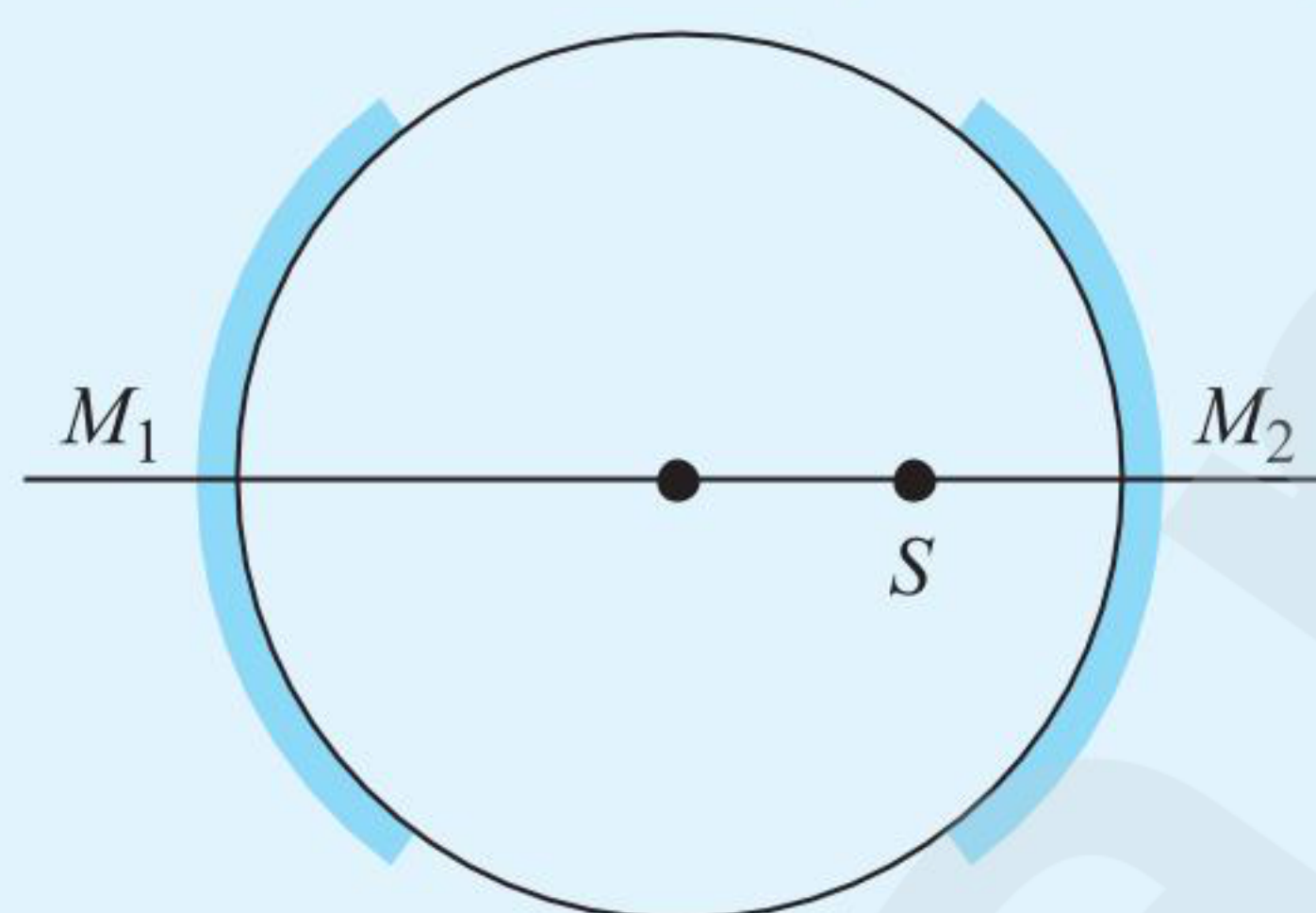
1. A plane mirror and a concave mirror are arranged as shown in figure and O is a point object. Find the position of image formed by two reflections, first one taking place at concave mirror.



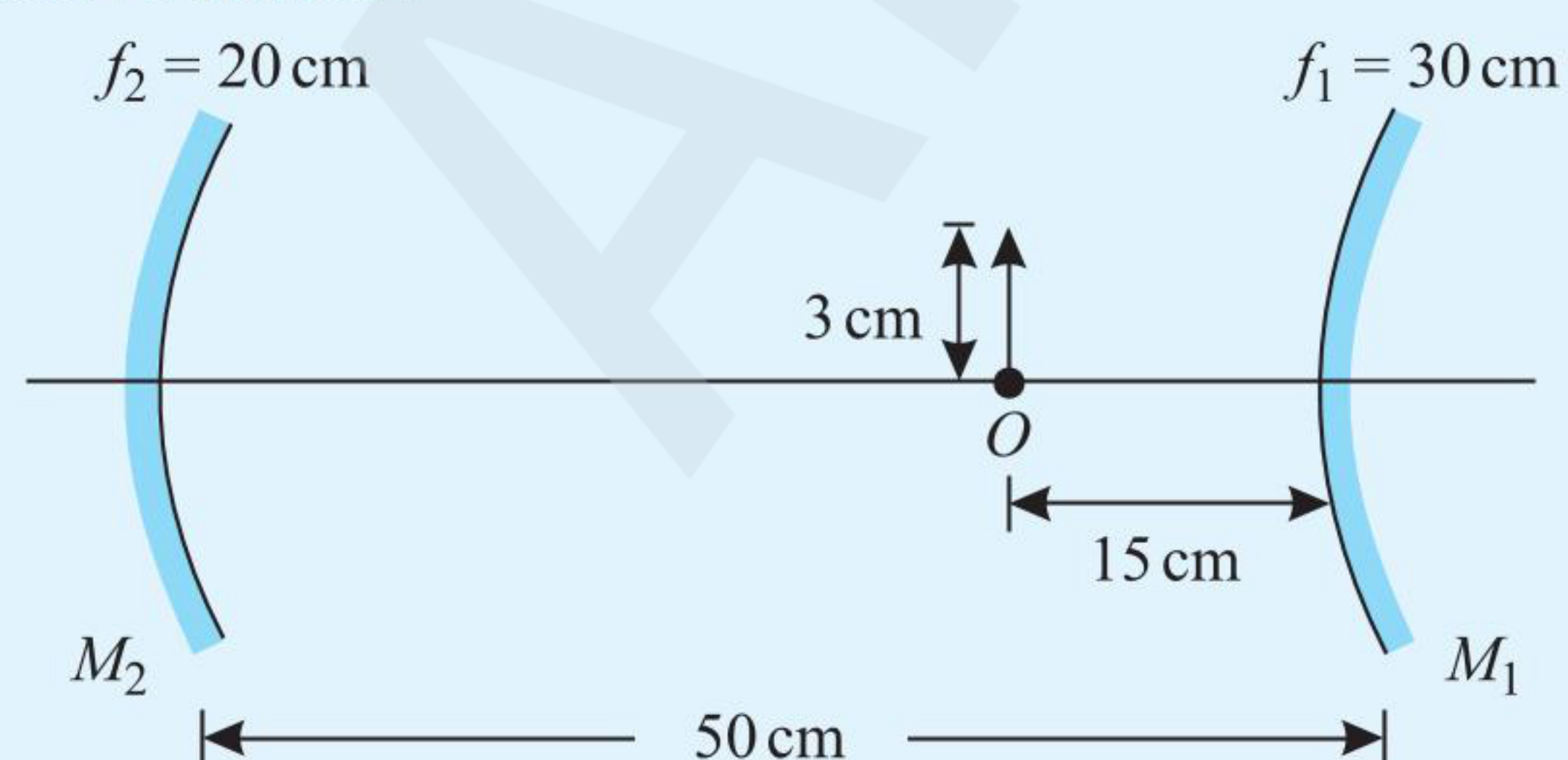
2. A small plane mirror strip is kept at a distance of 10 cm in front of a convex mirror, with its plane normal to the principal axis. An object is placed at a distance of 20 cm from the plane mirror, as shown in figure. Calculate the focal length of the convex mirror, if the image formed by the plane mirror and the convex mirror coincide, without parallax.



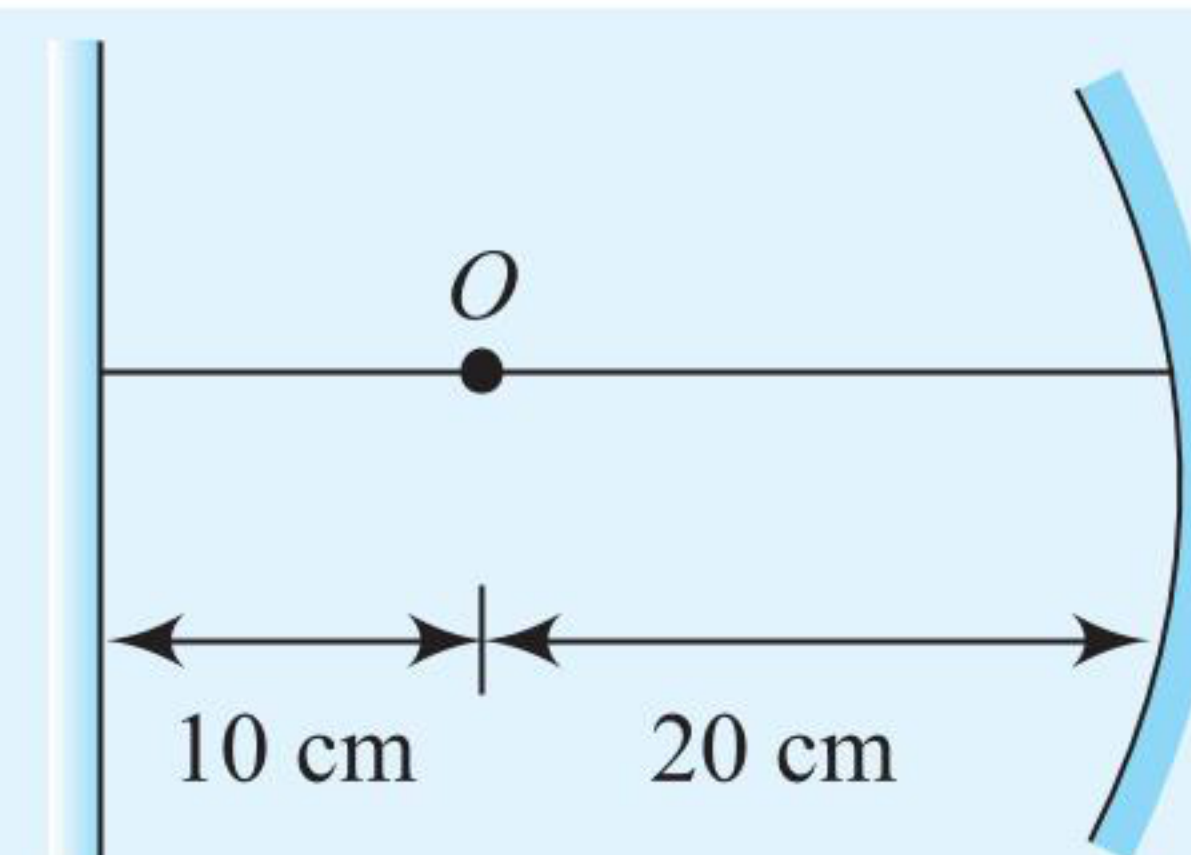
3. The inner surface of the wall of a sphere is perfectly reflecting. Radius of the sphere is R . A point source S is placed at a distance $R/2$ from the centre of the sphere. Consider the reflection of light from the farthest wall followed by reflection from the nearest wall. Where is the image of the source? Consider paraxial rays only.



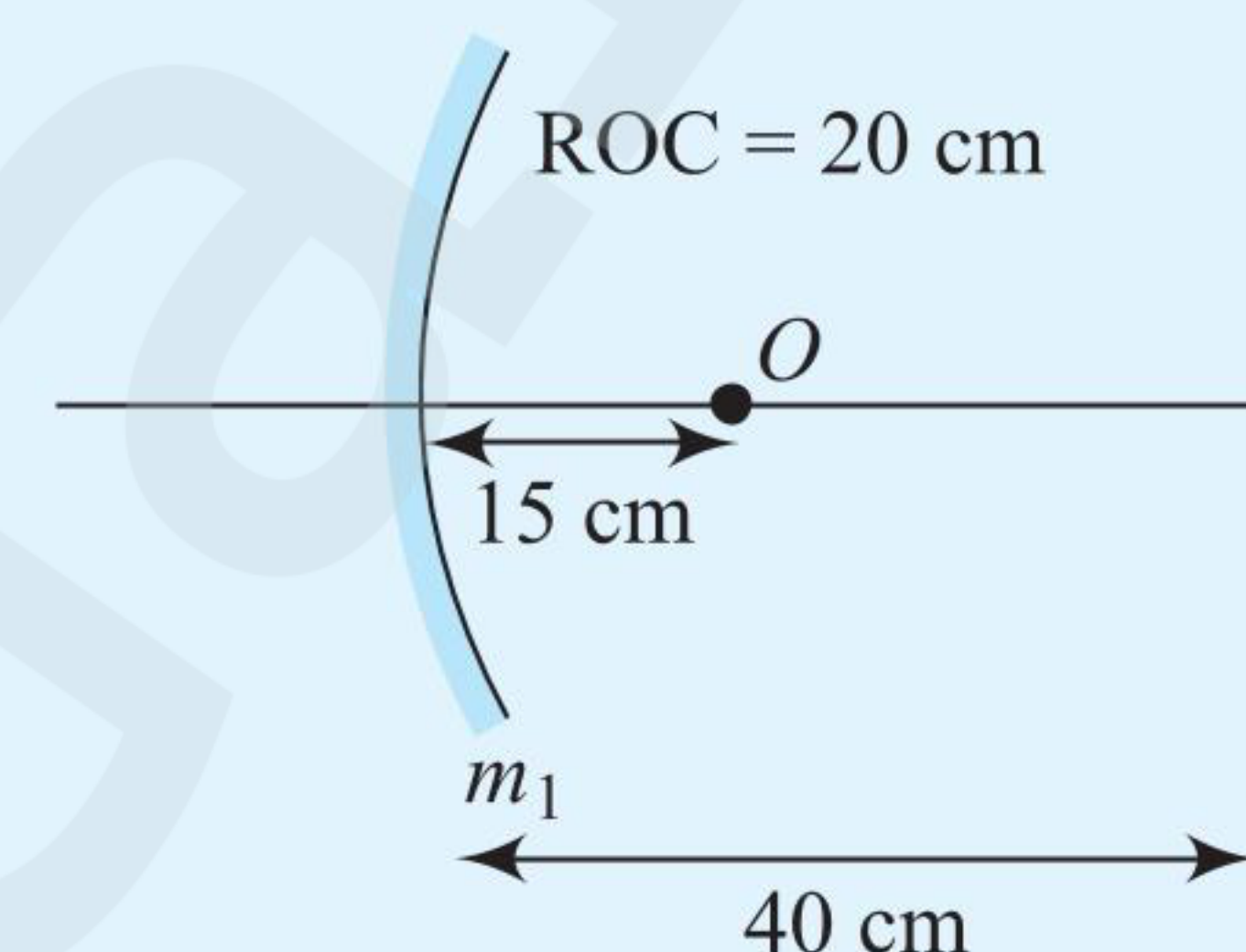
4. A concave mirror and a convex mirror of focal lengths 20 cm and 30 cm are placed at a distance of 50 cm. An object of height 3 cm is kept at a distance 15 cm from convex mirror. Find the position and height of the image formed by reflection, first at the convex and then at the concave mirror.



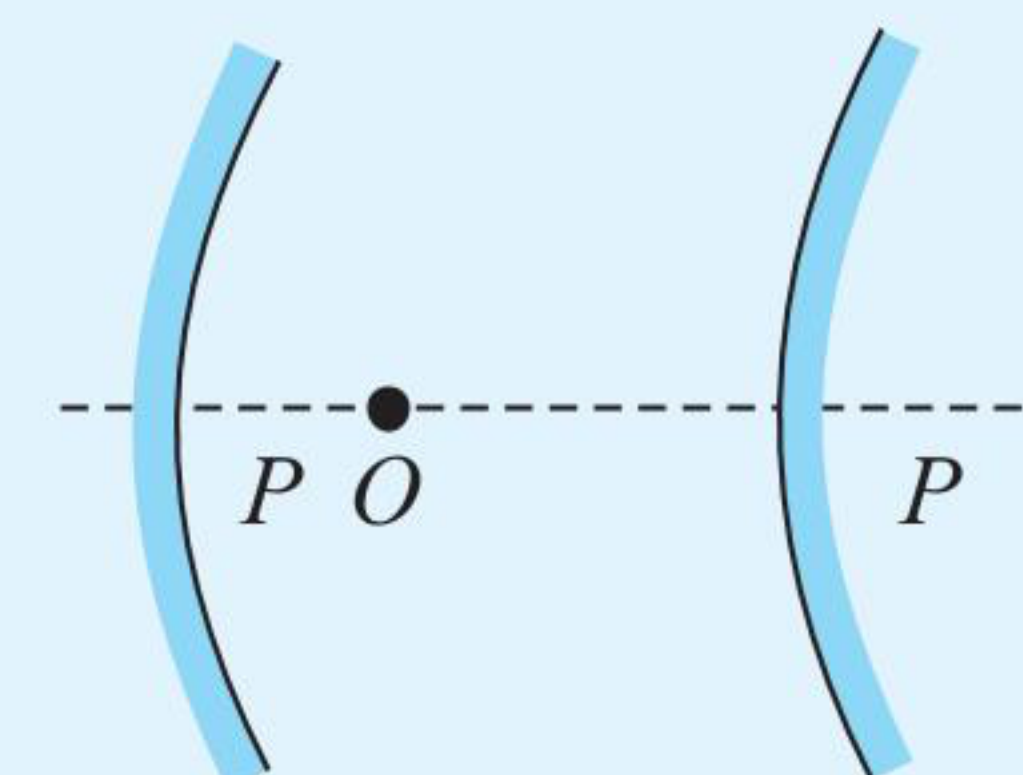
5. An object is placed between a plane mirror and a concave mirror of focal length 15 cm as shown in the figure. Find the positions of the images after two reflections.



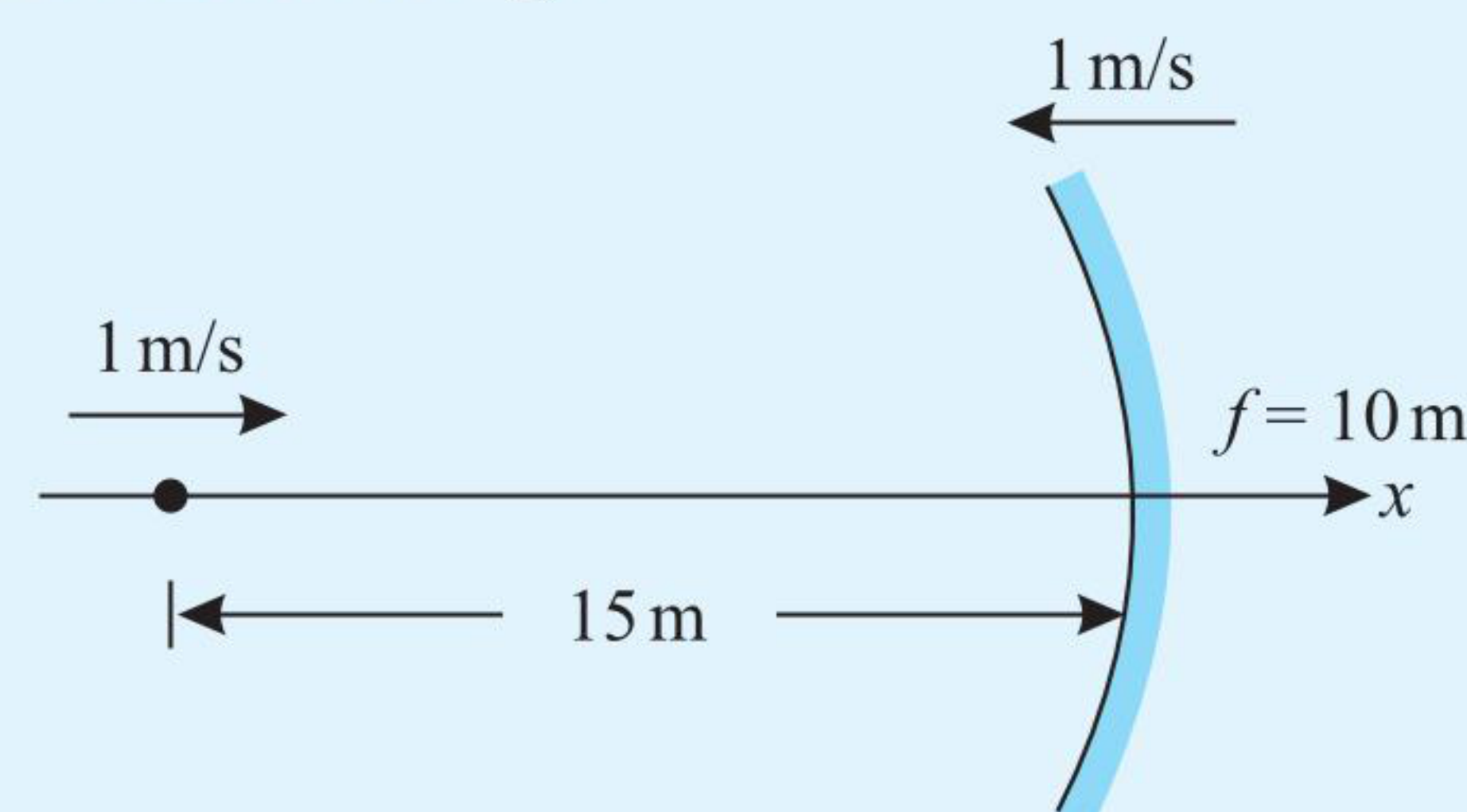
6. An object is placed midway between a concave mirror of focal length f and a convex mirror of focal length f . The distance between the two mirrors is $6f$. Trace the ray that is first incident on the concave mirror and then the convex mirror.
7. Find the position of the final image after three successive reflections taking the first reflection on m_1 .



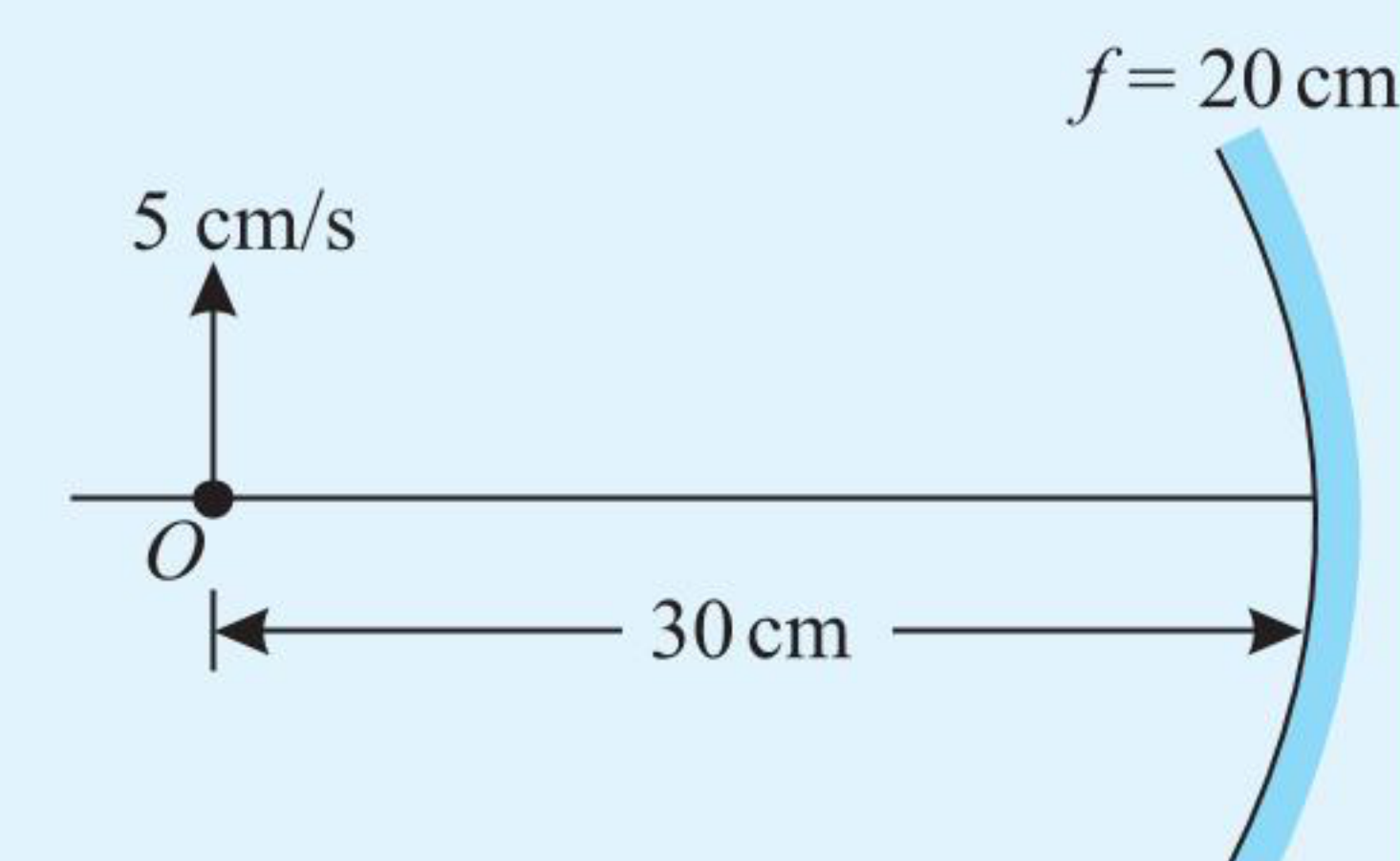
8. A concave mirror of focal length 10 cm is placed in front of a convex mirror of focal length 20 cm. The distance between the two mirrors is 20 cm. A point object is placed 5 cm from the concave mirror. Discuss the formation of image.



9. A point object moves in $+x$ -direction with $v = 1$ m/s along the principal axis of the concave mirror of focal length $f = 10$ m. When the mirror moves with a velocity $\vec{V}_m = -\hat{i}$ m/s and the object is at a distance of 15 m, find the speed of the image.



10. Find the velocity of image w.r.t. ground. The mirror is at rest and the object is moving perpendicular to the axis with 5 cm/s as in figure.



ANSWERS

2. 15 cm 3. $\frac{5R}{6}$ 4. 30 cm; 1 cm 5. 24 cm; 60 cm
7. 12.5 cm 9. 9 m/s 10. 10 cm/s

REFRACTION OF LIGHT

Deviation or bending of light rays from their original path while passing from one medium to another is called **refraction**. It is due to change in speed of light as light passes from one medium to another medium. If the light is incident normally then it goes to the second medium without bending, but still it is called refraction.

REFRACTIVE INDEX

Whenever light travels from vacuum to any other medium, its speed slows down. The speed of light is maximum in vacuum.

Why a light wave slows down when it enters an optical medium?

The answer can be found in electromagnetism where we find from Maxwell's equations that the speed of light in vacuum is given by,

$$c_{vac} = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

In a medium it is given by

$$c_{med} = \frac{1}{\sqrt{\epsilon_r \epsilon_0 \mu_r \mu_0}} = \frac{c_{vac}}{\sqrt{\epsilon_r \mu_r}}$$

Since ϵ_r and μ_r are both greater than 1, speed of light in a medium will be less than the speed of light in vacuum. The ratio of the speed of light in vacuum to the speed of light in the medium is called the refractive index of the medium, represented by μ .

$$\mu = \frac{c_{vac}}{c_{med}} = \frac{c}{v} = \frac{\text{speed of light in vacuum}}{\text{speed of light in medium}}$$

Refractive index of a medium is defined as the factor by which speed of light reduces as compared to the speed of light in vacuum. More (less) refractive index implies less (more) speed of light in that medium, which therefore is called denser (rarer) medium.

This μ (refractive index of medium w.r.t. vacuum) is called the **absolute (or standard) refractive index**. Absolute refractive index is commonly referred to simply as refractive index. Further, refractive index is a measure of speed of light in a transparent medium or technically a measure of the **optical density** of the material. For example, the speed of light in water is less than that in air, so water is said to be optically denser. Optical density in general correlates with mass density.

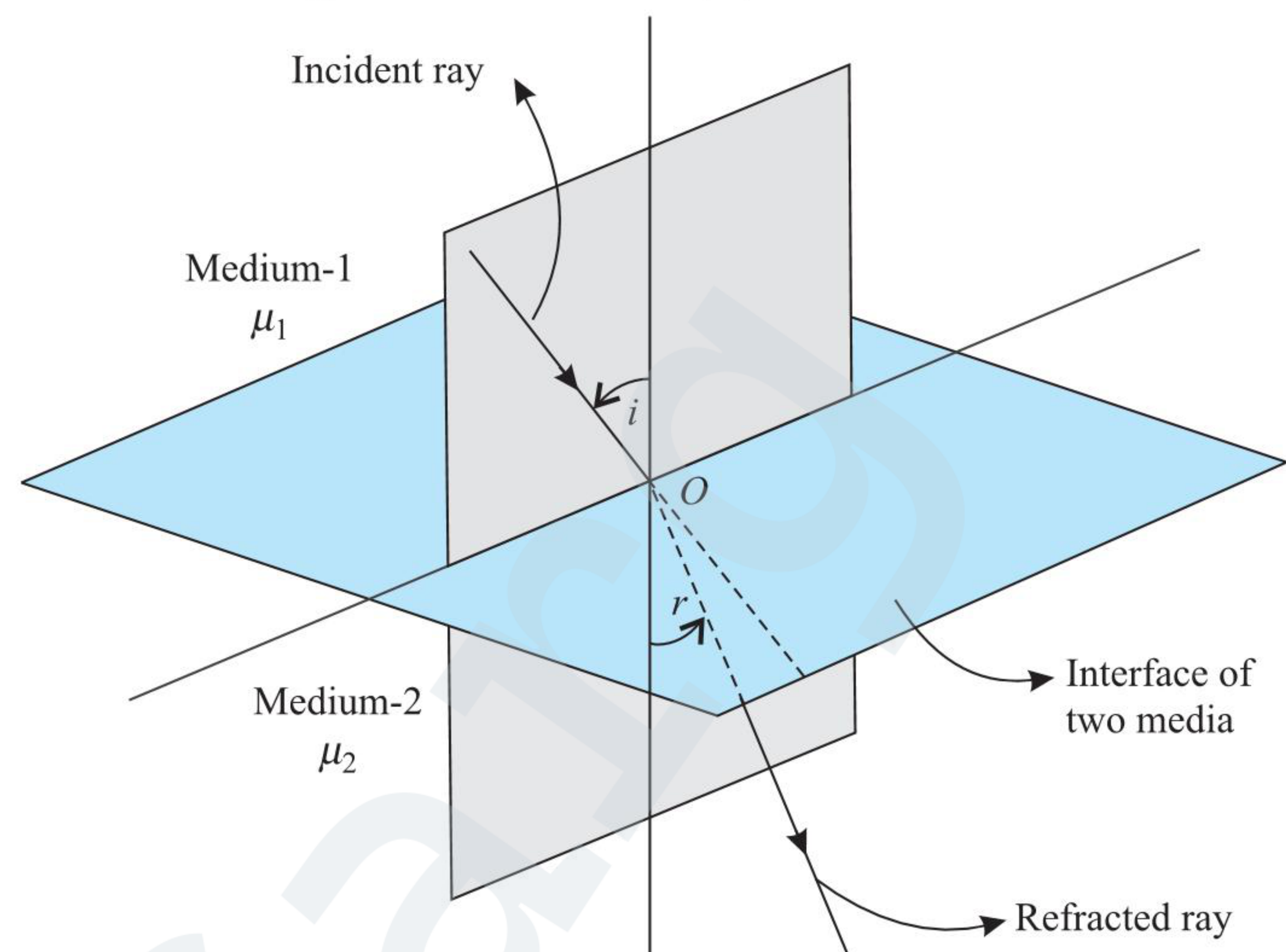
However, in some cases, a material with greater optical density than another can have a lower mass density, e.g., mass density of turpentine oil is less than that of water but its optical density is higher. Thus, greater the refractive index of a material, the greater is the material's optical density and the smaller is the speed of light in that material (as $v = c/\mu$).

LAWS OF REFRACTION

The bending or the change in direction of propagation of light occurs except when it strikes the interface normally, i.e., when angle of incidence, $i = 0$.

The incident ray, as earlier said, is described by the angle of incidence whereas the refracted ray is described by the angle

of refraction (r), both angles measured from the normal to the interface at the point of incidence (O).



Law 1: The incident ray, the normal to the interface at the point of incidence and the refracted ray, lie in the same plane.

Law 2: The ratio of the sine of the angle of incidence to the sine of the angle of refraction is always a constant (a different constant for different media),

For any two media the ratio of the sine of the angle of incidence to the sine of the angle of refraction is always constant.

$$\frac{\sin i}{\sin r} = \text{constant} = {}^1\mu_2 = \frac{\mu_2}{\mu_1} \quad \dots(i)$$

Equation (i) is known as **Snell's law**. The constant ${}^1\mu_2$ (or μ_{21}) is called the refractive index of medium-2 with respect to medium-1 when a ray of light travels from medium-1 to medium-2. In other words, ${}^1\mu_2$ (or μ_{21}) is the refractive index of the medium in which the refracted ray lies w.r.t. the medium in which the incident ray lies.

Equation (i) can be written as $\mu_1 \cdot \sin i = \mu_2 \cdot \sin r = \text{constant}$.

The product of the refractive index and the sine of angle made by the ray with the normal at incident is constant for a given ray in both the media.

PRINCIPLE OF REVERSIBILITY OF LIGHT RAYS

According to the principle of reversibility of light, if the final path of a ray of light, which has suffered a number of reflections and refractions is reversed, the entire ray retraces its path. Thus, if we place a plane mirror at an angle of 90° to the refracted ray, then it will become the incident ray and initial ray (incident ray) will be the refracted ray. In such a case, angle of incidence = r and angle of refraction = ' i '. Thus,

$$\frac{\sin r}{\sin i} = {}^2\mu_1 = \frac{\mu_1}{\mu_2} \quad \dots(ii)$$

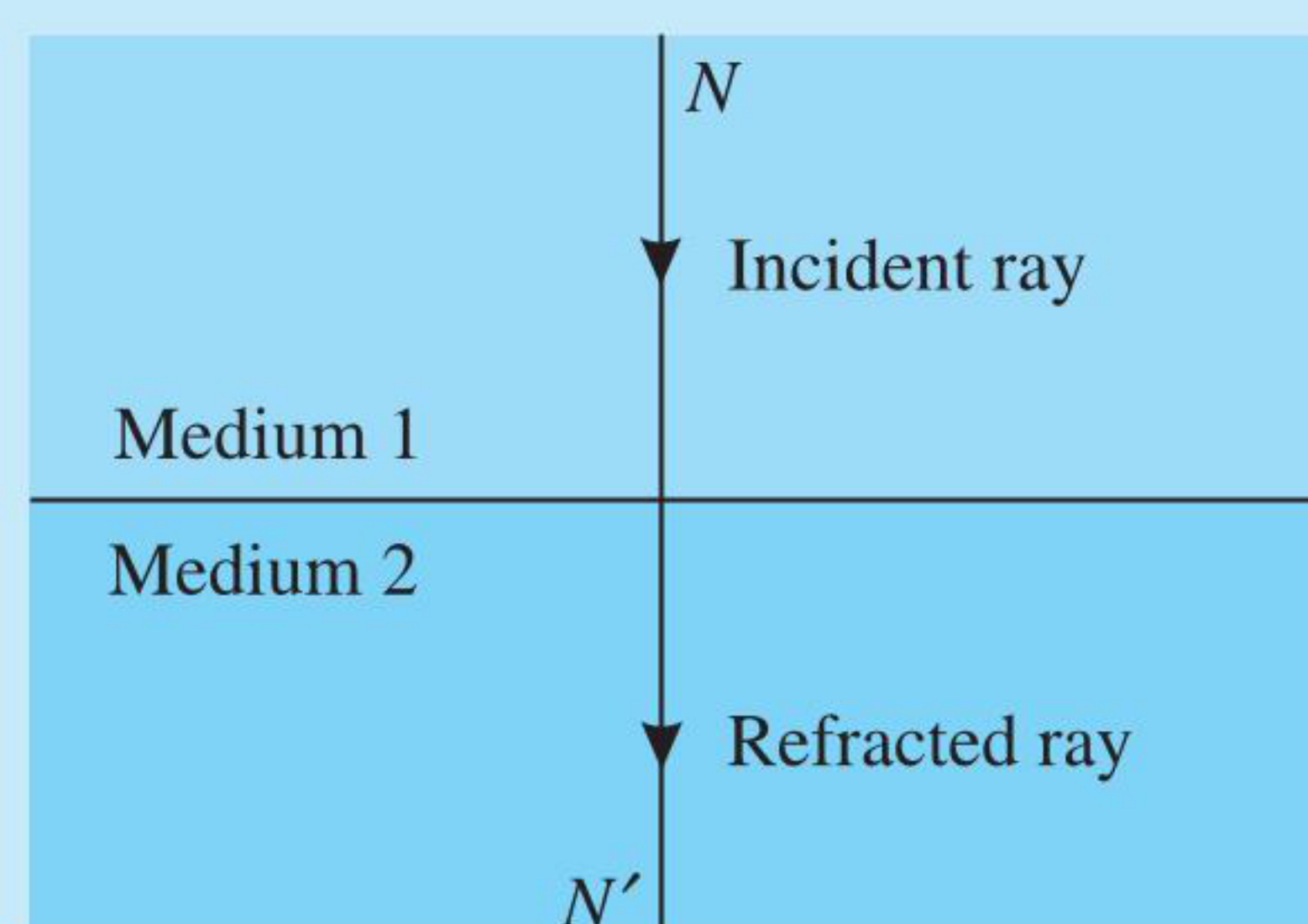
From Eqs. (i) and (ii),

$${}^1\mu_2 = \frac{1}{{}^2\mu_1}$$

$$\text{or } {}^2\mu_1 \times {}^1\mu_2 = 1 \quad \dots(iii)$$

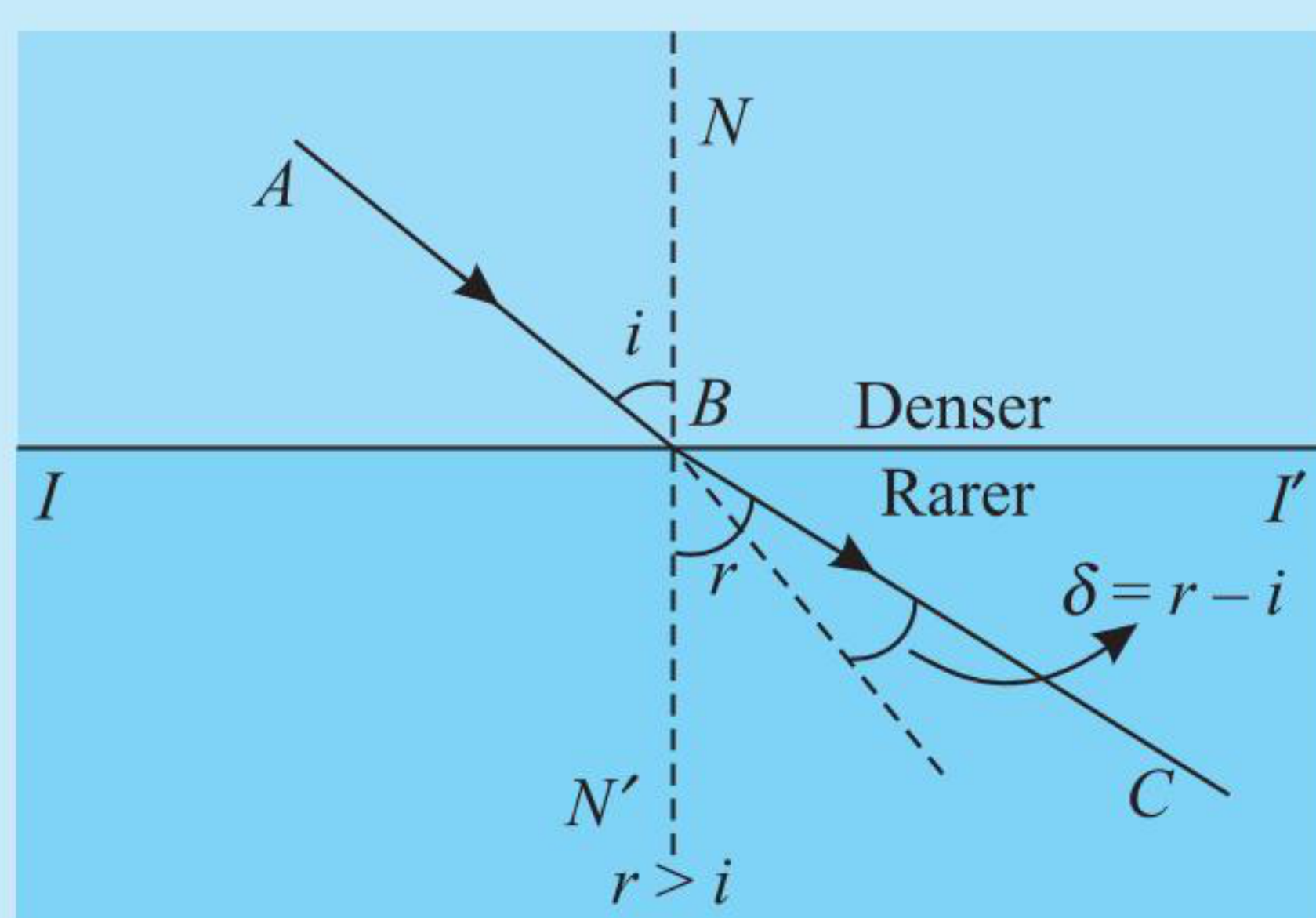
Special Cases

Case 1. Normal incidence: $i = 0$



From Snell's law: $r = 0$

Case 2. When light moves from denser to rarer medium, it bends away from the normal



Case 3. When light moves from rarer to denser medium, it bends towards the normal

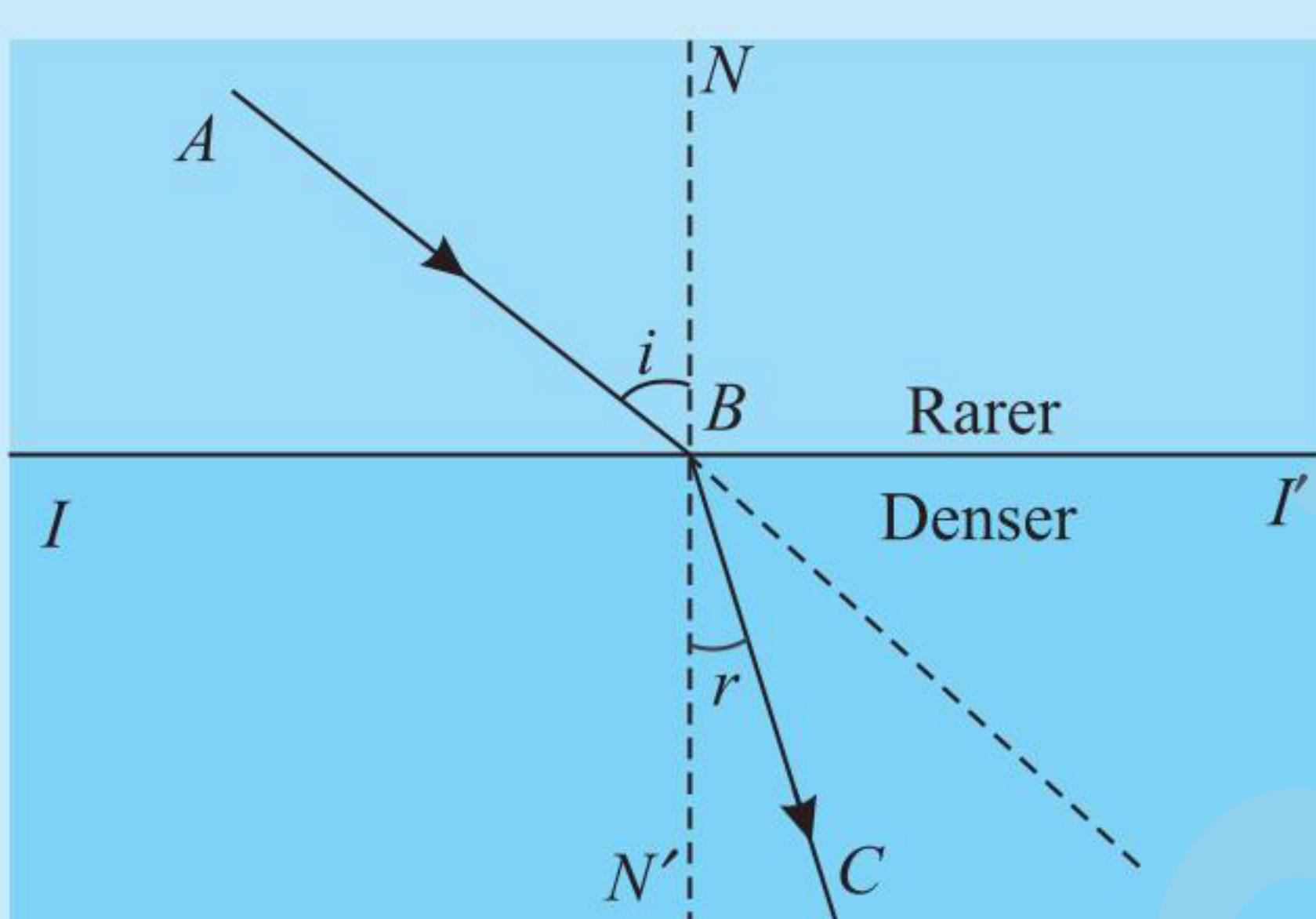


ILLUSTRATION 1.40

Determine the refractive index of glass with respect to water. Given that $\mu_g = 3/2$; $\mu_w = 4/3$.

Sol. Refractive index of glass with respect to water or relative refractive index of glass w.r.t. water:

$${}_w\mu_g = \mu_{gw} = \frac{\text{RI of glass}}{\text{RI of water}} = \frac{\mu_g}{\mu_w} = \frac{3/2}{4/3} = \frac{9}{8} = 1.125$$

ILLUSTRATION 1.41

Find the speed of light in medium 'a' if speed of light in medium 'b' is $c/3$, where c = speed of light in vacuum and light refracts from medium 'a' to medium 'b' making 45° and 60° , respectively, with the normal.

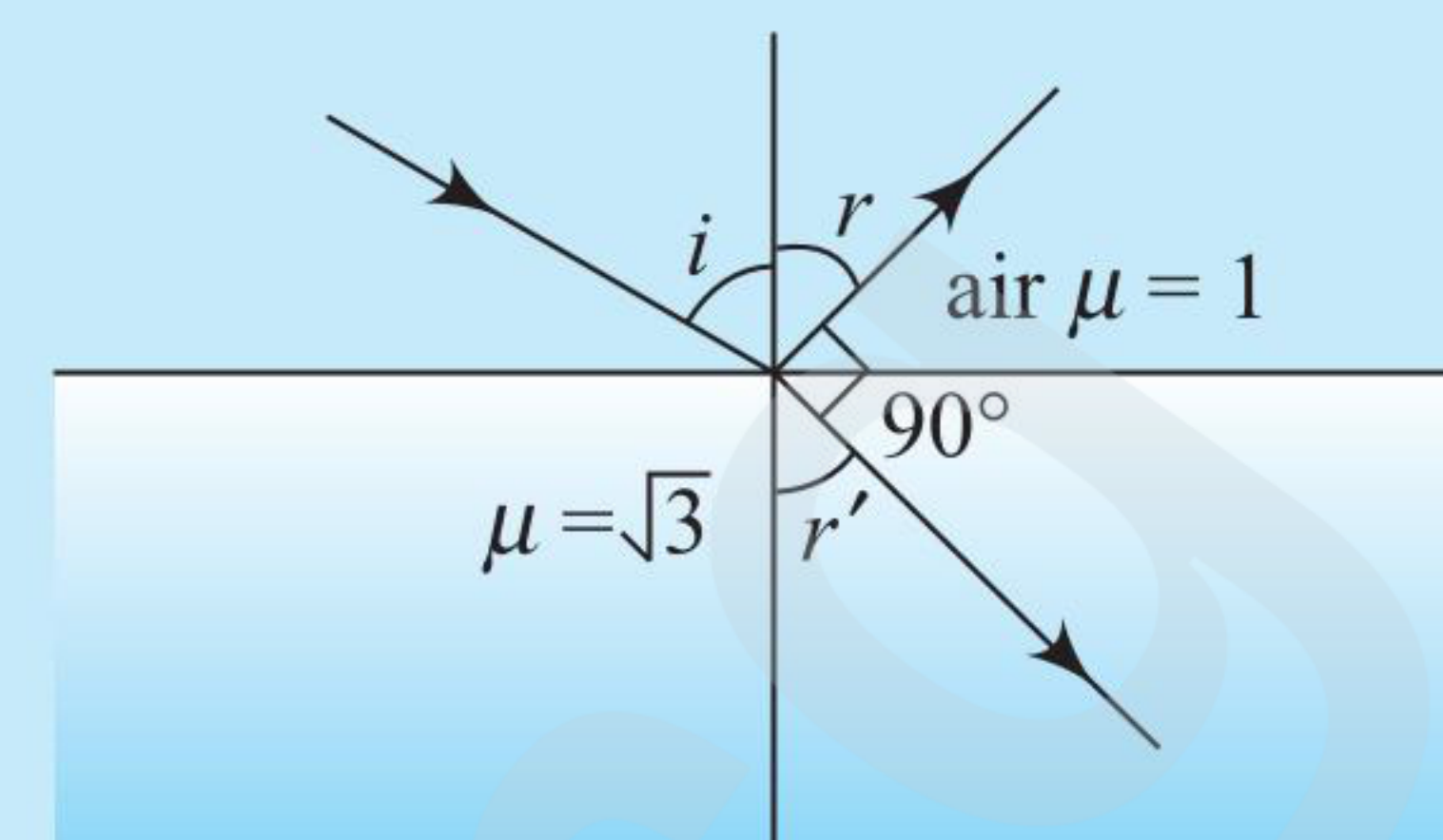
Sol. Snell's law:

$$\mu_a \sin \theta_a = \mu_b \sin \theta_b \Rightarrow \frac{c}{v_a} \sin \theta_a = \frac{c}{v_b} \sin \theta_b$$

$$\frac{c}{v_a} \sin 45^\circ = \frac{c}{c/3} \sin 60^\circ \Rightarrow v_a = \frac{c}{3} \sqrt{\frac{2}{3}}$$

ILLUSTRATION 1.42

A ray of light is incident on a transparent glass slab of refractive index $\sqrt{3}$. If the reflected and refracted rays are mutually perpendicular, what is the angle of incidence?



Sol. Let the angle of incidence, angle of reflection, and angle of refraction be i , r and r' , respectively.

Now, as per the equation $(90^\circ - r) + (90^\circ - r') = 90^\circ$

$r' = (90^\circ - i)$ (because $i = r$, in case of reflection according to Snell's law, $1 \sin i = \mu \sin r'$)

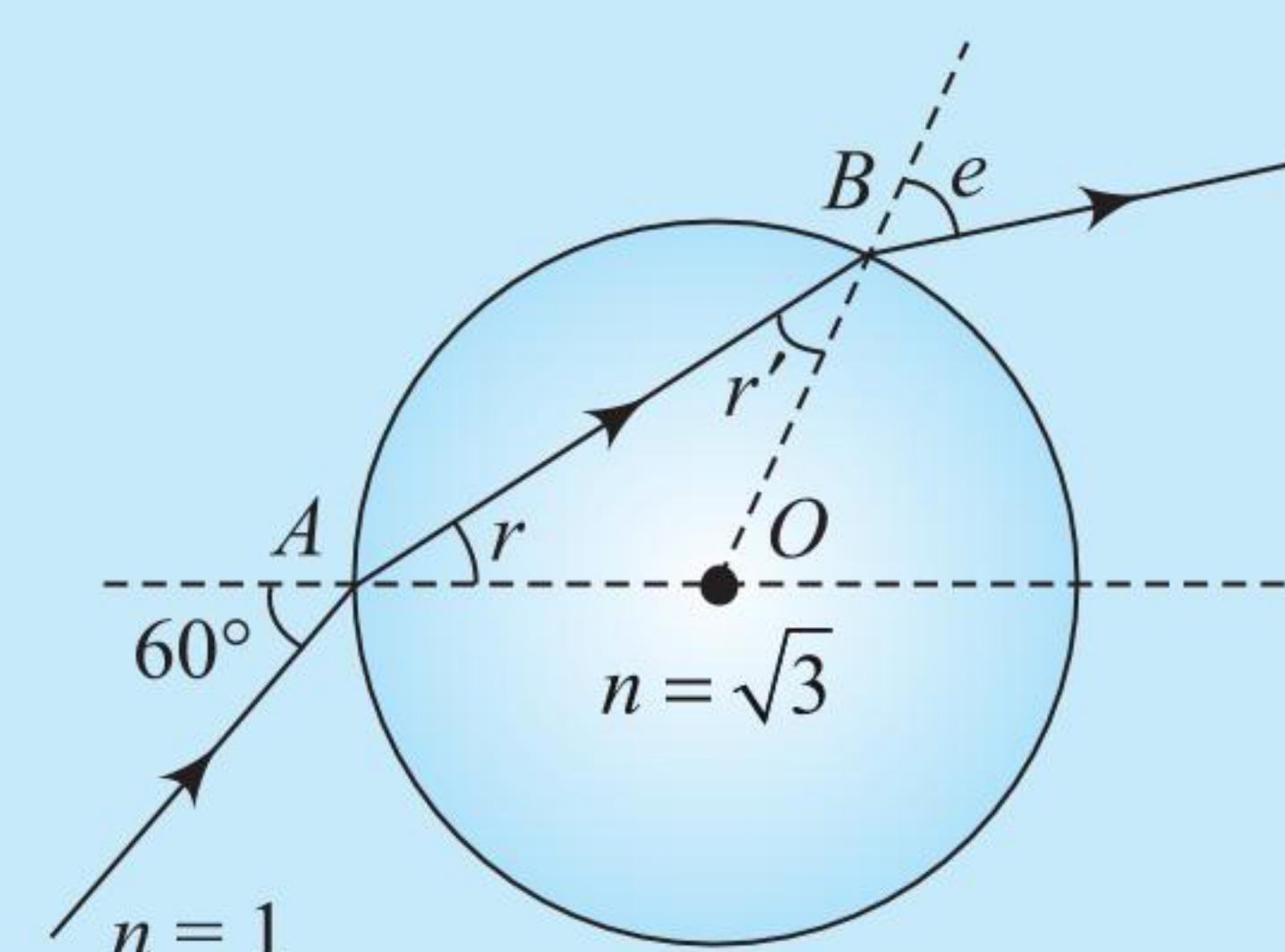
$$\text{or } \sin i = \mu \sin r'$$

$$\text{or } \sin i = \mu \sin (90^\circ - i) \Rightarrow \tan i = \mu$$

$$\text{or } i = \tan^{-1} \mu = \tan^{-1} \sqrt{3} = 60^\circ$$

ILLUSTRATION 1.43

A light ray is incident on a glass sphere of refractive index $\mu = \sqrt{3}$ at an angle of incidence 60° as shown in figure. Find the angles r , r' , e and the total deviation after two refractions.



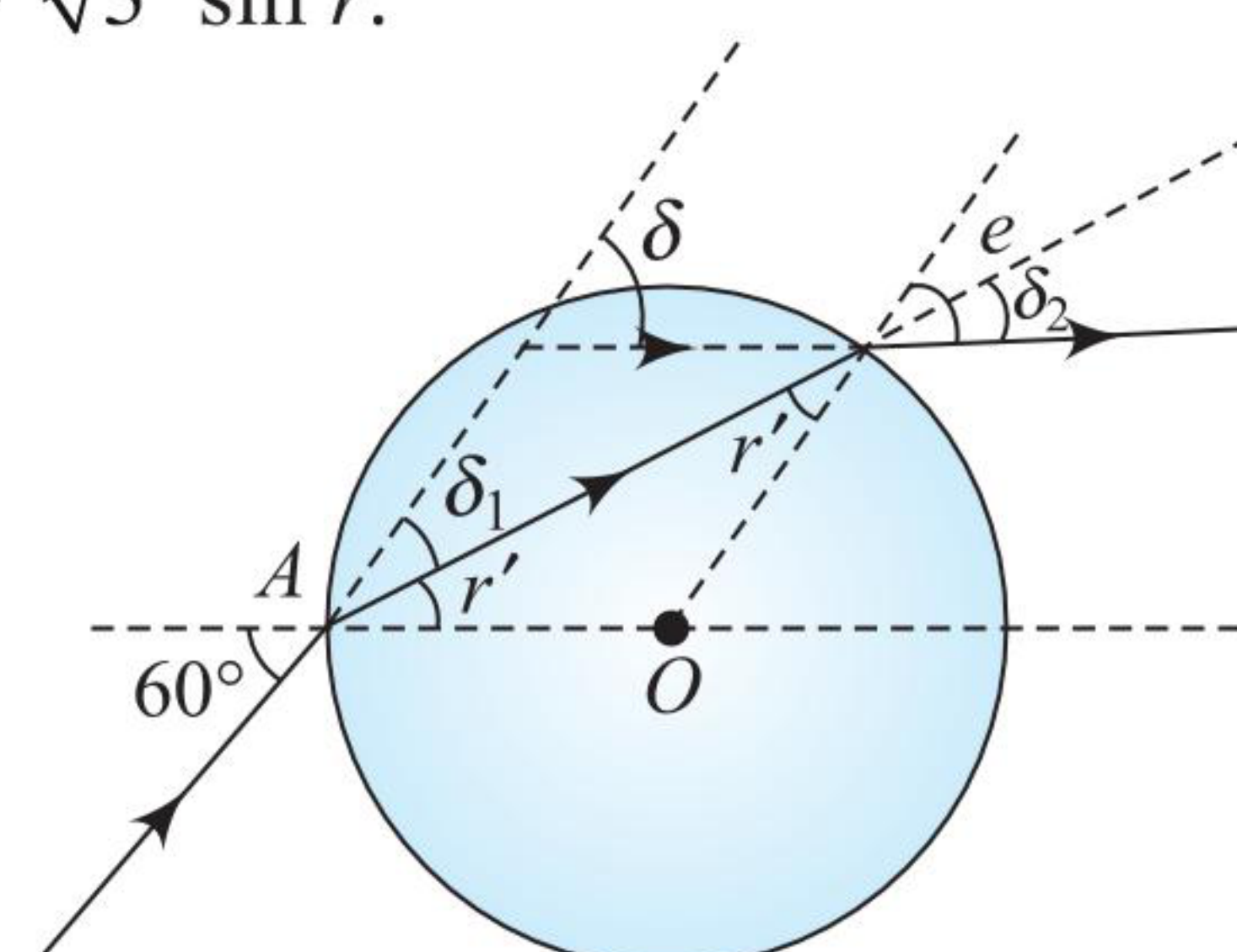
Sol. At point 'A': Applying Snell's law

$$1 \sin 60^\circ = \sqrt{3} \sin r \Rightarrow r = 30^\circ$$

From symmetry $r' = r = 30^\circ$.

Again applying Snell's law at second surface (at point 'B')

$$1 \sin e = \sqrt{3} \sin r.$$



$$\Rightarrow e = 60^\circ$$

Deviation at the first surface, $\delta_1 = i - r = 60^\circ - 30^\circ = 30^\circ$

Deviation at the second surface, $\delta_2 = e - r' = 60^\circ - 30^\circ = 30^\circ$

Therefore, total deviation = 60° .

ILLUSTRATION 1.44

A light beam passes from medium 1 to medium 2. Show that the emerging beam is parallel to the incident beam.

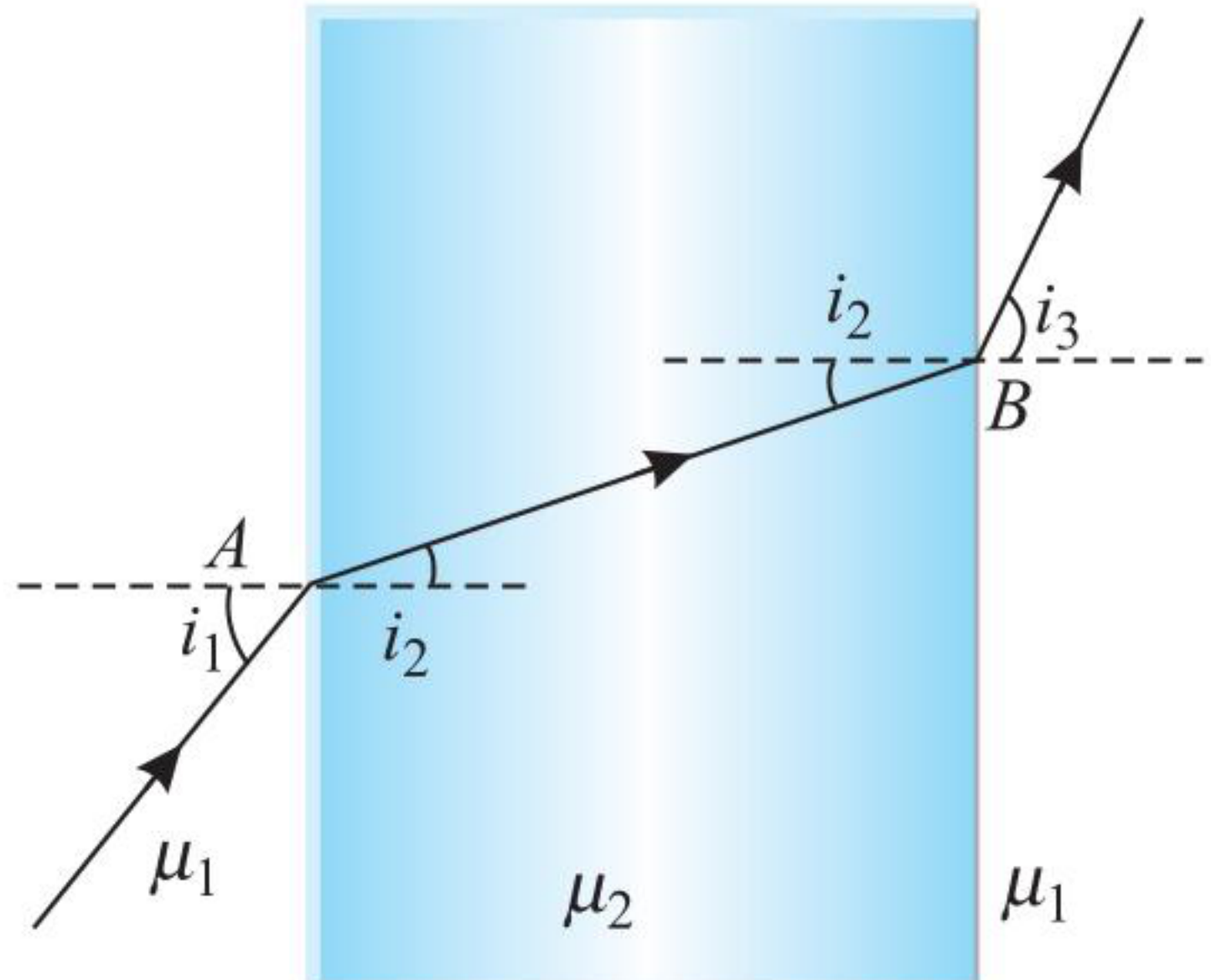
Sol. Applying Snell's law at A , $\mu_1 \sin i_1 = \mu_2 \sin i_2$

$$\text{or } \frac{\mu_1}{\mu_2} = \frac{\sin i_2}{\sin i_1} \quad \dots(i)$$

Similarly at B , $\mu_2 \sin i_2 = \mu_1 \sin i_3$

$$\therefore \frac{\mu_1}{\mu_2} = \frac{\sin i_2}{\sin i_3} \quad \dots(ii)$$

From Eqs. (i) and (ii), we have $i_3 = i_1$ i.e., the emergent ray is parallel to incident ray.

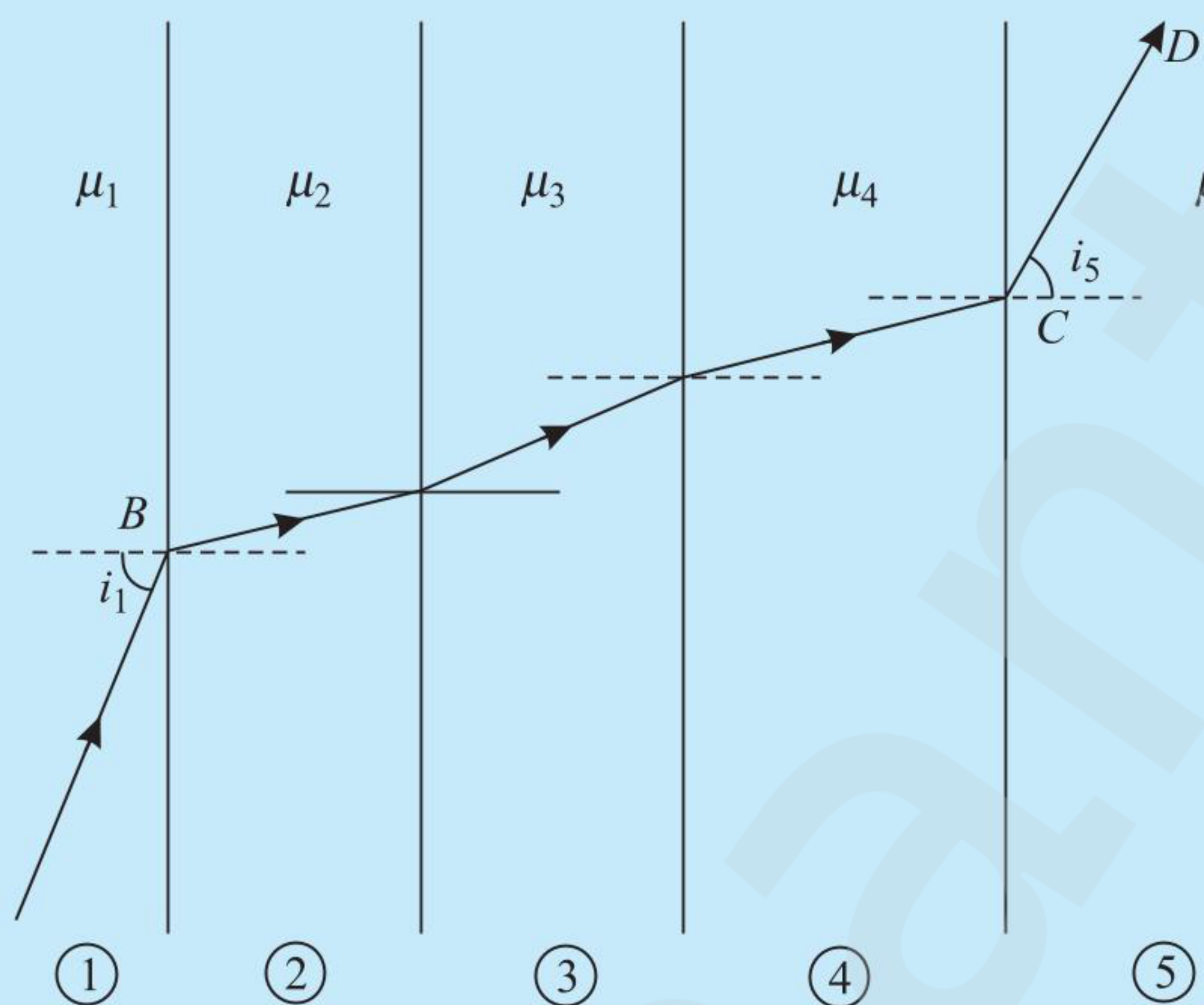


Important Point:

If the boundaries of the media are parallel, the emergent ray although laterally displaced, is parallel to the incident ray, if $\mu_1 = \mu_5$. We can also directly apply the Snell's law ($\mu \sin I = \text{constant}$) in medium 1 and 5, i.e.,

$$\mu_1 \sin i_1 = \mu_5 \sin i_5$$

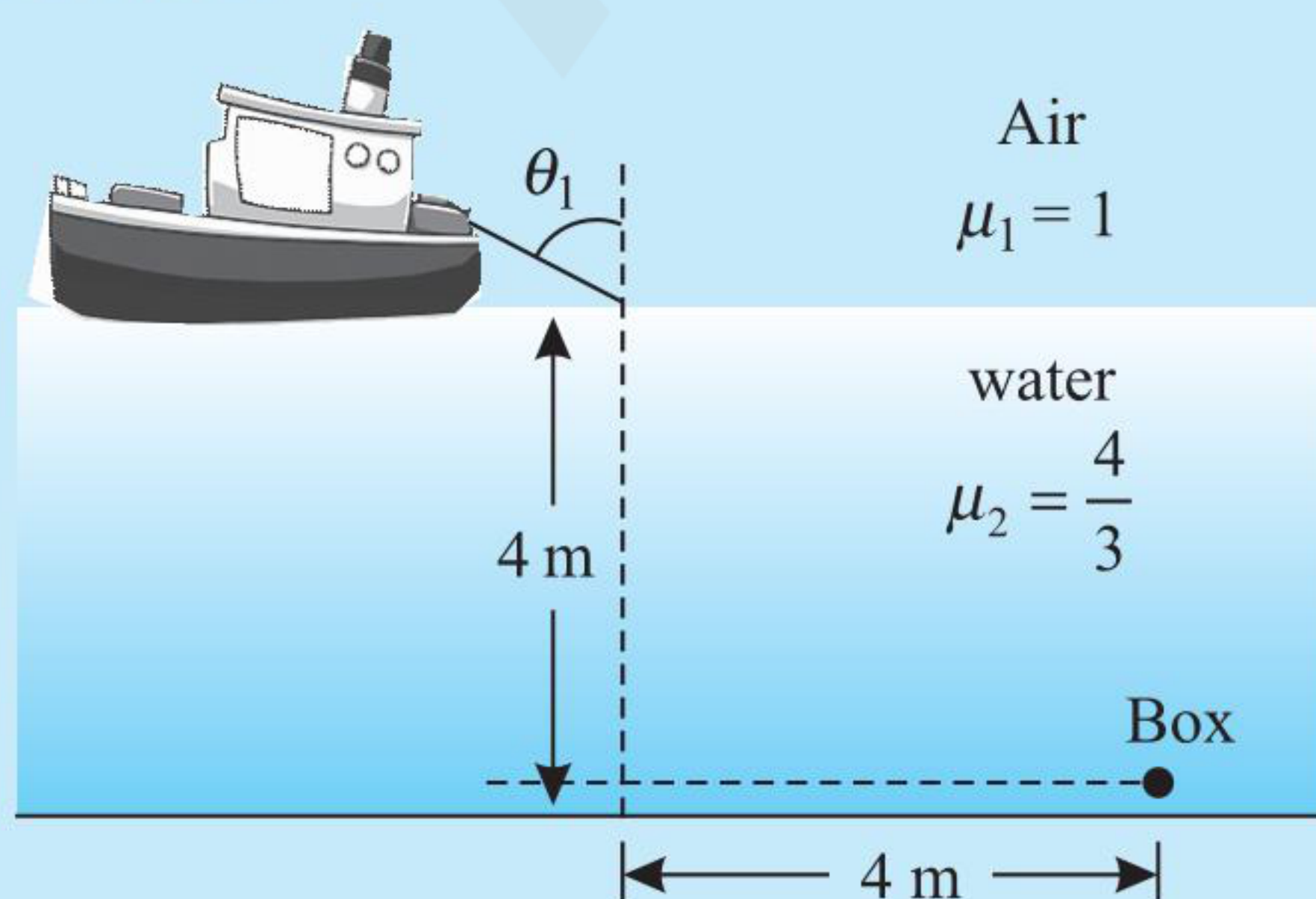
So, $i_1 = i_5$ if $\mu_1 = \mu_5$



If any of the boundary is not parallel we cannot use this law directly by jumping the intervening media.

ILLUSTRATION 1.45

A searchlight on a yacht is being used at night to illuminate a sunken box, as in figure. At what angle of incidence θ_1 should the light be aimed?



Sol. $\tan \theta_2 = \frac{4}{3} \Rightarrow \theta_2 = 45^\circ$; Given: $\mu_1 = 1$ for air and $\mu_2 = \frac{4}{3}$ for water; Snell's law gives $\mu_1 \cdot \sin \theta_1 = \mu_2 \cdot \sin \theta_2$

$$1 \cdot \sin \theta_1 = \frac{4}{3} \cdot \sin 45^\circ \Rightarrow \sin \theta_1 = \frac{4}{3} \cdot \frac{1}{\sqrt{2}}$$

$$\Rightarrow \sin \theta_1 = \frac{2\sqrt{2}}{3} \Rightarrow \theta_1 = \sin^{-1} \left(\frac{2\sqrt{2}}{3} \right)$$

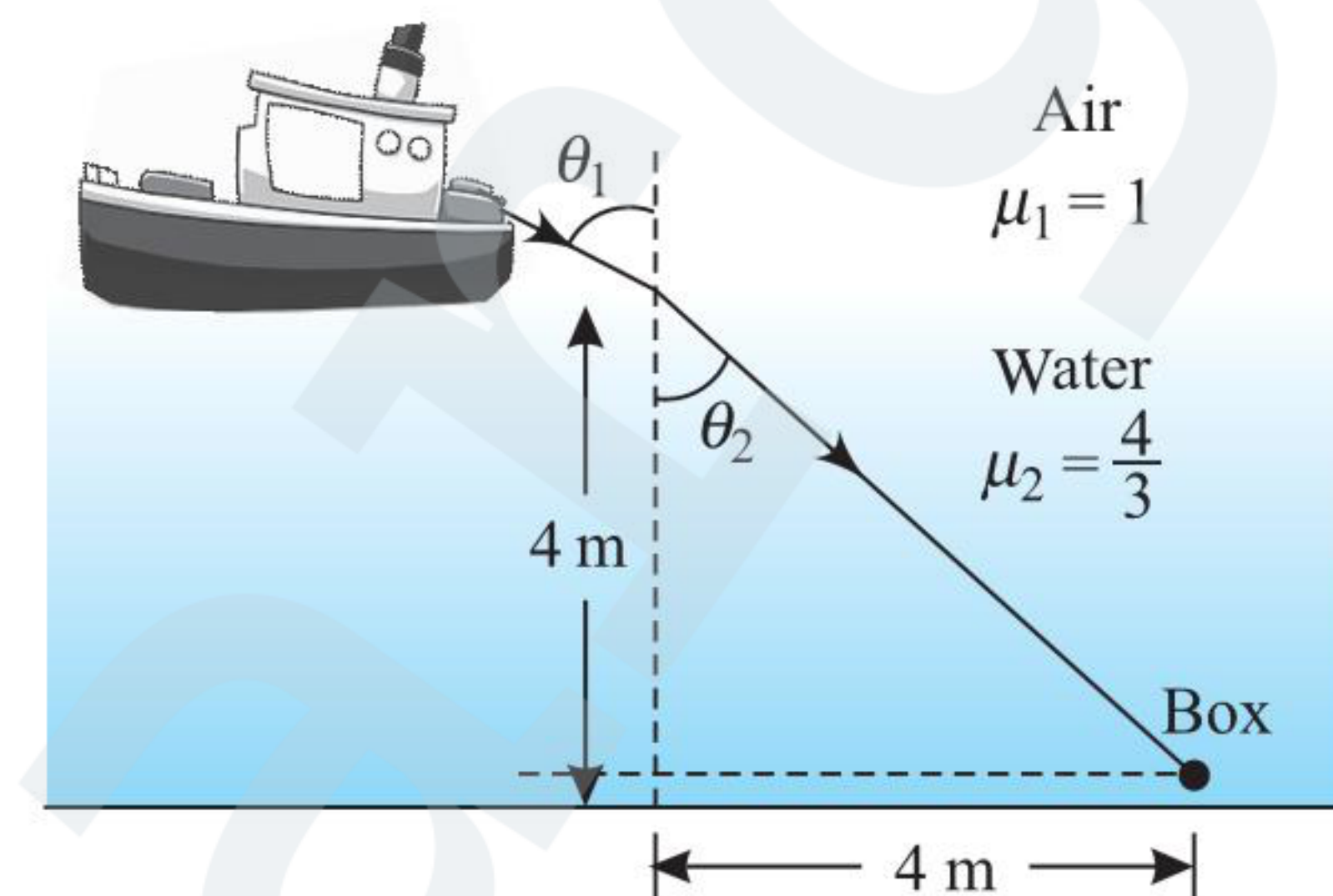
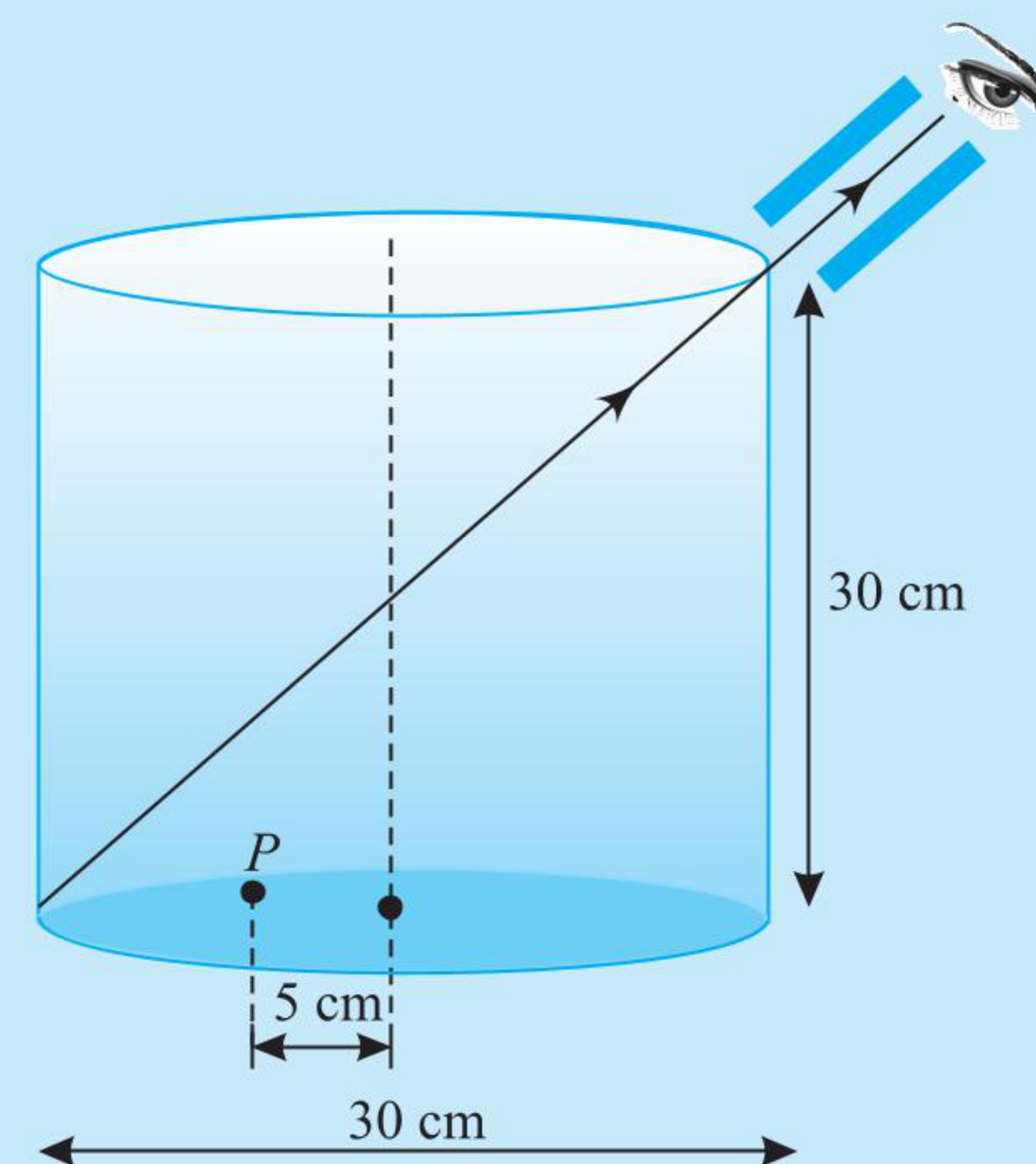


ILLUSTRATION 1.46

A cylindrical vessel, whose diameter and height both are equal to 30 cm, is placed on a horizontal surface and a small particle P is placed in it at a distance of 5.0 cm from the centre. An eye is placed at a position such that the edge of the bottom is just visible. The particle P is in the plane of drawing. Up to what minimum height should water be poured in the vessel to make the particle P visible?



Sol. If we pour water in vessel, refraction will take place at air and water interface.

Applying Snell's law at A , we get $1 \cdot \sin 45^\circ = \frac{4}{3} \sin \theta$

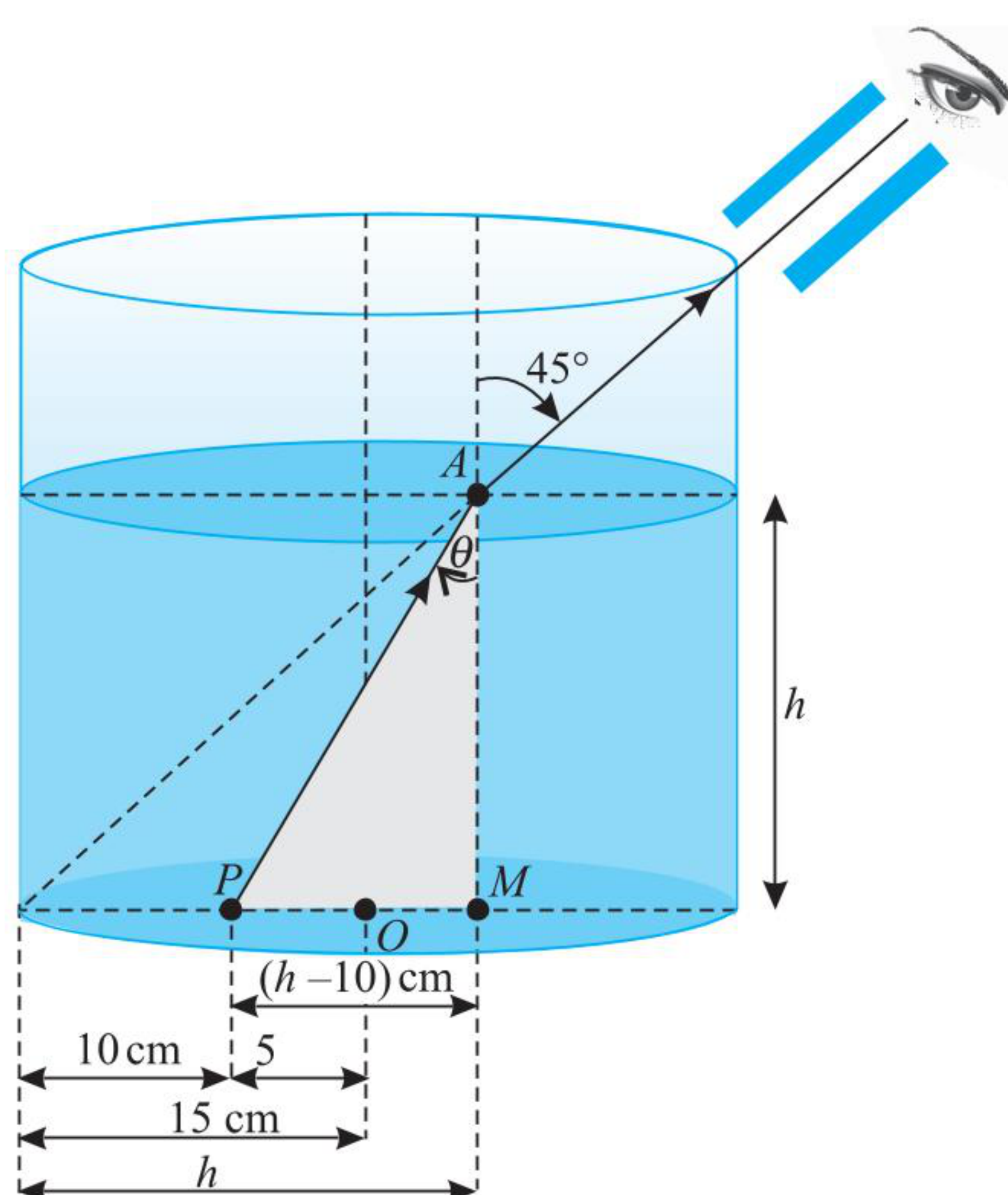
Here, θ is the angle which the incidence ray of light makes with the normal.

$$\Rightarrow 1 \left(\frac{1}{\sqrt{2}} \right) = \frac{4}{3} \sin \theta \Rightarrow \sin \theta = \frac{3}{4\sqrt{2}}$$

$$\tan \theta = \frac{3}{\sqrt{16 \times 2 - 9}} = \frac{3}{\sqrt{23}}$$

$\Rightarrow$ In $\triangle APM$,

$$\tan \theta = \frac{3}{\sqrt{23}} = \frac{PM}{AM} = \frac{5+x}{h} = \frac{5+h-15}{h}$$



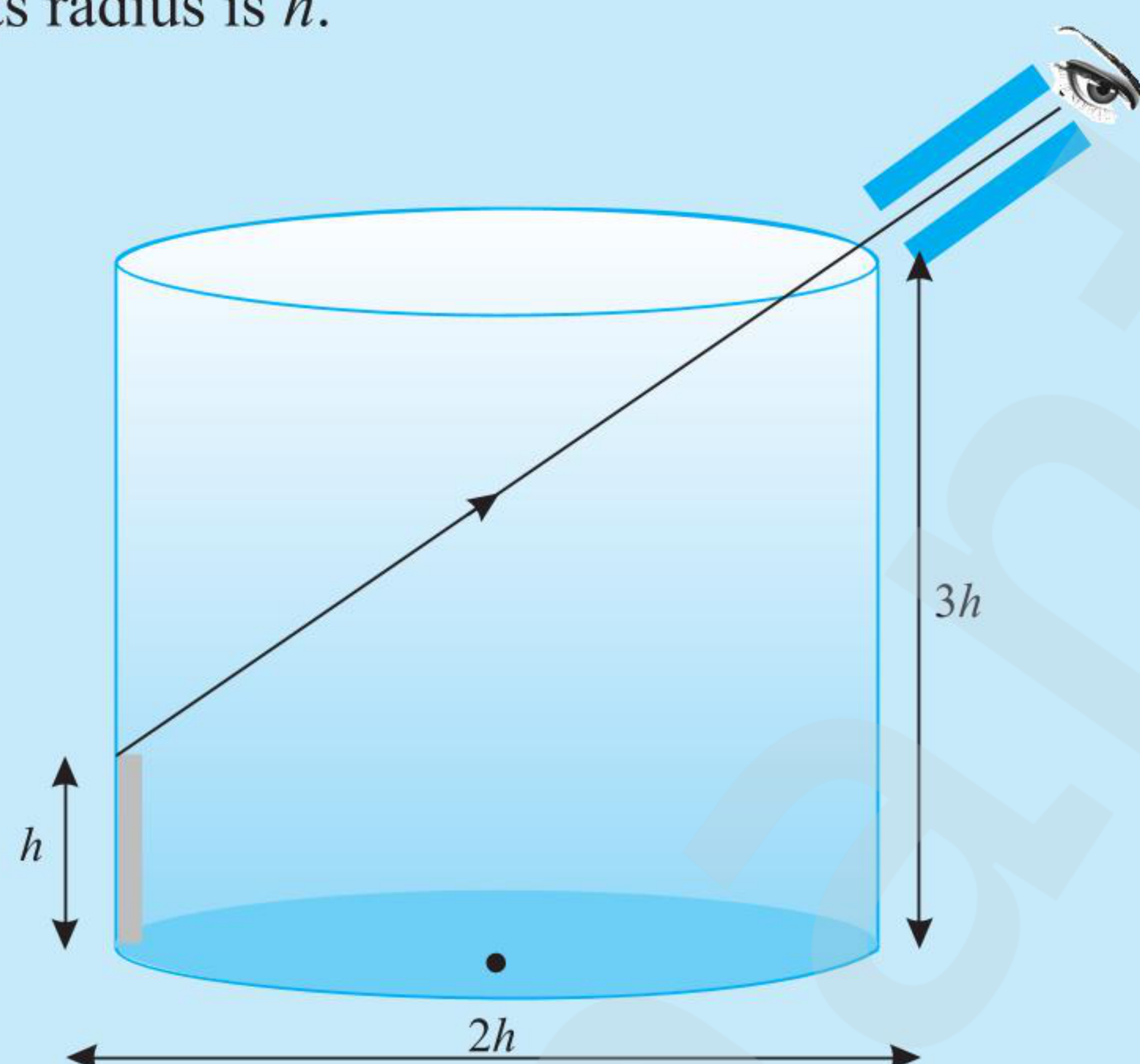
$$\frac{3}{\sqrt{23}} = \frac{h-10}{h} \Rightarrow 3h = h\sqrt{23} - 10\sqrt{23}$$

$$h = \left(\frac{10\sqrt{23}}{\sqrt{23}-3} \right) \text{ cm}$$

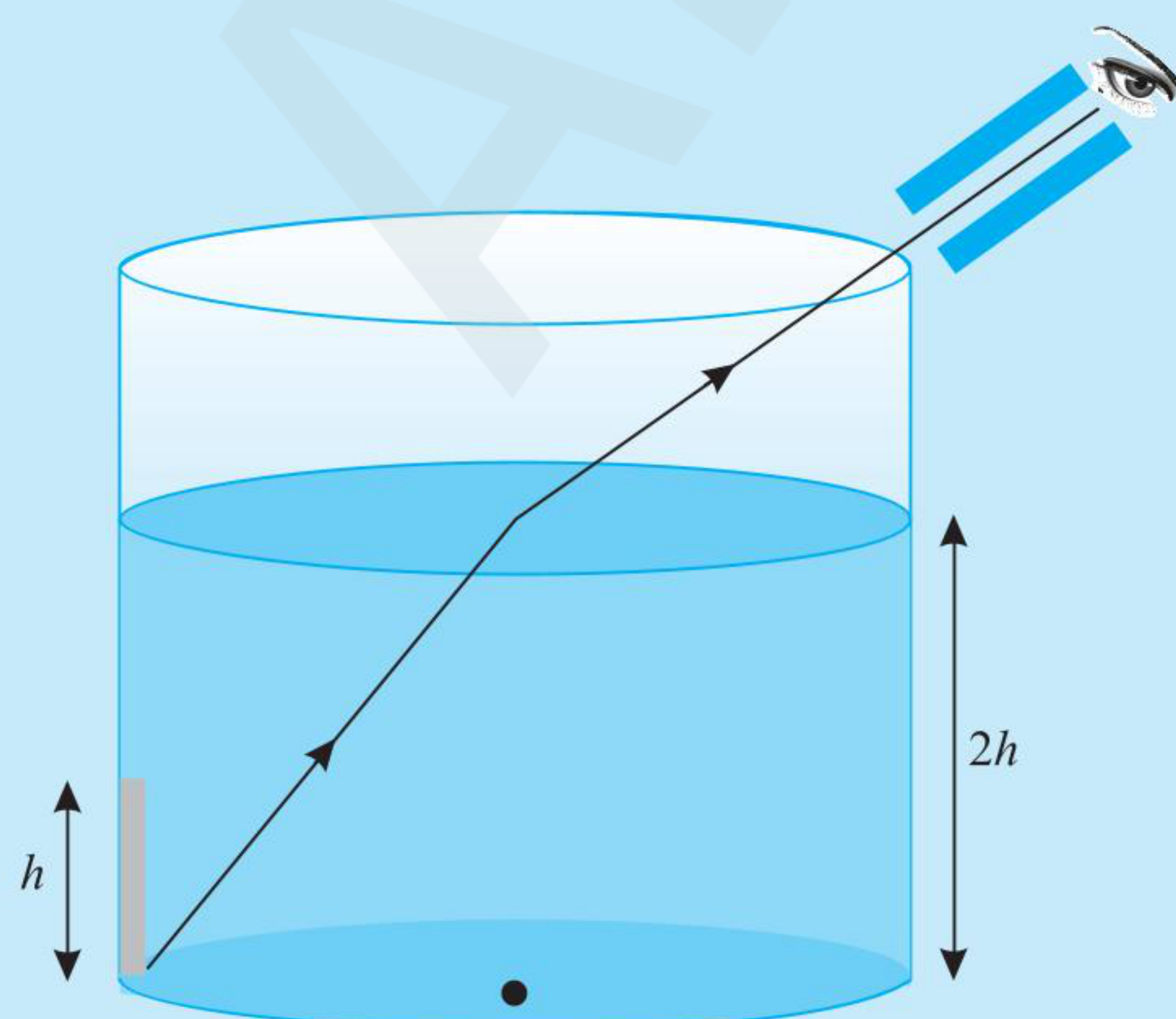
Hence, water should be poured up to height $\frac{10\sqrt{23}}{\sqrt{23}-3}$ cm to make the particle 'P' visible.

ILLUSTRATION 1.47

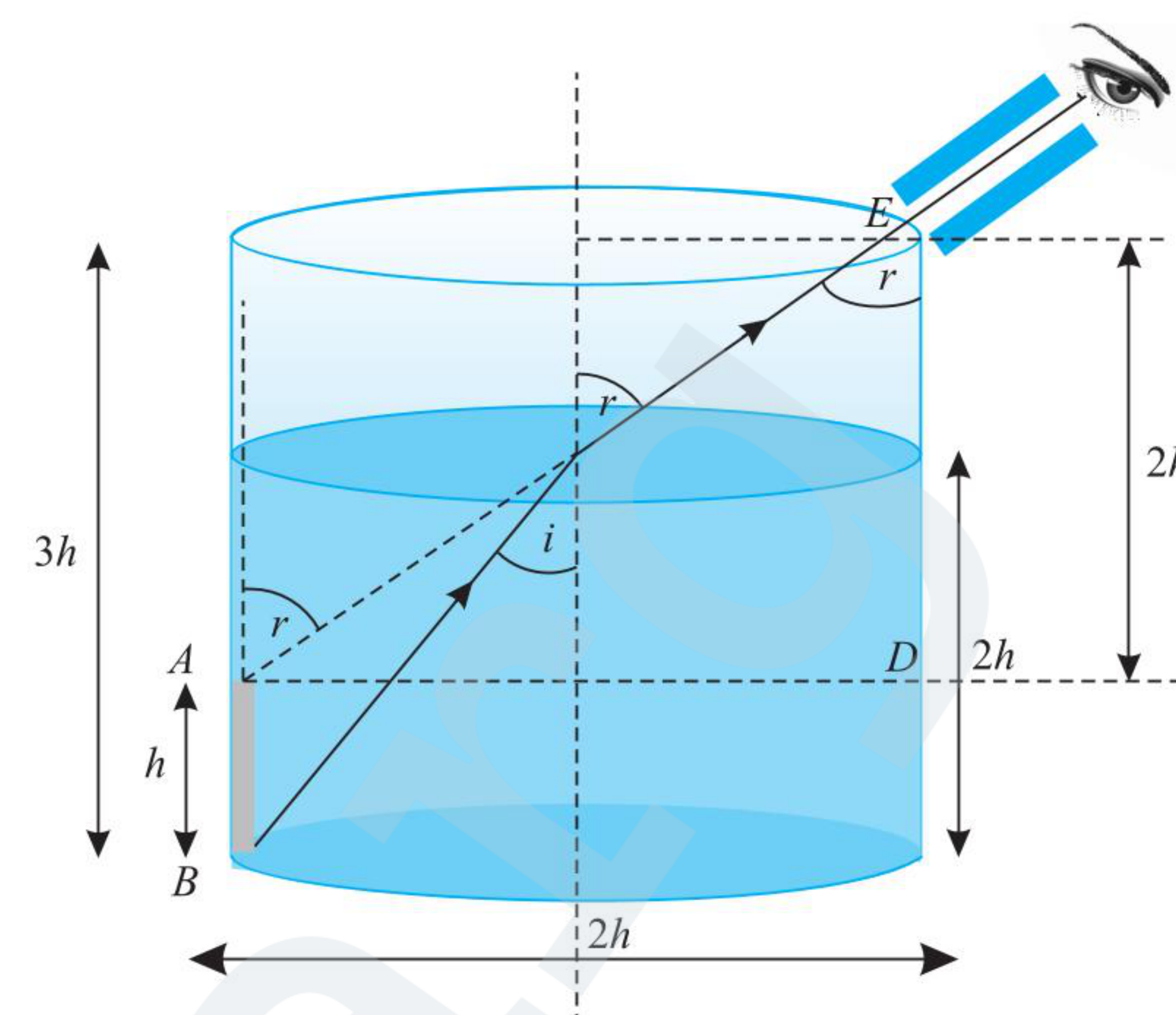
An observer can see through a pin hole, the top of a thin rod of height h , placed as shown in figure. The beaker's height is $3h$ and its radius is h .



When the beaker is filled with a liquid upto a height $2h$, he can see the lower end of the rod. Find the refractive index of liquid.



Sol. Considering refraction from top surface, applying Snell's law at top



$$n \cdot \sin i = 1 \sin r \quad \dots(i)$$

$$\text{Since } \tan r = \frac{2h}{2h} = 1 \Rightarrow r = 45^\circ$$

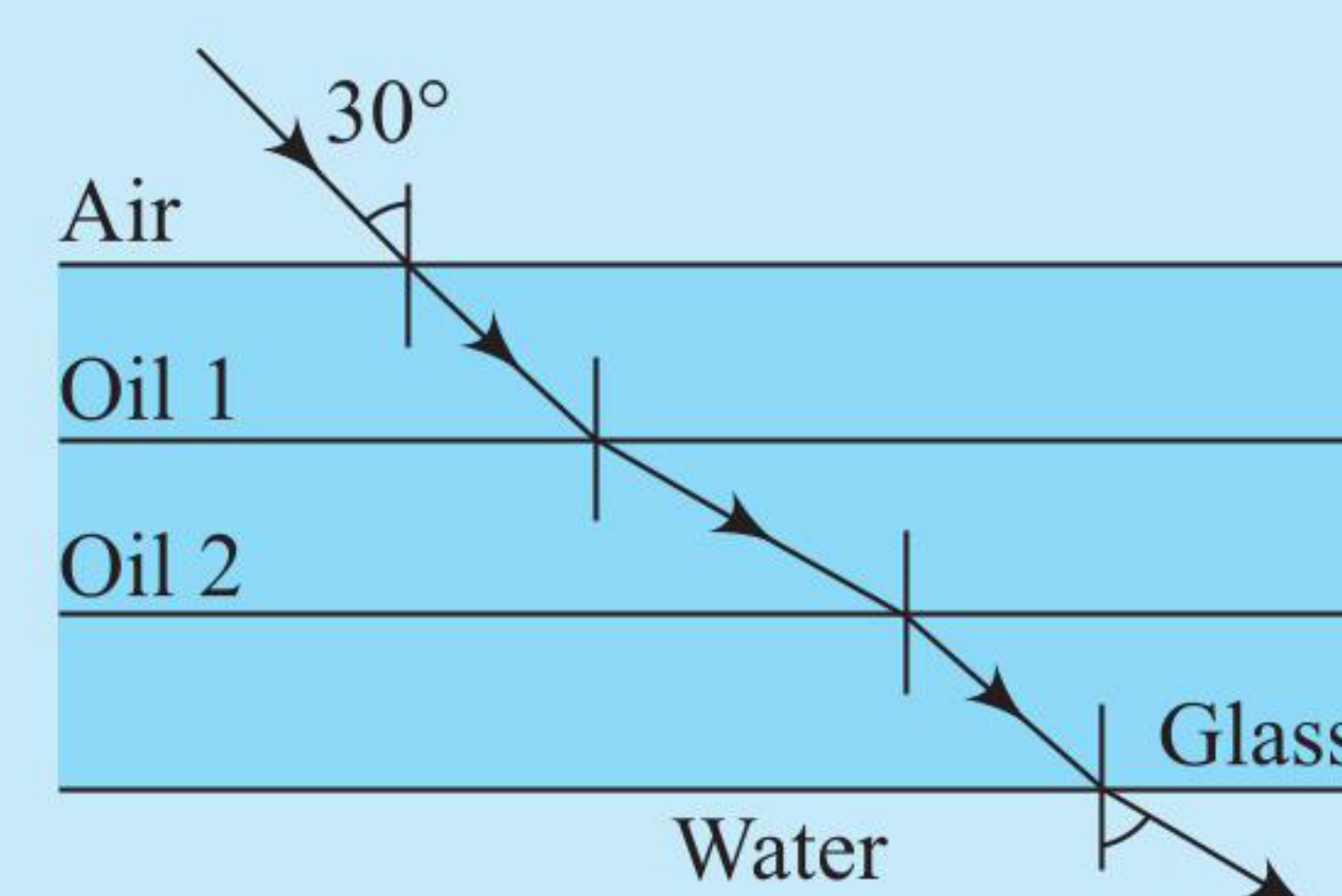
$$\text{And } \sin i = \frac{h}{h\sqrt{5}} \Rightarrow \sin i = \frac{1}{\sqrt{5}}$$

Substituting the values of i and r in Eq. (i)

$$\therefore n \cdot \frac{1}{\sqrt{5}} = 1 \cdot \frac{1}{\sqrt{2}} \Rightarrow n = \sqrt{\frac{5}{2}}$$

ILLUSTRATION 1.48

Light is incident from air on an oil layer at an incident angle of 30° . After moving through the oil 1, oil 2, and glass it enters water. If the refractive index of glass and water are 1.5 and 1.3, respectively. Find the angle which the ray makes with the normal in water.



Sol. As we know that $\mu \sin i = \text{constant}$

$$\Rightarrow \mu_{\text{air}} \sin i_{(\text{air})} = \mu_{\text{glass}} \sin r_{(\text{glass})}$$

$$\sin i_{(\text{glass})} = \frac{\mu_{\text{air}}}{\mu_{\text{glass}}} \sin i_{(\text{air})} \quad \dots(i)$$

$$\text{Again } \mu_{\text{glass}} \sin i_{\text{glass}} = \mu_{\text{water}} \sin r_{\text{water}} \quad \dots(ii)$$

From Eqs. (i) and (ii), $\sin 30^\circ = 1.3 \sin r$

$$\Rightarrow \sin r = \frac{1}{2 \times 1.3} = \frac{1}{2.6}; r = \sin^{-1} \frac{1}{2.6}$$

Note: For parallel layers, we can apply Snell's law directly at initial and final parts.

VECTOR REPRESENTATION OF A LIGHT RAY

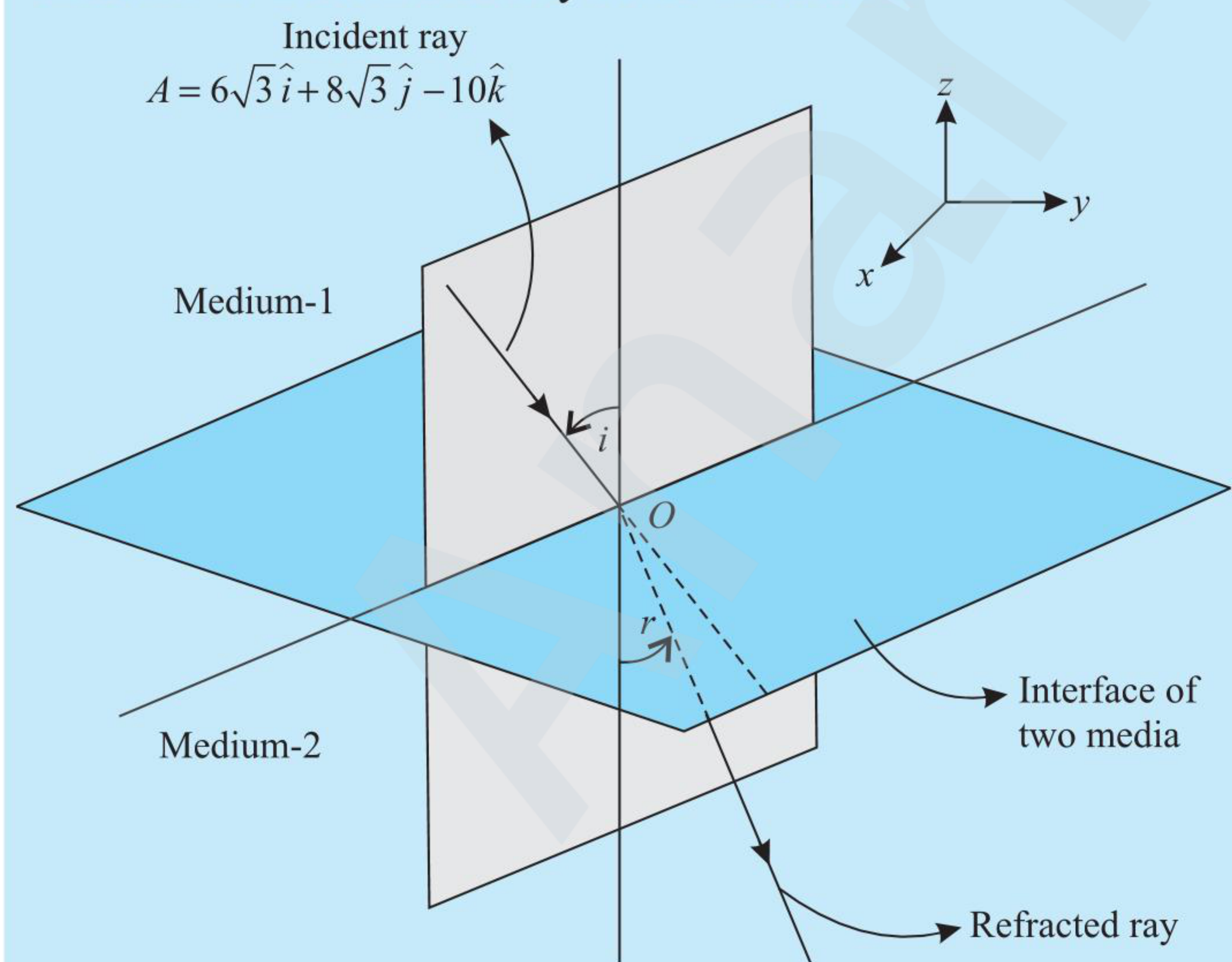
The angle of incidence of a light ray on an interface is usually determined geometrically by simply drawing the ray diagram. However, in some situations, the vector representation of the line along which the light ray travels is given. The ray may then reflect or refract at an interface and emerge. How do we find the vector representation of the line along which the emergent ray travels? Following are the steps for finding this:

- Determine the unit vector representation of the normal to the interface where the incident ray strikes the surface, say $\hat{e}_n$.
- Find the component of the incident ray along the normal.
- Subtract this component from the original ray to compute the plane in which the incident ray travels. Calculate the unit vector that represents the plane of the incident ray and the normal, say $\hat{e}_p$.
- Using the vector dot product, calculate the angle between the incident ray and the normal.
- From the governing equation calculate the angle of reflection/refraction, say r .
- We know that the emergent ray will be in the same plane as that of the incident ray and the normal. Thus the unit vector representing the direction of the emergent beam is simply $\cos(r)\hat{e}_n + \sin(r)\hat{e}_p$.

Let us learn to apply these steps through following illustrations.

ILLUSTRATION 1.49

The x - y plane is the boundary between two transparent media. Medium-1 with $z \geq 0$ has refractive index $\sqrt{2}$ and medium-2 with $z \leq 0$ has a refractive index $\sqrt{3}$. A ray of light in medium-1 given by the vector $\vec{A} = 6\sqrt{3}\hat{i} + 8\sqrt{3}\hat{j} - 10\hat{k}$ is incident on the plane of separation. Find the unit vector in the direction of the refracted ray in the medium 2.



Sol. Unit vector representing the normal to the plane $\hat{e}_n = \hat{k}$. Component of the incident ray along the normal is $-10\hat{k}$. The unit vector that represents the plane of the incident ray and the normal also in the direction of q .

$$\begin{aligned} \text{Unit vector in the direction of } q, \hat{e}_p = \hat{q} &= \frac{6\sqrt{3}\hat{i} + 8\sqrt{3}\hat{j}}{\sqrt{(6\sqrt{3})^2 + (8\sqrt{3})^2}} \\ &= \frac{6\sqrt{3}\hat{i} + 8\sqrt{3}\hat{j}}{\sqrt{36 \times 3 + 64 \times 3}} = \frac{6\sqrt{3}\hat{i} + 8\sqrt{3}\hat{j}}{10\sqrt{3}} \\ \Rightarrow \hat{q} &= \left(\frac{3}{5}\hat{i} + \frac{4}{5}\hat{j} \right) \end{aligned}$$

Angle made by ray with z -axis = angle made by ray with normal

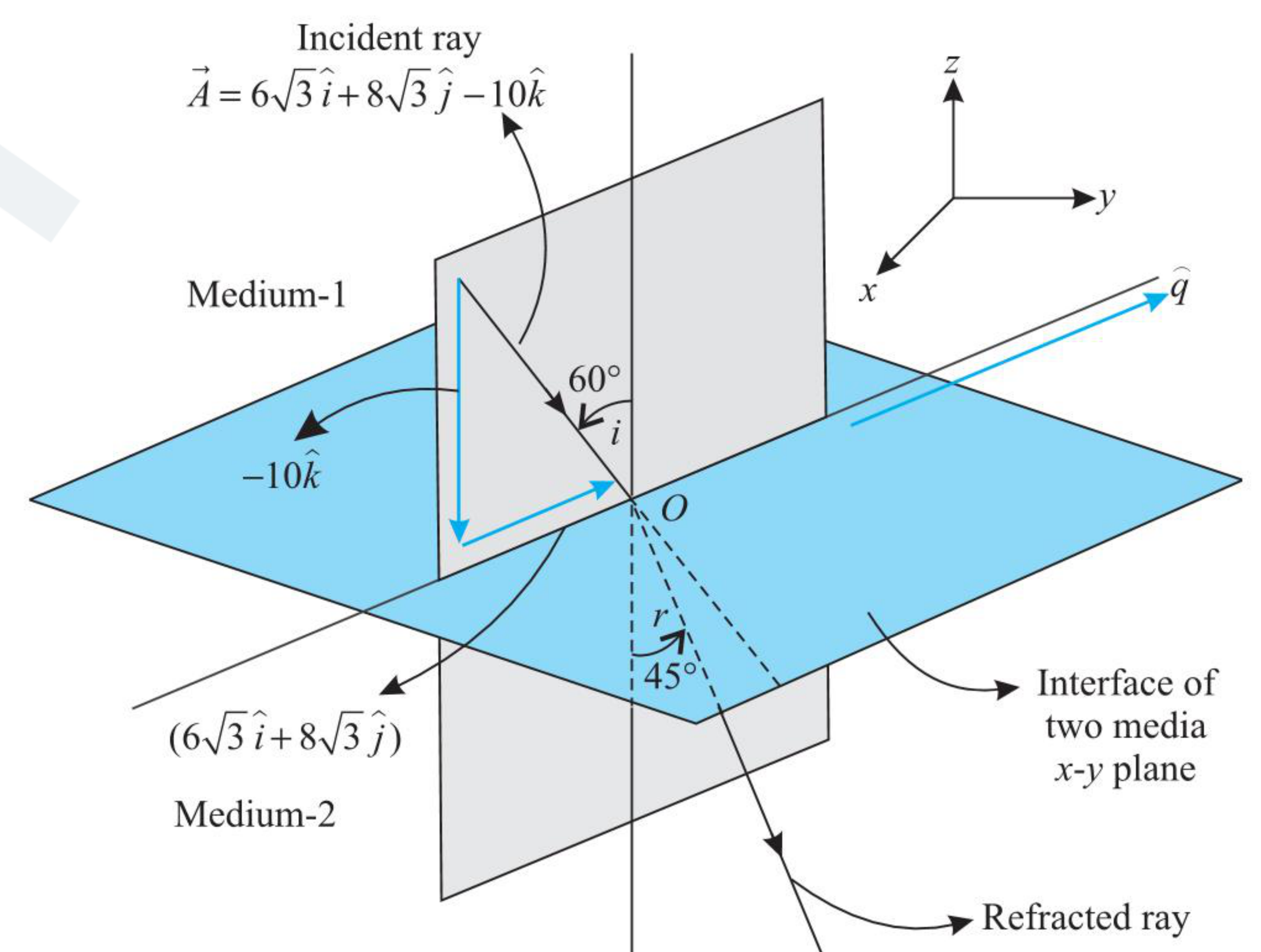
$$\begin{aligned} |\cos \gamma| &= \frac{A_z}{|A|} = \frac{-10}{\sqrt{(6\sqrt{3})^2 + (8\sqrt{3})^2 + (10)^2}} \\ &= \frac{-10}{\sqrt{400}} = -\frac{1}{2} = \cos \theta_z \Rightarrow \theta_z = 120^\circ \end{aligned}$$

The angle of incidence is $i = 180^\circ - 120^\circ = 60^\circ$

Angle made by refracted ray with z -axis = angle made by refracted ray with normal

Applying Snell law $n_1 \sin 60^\circ = n_2 \sin r$

$$\begin{aligned} \sqrt{2} \cdot \frac{\sqrt{3}}{2} &= \sqrt{3} \cdot \sin r \\ \Rightarrow \sin r &= \frac{1}{\sqrt{2}} \Rightarrow r = 45^\circ \end{aligned}$$



Unit vector in the direction of refracted ray in the medium-2,

The equation of the emergent ray is $\cos(r)\hat{e}_n + \sin(r)\hat{e}_p$

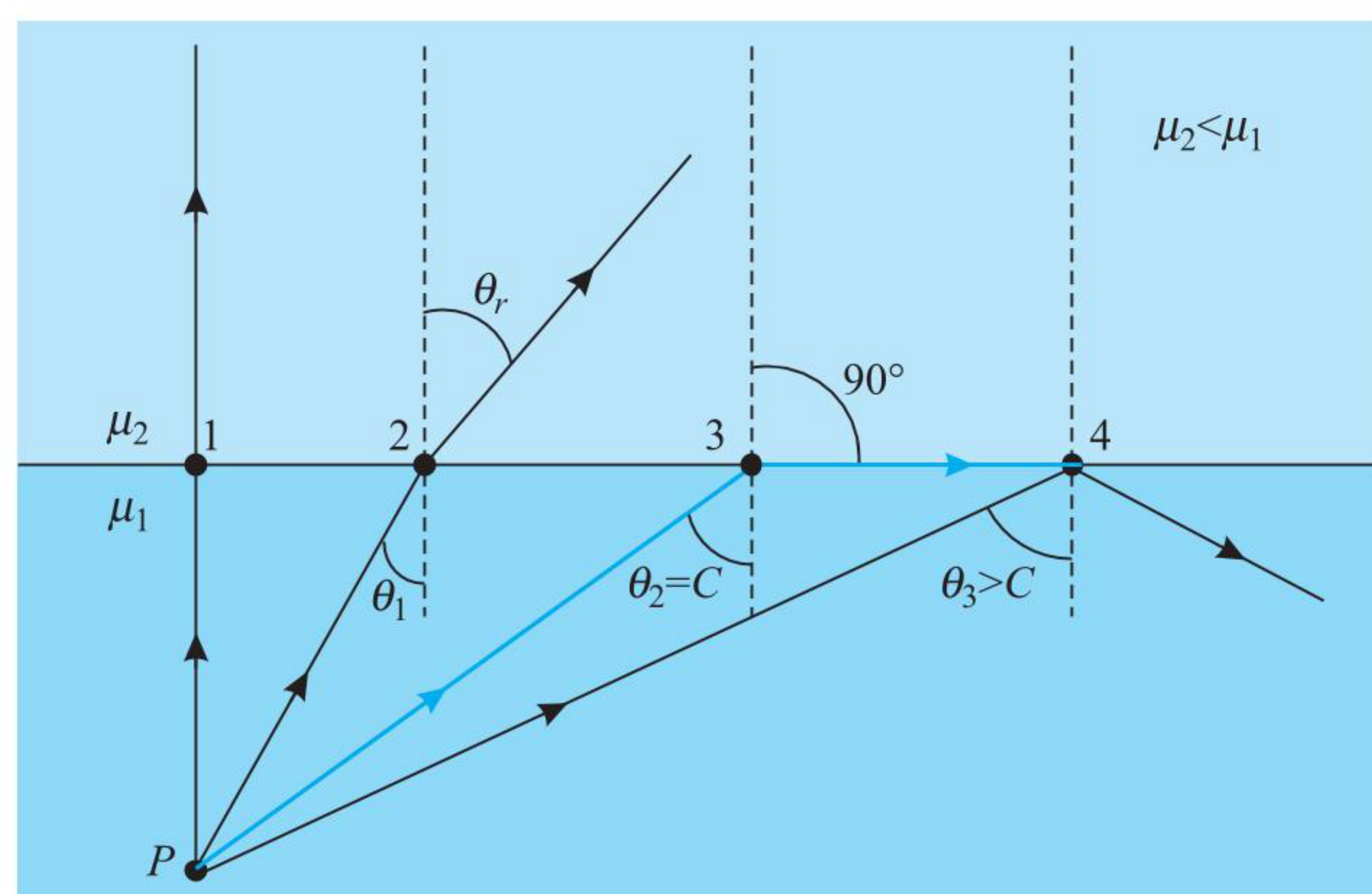
$$\begin{aligned} \hat{r} &= \cos 45^\circ (-\hat{k}) + \sin 45^\circ \cdot \hat{q} \\ &= \frac{1}{\sqrt{2}} \cdot (-\hat{k}) + \frac{1}{\sqrt{2}} \cdot \left(\frac{3}{5}\hat{i} + \frac{4}{5}\hat{j} \right) \\ \Rightarrow \hat{r} &= \frac{1}{\sqrt{2}} \cdot \left(\frac{3}{5}\hat{i} + \frac{4}{5}\hat{j} \right) - \frac{1}{\sqrt{2}} \cdot \hat{k} \end{aligned}$$

CRITICAL ANGLE AND TOTAL INTERNAL REFLECTION

Now let's consider a ray of light travelling in an optical medium with index of refraction μ_1 that crosses a boundary with another optical medium with a lower index of refraction μ_2 such that $\mu_2 < \mu_1$. In this case, the light is bent away from the normal. As the angle of incidence increases in the denser medium the angle of refraction in the rarer medium increases (see figure).

The angle of incidence for which the angle of refraction becomes 90° is called **critical angle** (θ_c or C).

When the angle of incidence of a ray travelling from a denser medium to rarer medium is greater than the critical angle, no refraction occurs. The incident ray is totally reflected back into the same medium, **total internal reflection** takes place instead of refraction. Here, the laws of reflection hold good. Some light is also reflected before the critical angle is achieved, but not totally.



Applying Snell's law at the position of critical angle

$$\frac{\sin C}{\sin 90^\circ} = {}_1\mu_2 = \frac{\mu_2}{\mu_1} \Rightarrow \sin C = \frac{1}{{}_2\mu_1} = \frac{\mu_2}{\mu_1}$$

or $\sin C = \frac{\text{RI of rarer medium}}{\text{RI of denser medium}}$

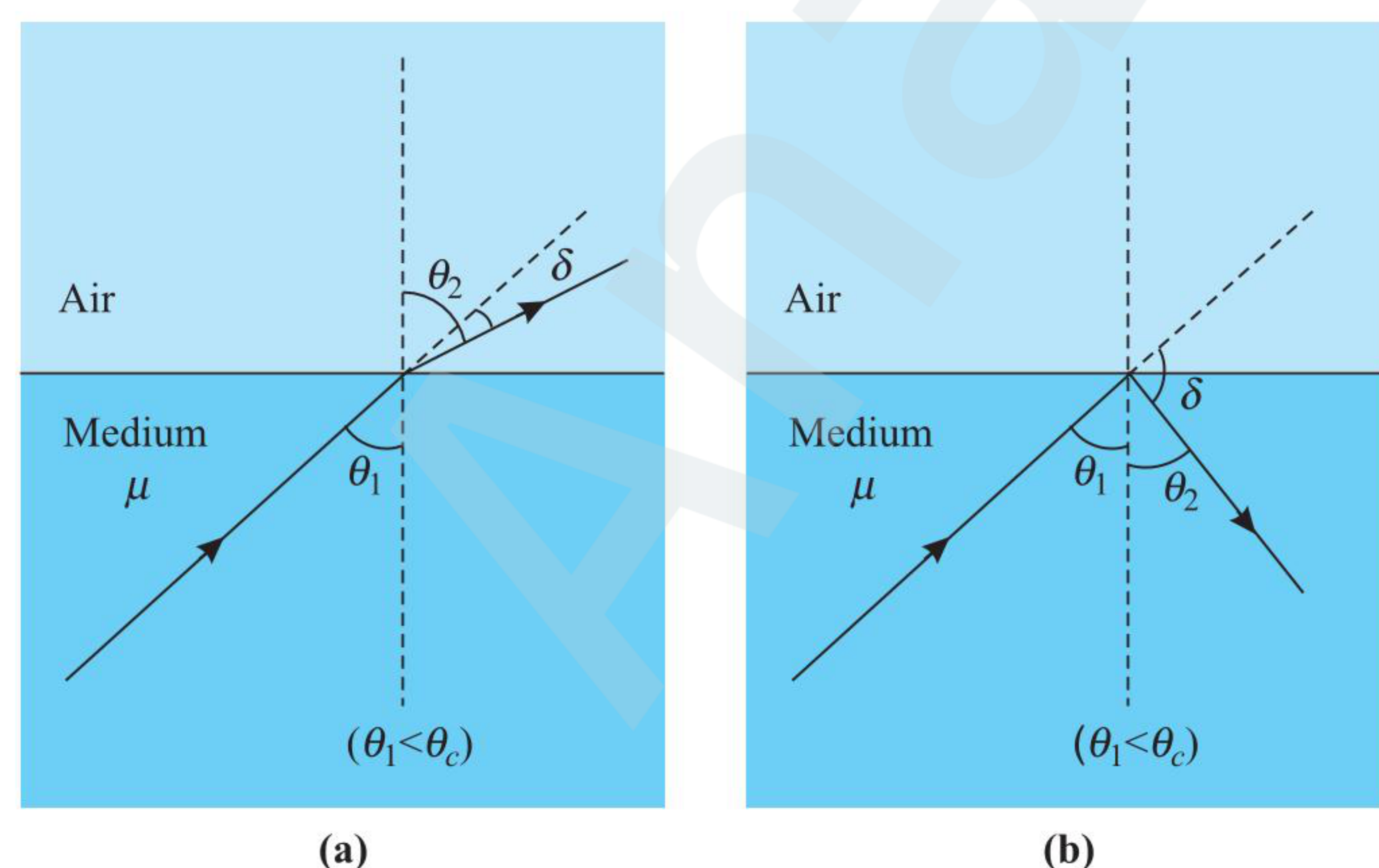
DEVIATION OF RAY DUE TO REFRACTION

The figure shows a light ray travelling from a denser to rarer medium at an angle θ_1 smaller than the critical angle C . The deviation of the light ray is given by $\delta = \theta_2 - \theta_1$. Since $\mu_{\text{denser}} \sin \theta_1 = \mu_{\text{rarer}} \sin \theta_2$, therefore,

$$\sin \theta_2 = \frac{\mu_D}{\mu_R} \sin \theta_1 = \mu \sin \theta_1$$

$$\theta_2 = \sin^{-1}(\mu \sin \theta_1)$$

$$\delta = \sin^{-1}(\mu \sin \theta_1) - \theta_1$$



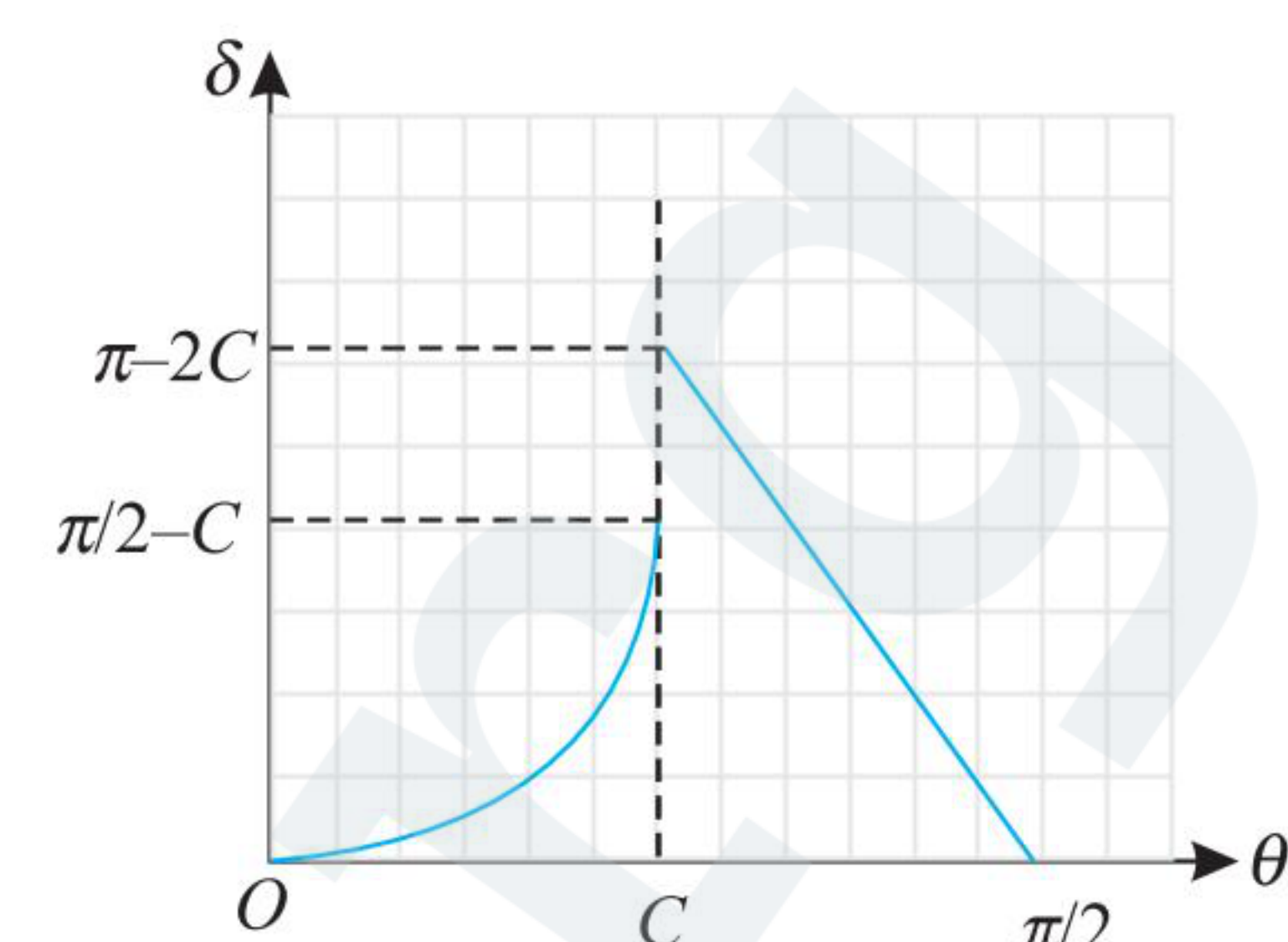
This is a non-linear increasing equation. The maximum value of δ occurs when $\theta_1 = C$, and is equal to

$$\delta_{\max} = \frac{\pi}{2} - C$$

If the light is incident at angle $\theta_1 > C$, then the angle of deviation is given by, $\delta = \pi - 2\theta_1$.

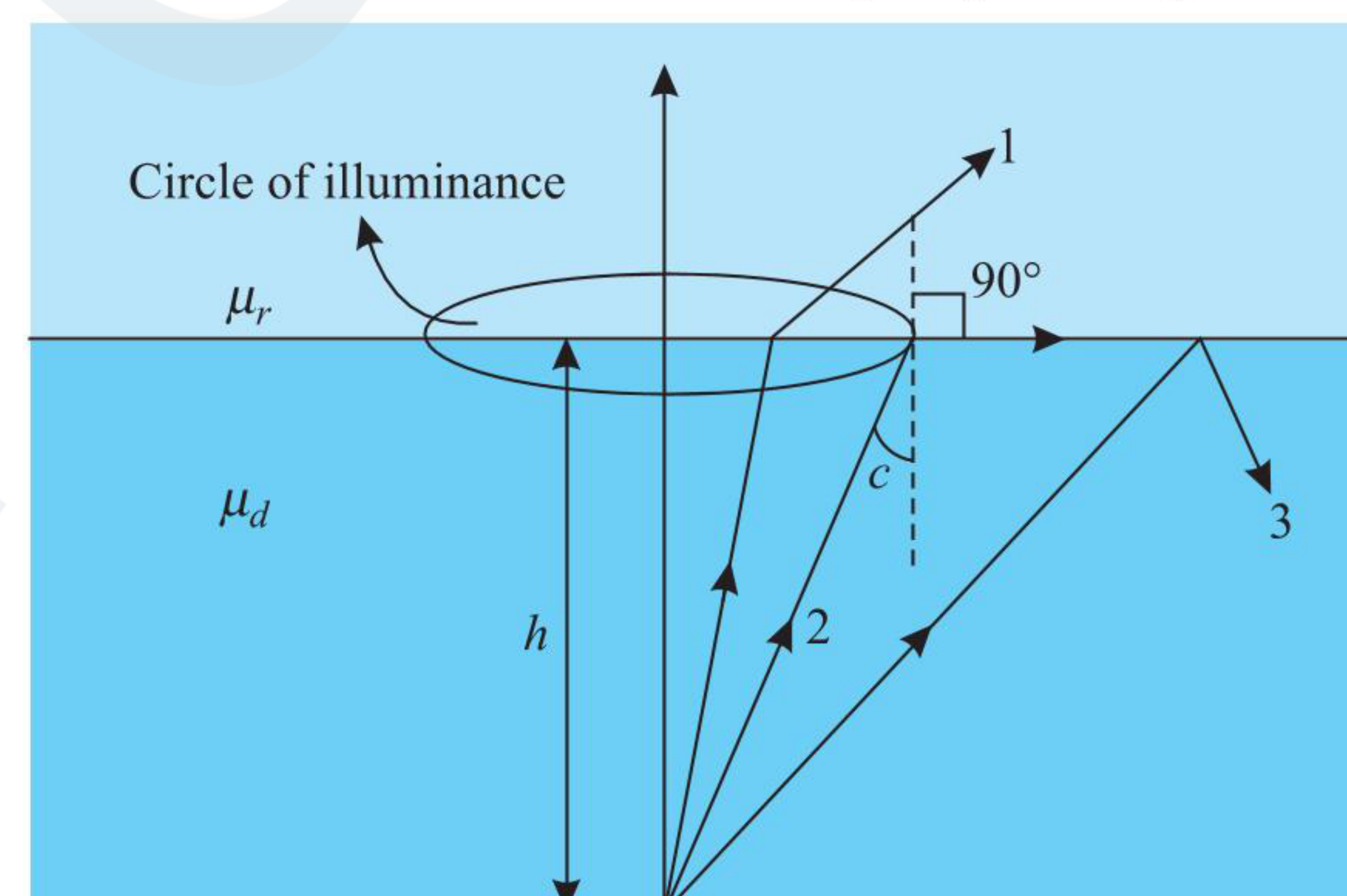
This is a linearly decreasing function. The maximum value of δ occurs when $\theta_1 = C$ and is equal to $\delta_{\max} = \pi - 2C$.

The variation of δ with the angle of incidence θ_1 is plotted in figure.



Circle of Illuminance

For total internal reflection, angle of incidence should be greater than the critical angle ($i > C$). Figure shows a luminous object placed in denser medium at a distance h from an interface separating two media of refractive indices μ_r and μ_d . Subscripts r and d stand for rarer and denser media, respectively.



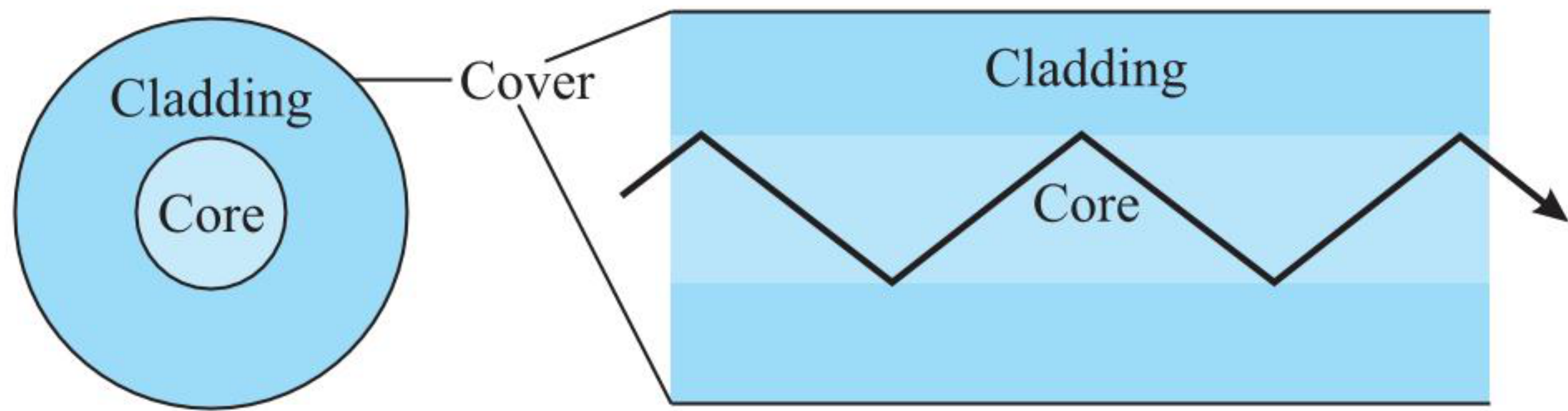
In the figure, ray 1 strikes the surface at an angle less than critical angle C and gets refracted in rarer medium. Ray 2 strikes the surface at critical angle and grazes the interface. Ray 3 strikes the surface making an angle greater than the critical angle and gets internally reflected. The locus of points where ray strikes at critical angle is a circle, called **circle of illuminance (COI)**.

All light rays striking inside the circle of illuminance get refracted in the rarer medium. If an observer is in the rarer medium, he/she will see light coming out only from within the circle of illuminance. If a circular opaque plate covers the circle of illuminance, no light will get refracted in the rarer medium and then the object cannot be seen from the rarer medium. Radius of COI can be easily found.

Optical Fiber

An important application of total internal reflection is the transmission of light in **optical fibers**. Light is injected into a fiber so that the angle of incidence at the outer surface of the fiber is greater than the critical angle for total internal reflection. The light is then transported the length of the fiber as it bounces repeatedly from the fiber surface. Thus, optical fibers can be used to transport light from a source to a destination. Figure shows a bundle of optical fibers with the end of the fibers open to the

camera. The other end of the fibers is optically connected to a light source.



Note that optical fibers can transport light in directions other than a straight line. The fiber may be bent, as long as the radius of curvature of the bend is not small enough to allow the light traveling in the optical fiber to have angles of incidence θ_i less than θ_c .

ILLUSTRATION 1.50

Find the angle of refraction in a medium ($\mu = 2$) if light is incident in vacuum, making an angle equal to twice the critical angle.

Sol. Since the incident light is in rarer medium, total internal reflection cannot take place.

$$C = \sin^{-1} \frac{1}{\mu} = 30^\circ$$

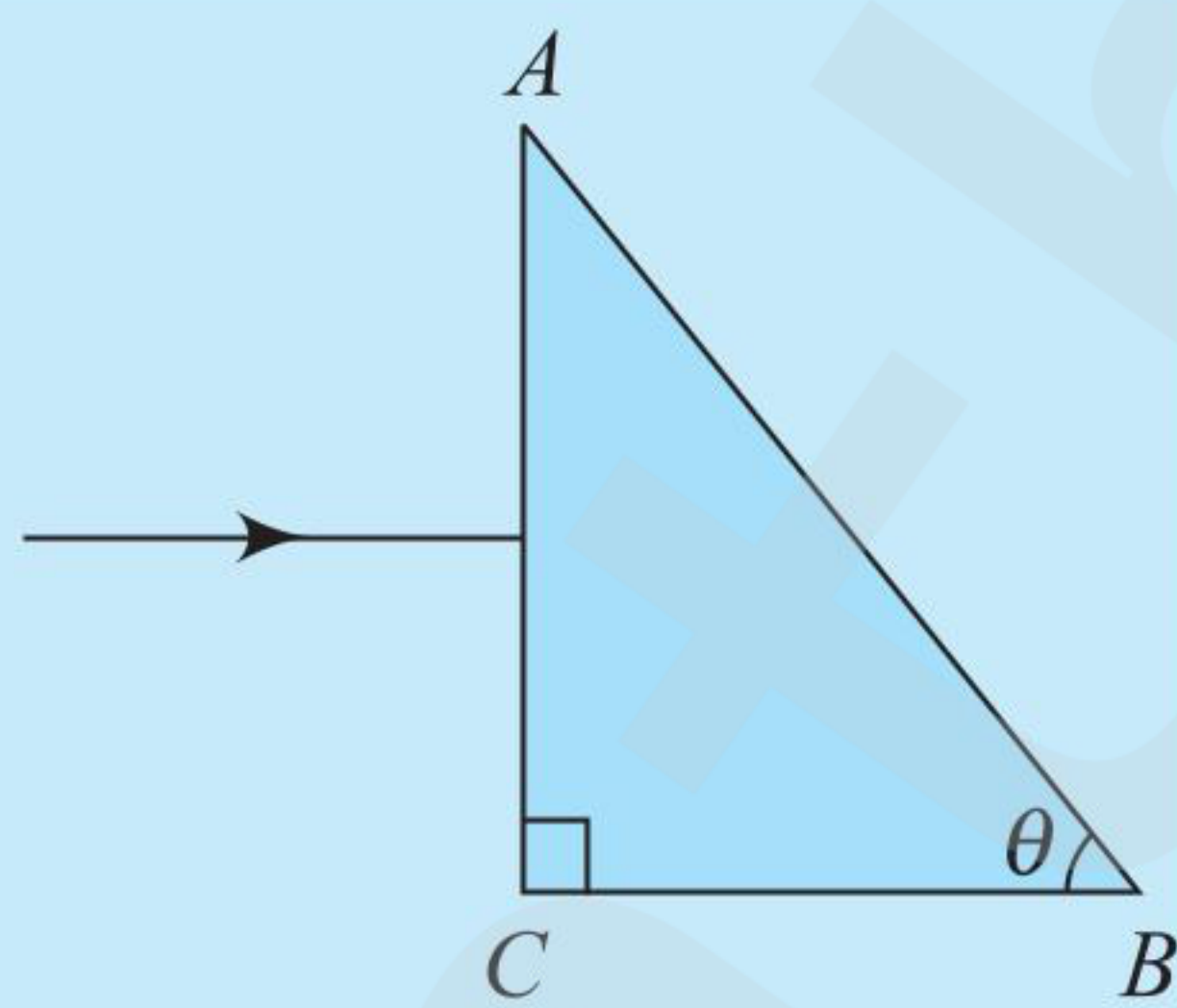
$$\therefore i = 2C = 60^\circ$$

Applying Snell's law, $1 \sin 60^\circ = 2 \sin r$

$$\sin r = \frac{\sqrt{3}}{4} \Rightarrow r = \sin^{-1} \left(\frac{\sqrt{3}}{4} \right)$$

ILLUSTRATION 1.51

What should be the value of angle θ so that light entering normally through the surface AC of a prism ($n = 3/2$) does not cross the second refracting surface AB ?



Sol. Light ray will pass the surface AC without bending since it is incident normally. Suppose it strikes the surface AB at an angle of incidence i .

$$i = 90^\circ - \theta$$

For the required condition: $i > C$

$$\Rightarrow 90^\circ - \theta > C$$

$$\text{or } \sin(90^\circ - \theta) > \sin C$$

$$\text{or } \cos \theta > \sin C = \frac{1}{3/2} = \frac{2}{3}$$

$$\text{or } \theta < \cos^{-1} \frac{2}{3}$$

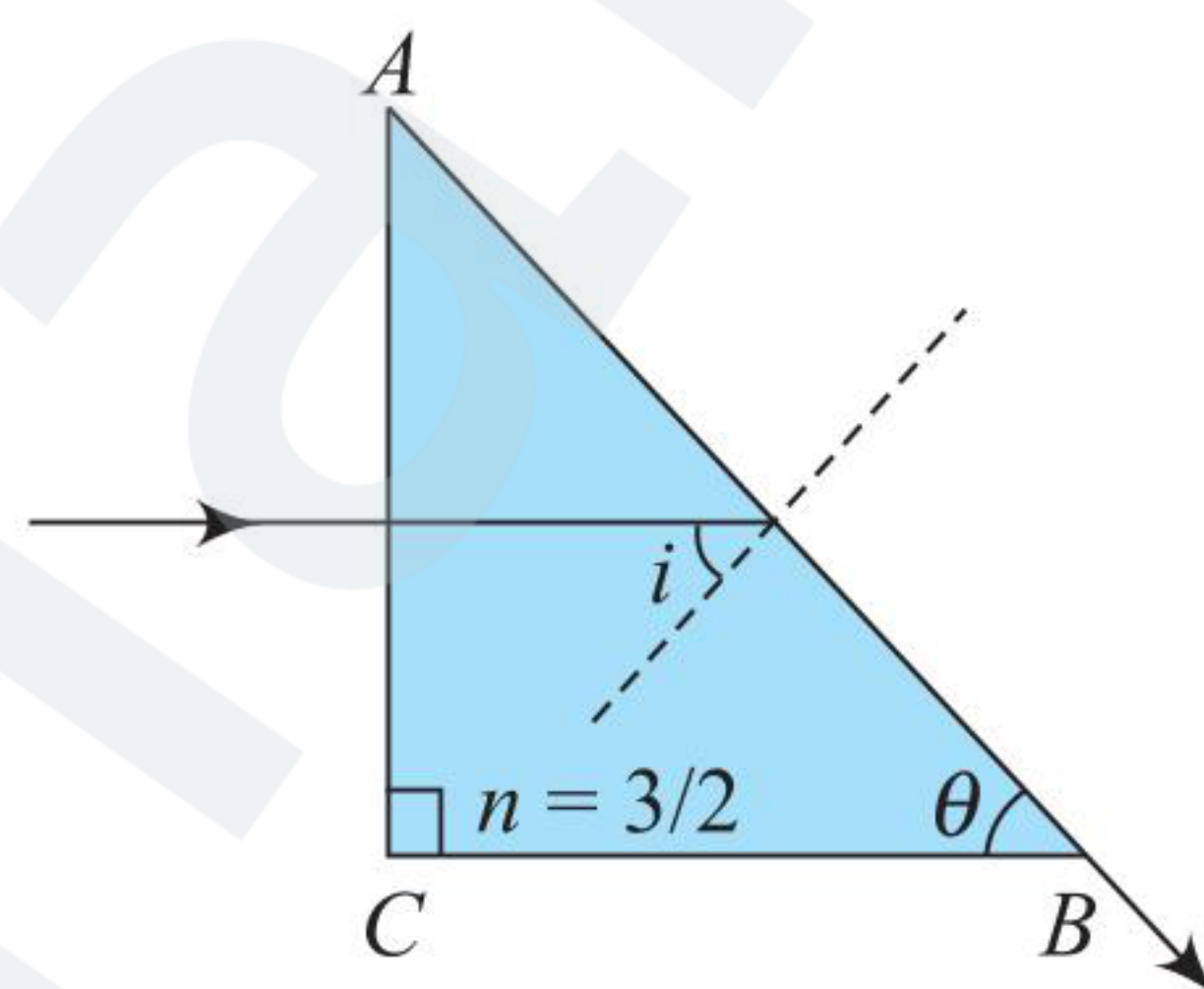
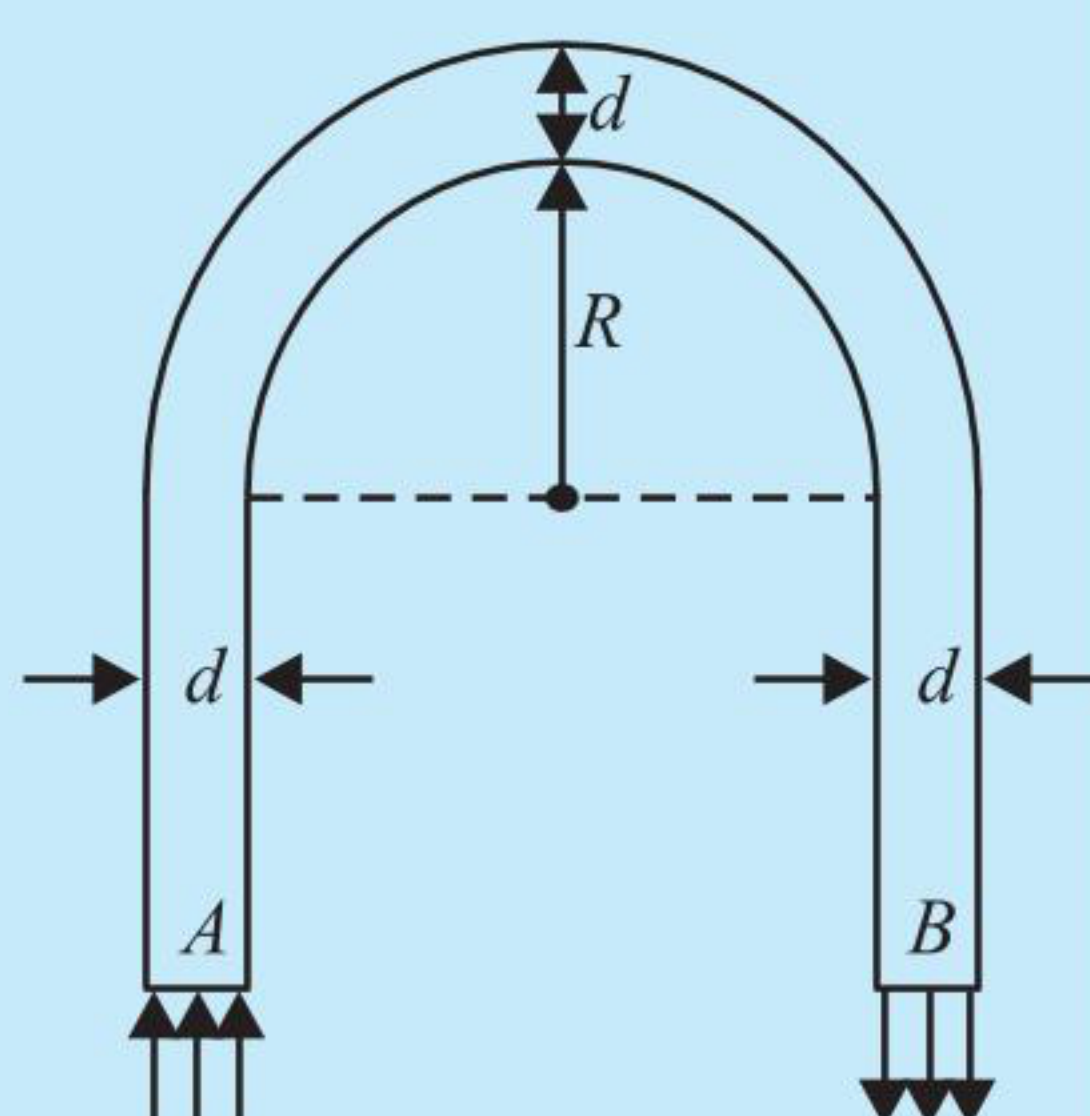


ILLUSTRATION 1.52

A glass rod having square cross-section is bent into the shape as shown in the figure. The radius of the inner semi-circle is R and width of the rod is d . Find the minimum value of d/R so that the light that enters at A will emerge at B . Refractive index of glass is $\mu = 1.5$



Sol. Consider the figure. If smallest angle of incidence θ is greater than critical angle then all light will emerge out at B .

$$\Rightarrow \theta \geq \sin^{-1} \left(\frac{1}{\mu} \right) \Rightarrow \sin \theta \geq \frac{1}{\mu}$$

$$\text{From figure, } \sin \theta = \frac{R}{R+d}$$

$$\Rightarrow \frac{R}{R+d} \geq \frac{1}{\mu} \Rightarrow \left(1 + \frac{d}{R} \right) \leq \mu$$

$$\Rightarrow \frac{d}{R} \leq \mu - 1 \Rightarrow \left(\frac{d}{R} \right)_{\max} = 0.5$$

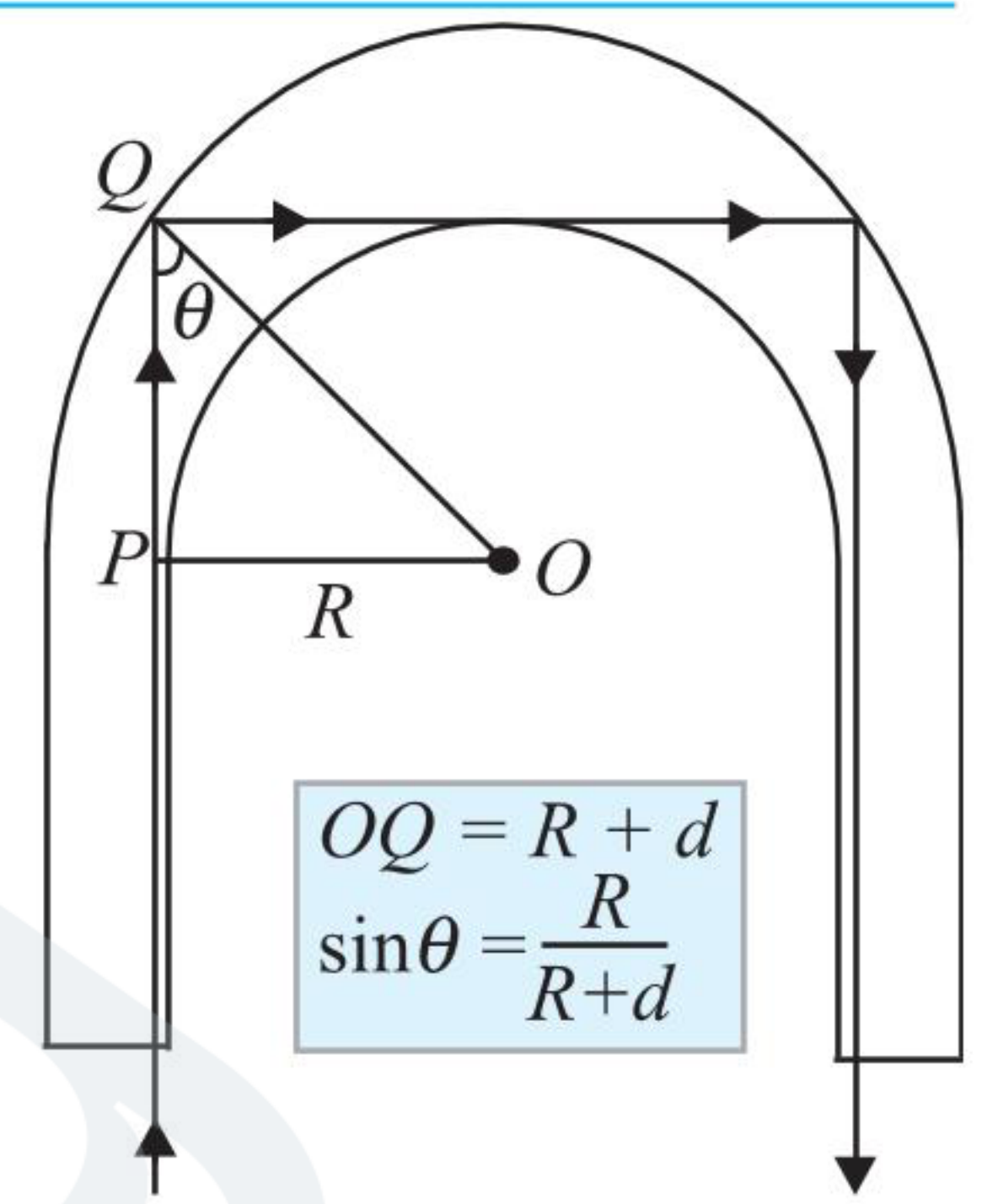
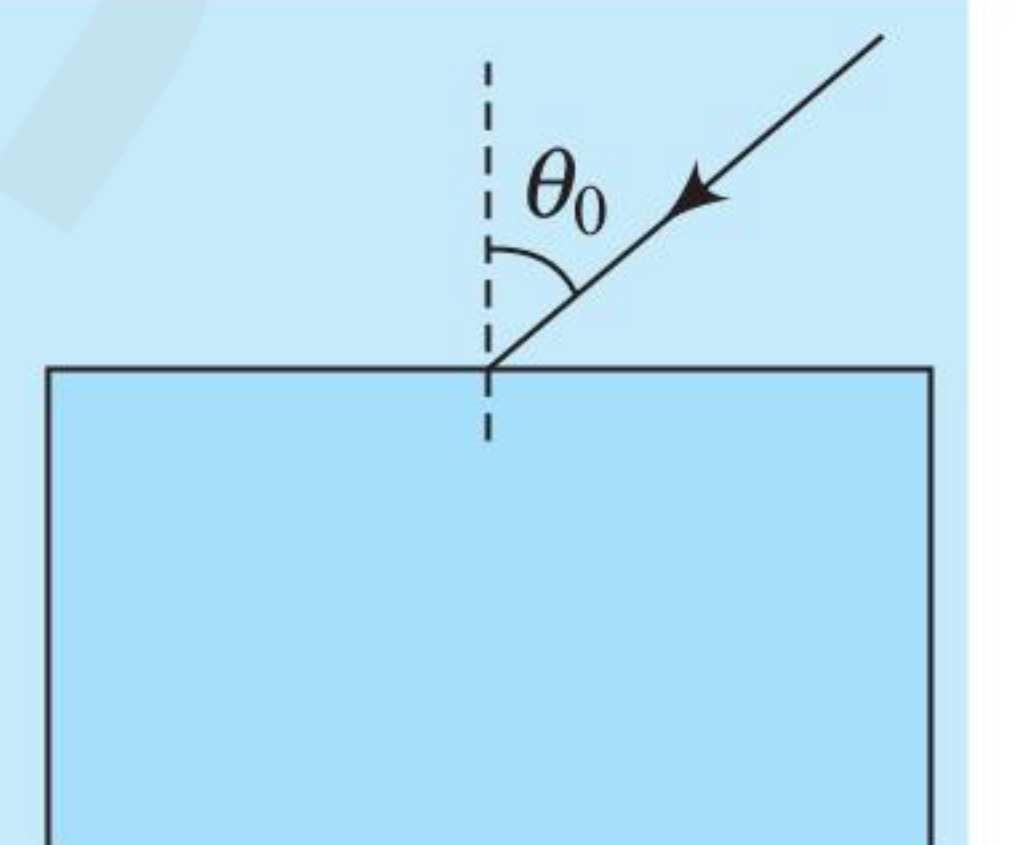


ILLUSTRATION 1.53

A slab of refractive index μ is placed in air and light is incident at maximum angle θ_0 from vertical. Find minimum value of μ for which total internal reflection takes place at the vertical surface.



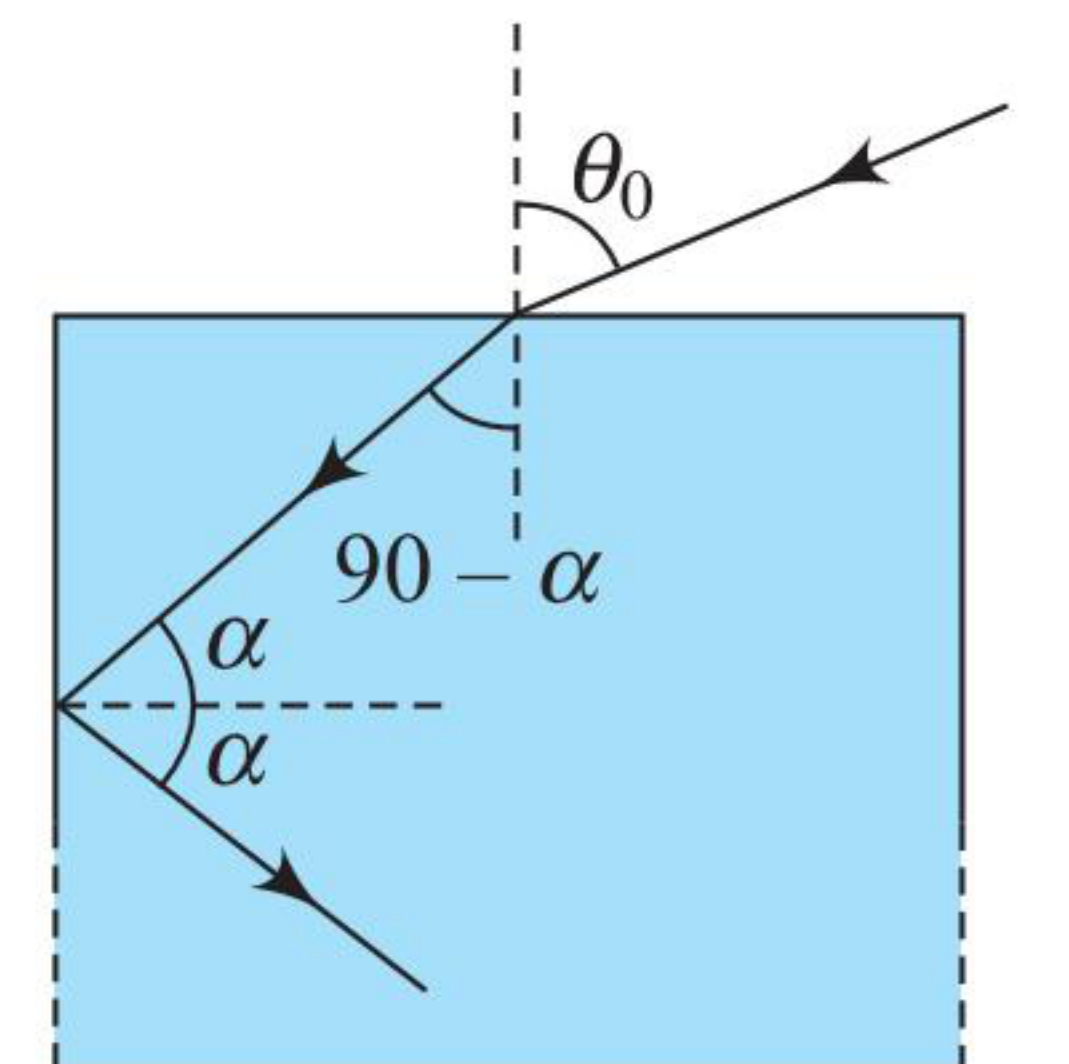
Sol. For vertical surface, $\alpha > C$

$$\Rightarrow \sin \alpha > \sin C$$

$$\Rightarrow \sin \alpha > \frac{1}{\mu} \quad \dots(i)$$

For horizontal surface $\sin \theta_0 = \mu \cos \alpha$

$$\Rightarrow \sin \alpha = \frac{\sqrt{\mu^2 - \sin^2 \theta_0}}{\mu} \quad \dots(ii)$$



From Eqs. (i) and (ii), we get

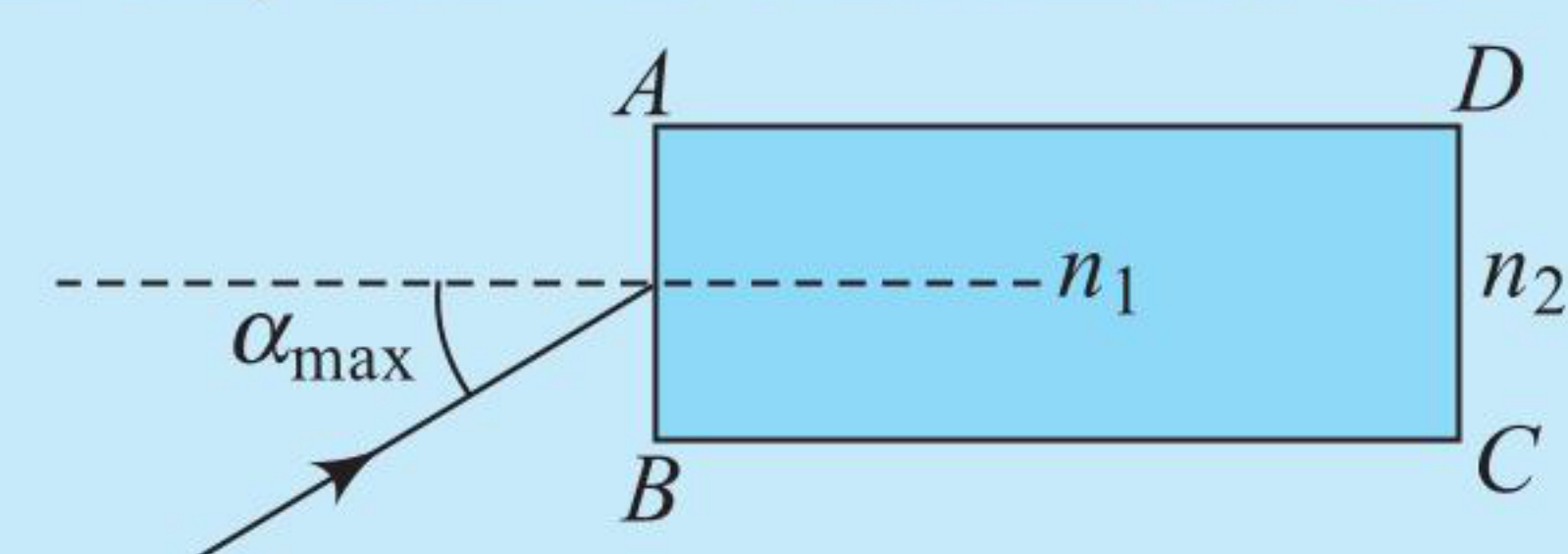
$$\frac{\sqrt{\mu^2 - \sin^2 \theta_0}}{\mu} > \frac{1}{\mu} \Rightarrow \mu^2 - \sin^2 \theta_0 > 1$$

$$\Rightarrow \mu > \sqrt{1 + \sin^2 \theta_0}$$

So, minimum value of $\mu = \sqrt{1 + \sin^2 \theta_0}$

ILLUSTRATION 1.54

A rectangular slab $ABCD$, of refractive index n_1 , is immersed in water of refractive index n_2 ($n_1 > n_2$). A ray of light is incident at the surface AB of the slab as shown in figure. Find the maximum value of angle of incidence $\alpha_{\max}$, such that the ray comes out only from the other surface CD .



Sol. For a maximum angle of incidence at surface AB there will be a minimum angle of incidence at the surface AD . A ray to pass through the face CD as it should not pass beyond AD i.e. it should not refract at AD . Hence, the angle θ should be the critical angle.

$$\text{By Snell's law, } \sin \theta = \frac{n_2}{n_1}$$

$$n_2 \sin \alpha_{\max} = n_1 \sin(90^\circ - \theta)$$

$$\sin \alpha_{\max} = \frac{n_1}{n_2} \cos \theta \Rightarrow \alpha_{\max} = \sin^{-1} \left[\frac{n_1}{n_2} \cos \left(\sin^{-1} \frac{n_2}{n_1} \right) \right]$$

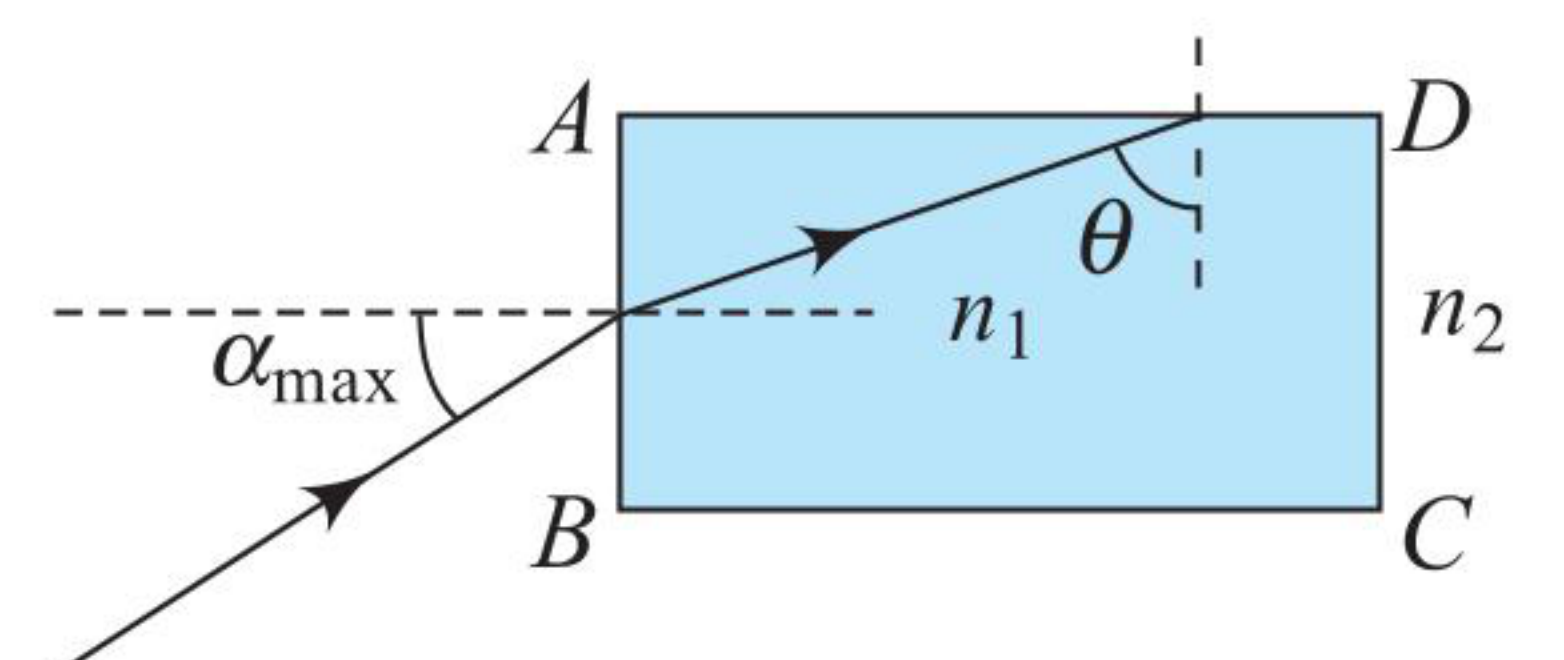


ILLUSTRATION 1.55

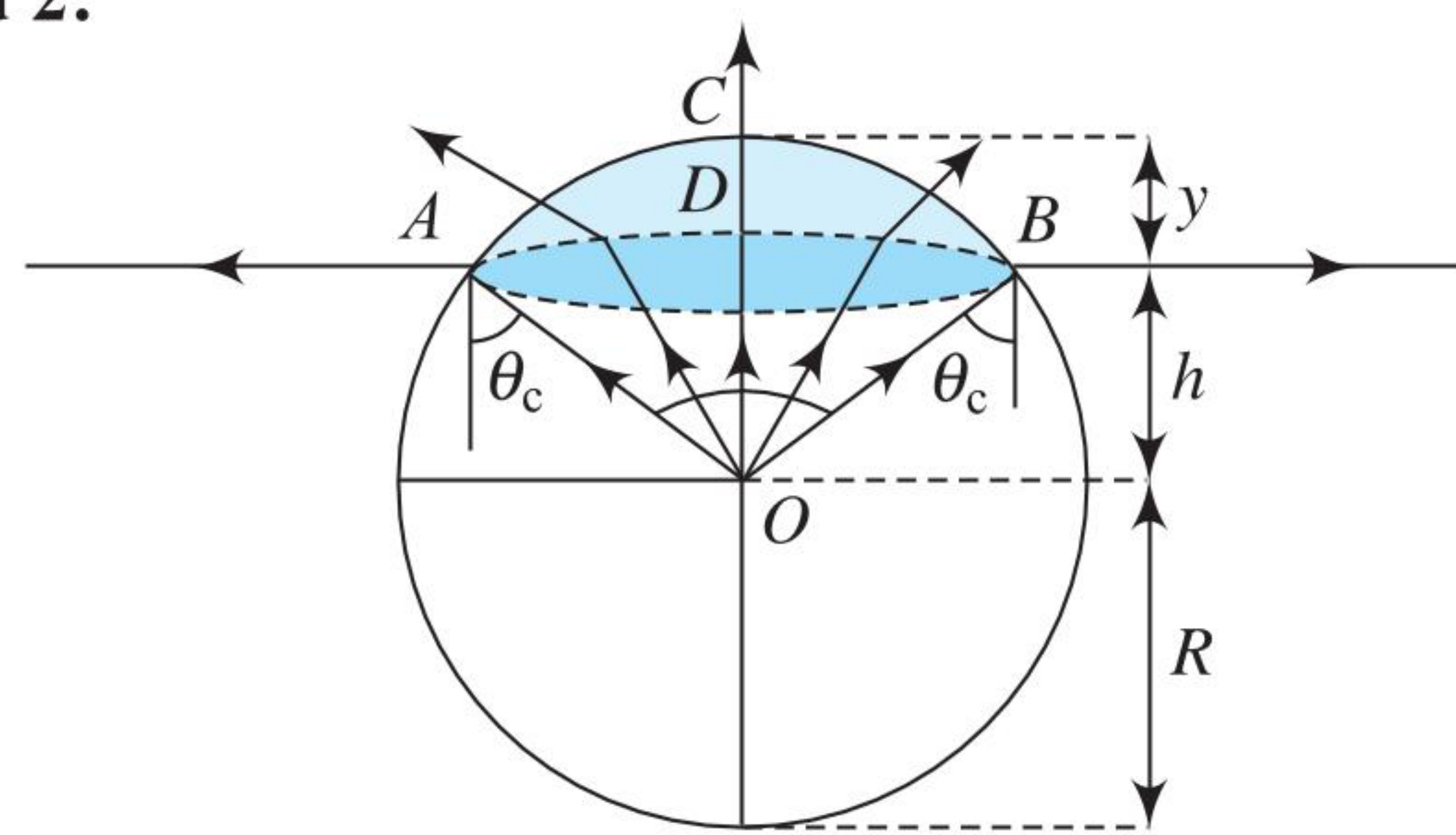
A point source of light is placed at distance h below the surface of a large and deep lake. What fraction of light will escape through the surface of water?

Sol. Method 1: Due to total internal reflection some of the light rays incident at the interface will return back into water. So, only that portion of light will escape for which the angle of incidence at the interface of the medium is less than the critical angle.

If the critical angle is θ_c , then the light rays that reach beyond the base of the cone whose vertical angle is $2\theta_c$ will suffer total internal reflection.

Hence, only the light incident on the base of the cone refracts and escapes.

Method 2:



The fraction of light escaping,

$$f = \frac{\text{Area of the cap}}{\text{Area of sphere}} = \frac{2\pi Ry}{4\pi R^2}, \text{ i.e.,}$$

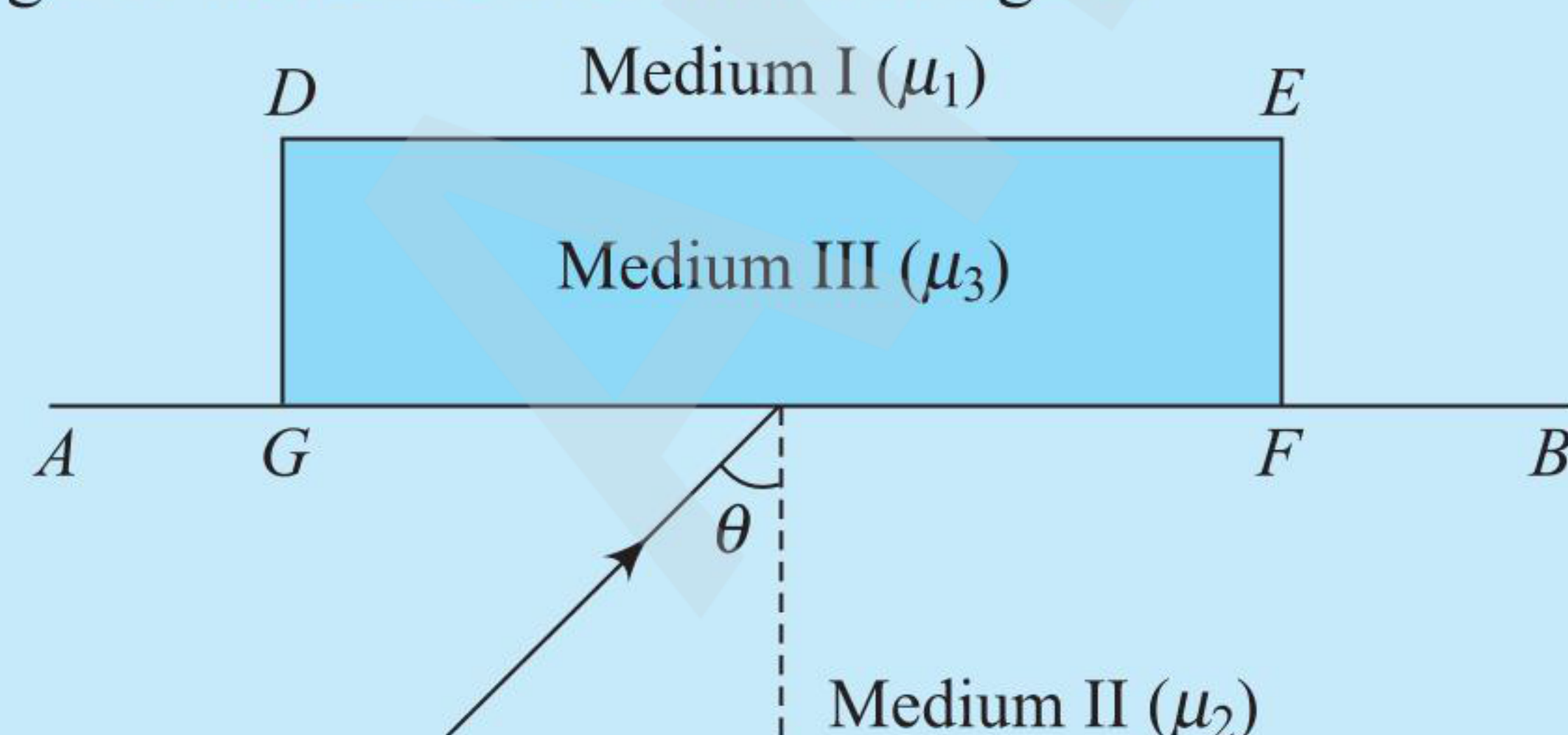
$$f = \frac{1}{2} \left[\frac{y}{R} \right] = \frac{1}{2} \left[\frac{R-h}{R} \right]$$

Area of cap $ABCD$ can be calculated by using method of integration,

$$\begin{aligned} \text{i.e., } f &= \frac{1}{2} \left[1 - \frac{h}{R} \right] = \frac{1}{2} [1 - \cos \theta_c] \\ &= \frac{1}{2} \left[1 - \sqrt{1 - \sin^2 \theta_c} \right] = \frac{1}{2} \left[1 - \sqrt{1 - \frac{1}{n^2}} \right] \end{aligned}$$

ILLUSTRATION 1.56

A monochromatic light is incident on the plane interface AB between two media of refractive indices μ_1 and μ_2 ($\mu_2 > \mu_1$) at an angle of incidence θ as shown in figure.



The angle θ is infinitesimally greater than the critical angle for the two media so that total internal reflection takes place. Now, if a transparent slab $DEFG$ of uniform thickness and of refractive index μ_3 is introduced on the interface (as shown in the figure), show that for any value of μ_3 all light will ultimately be reflected back into medium II.

Sol. We will use the symbol $\leq$ to mean 'infinitesimally greater than'.

When the slab is not inserted,

$$\theta \geq \theta_c = \sin^{-1}(\mu_1/\mu_2) \text{ or } \sin \theta \geq \mu_1/\mu_2$$

When the slab is inserted, we have two cases

$$\mu_3 \leq \mu_1 \text{ and } \mu_3 > \mu_1$$

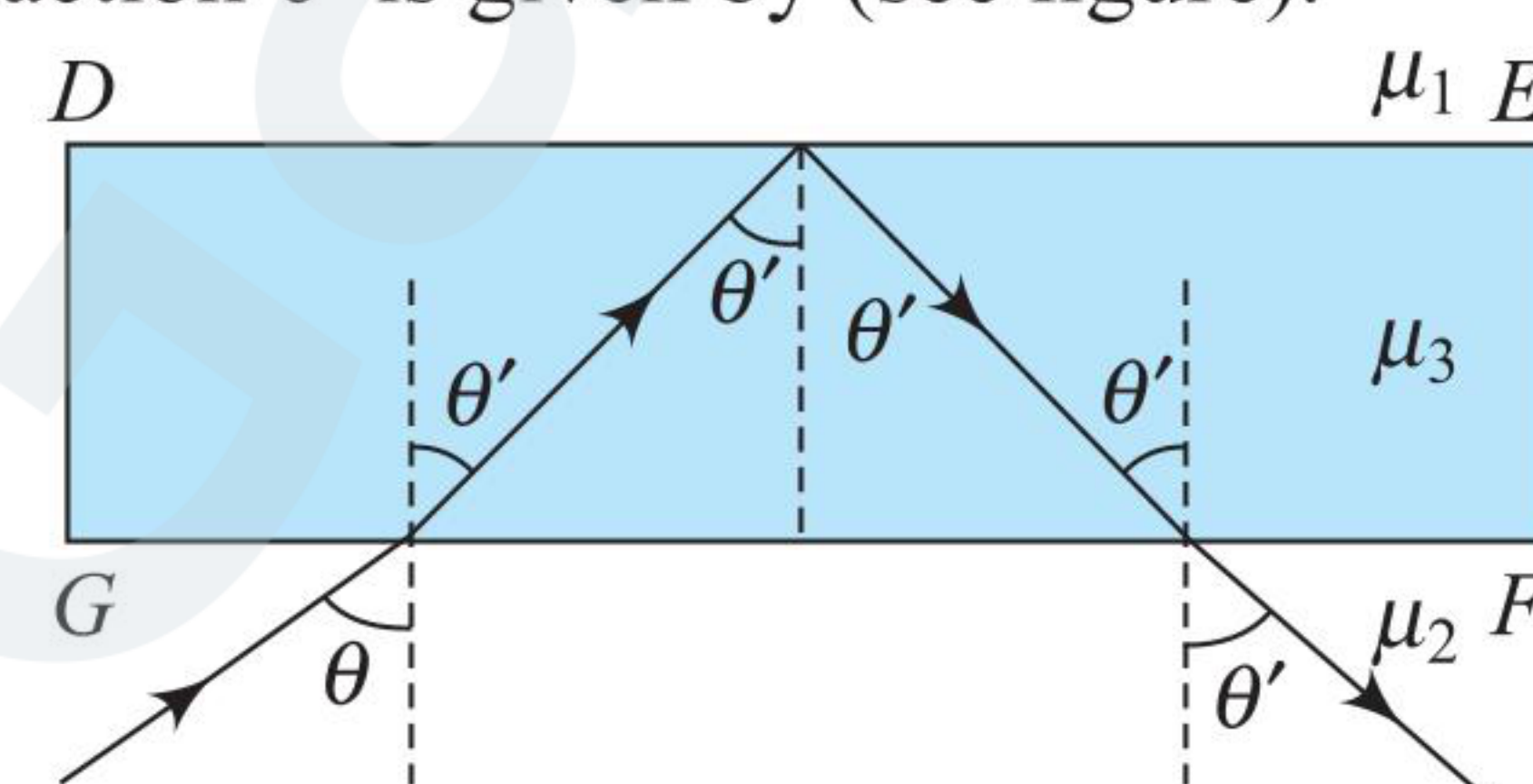
Case I: $\mu_3 < \mu_1$. We have $\sin \theta \geq \mu_1/\mu_2 \geq \mu_3/\mu_2$

Thus, the light is incident on AB at an angle greater than the critical angle $\sin^{-1}(\mu_3/\mu_2)$. It suffers total internal reflection and goes back to medium II.

Case II: $\mu_3 > \mu_1$

$$\sin \theta \geq \mu_1/\mu_2 < \mu_3/\mu_2$$

Thus, the angle of incidence θ may be smaller than the critical angle $\sin^{-1}(\mu_3/\mu_2)$ and hence it may enter medium III. The angle of refraction θ' is given by (see figure).



$$\frac{\sin \theta}{\sin \theta'} = \frac{\mu_3}{\mu_2} \quad \dots(i)$$

$$\Rightarrow \sin \theta' = \frac{\mu_2}{\mu_3} \sin \theta \leq \frac{\mu_2}{\mu_3} \cdot \frac{\mu_1}{\mu_2}$$

$$\text{Thus, } \sin \theta' \geq \frac{\mu_1}{\mu_3} \Rightarrow \theta' \geq \sin^{-1} \left(\frac{\mu_1}{\mu_3} \right) \quad \dots(ii)$$

As the slab has parallel faces, the angle of refraction at the face FG is equal to the angle of incidence at the face DE . Equation (ii) shows that this angle is infinitesimally greater than the critical angle here. Hence, the light suffers total internal reflection and falls at the surface FG at an angle of incidence θ' . At this face, it will refract into medium II and the angle of refraction will be θ as shown by Eq. (i). Thus, the total light energy is ultimately reflected back into medium II.

ILLUSTRATION 1.57

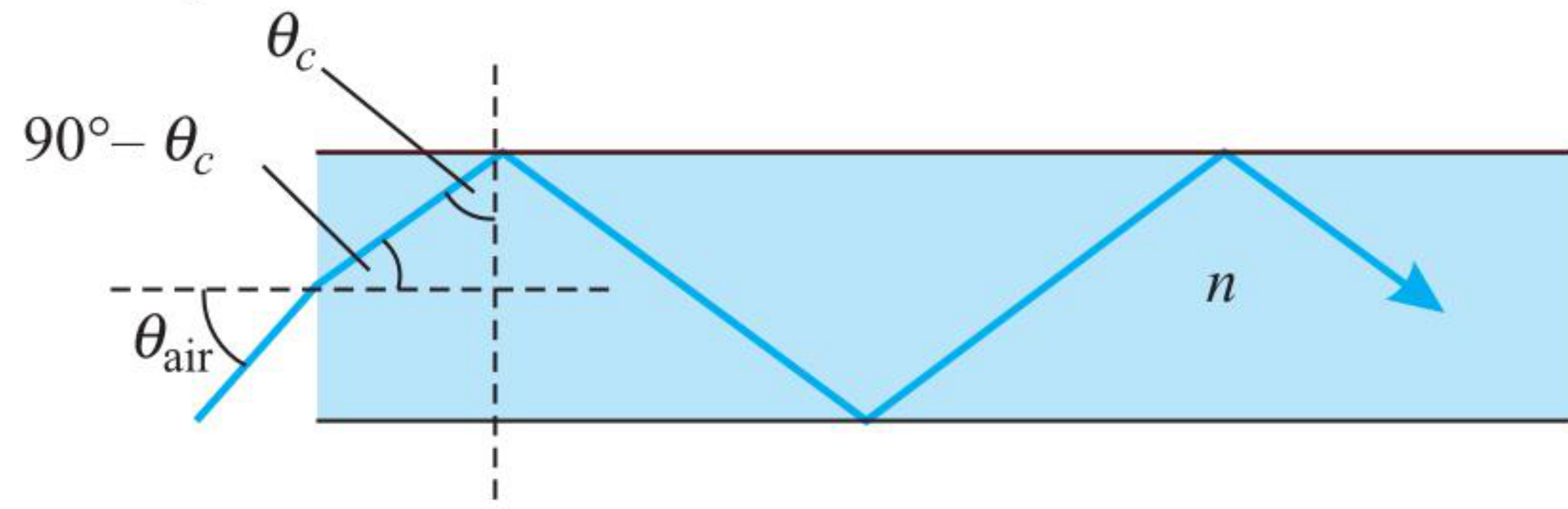
Consider a long optical fiber with index of refraction n , that is surrounded by air. (There is no cladding.) The end of the fiber is polished to be flat and is perpendicular to the length of the fiber. A light ray from a laser is incident from air onto the centre of the circular face of the optical fiber at angle θ_{air} .



What is the maximum angle of incidence for this light ray such that it will be confined and transported by the optical fiber? (Neglect any reflection as the light ray enters the fiber.)

Sol. The light ray will refract as it enters the optical fiber. Once inside the fiber, if the angle of incidence on the surface of the optical fiber is greater than the critical angle for total internal reflection, then the light is transmitted without loss.

Figure shows a sketch of the light ray entering the optical fiber and reflecting off its inner surface.



The critical angle for total internal reflection θ_c in the fiber for light entering from air is given by

$$\sin \theta_c = \frac{1}{n} \Rightarrow \theta_c = \sin^{-1} \left(\frac{1}{n} \right) \quad \dots(i)$$

where n is the index of refraction of the optical fiber. For the light entering the optical fiber, Snell's law tells us that since $n_{\text{air}} = 1$,

$$\sin \theta_{\text{air}} = n \sin \theta_{\text{medium}} \quad \dots(ii)$$

where θ_{medium} is the angle of the refracted light ray in the fiber. From figure we can see that

$$\theta_{\text{medium}} = 90^\circ - \theta_c \quad \dots(iii)$$

We can solve Eq. (ii) to obtain the maximum angle of incidence,

$$\theta_{\text{air}} = \sin^{-1} (n \sin \theta_{\text{medium}})$$

Using Eq. (iii) we can write

$$\theta_{\text{air}} = \sin^{-1} (n \sin (90^\circ - \theta_c)) = \sin^{-1} (n \cos \theta_c)$$

We can then use Eq. (i) to obtain

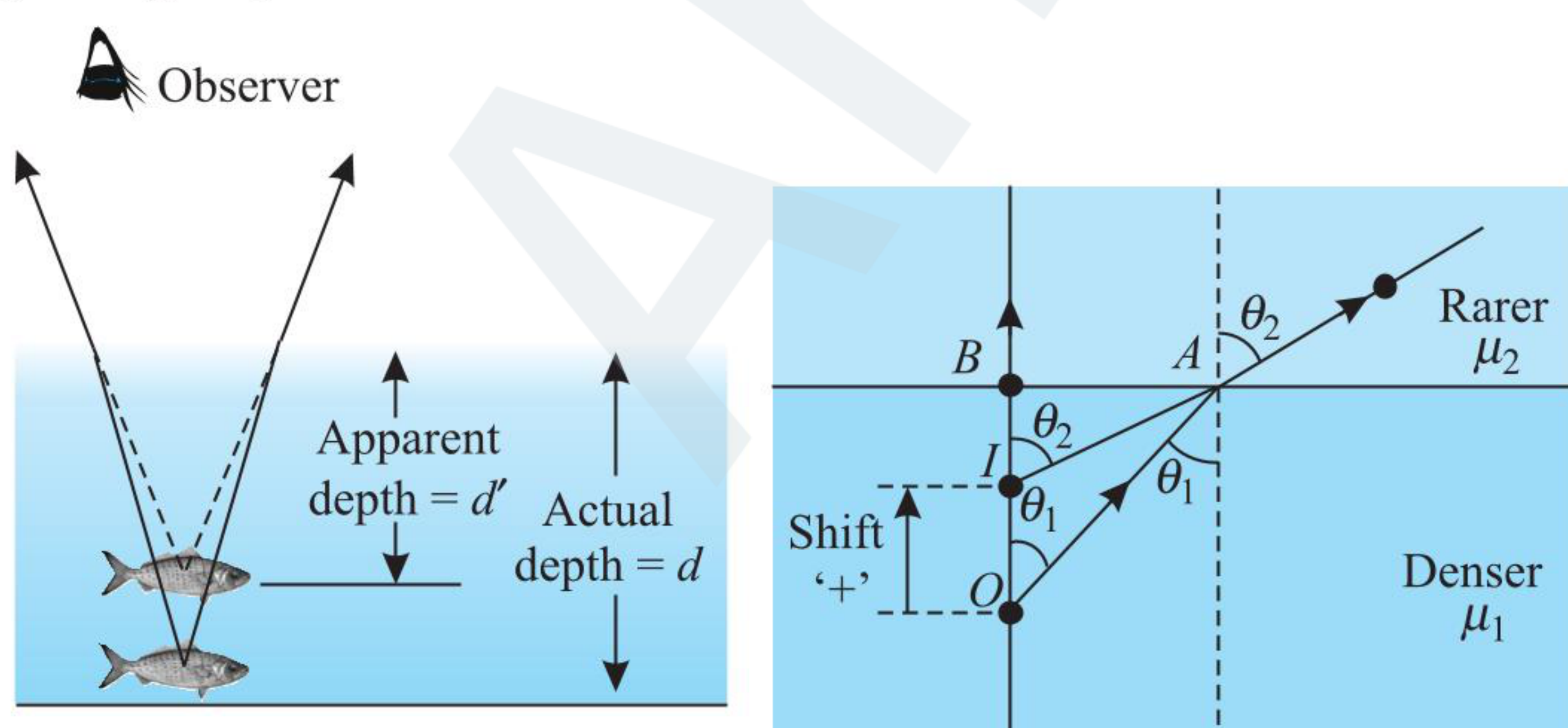
$$\theta_{\text{air}} = \sin^{-1} \left(n \cos \left(\sin^{-1} \left(\frac{1}{n} \right) \right) \right)$$

APPARENT SHIFT OF AN OBJECT DUE TO REFRACTION

Due to bending of light at the interface of two different media, the image formed due to refraction appears at a place other than the object position. This image formation due to refraction creates illusion of shifting of the object position.

Locating the position of image formed becomes much simpler if we restrict ourselves to nearly normal incident rays.

Consider an object O in the medium (RI = μ_1). After refraction, the ray at the interface bend. When the bent ray falls in our eye our eye perceives it along a straight line and it appears at I . (see figure).



For nearly normal incident rays, θ_1 and θ_2 will be very small.

$$\tan \theta_1 \simeq \sin \theta_1 = \frac{AB}{\text{object distance from the refracting surface}}$$

$$\tan \theta_2 \simeq \sin \theta_2 = \frac{AB}{\text{image distance from the refracting surface}}$$

$$\Rightarrow \frac{\sin \theta_1}{\sin \theta_2} = \frac{\text{Image distance from the refracting surface}}{\text{Object distance from the refracting surface}} = \frac{\tan \theta_1}{\tan \theta_2}$$

$$\text{Using Snell's law, } \mu_1 \sin \theta_1 = \mu_2 \sin \theta_2 \Rightarrow \frac{\sin \theta_1}{\sin \theta_2} = \frac{\mu_2}{\mu_1} = \frac{\tan \theta_1}{\tan \theta_2}$$

$$\Rightarrow \frac{\frac{AB}{OB}}{\frac{BA}{BI}} = \frac{\mu_2}{\mu_1} \Rightarrow \frac{BI}{OB} = \frac{\text{Apparant depth}}{\text{Real depth}} = \frac{\mu_2}{\mu_1}$$

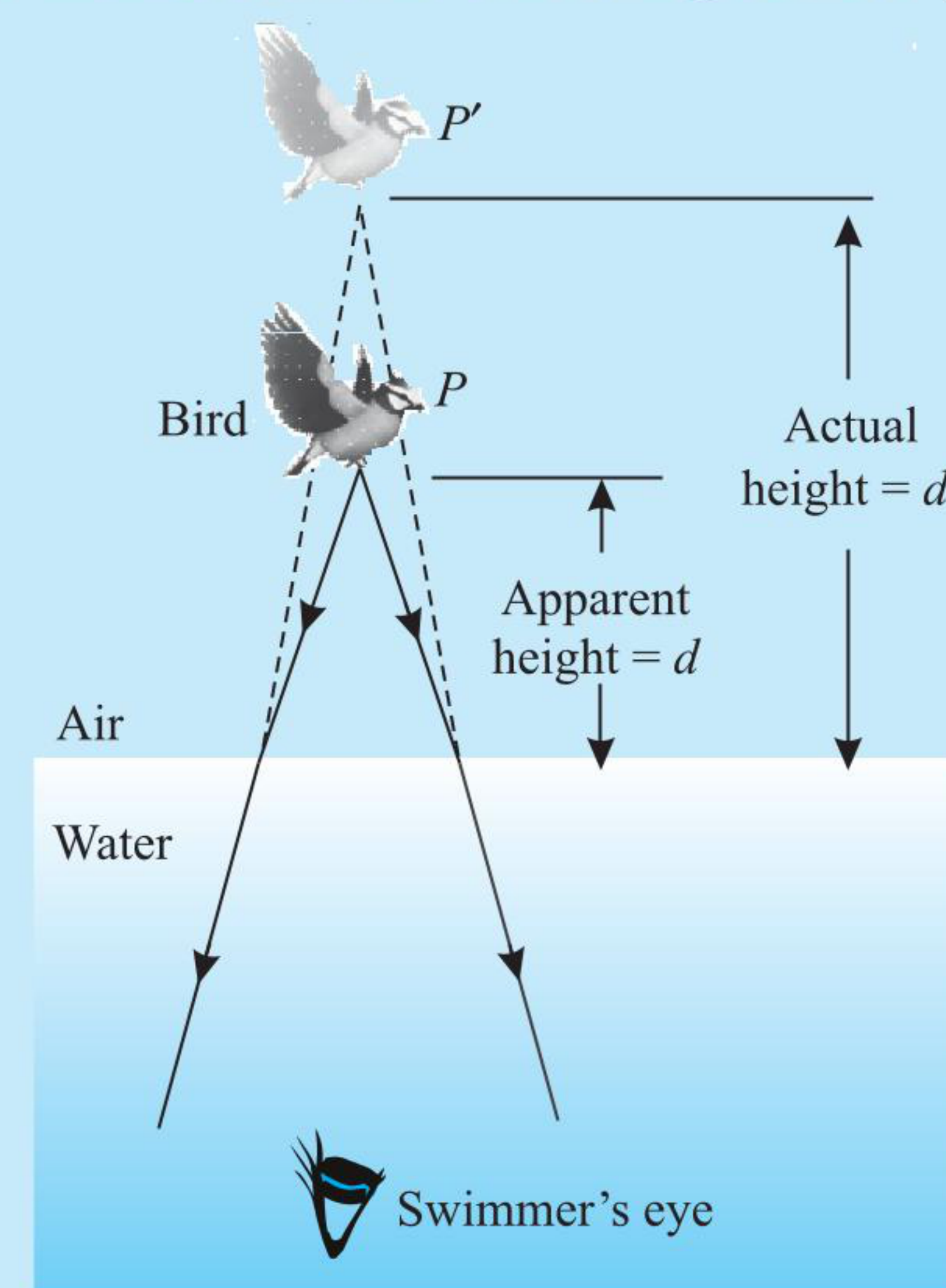
$$\Rightarrow \text{Apparant depth} = \frac{\text{Real depth}}{\mu_1 / \mu_2} = \frac{\text{Real depth}}{\mu_{\text{relative}}}$$

$$\mu_{\text{relative}} = \frac{\mu_i (\text{RI of medium of incidence})}{\mu_r (\text{RI of medium of refraction})}$$

So, Apparent shift = Real depth – Apparent depth

$$\Rightarrow s = \text{Real depth} \left(1 - \frac{1}{\mu_{\text{relative}}} \right)$$

Note: If $\mu_1 < \mu_2$, shift becomes negative (in the direction opposite to initial, ray travelling). In the given figure, the refractive index of medium of incident (μ_1 , the medium where the bird is located, *air*) is less than the refractive index of medium of refraction (μ_2 , the medium where the observer is located, *water*). In this case, Image distance > object distance, i.e., image is farther from the refracting surface (see figure).



Note: Apparent depth and shift of submerged object:

At near normal incidence (small angle of incidence i), apparent

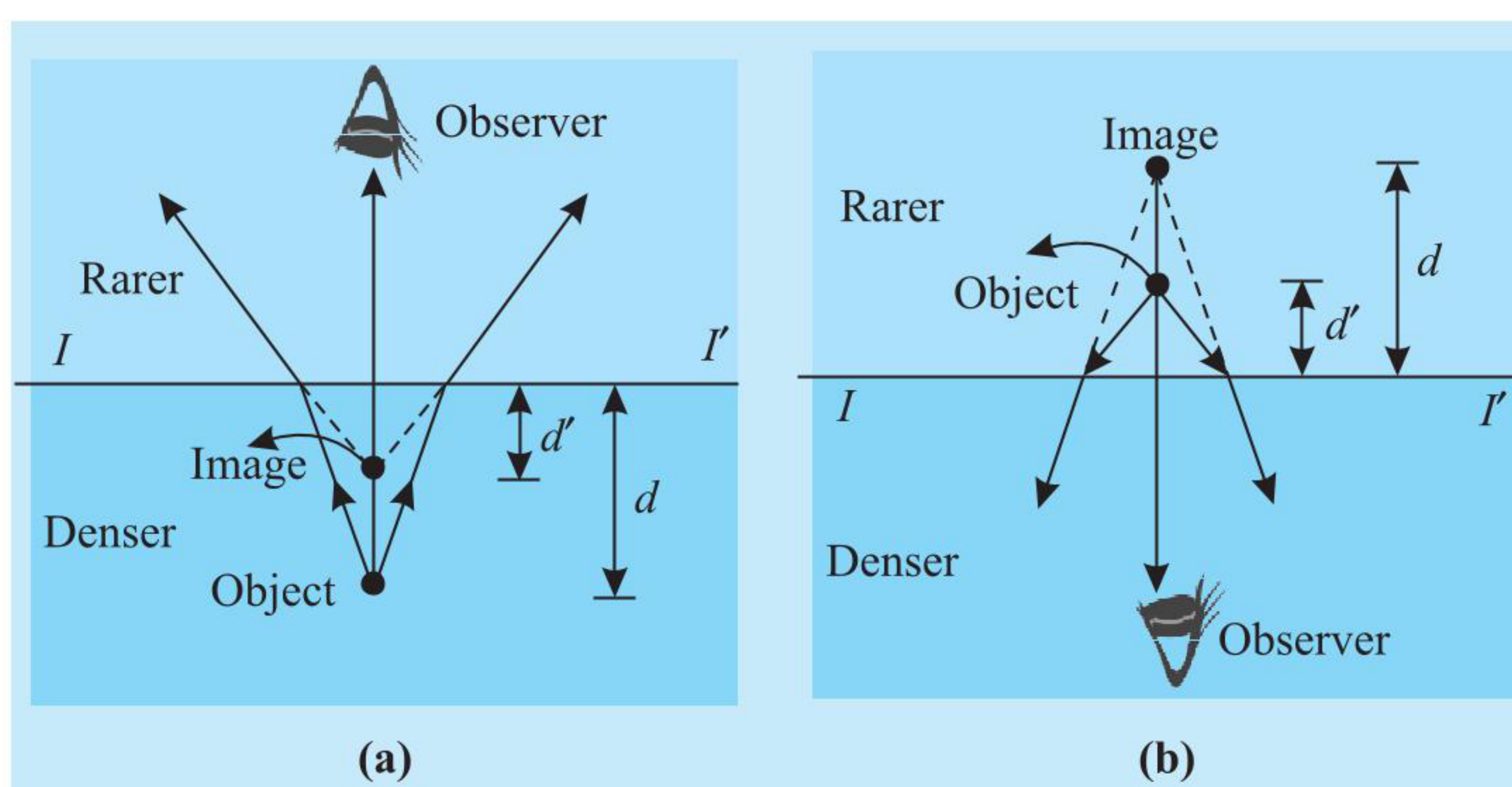
$$\text{depth } (d') \text{ is given by: } d' = \frac{d}{n_{\text{relative}}}$$

$$\text{where } n_{\text{relative}} = \frac{n_i (\text{RI of medium of incidence})}{n_r (\text{RI of medium of refraction})}$$

d = distance of object from the interface = real depth

d' = distance of image from the interface = apparent depth

$$\text{In general, we can write } \frac{n_i}{d} = \frac{n_r}{d'}$$

**ILLUSTRATION 1.58**

An object lies 100 cm inside water ($\mu = 4/3$). It is viewed from air nearly normally. Find the apparent depth of the object.

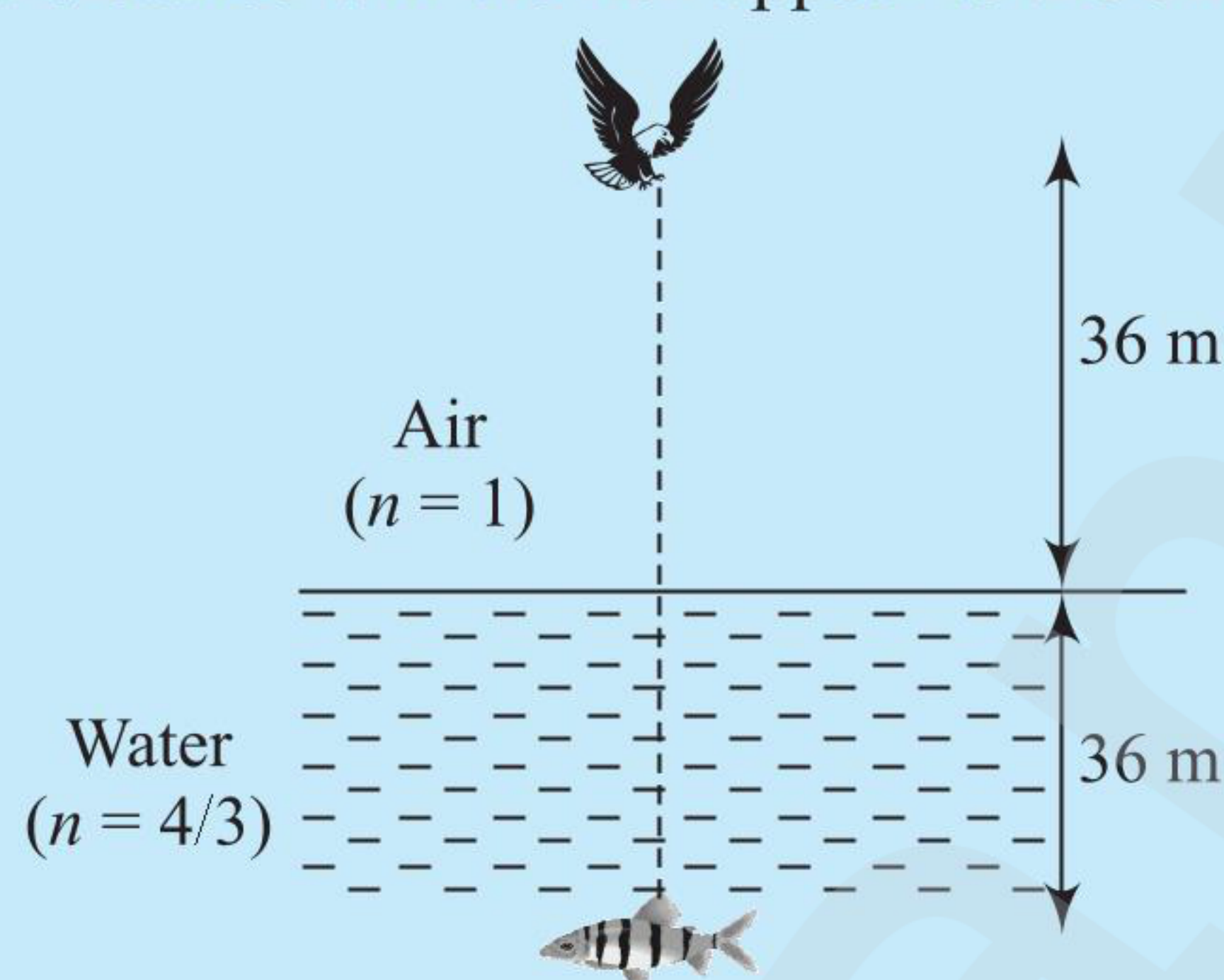
Sol. Here object is placed inside water and the observer is situated in air. Hence, apparent depth

$$d' = \frac{d}{(n_i/n_r)} = \frac{d}{\left(\frac{n_{\text{water}}}{n_{\text{air}}}\right)} = \frac{d}{n_{\text{relative}}} = \frac{100}{4/3} = 75 \text{ cm}$$

ILLUSTRATION 1.59

See figure and answer the following questions.

- Find apparent height of the bird.
- Find apparent depth of the fish.
- At what distance will the bird appear to the fish?
- At what distance will the fish appear to the bird?



Sol.

- Here bird is an object and fish is an observer. Hence, apparent height observed by the fish

$$d'_B = \frac{d}{n_{\text{rel}}} = \frac{d}{\left(\frac{n_{\text{air}}}{n_{\text{water}}}\right)} \Rightarrow d'_B = \frac{36}{\frac{1}{4/3}} = \frac{36}{3/4} = 48 \text{ m}$$

- Here the fish is an object and the bird is an observer. Hence, apparent height observed by the bird

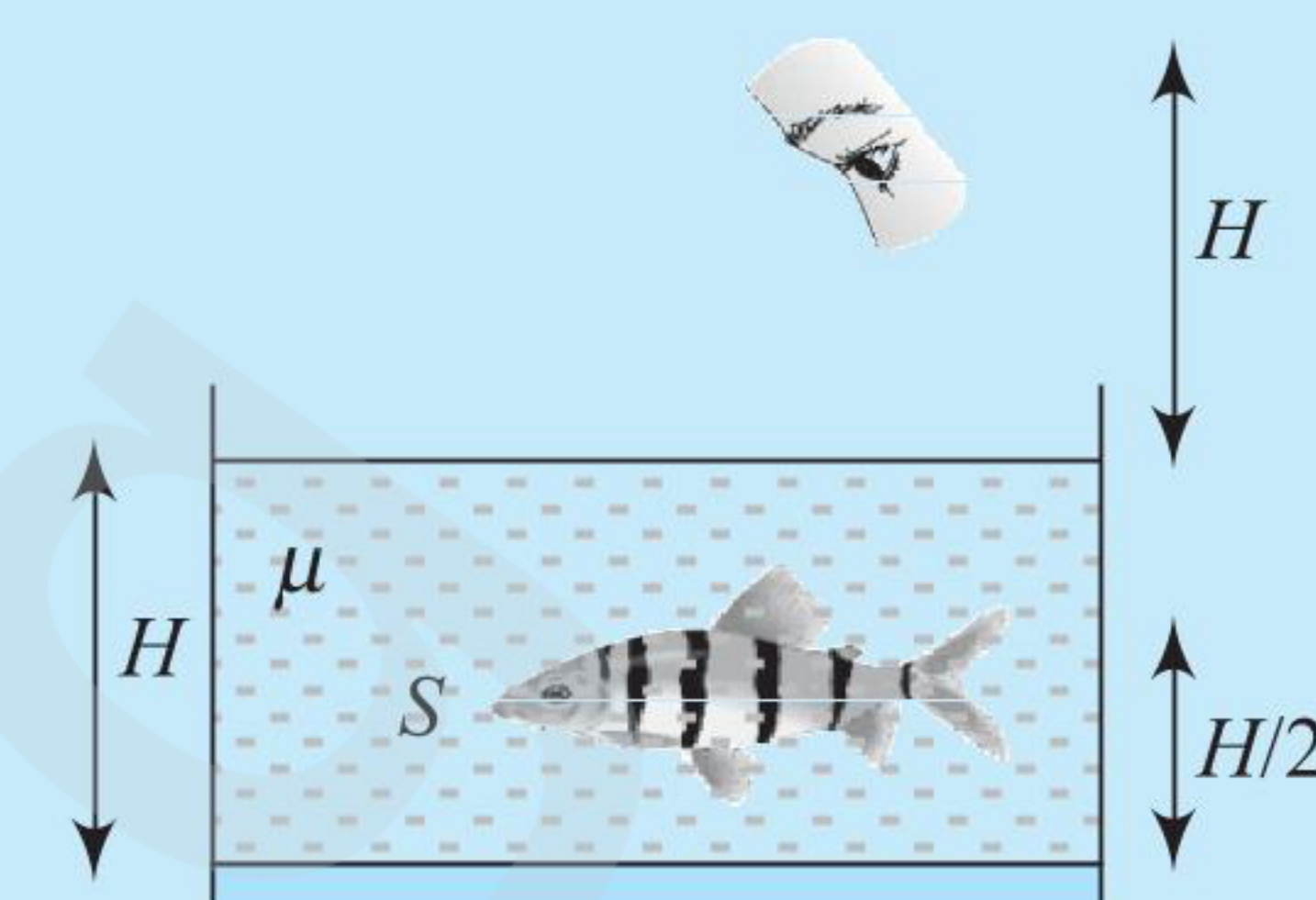
$$d'_F = \frac{d}{n_{\text{rel}}} = \frac{d}{\left(\frac{n_{\text{water}}}{n_{\text{air}}}\right)} \Rightarrow d'_F = \frac{36}{4/3} = 27 \text{ m}$$

- For the fish, the bird will be observed at a distance d'_B from the fish: $d_B = 36 + 48 = 84 \text{ m}$
- For the bird, the fish will be observed at a distance d'_F from the bird: $d_F = 27 + 36 = 63 \text{ m}$

ILLUSTRATION 1.60

Consider the situation in figure. The bottom of the pot is a reflecting plane mirror, S is a small fish, and T is a human eye. Refractive index of water is μ .

- At what distance(s) from itself will the fish see the image(s) of the eye?
- At what distance(s) from itself will the eye see the image(s) of the fish?



Sol.

- The fish will observe the images of eye one from direct observation and the other reflected image from the plane mirror.

(i) Direct observation of eye from fish:

$$\text{Apparent height, } H' = \frac{H}{n_{\text{rel}}} = \frac{H}{\left(\frac{n_{\text{air}}}{n_{\text{water}}}\right)} = \frac{H}{\left(\frac{1}{\mu}\right)}$$

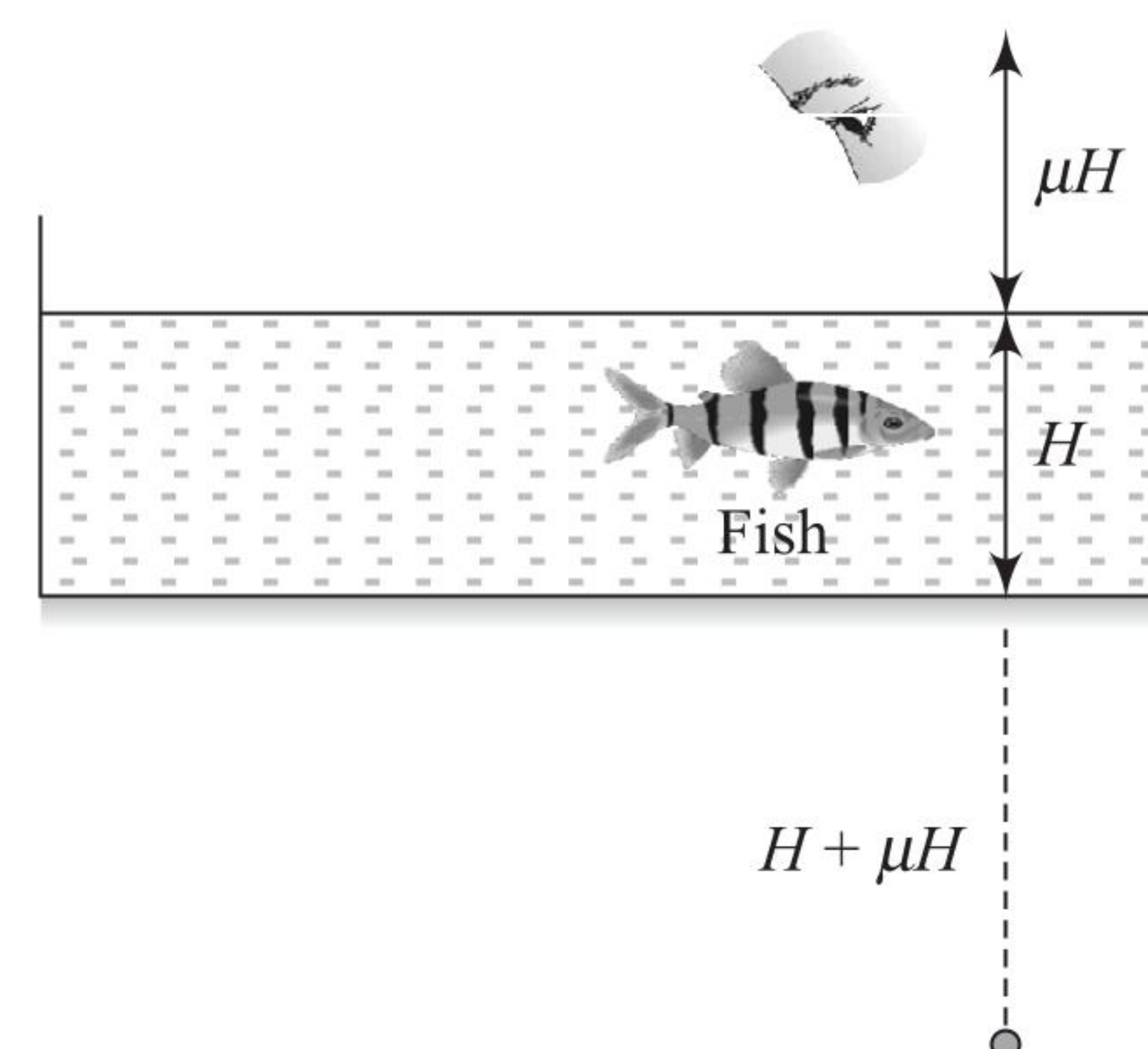
Hence, $H' = \mu H$

Distance of image of eye from fish

$$d = \frac{H}{2} + \mu H = H \left(\frac{1}{2} + \mu \right)$$

(ii) Observation of reflected image of the eye from the fish:

For mirror, the distance of eye from it will be $(H + \mu H)$. Hence, the image of eye from mirror will be $(H + \mu H)$ behind the mirror. Hence, distance of image of eye from the fish



$$d' = \frac{H}{2} + H + \mu H = \frac{3}{2}H + \mu H$$

$$\Rightarrow d' = H \left(\frac{3}{2} + \mu \right)$$

- The eye will also observe two images of the fish, one from direct observation and the other reflected image from the mirror.

(i) Direct observation of fish from eye:

Apparent depth of the fish observed by the eye

$$H' = \frac{H/2}{n_r} = \frac{H/2}{\left(\frac{n_{\text{water}}}{n_{\text{air}}}\right)} = \frac{H/2}{\mu} = \frac{H}{2\mu}$$

Distance of image of the fish from the eye,

$$d = H + \frac{H}{2\mu} = H \left(1 + \frac{1}{2\mu}\right)$$

(ii) Eye observing image of fish:

The eye will observe the image of fish reflected from the mirror.

Apparent depth of image of the fish from air and water interface.

$$H' = \frac{\text{Real depth}}{n_{\text{rel}}} = \frac{\frac{3}{2}H}{\left(\frac{n_{\text{water}}}{n_{\text{air}}}\right)}$$

Here real depth from top surface of water = $H + \frac{H}{2} = \frac{3}{2}H$

$$H' = \frac{3H}{2\mu}$$

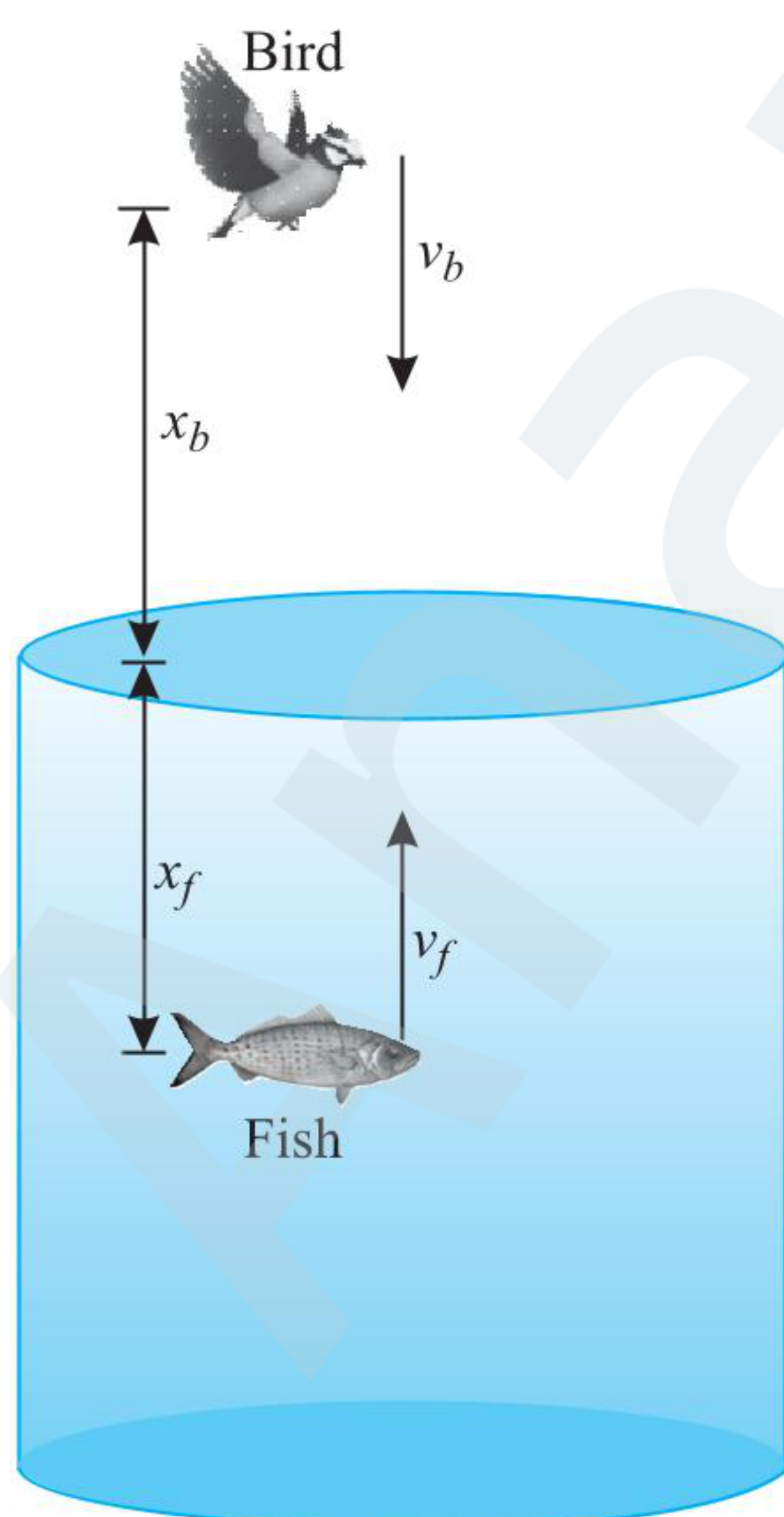
Hence, distance between this image and the eye,

$$d' = H + \frac{3H}{2\mu} = H \left(1 + \frac{3}{2\mu}\right)$$

ILLUSTRATION 1.61

A fish in an aquarium approaches the top surface of the water at a rate of 3 m s^{-1} observes a bird approaching it at 8 ms^{-1} . If the refractive index of water is $(4/3)$, find the actual velocity of the bird.

Sol. The fish (in denser medium) observing the bird (in rarer medium).



The distance of the bird as seen from fish,

$$h_{b,f} = x_f + \mu x_b \quad \dots(i)$$

For finding speed we need to differential Eq. (i) w.r.t. time

$$\frac{dh_{b,f}}{dt} = \frac{dx_f}{dt} + \mu \frac{dx_b}{dt}$$

$$v_{bf} = (-v_f) + \mu(-v_b) = -(v_f + \mu v_b) \quad \dots(ii)$$

Here $\frac{dx_f}{dt} = -v_f$ (as x_f is decreasing with time)

and $\frac{dx_b}{dt} = -v_b$ (x_b is decreasing with time)

Now in Eq. (ii) substituting $v_{bf} = -8 \text{ m/s}$

v_{bf} should be written with negative sign as bird is approaching towards fish i.e., the separation between the fish and bird is decreasing.

$$v_f = 3 \text{ m/s and } \mu = \frac{4}{3}$$

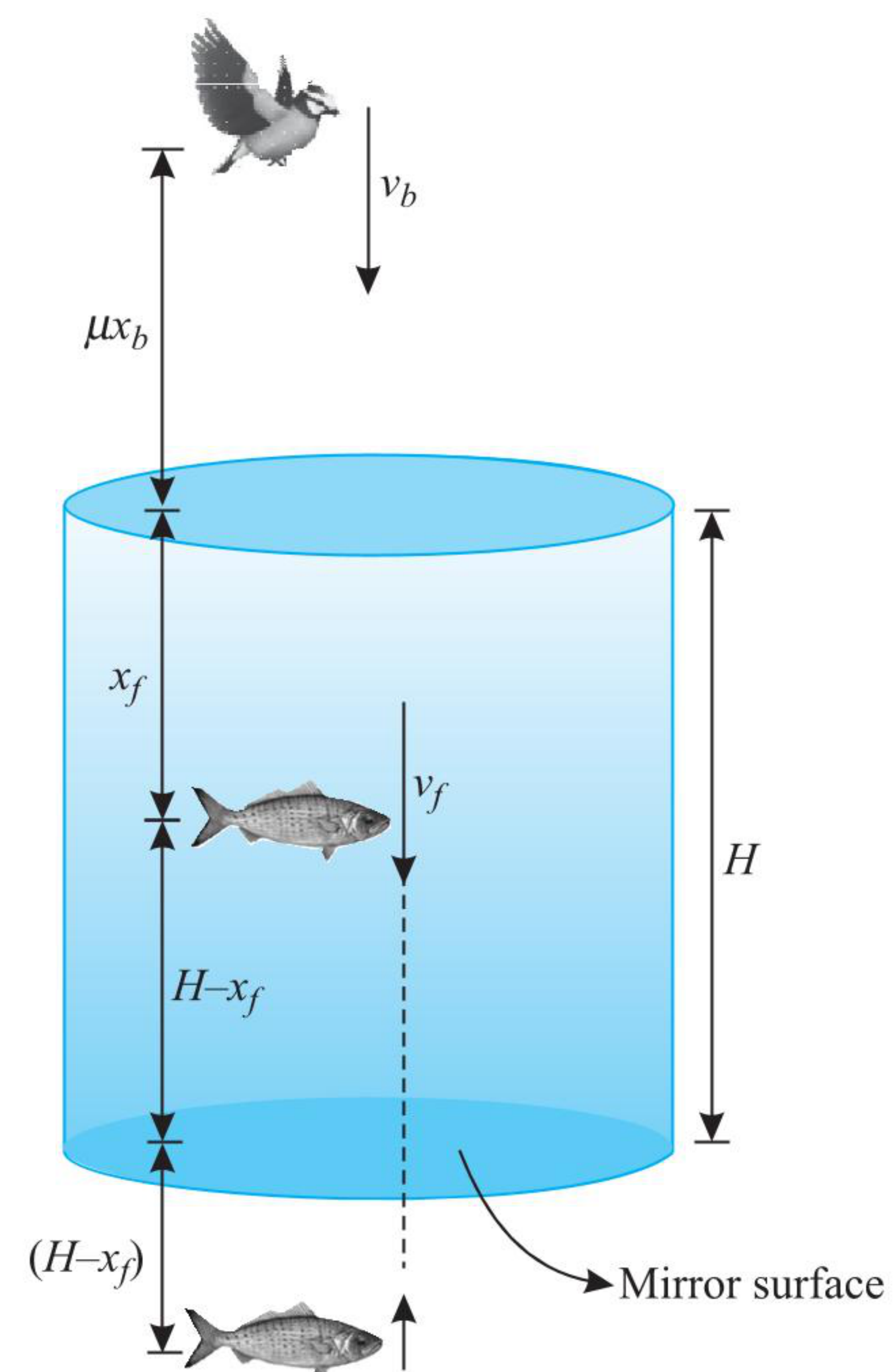
We get $-8 = -\left(3 + \frac{4}{3} \times v_b\right)$

$$8 = 3 + \frac{4}{3} \times v_b \Rightarrow v_b = \frac{15}{4} \text{ m/s}$$

ILLUSTRATION 1.62

A bird is moving towards an aquarium with speed 4 m/s . A fish in the aquarium is moving down with speed 2 m/s . Suppose the base of the aquarium is silvered. Find the speed of the bird as seen by fish in mirror.

Sol. Distance of the bird as seen by fish through mirror,

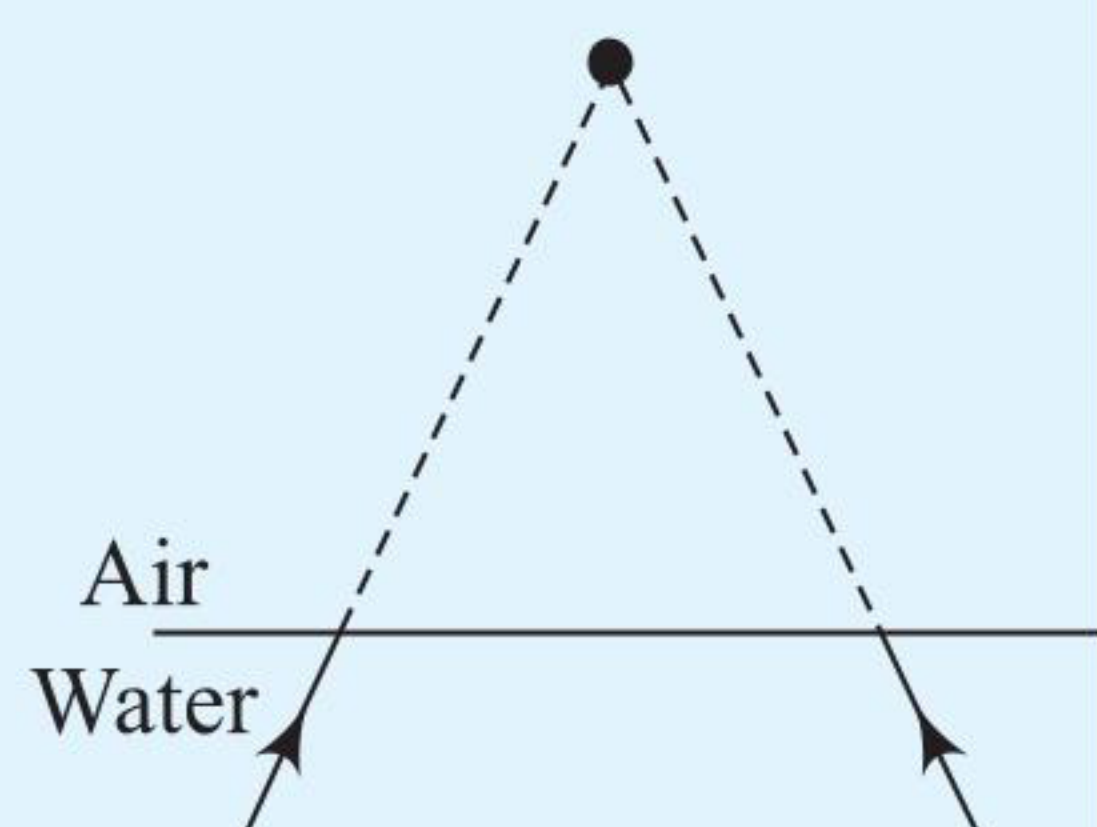


$$h_{b,f} = \mu x_b + H + H - x_f$$

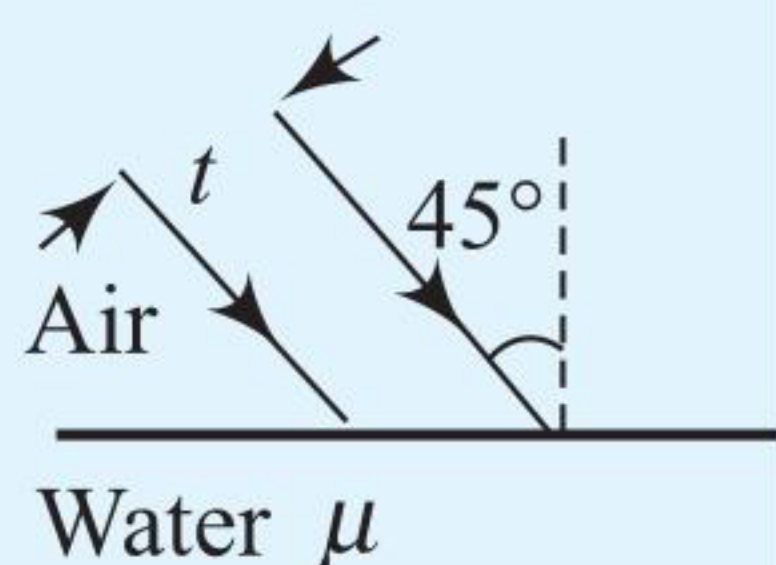
$$\begin{aligned} \frac{dx_{bf}}{dt} &= \mu \frac{dx_b}{dt} - \frac{dx_f}{dt} \\ &= \mu(-v_b) - v_f = -(\mu v_b + v_f) \\ &= -\left(\frac{4}{3} \times 4 - 2\right) = -\frac{22}{3} \text{ m/s} \end{aligned}$$

CONCEPT APPLICATION EXERCISE 1.4

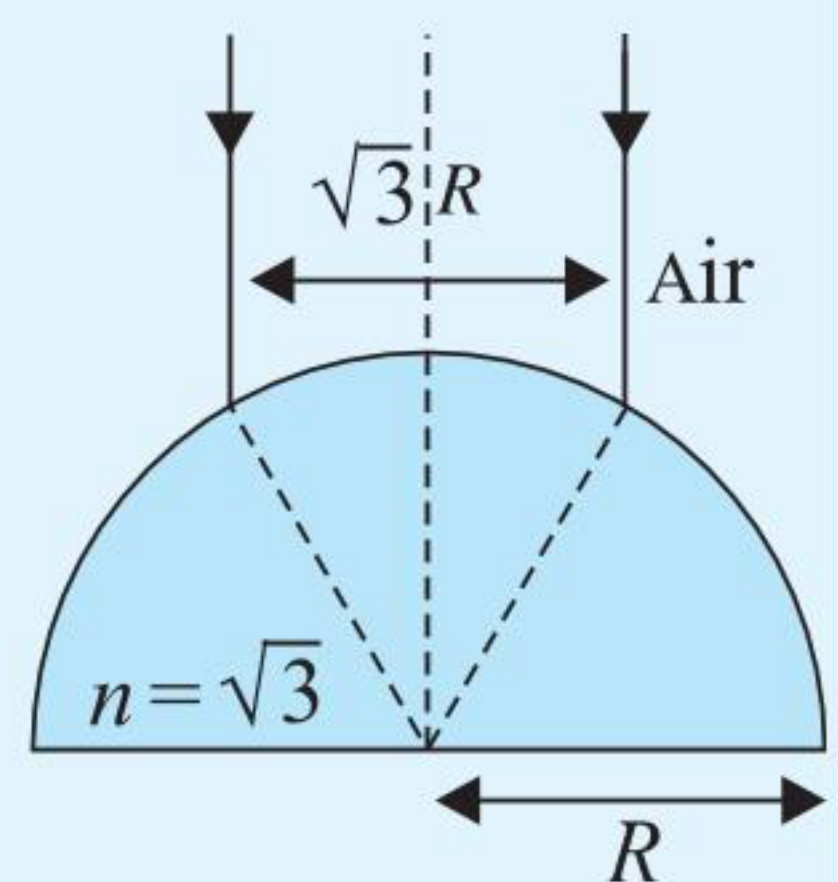
1. A converging set of rays, travelling from water to air, is incident on a plane interface. In the absence of the interface, the rays would have converged to a point O , 60 cm above the interface. However, due to refraction the rays will bend. At what distance above the interface will the rays actually converge?



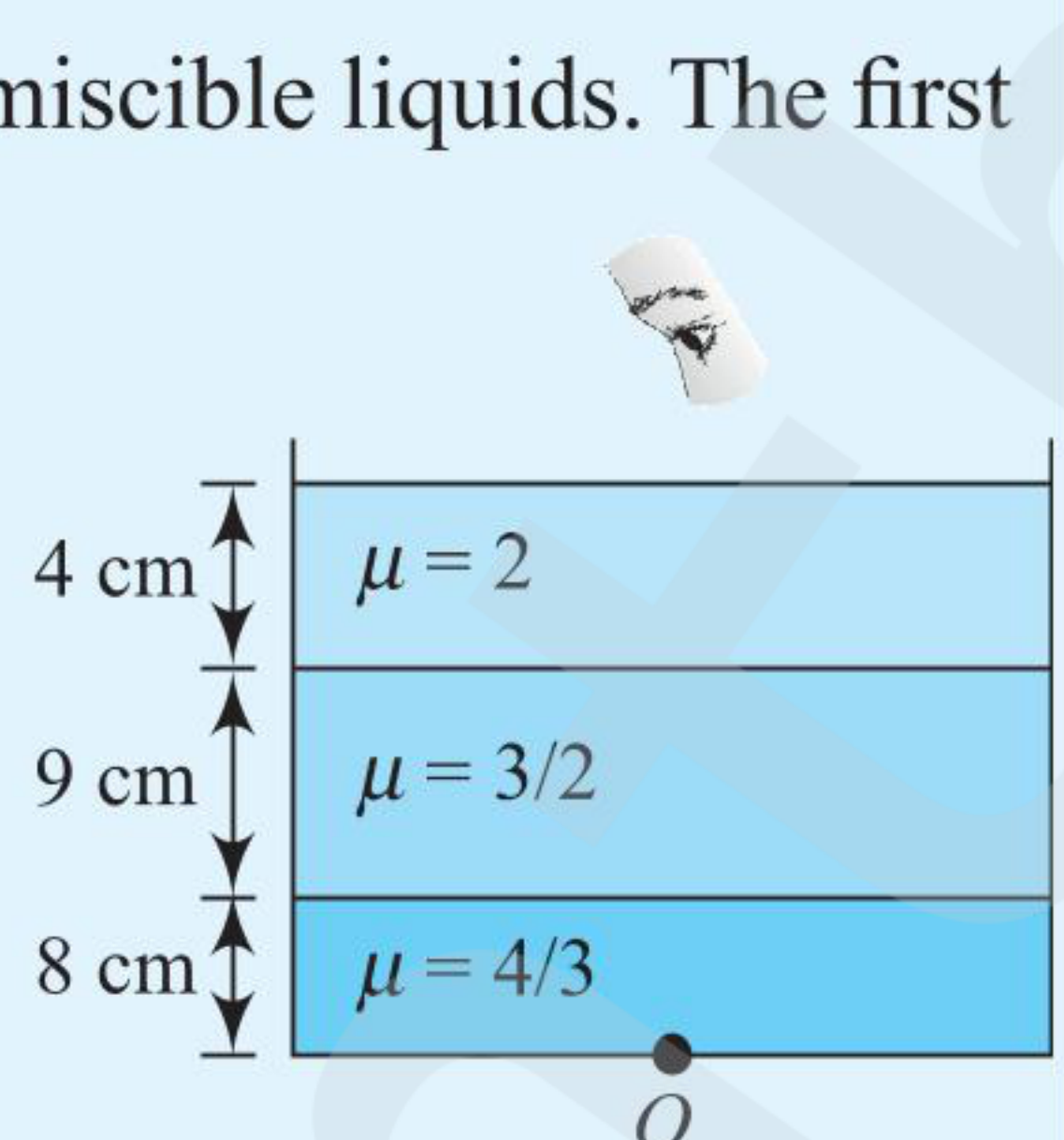
2. A beam of width t is incident at 45° on an air–water boundary. The width of the beam in water is _____.



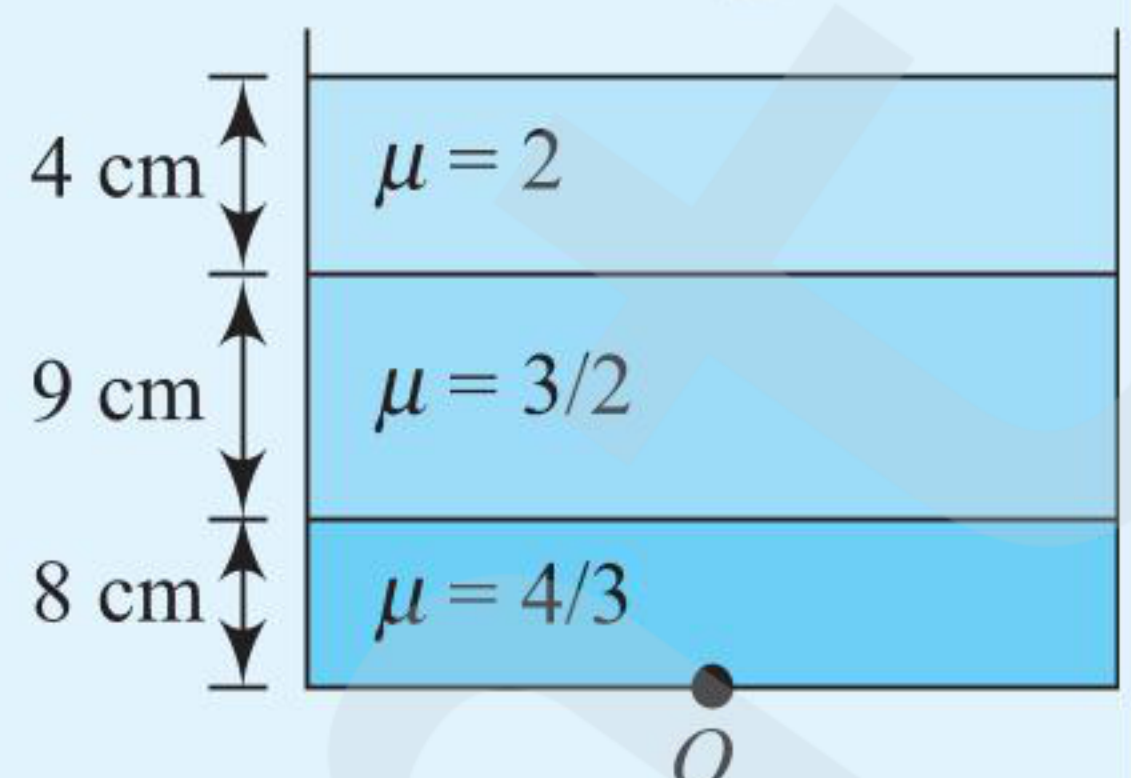
3. A fish is 60 cm under water ($\mu = 4/3$). A bird directly overhead looks at the fish. If the bird is at a distance of 120 cm from the water surface, (a) what is the apparent position of the fish as seen by the bird and (b) what is the apparent position of the bird as seen by the fish?



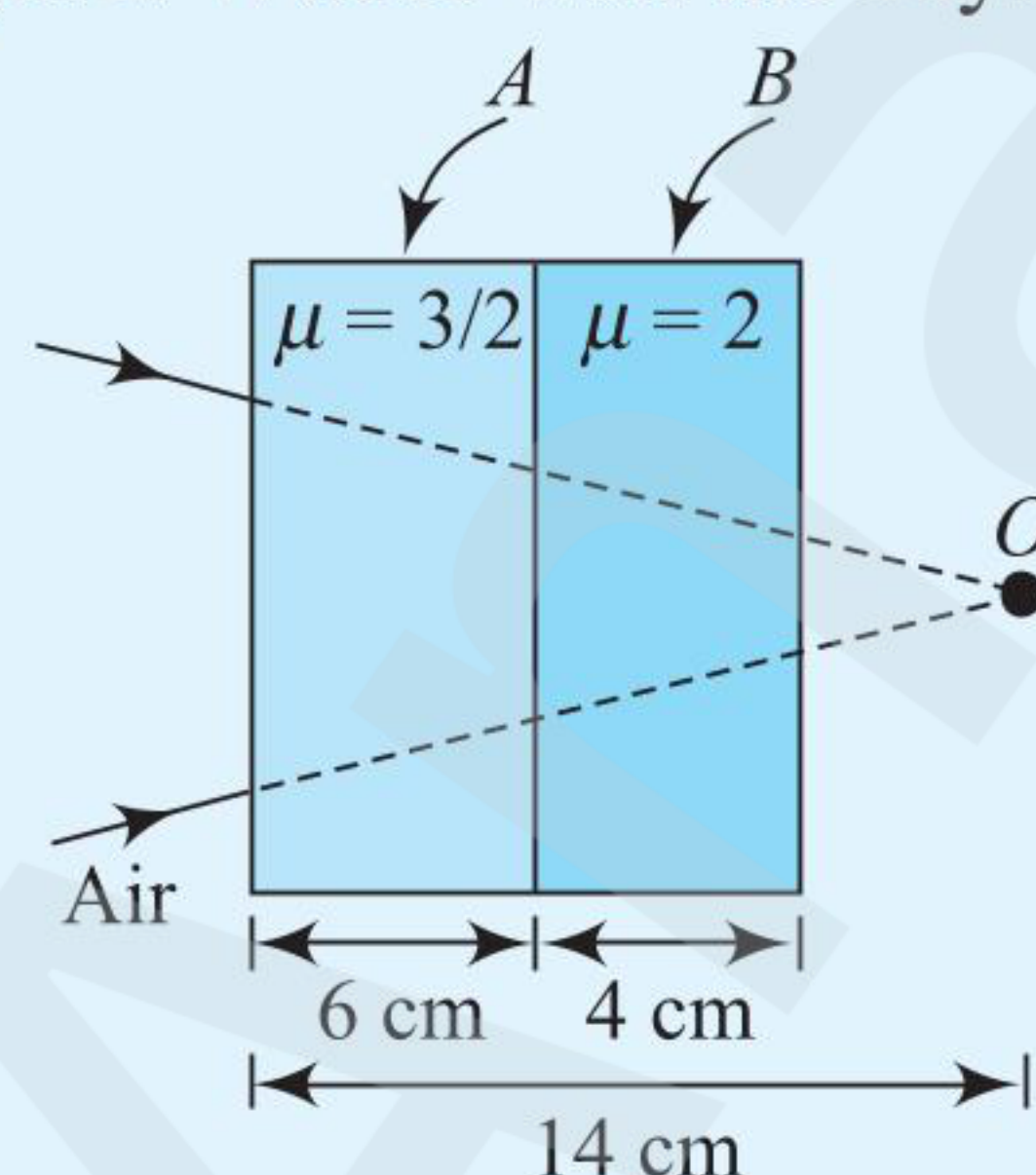
4. A light beam of diameter $\sqrt{3}R$ is incident symmetrically on a glass hemisphere of radius R and of refractive index $n = \sqrt{3}$. Find radius of the beam at the base of hemisphere.



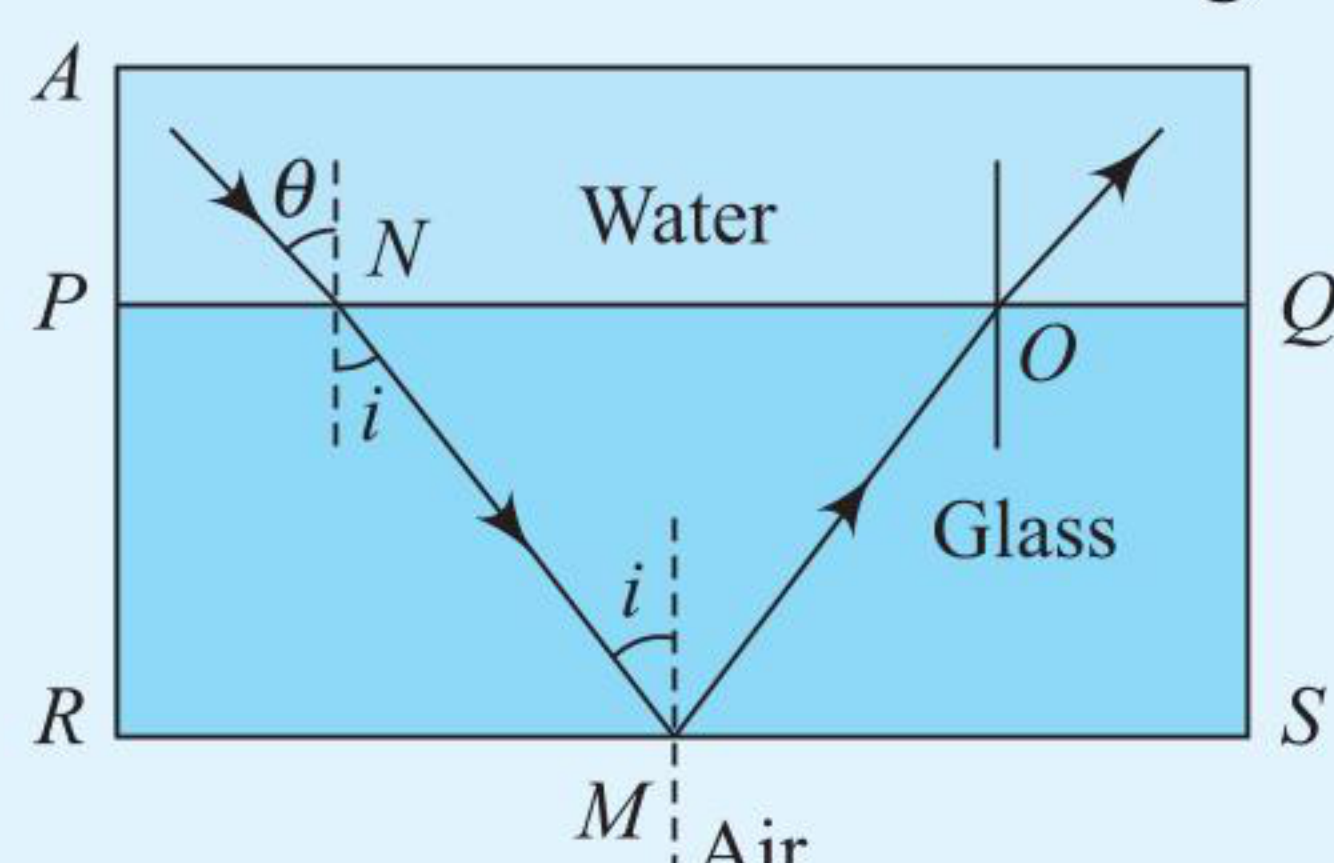
5. A tank contains three layers of immiscible liquids. The first layer is of water with refractive index $4/3$ and thickness 8 cm. The second layer is of oil with refractive index $3/2$ and thickness 9 cm while the third layer is of glycerine with refractive index 2 and thickness 4 cm. Find the apparent depth of the bottom of the container.



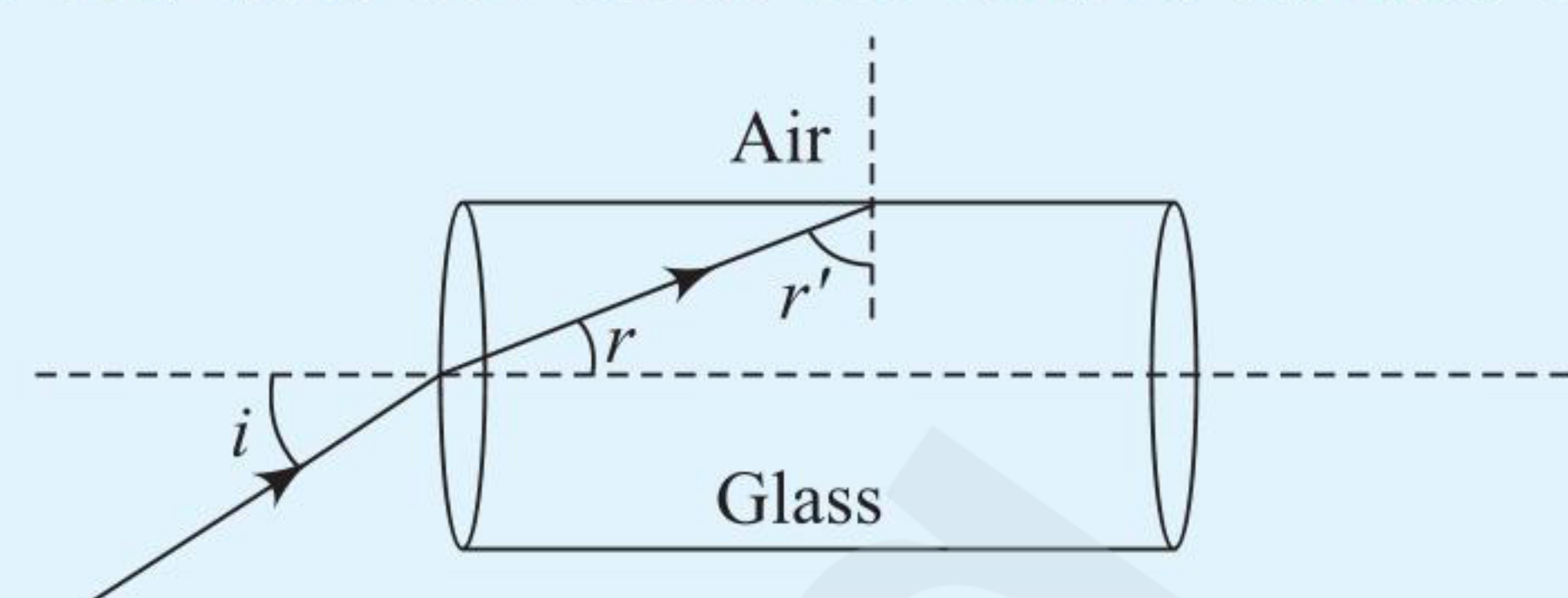
6. A convergent beam is incident on two slabs placed in contact as shown in figure. Where will the rays finally converge?



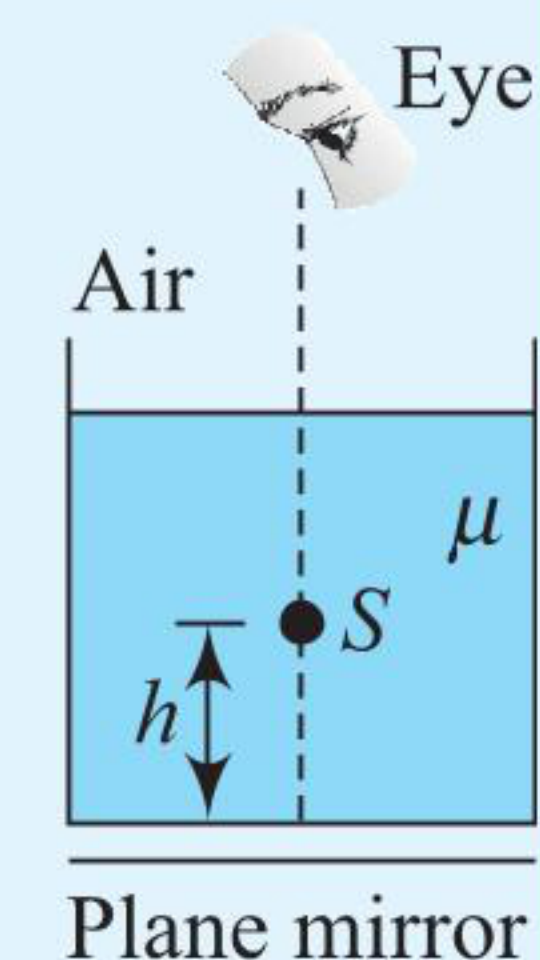
7. A slab of water is on the top of a glass slab of refractive index 2. At what angle to the normal must a ray be incident on the top surface of the glass slab so that it is reflected from the bottom surface as shown in figure?



8. What should be the value of refractive index μ of a glass rod placed in air, so that the light entering through the flat surface of the rod does not cross the curved surface of the rod?



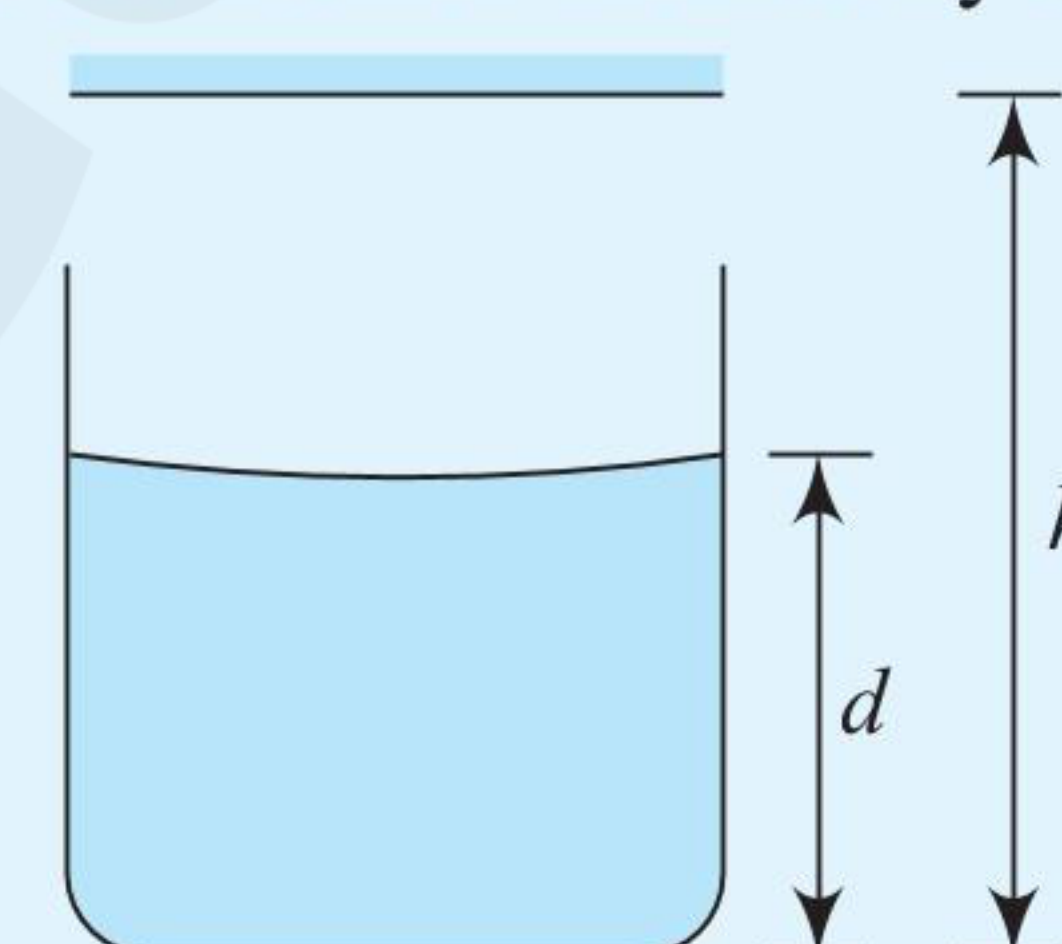
9. In figure, a point source S is placed at a height h above the plane mirror in a medium of refractive index μ .



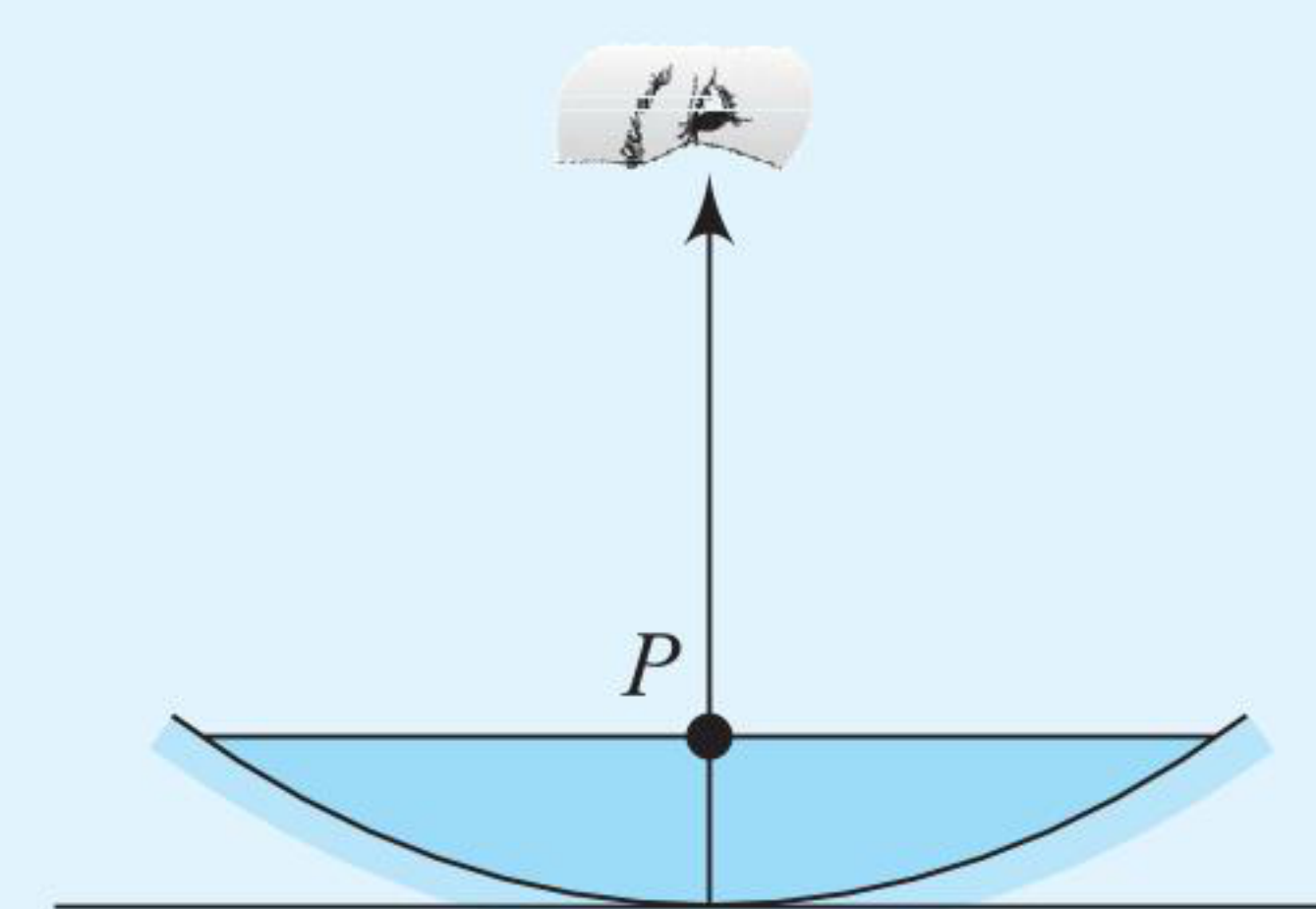
- (a) Find the number of images seen for normal view.

- (b) Find the distance between the images.

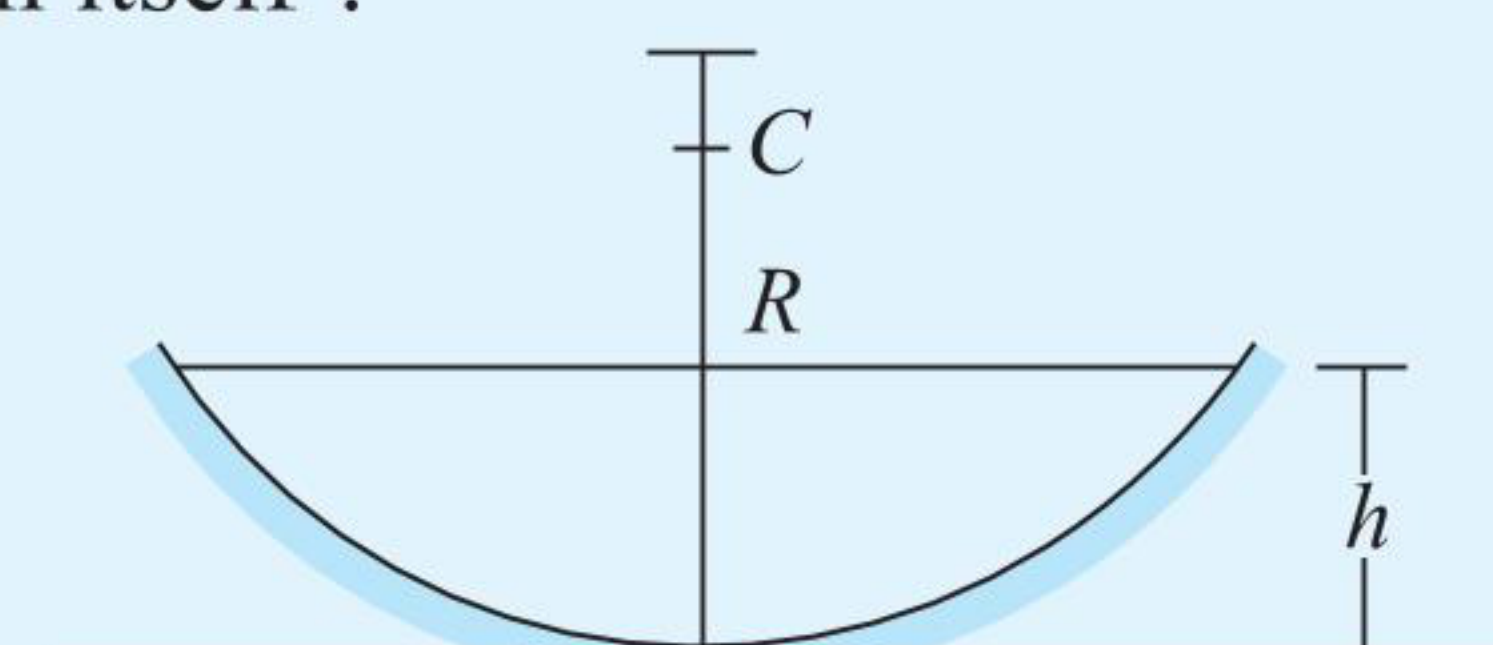
10. Consider the situation shown in figure. A plane mirror is fixed at a height h above the bottom of a beaker containing water (refractive index μ) up to a height d . Find the position of the image of the bottom formed by the mirror.



11. A concave mirror of radius 40 cm lies on a horizontal table and water is filled in it up to a height of 5.00 cm. A small dust particles floats on the water surface at a point P vertically above the point of contact of the mirror with the table. Locate the image of the dust particle as seen from a point directly above it. The refractive index of water is 1.33.



12. A concave mirror of radius R is kept on a horizontal table. Water (refractive index $= \mu$) is poured into it up to a height h . Where should an object be placed so that its image is formed on itself?



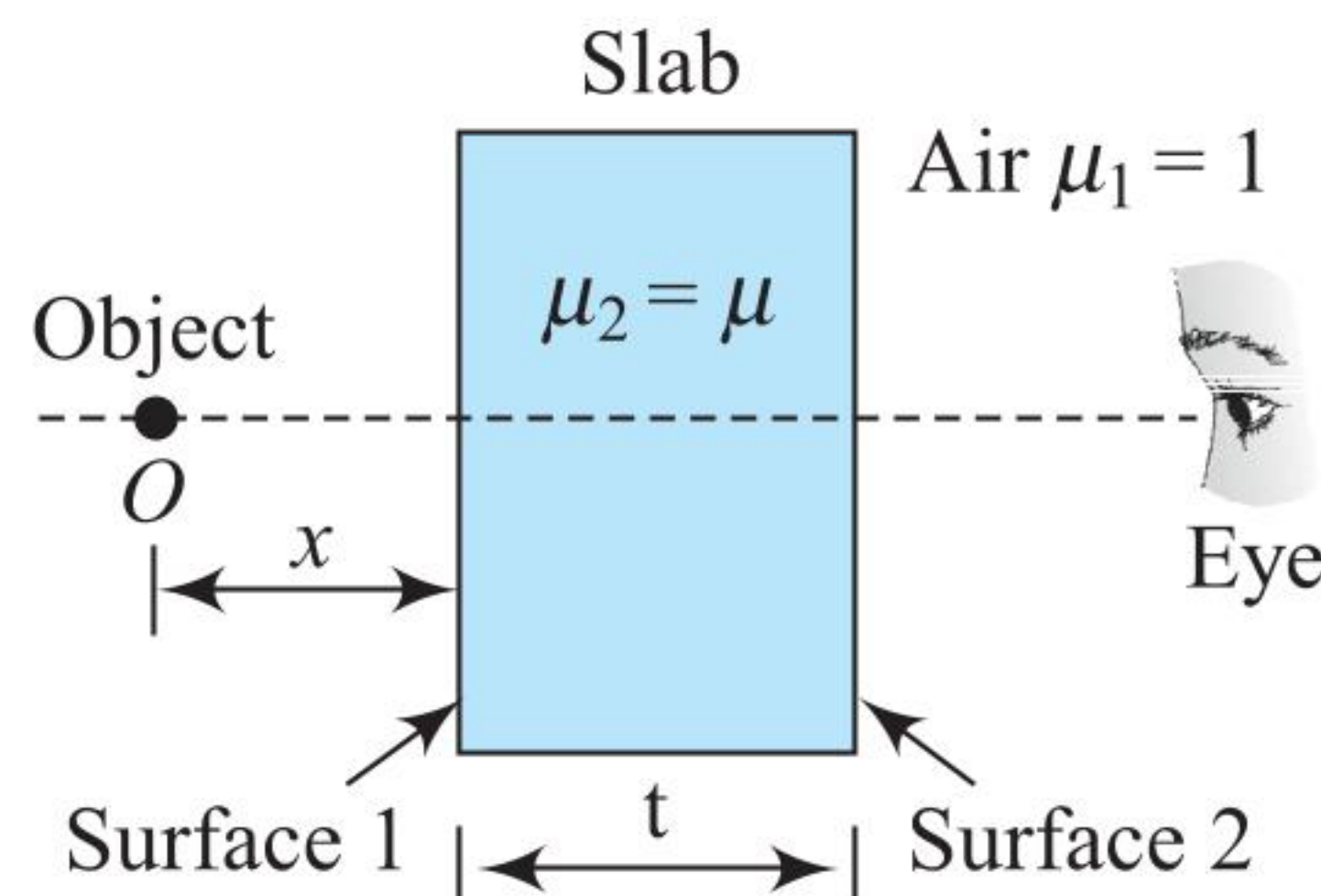
ANSWERS

1. 45 cm 2. $\frac{t\sqrt{2\mu^2-1}}{\mu}$ 3. 45 cm; 160 cm 4. $\frac{R}{\sqrt{3}}$
 5. 14 cm 6. 18 cm (left) 7. $\sin^{-1}\left(\frac{3}{4}\right)$
 9. (a) Two; (b) $\frac{2h}{\mu}$ 10. $h - d + (d/\mu)$ 11. 8.75 cm
 12. $\frac{R-h}{\mu}$

REFRACTION THROUGH A PARALLEL SLAB

A slab is formed when a medium is isolated from its surroundings by two plane surfaces parallel to each other. In this section, we will determine the position and nature of the image formed when a slab is placed in front of an object.

Consider an object O placed a distance x in front of a glass slab of thickness " t " and refractive index μ . The observer is on the other side of the slab. A ray of light from the object first refracts at surface (1) then refracts at surface (2) before reaching the observer (see figure). Let us analyse the location of image as seen by observer taking one step at a time.

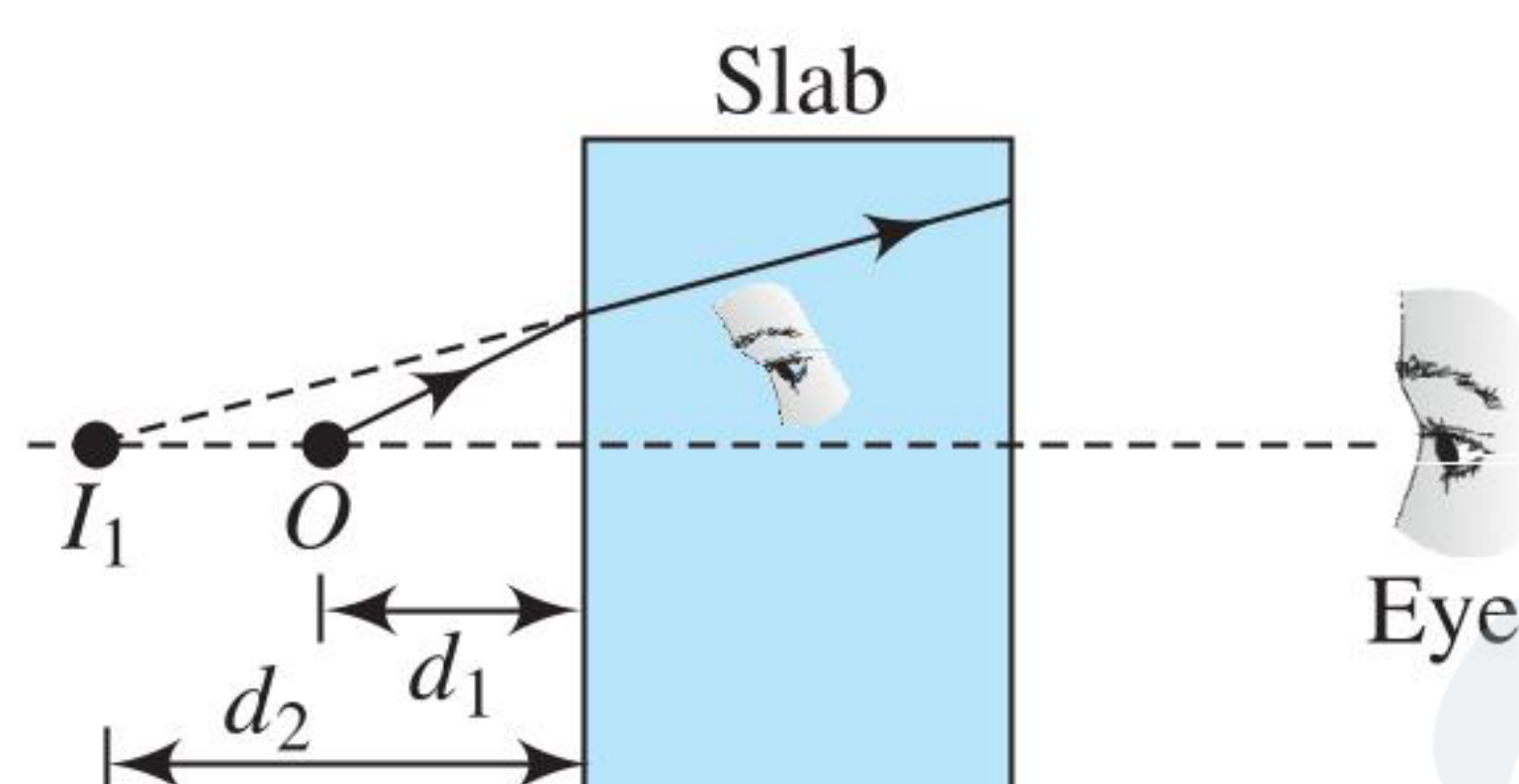


All objects viewed through a transparent glass slab

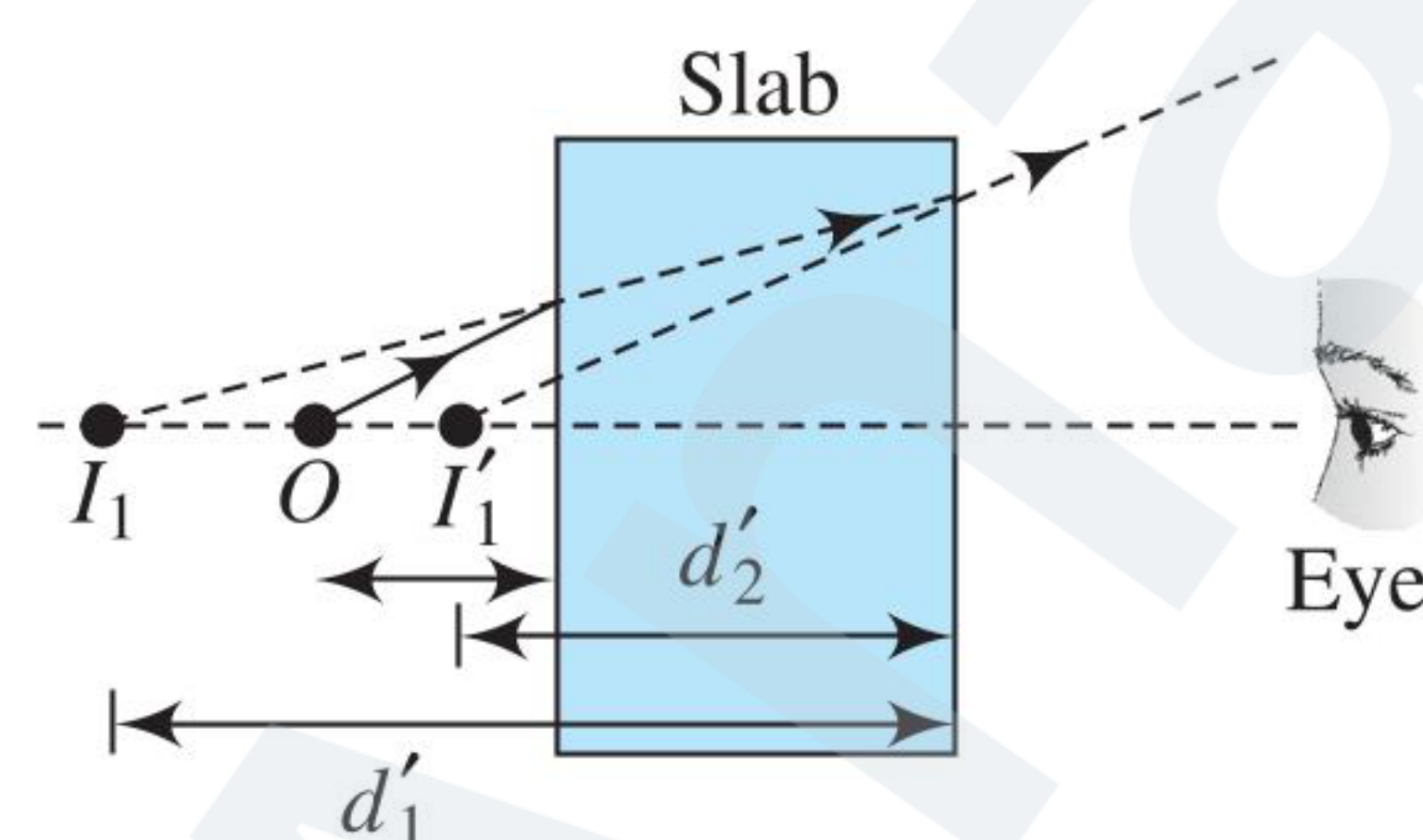
Refraction at surface 1:

Here, $\mu_1 = 1$, $\mu_2 = \mu$, $d_1 = \text{real depth}$, $d_2 = \text{apparent depth}$

$$\text{Apparent depth, } d_2 = \frac{d_{\text{real}}}{n_{\text{relative}}} = \frac{d_1}{\left(\frac{n_{\text{incident}}}{n_{\text{refraction}}}\right)} = \frac{d_1}{\left(\frac{\mu_1}{\mu_2}\right)}$$



(a) The apparent position of the object after refraction at the first surface



(b) Final position of the object after refraction at both surfaces

$$d_2 = \frac{x}{(1/\mu)}$$

$$d_2 = \mu x \quad \dots(i)$$

Thus, the first image is formed behind the object at the point I_1 . I_1 now serves as the object for the second surface [see Fig. (a)].

Refraction at surface 2:

Here, $n_{\text{incident}} = \mu_2 = \mu$
 $n_{\text{refraction}} = \mu_1 = 1$

$$\text{Apparent depth, } d_2' = \frac{d_{\text{real}}}{n_{\text{real}}}$$

$$d_2' = \frac{d_1'}{\left(\frac{\mu}{1}\right)} \text{ but } d_1' = (d_2 + t)$$

$$d_2' = \frac{(\mu x + t)}{\mu} \quad \dots(ii)$$

Thus, the final image I_1' is at a distance $x + (t/\mu)$ behind the second interface. The original object was at a distance $x + t$ behind the second interface. Therefore, the image appears shifted by

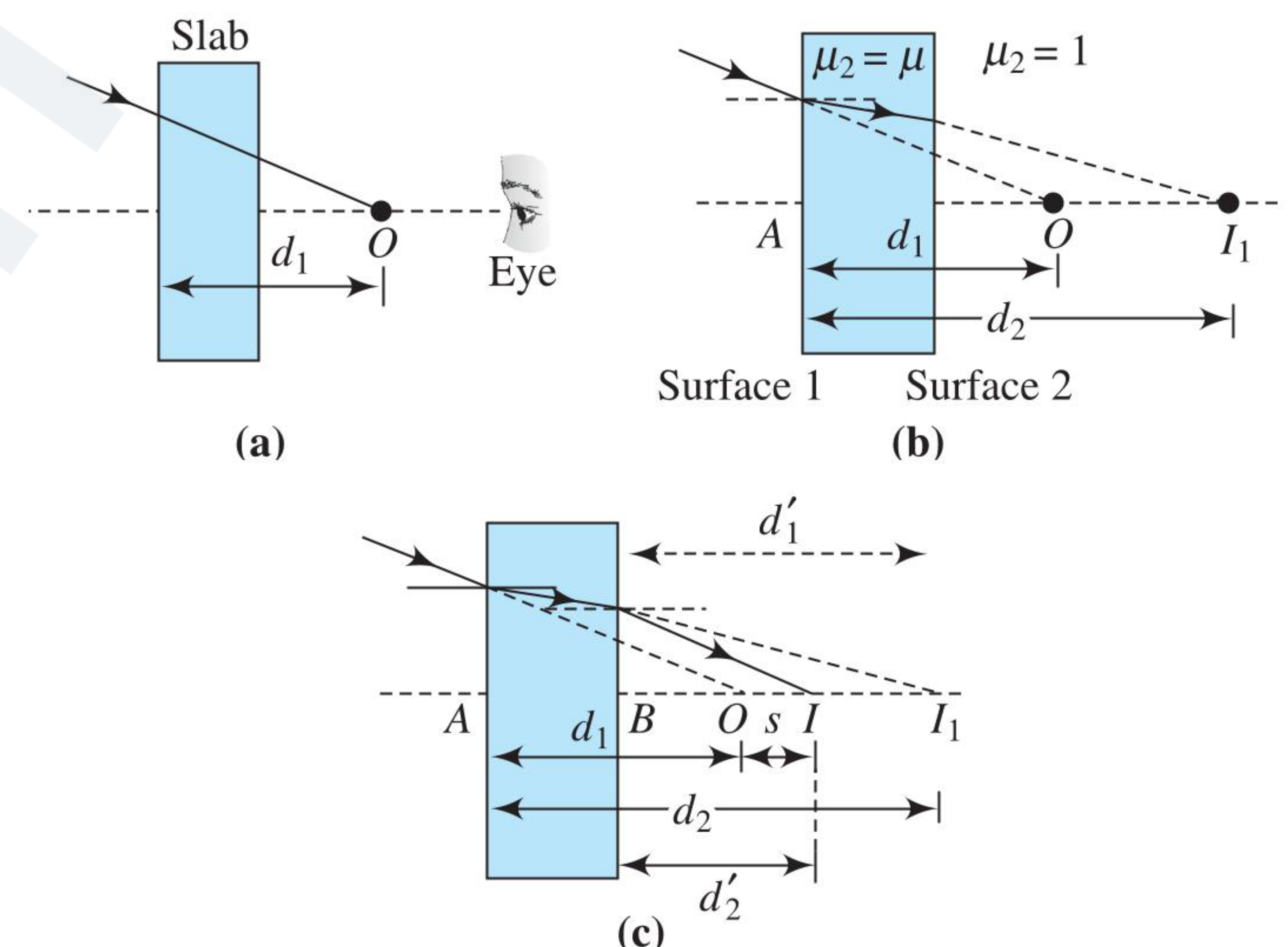
$$s = \left[x + \frac{t}{\mu} \right] - [x + t] = t \left[1 - \frac{1}{\mu} \right] \quad \dots(iii)$$

We can say a slab appears to shift the object along the perpendicular to the slab by a distance $t[1 - (1/\mu)]$ in the direction of the travelling ray [see Fig. (b)].

How about the direction of the shift?

As discussed in the previous section the object is shifted towards the glass slab. Is it always true?

Consider a converging set of rays incident on a glass slab as shown in Fig. (a). Here, we have a virtual object to the right of the slab. Following the same procedure as that in earlier, we can show that the net shift is same as given by Eq. (iii). However, let us look a little closely at the direction of the shift.



Here, μ is the refractive index of the slab with respect to surrounding medium ($\mu = \mu_2/\mu_1$). If the refractive index of medium is greater than the refractive index of slab, then shifting (s) will be negative. In this case, the shifting will be in the direction opposite to the travelling ray.

Refraction at surface 1:

The converging rays will be focused at O . The point O will act as virtual object for refraction through surface 1.

For surface 1: Real depth for surface 1, $AO = d_1$

Here, $n_{\text{incident}} = \mu_1 = 1$

$n_{\text{refraction}} = \mu_2 = \mu$

$$\text{Apparent depth, } d_2 = \frac{d_{\text{real}}}{n_{\text{relative}}} = \frac{d_1}{\left(\frac{n_{\text{incident}}}{n_{\text{refraction}}}\right)}$$

$$d_2 = \frac{d_1}{\left(\frac{1}{\mu}\right)} = \mu d_1$$

Refraction at surface 2:

After refraction from surface 1, the image of O will form at I_1 . For surface 2, I_1 will act as an object.

For surface 2: Real depth for surface $BI_1 = d'_1 = (d_2 - t) = (\mu d_1 - t)$

Here, $n_{\text{incident}} = \mu_2 = \mu$

$n_{\text{refraction}} = \mu_1 = 1$

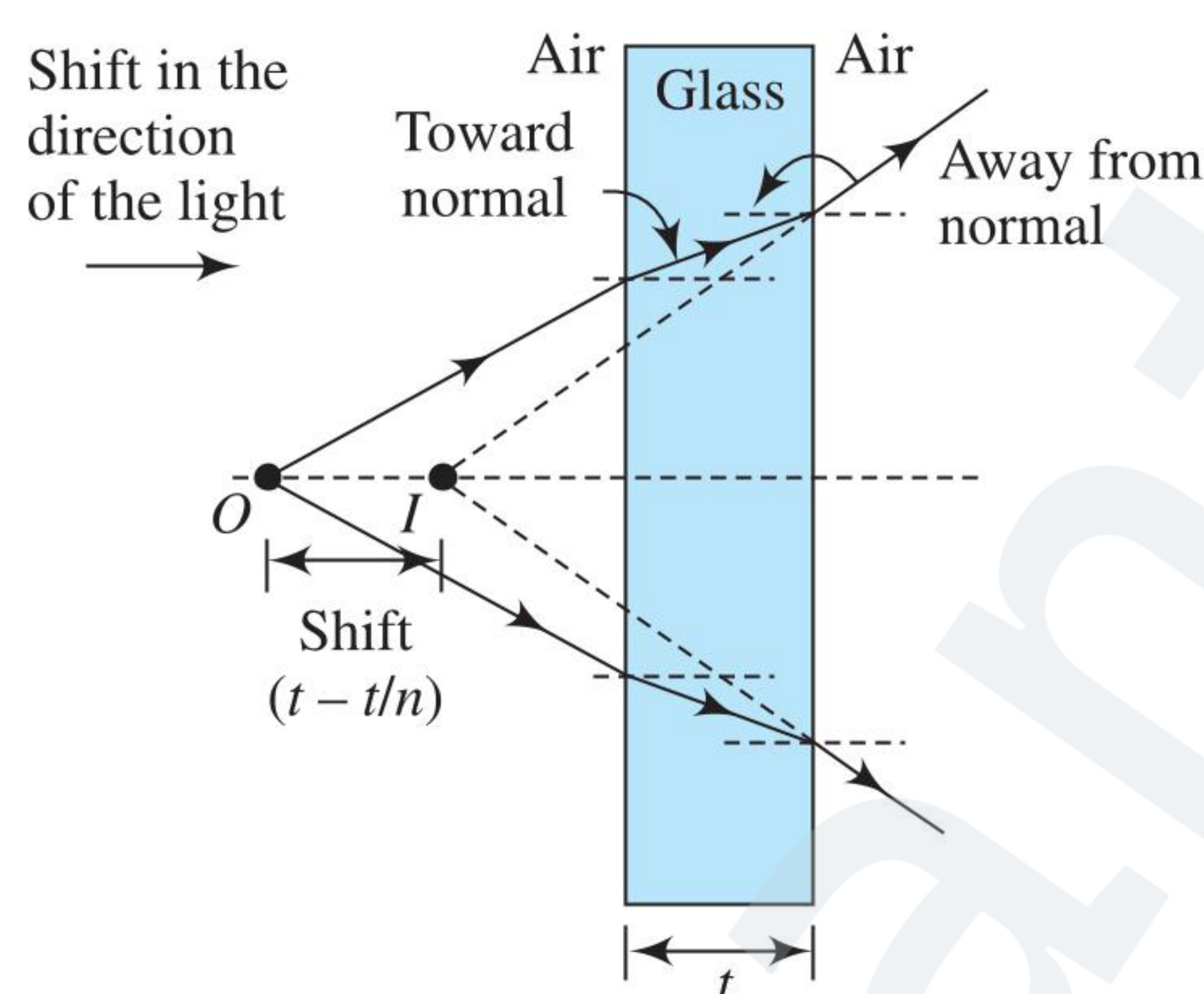
$$\begin{aligned} \text{Apparent depth, } d'_2 &= \frac{d_{\text{real}}}{n_{\text{relative}}} = \frac{d'_1}{\left(\frac{n_{\text{incident}}}{n_{\text{refraction}}}\right)} \\ &= \frac{d'_1}{\left(\frac{\mu}{1}\right)} = \frac{(\mu d_1 - t)}{\mu} \end{aligned}$$

$$d'_2 = \frac{(\mu d_1 - t)}{\mu} = \left(d_1 - \frac{t}{\mu}\right)$$

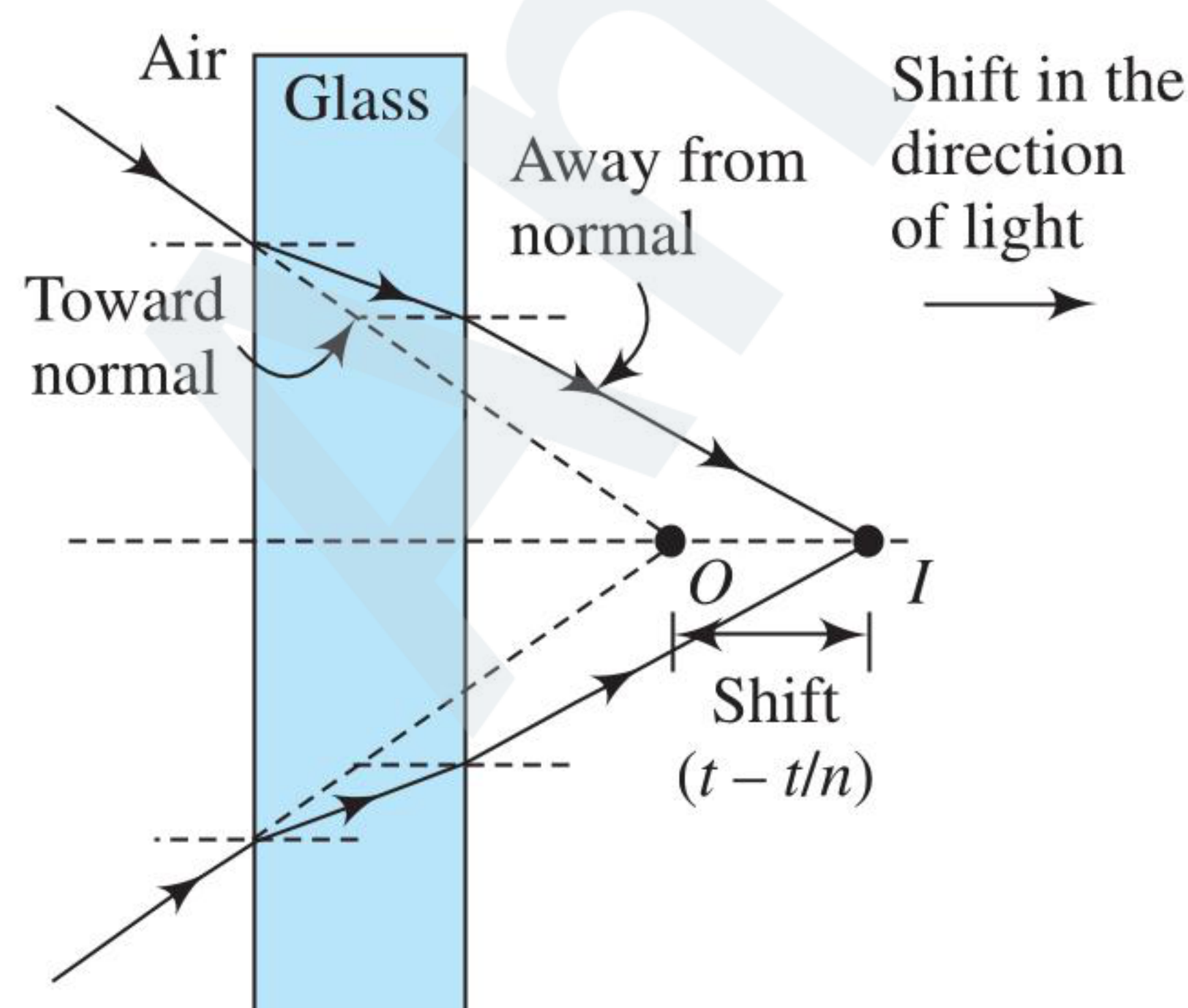
Hence, shifting of object position from slab

$$OI = s = BI - BO = d'_2 - (d_1 - t)$$

$$s = \left(d_1 - \frac{t}{\mu}\right) - (d_1 - t) = \left(t - \frac{t}{\mu}\right) = t \left(1 - \frac{1}{\mu}\right)$$

Image formation by a slab:

(a) Image formed is virtual

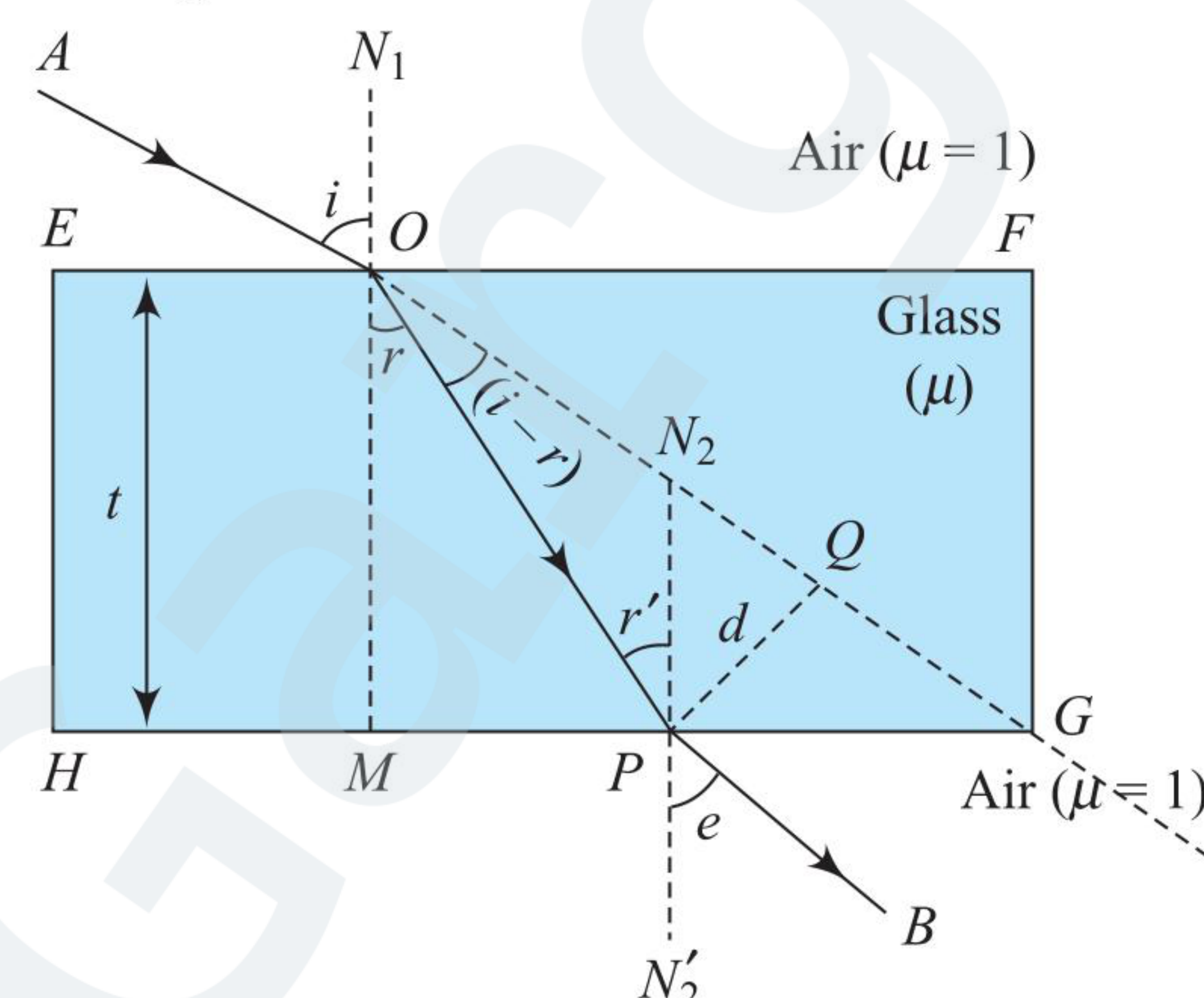


(b) Image formed is real

In both cases, shifting is in the direction of the travelling ray.

LATERAL DISPLACEMENT OF EMERGENT BEAM THROUGH A GLASS SLAB

Figure represents the refraction of a ray AO incident on the slab at an angle of incidence i through the glass slab $EFGH$. At face EF , the incident ray AO is refracted along OP , the angle of refraction being r .



1. If e is angle of emergence, then from Snell's law at faces EF and HG ,

$$\mu_a \sin i = \mu \sin r \quad \text{and} \quad \mu \sin r' = \mu_a \sin e$$

$$r' = r \quad \text{and} \quad \mu_a = 1, \text{ we have}$$

$$\sin i = \sin e \text{ or } e = i$$

That is the emergent ray is parallel to the incident ray.

2. Let PQ be the perpendicular dropped from P on incident ray produced.

Then, the lateral displacement caused by the plate,

$$\begin{aligned} d = PQ &= OP \sin(i - r) = \frac{OM}{\cos r} \sin(i - r) \\ &= \frac{t \sin(i - r)}{\cos r} \end{aligned}$$

Special Cases

If i is very small, r is also very small, then

$\sin i \rightarrow i$, $\sin r \rightarrow r$ and $\cos r \rightarrow 1$ so that

$$\frac{\sin i}{\sin r} = \mu \text{ takes the form } \frac{i}{r} = \mu.$$

Therefore, the expression for lateral displacement takes the form

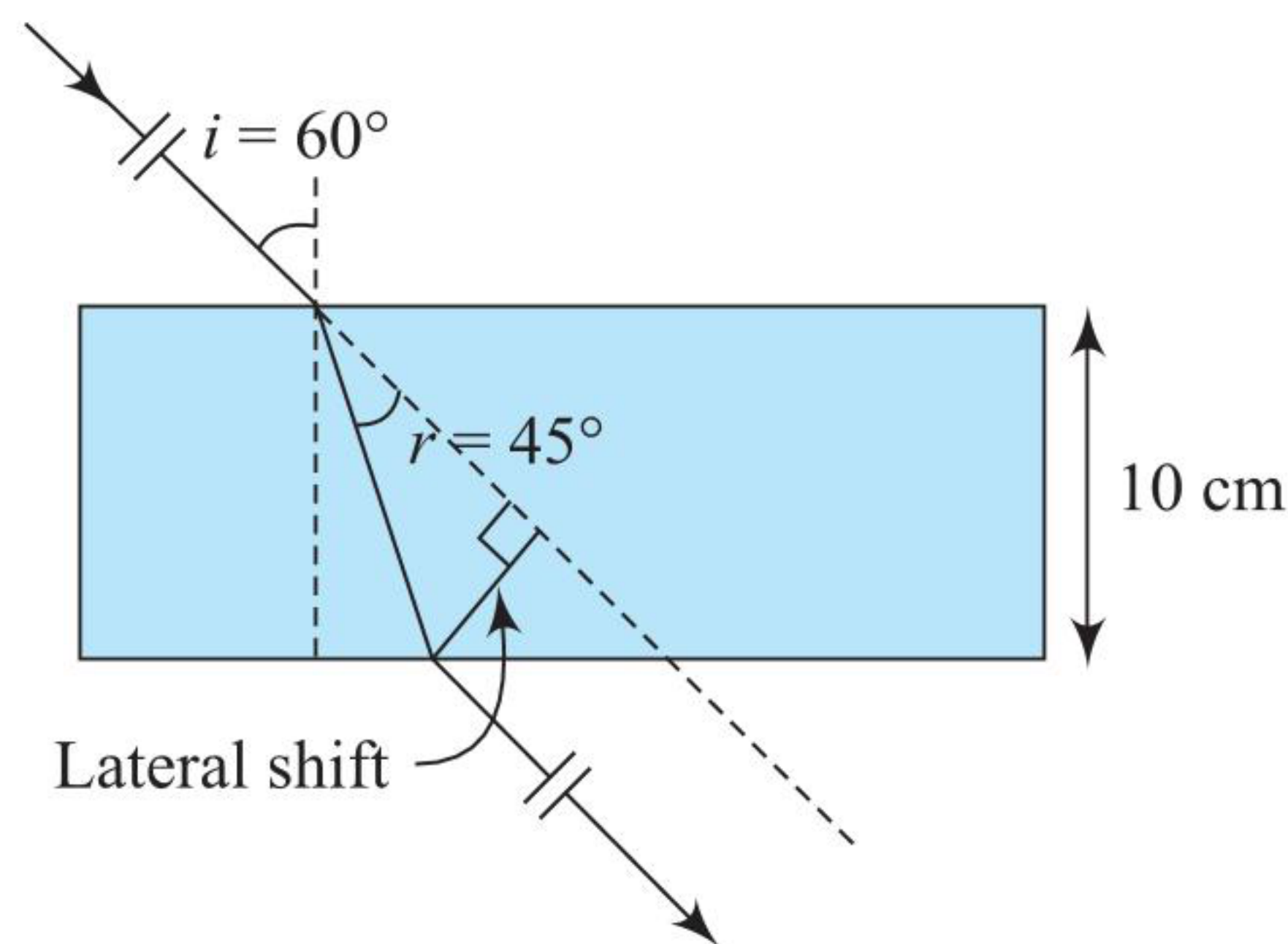
$$d = t(i - r) = ti \left(1 - \frac{r}{i}\right) = ti \left(1 - \frac{1}{\mu}\right)$$

ILLUSTRATION 1.63

Find the lateral shift of a light ray while it passes through a parallel glass slab of thickness 10 cm placed in air. The angle of incidence in air is 60° and the angle of refraction in glass is 45° .

Sol.
$$d = \frac{t \sin(i - r)}{\cos r} = \frac{10 \sin(60^\circ - 45^\circ)}{\cos 45^\circ}$$

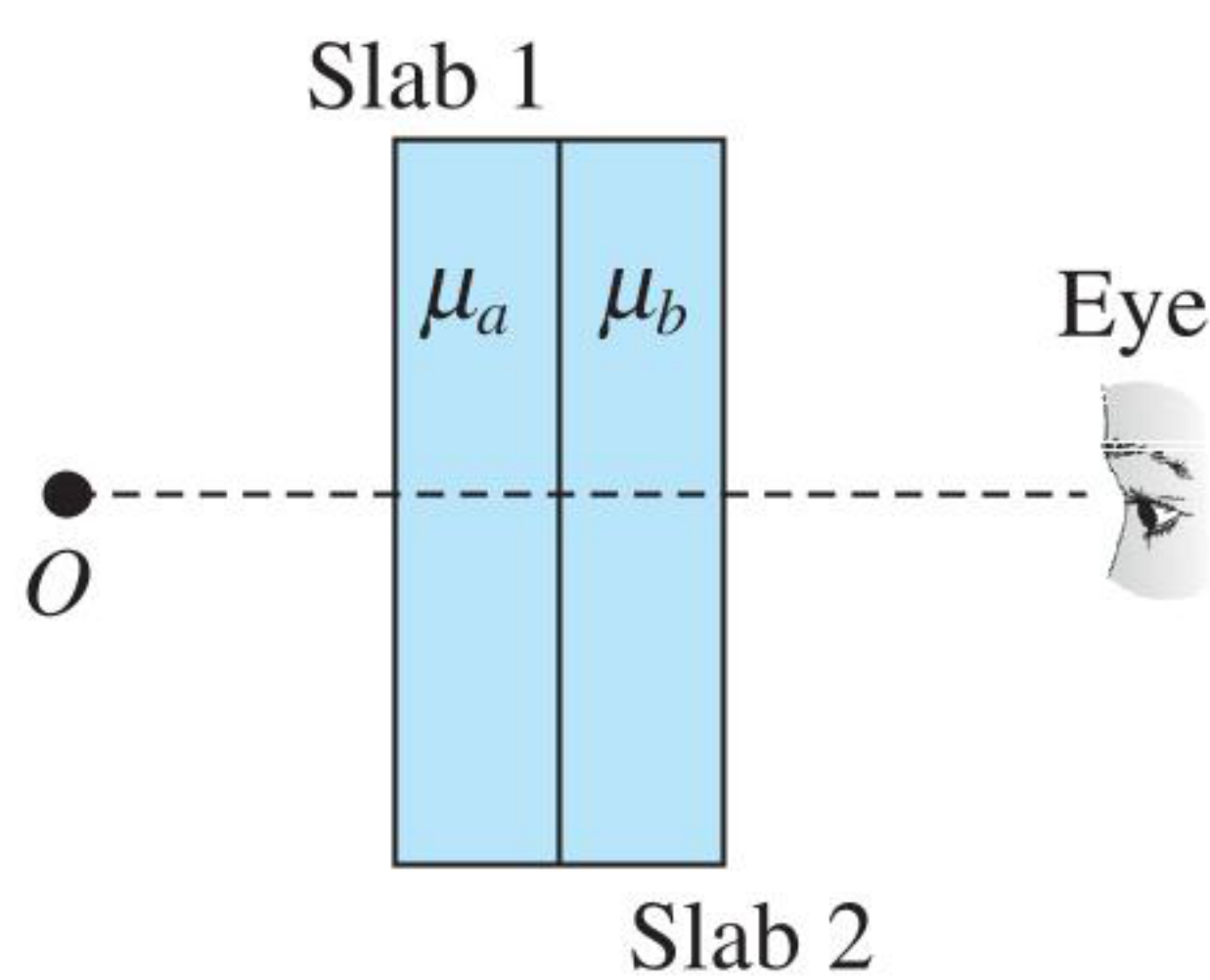
$$= \frac{10 \sin 15^\circ}{\cos 45^\circ} = 10\sqrt{2} \sin 15^\circ$$



REFRACTION ACROSS MULTIPLE SLABS

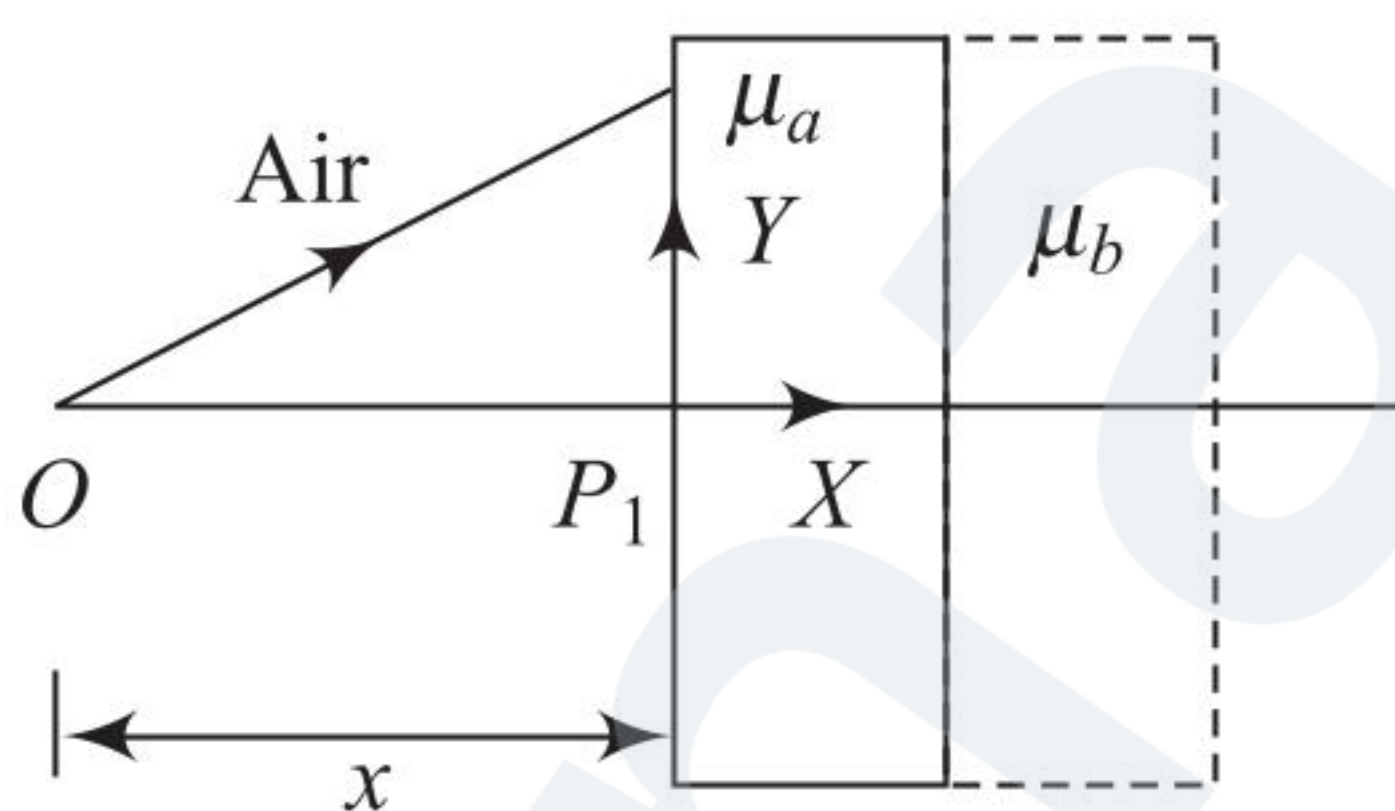
In figure, an object is placed in front of two slabs in contact. The thickness and refractive indices of the slabs are t_1, μ_a and t_2, μ_b , respectively. Where will the final image of the object appear to be?

A light ray emerging from O now refracts at three surfaces. The first is between air and μ_a the second between μ_a and μ_b while the third is between μ_b and air. Let us solve the problem taking one step at a time.



An object placed in front of two glass slabs in contact

First interface (see figure): Here, $\mu_1 = 1, \mu_2 = \mu_a$



$$d_1 = x, d_2 = ?$$

$$d_2 = \frac{d_1}{\mu_{\text{relative}}} = \frac{d_1}{\mu_1/\mu_2}$$

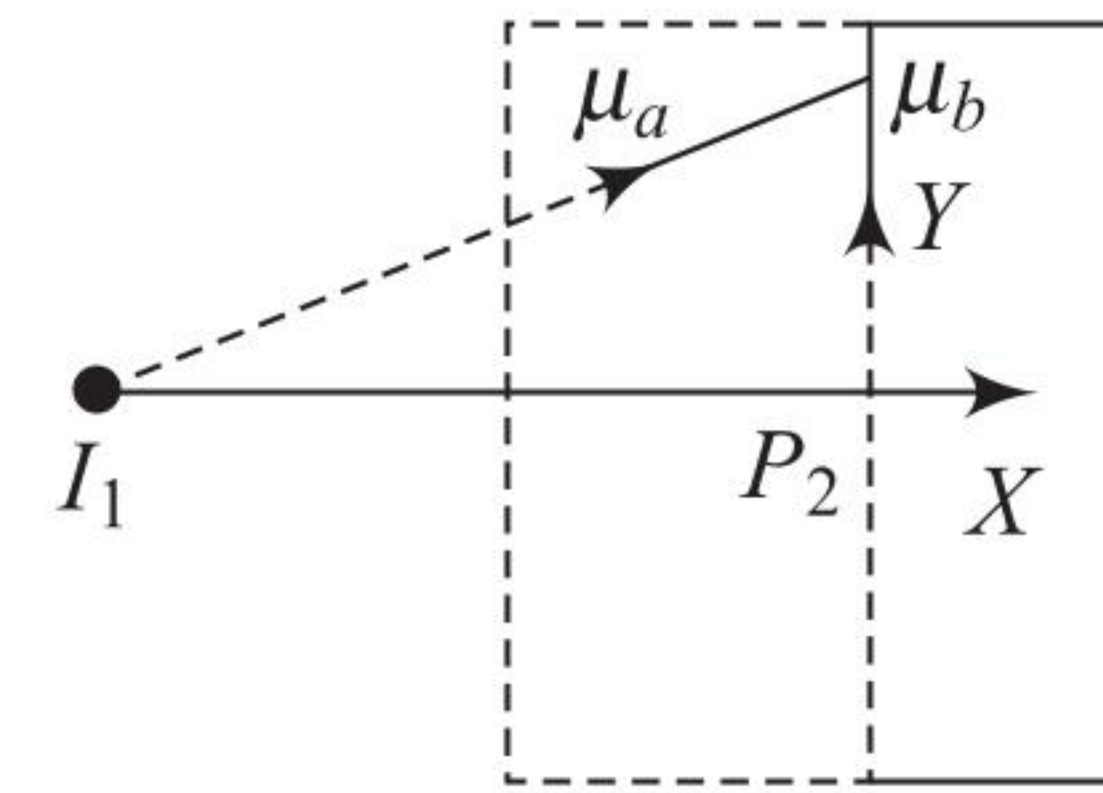
$$\Rightarrow \frac{\mu_1}{d_1} = \frac{\mu_2}{d_2}$$

$$\text{or } d_2 = \frac{\mu_2}{\mu_1} d_1 = \mu_a d_1$$

Therefore, the image distance, $d_2 = -\mu_a x$... (i)

Second interface (see figure):

Here, $\mu_1 = \mu_a, \mu_2 = \mu_b$

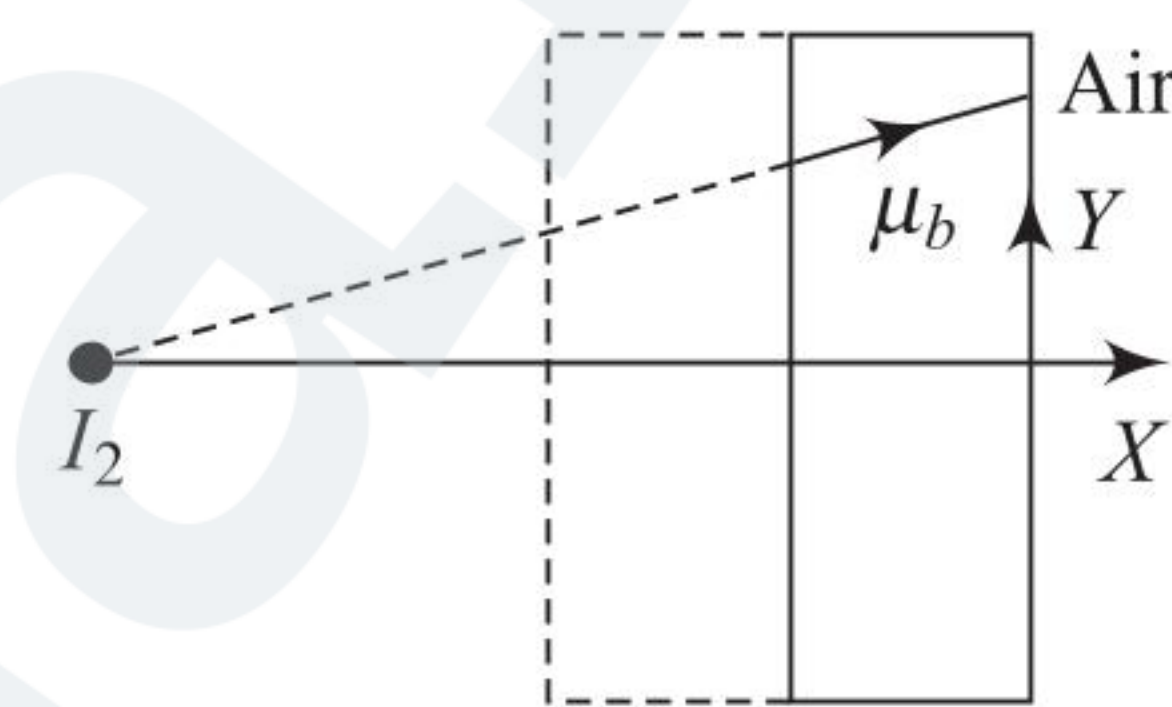


$$d_1 = -(\mu_a x + t_1), d_2 = ?$$

$$\text{Since } \frac{\mu_1}{d_1} = \frac{\mu_2}{d_2}$$

$$\text{The image distance, } d_2 = -\mu_b \left(x + \frac{t_1}{\mu_a} \right) \quad \dots (ii)$$

Third interface (see figure):



Here, $\mu_1 = \mu_b, \mu_2 = 1$

$$d_1 = -\mu_b \left(x + \frac{t_1}{\mu_a} \right) + t_2, d_2 = ?$$

and the final image distance from the 3rd interface is

$$d_2 = - \left[x + \frac{t_1}{\mu_a} + \frac{t_2}{\mu_b} \right] \quad \dots (iii)$$

Therefore, the net shift in the position of the image is

$$s = - \left(x + \frac{t_1}{\mu_a} + \frac{t_2}{\mu_b} \right) - [-(x + t_1 + t_2)]$$

$$= t_1 \left(1 - \frac{1}{\mu_a} \right) + t_2 \left(1 - \frac{1}{\mu_b} \right) \quad \dots (iv)$$

Looking at the above result, we realize that the net shift in the position of the image is simply the sum of the individual shifts at each of the slabs if they were independently placed in air.

Thus, the simple problem of refraction in a glass slab can be tackled in two ways:

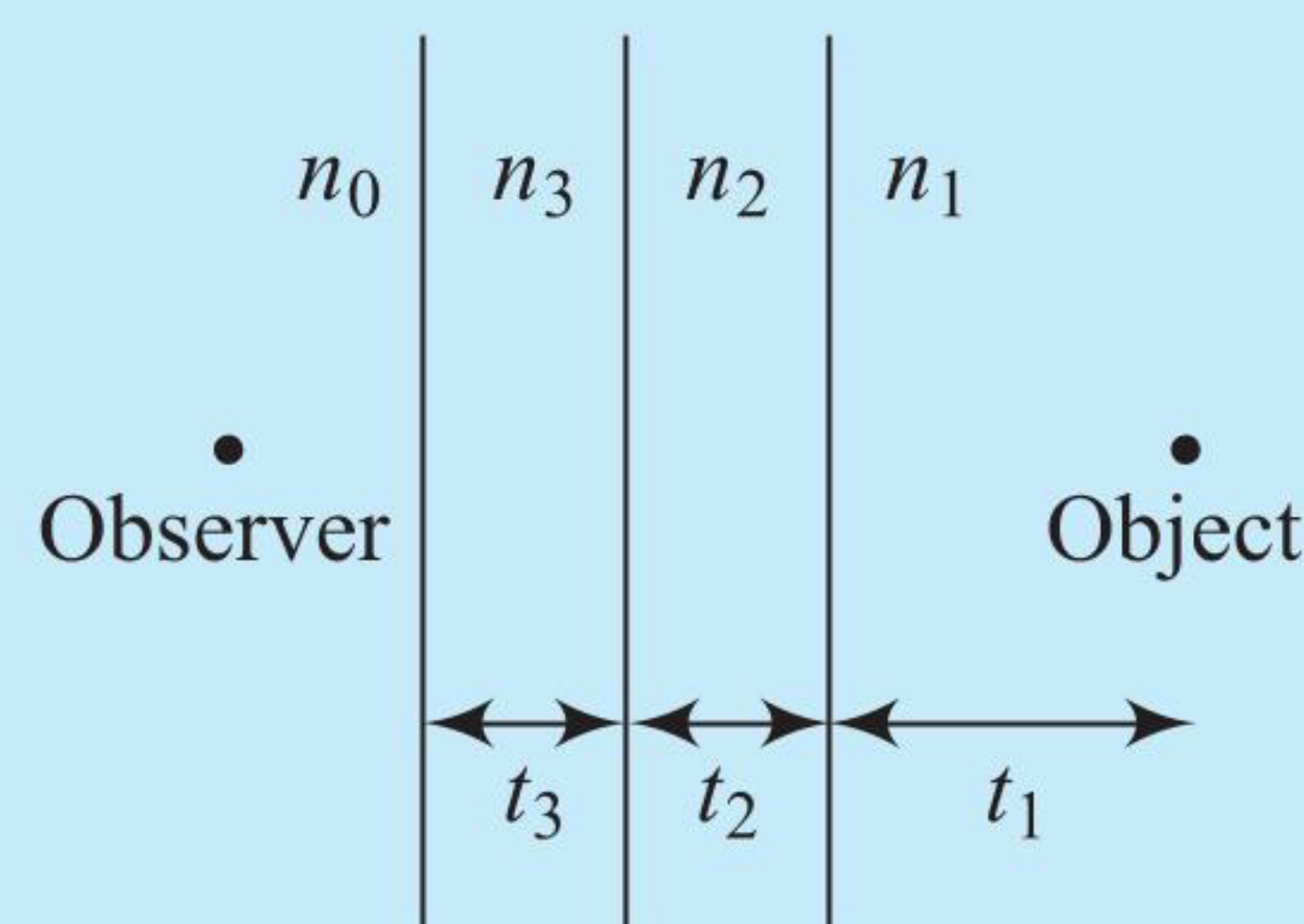
1. By the method of interfaces: Here, the refraction formula, equation $(\mu_1/d_1 = \mu_2/d_2)$, is applied at each interface.
2. By the method of elements: Here, the slab itself is an element with a governing equation $t \left[1 - \frac{1}{\mu} \right]$.

Most problems in ray optics can be solved by either of the two methods. In the following problems, we shall solve problems in both ways and highlight the method that gives a quicker solution.

Note: Refraction through a composite slab (or refraction through a number of parallel media, as seen from a medium of RI n_0)

- Apparent depth (distance of final image from final surface)

$$= \frac{t_1}{n_{1rel}} + \frac{t_2}{n_{2rel}} + \frac{t_3}{n_{3rel}} + \dots + \frac{t_n}{n_{nrel}}$$



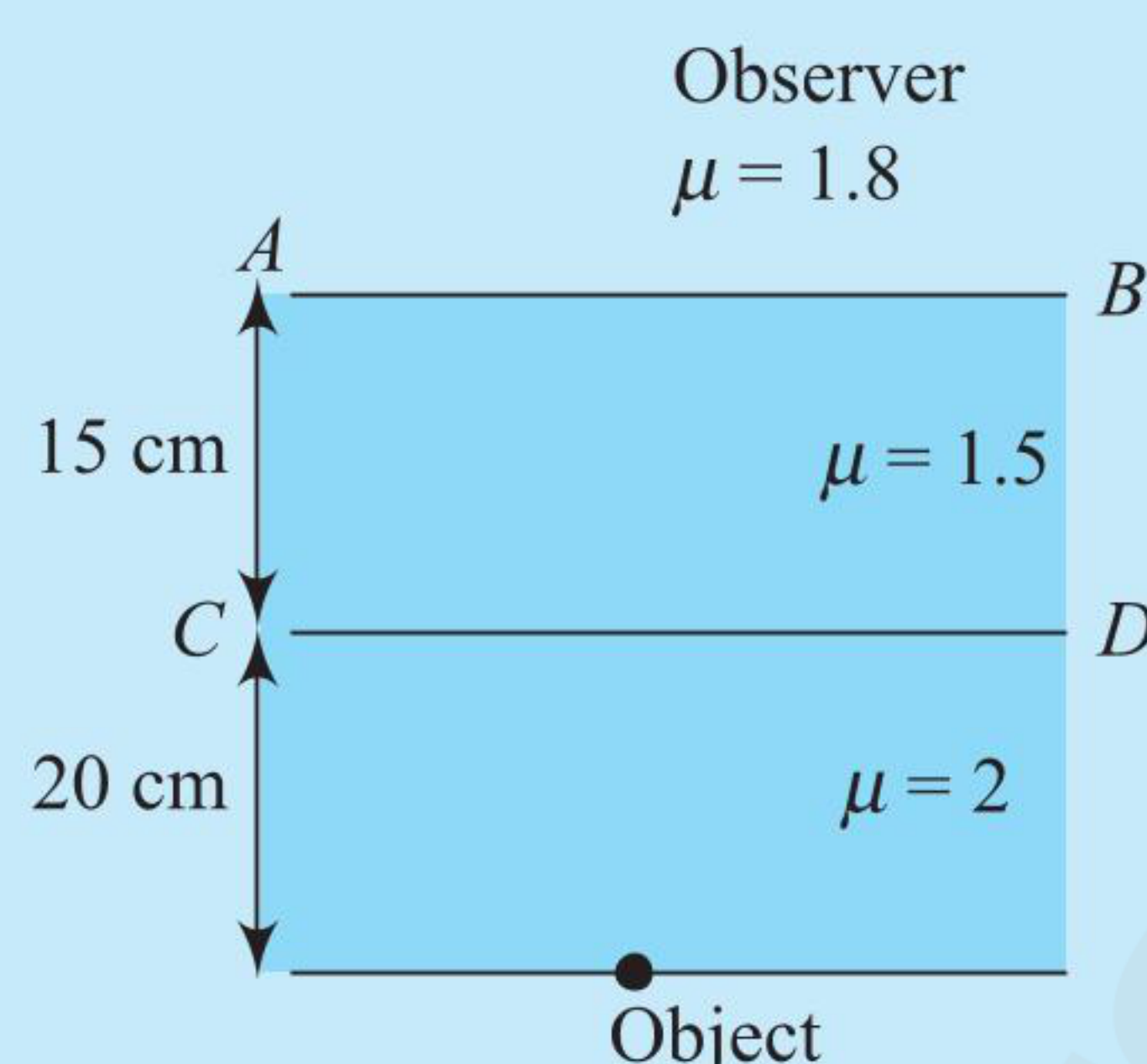
- Apparent shift

$$= t_1 \left[1 - \frac{1}{n_{1rel}} \right] + t_2 \left[1 - \frac{1}{n_{2rel}} \right] + \dots + t_n \left[1 - \frac{1}{n_{nrel}} \right]$$

where t represents thickness and n represents the RI of the respective media, relative to the medium of observer (i.e., $n_{1rel} = n_1/n_0$, $n_{2rel} = n_2/n_0$, etc.)

ILLUSTRATION 1.64

In figure, find the apparent depth of the object seen below surface AB .



Sol. $D_{app} = \sum \frac{d}{\mu} = \frac{20}{\left(\frac{2}{1.8}\right)} + \frac{15}{\left(\frac{1.5}{1.8}\right)} = 18 + 18 = 36 \text{ cm}$

ILLUSTRATION 1.65

A layer of oil 3 cm thick is floating on a layer of coloured water 5 cm thick. Refractive index of coloured water is $5/3$ and the apparent depth of the two liquids appears to be $36/7$ cm. Find the refractive index of oil.

Sol. Apparent depth (AI) $= \frac{t_1}{\mu_1} + \frac{t_2}{\mu_2}$

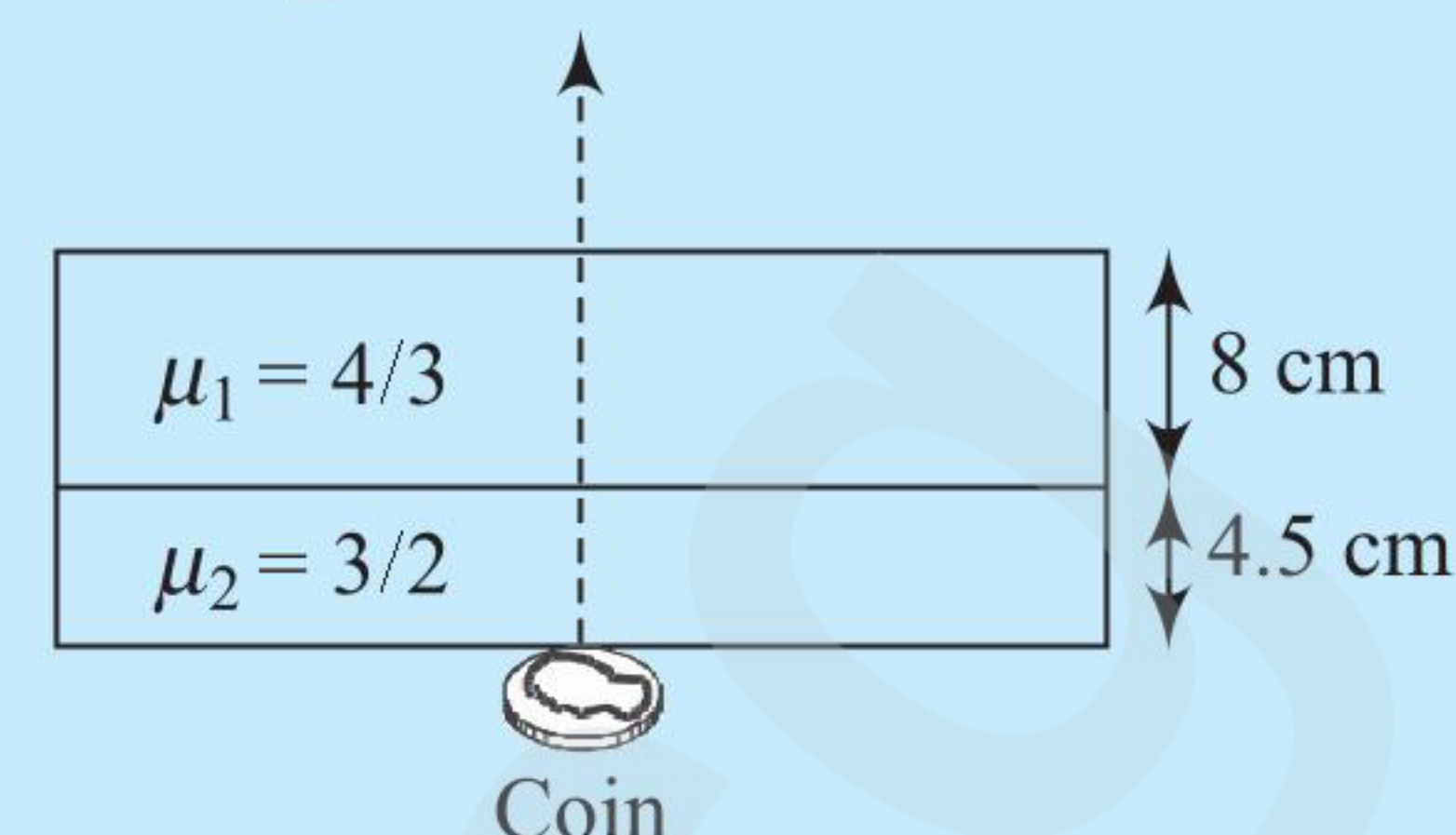
$$\therefore \frac{36}{7} = \frac{5}{5/3} + \frac{3}{\mu_2}$$

$$\Rightarrow \frac{3}{\mu_2} = \frac{36}{7} - 3 = \frac{15}{7}$$

$$\Rightarrow \mu_2 = \frac{7}{5} = 1.4$$

ILLUSTRATION 1.66

In figure, determine the apparent shift in the position of the coin. Also, find the effective refractive index of the combination of the glass and water slab.



Sol. Total apparent shift is

$$s = t_1 \left(1 - \frac{1}{\mu_1} \right) + t_2 \left(1 - \frac{1}{\mu_2} \right) = 8 \left(1 - \frac{1}{\frac{4}{3}} \right) + 4.5 \left(1 - \frac{1}{\frac{3}{2}} \right)$$

$$= 2 + 1.5 = 3.5 \text{ cm}$$

The apparent depth of the coin from the top is $t = (8 + 4.5) - 3.5 = 9 \text{ cm}$ and, the real depth of the coin is

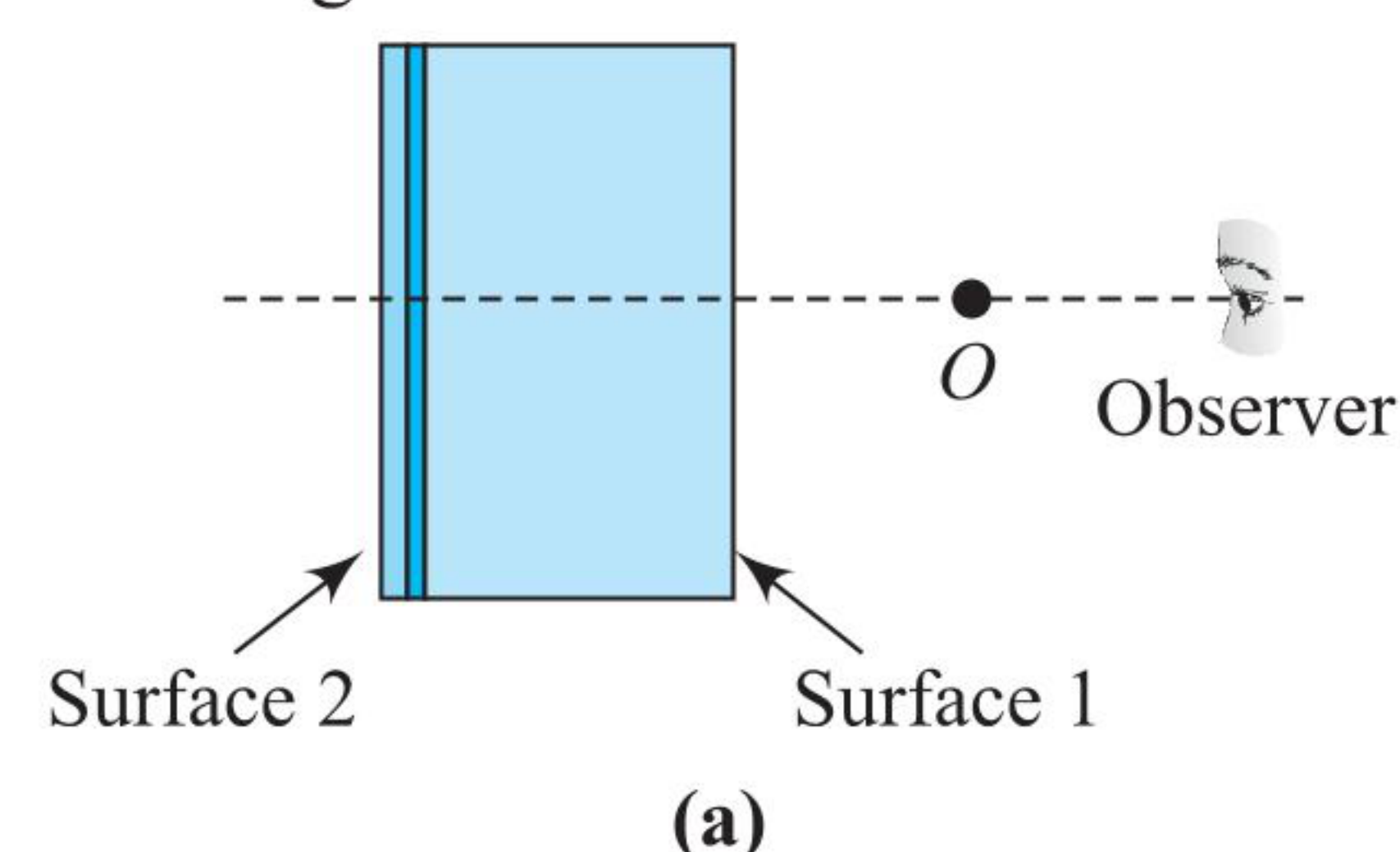
$$t_1 + t_2 = 8 + 4.5 = 12.5$$

Therefore, the effective refractive index is

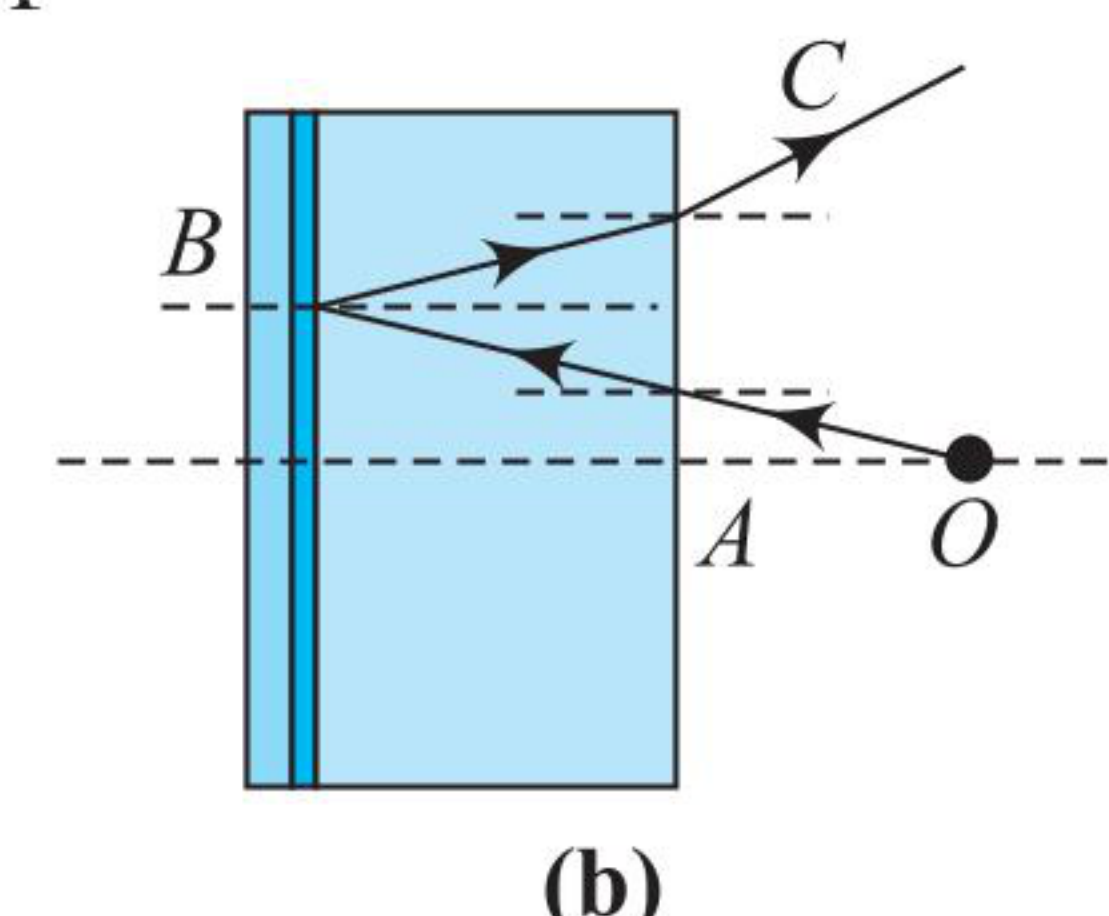
$$\mu_{eff} = \frac{\text{Real depth}}{\text{Apparent depth}} = \frac{t_1 + t_2}{t} = \frac{12.5}{9} = 1.39$$

SLAB AND MIRROR COMBINED

Let us observe what happens if one surface of the slab is silvered. Consider the silvered slab shown in Fig. (a). An object is placed in front of a silvered glass slab.



Here, a ray of light from the object first refracts at surface 1. It is then reflected from surface 2 before refracting again at surface 1 and emerging [see Fig. (b)]. So, we can consider a silvered slab as a combination of



1. A refracting surface,
2. A reflecting surface, and
3. A refracting surface again.

The above situation can be considered as a combination of a slab and a plane mirror placed together. Thus, a silvered slab is a combination of

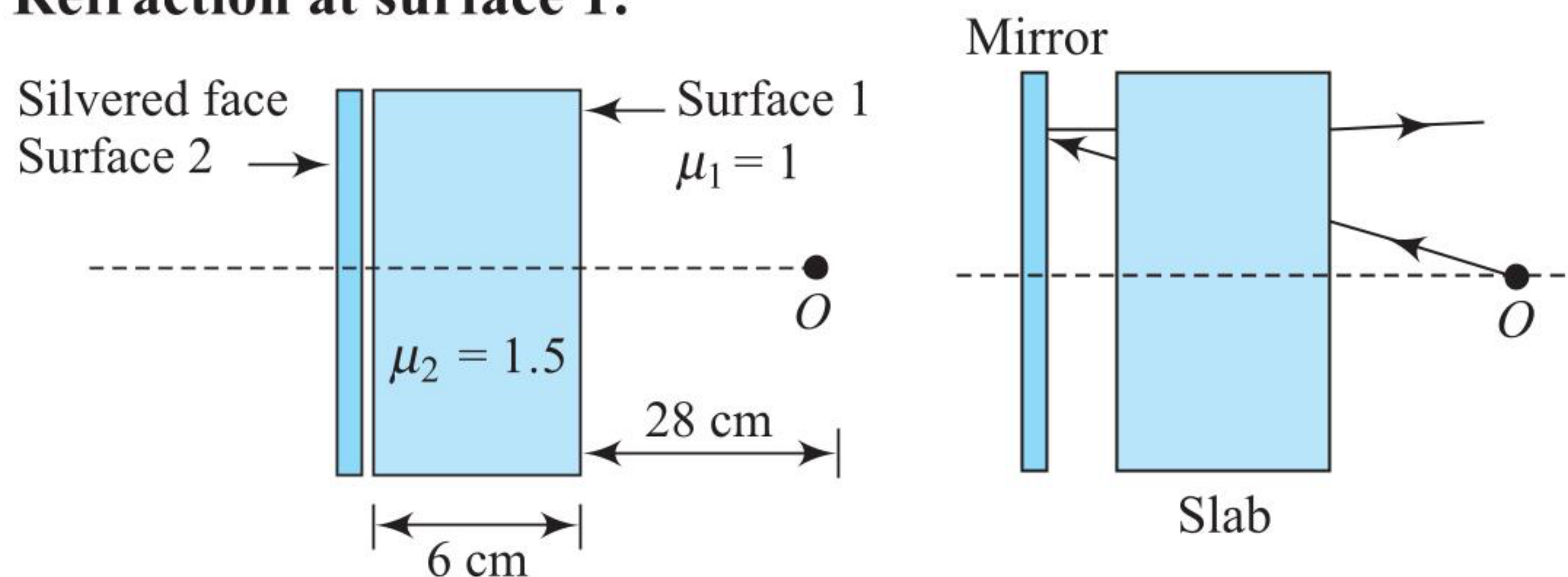
1. A glass slab,
2. A plane mirror, and
3. A glass slab again.

To learn the concept, we will discuss the situation through an illustration.

ILLUSTRATION 1.67

An object is placed in front of a slab ($\mu = 1.5$) of thickness 6 cm at a distance 28 cm from it. Other face of the slab is silvered. Find the position of final image.

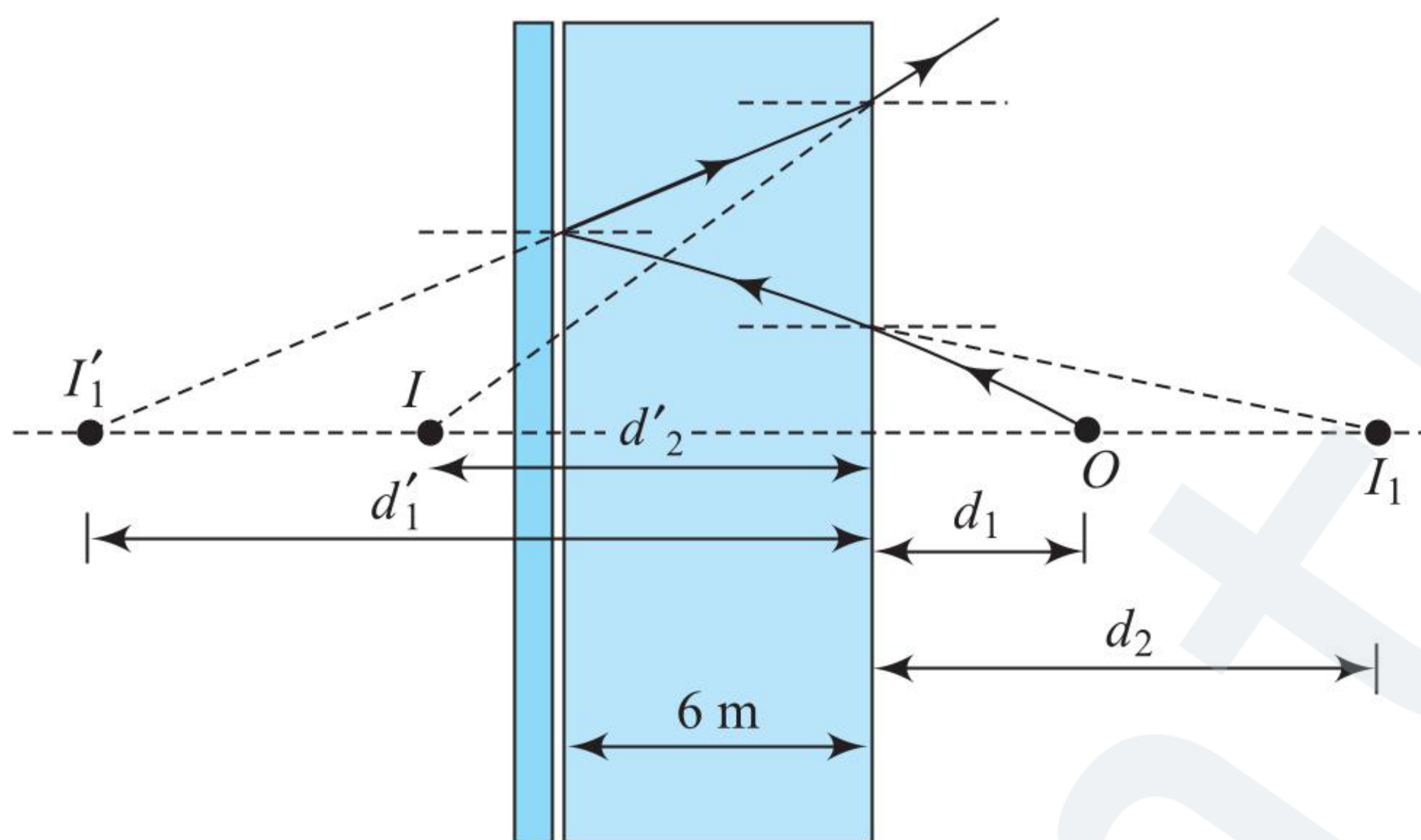
Sol. Method 1. Method of interface: A ray of light from the object O undergoes refraction, reflection and then refraction.

Refraction at surface 1:

Here, $\mu_1 = 1$, $\mu_2 = 1.5$, $d_1 = 28$ cm, $d_2 = ?$

$$\text{Since } d_2 = \frac{d_1}{n_{\text{relative}}} = \frac{d_1}{(1/\mu)}; \left[n_{\text{rel}} = \frac{n_{\text{incident}}}{n_{\text{refracted}}} = \frac{1}{\mu} \right]$$

$$\Rightarrow d_2 = \mu d_1 = 1.5 \times 28 = 42 \text{ cm}$$



Therefore, $d_2 = 42$ cm from the first interface. The first image I_1 is formed 42 cm in front of the slab.

Reflection at surface 2:

The object for reflection at the second surface is the image from refraction at the first.

Therefore, object distance from the mirror is $= 42 + 6 = 48$ cm.

As a result of reflection, the image will be formed as far behind the mirror as the object is in front of it. Therefore, the second image I_1' is formed 48 cm behind the mirror.

Second refraction at surface 1:

Object distance from surface 1,

$$d_1 = 48 + 6 = 54 \text{ cm}$$

$$d_2' = \frac{d_1'}{n_{\text{relative}}} = \frac{d_1'}{\left(\frac{n_{\text{incident}}}{n_{\text{refraction}}} \right)} = \frac{54}{(1.5/1)} = 36 \text{ cm}$$

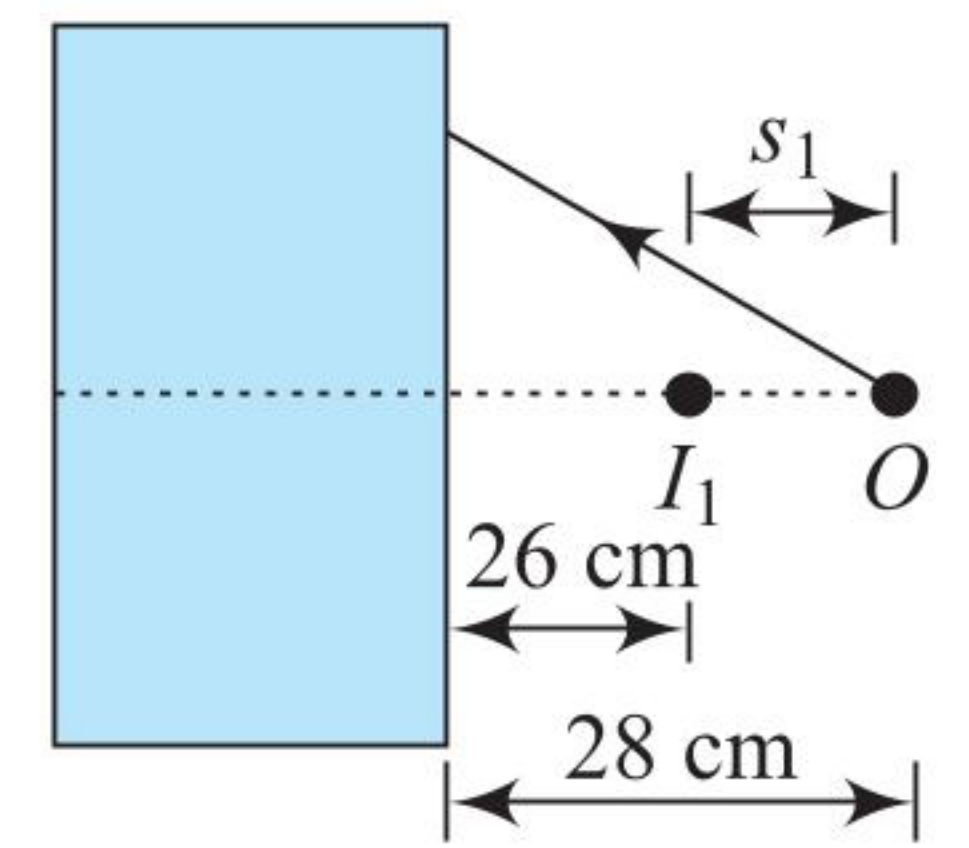
So, the final image is at a distance $d_2 = 36$ cm behind the first interface.

Hence, final image is formed 36 cm behind surface 1 or 30 cm behind surface 2.

Method 2. Shifting of object: A ray of light from the object first encounters a glass slab, then a mirror, and finally a glass slab again.

Glass slab: A slab simply shifts the object along the axis by a distance

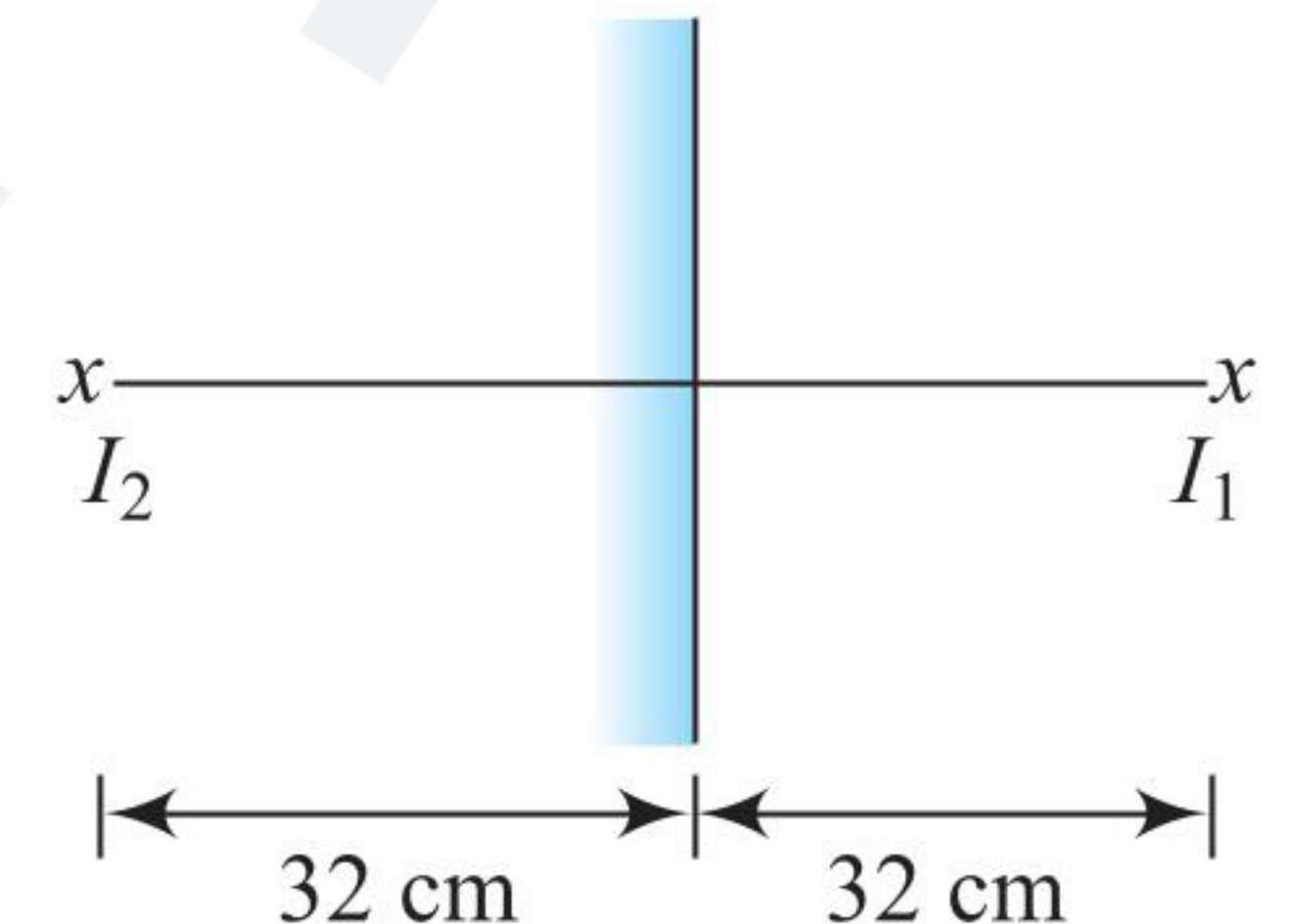
$$s_1 = t \left(1 - \frac{1}{\mu} \right) = 2 \text{ cm}$$



Direction of shift of object is towards left. Therefore, the object appears to be at I_1 which is $28 - 2 = 26$ cm from the slab.

For mirror, the object for the mirror is the image I_1 formed after shift due to the slab.

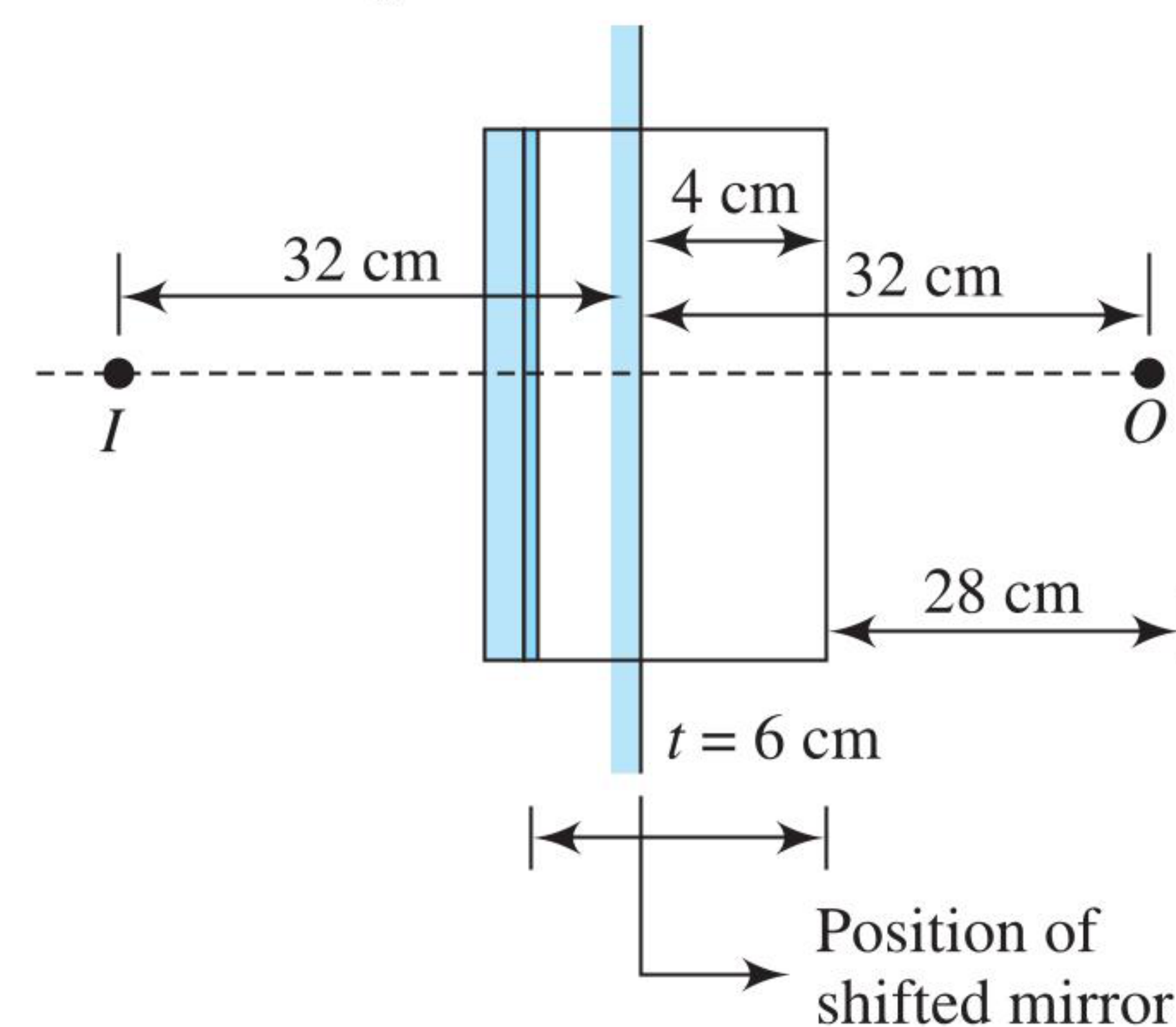
Therefore, object distance from the mirror is $26 + 6 = 32$ cm. The image will now be formed 32 cm behind the mirror. Now, reflected rays are travelling from left to right.



The ray now travels through the slab again but this time from left to right. Therefore, it is shifted again by a distance of 2 cm, but towards the right. Thus, final position of the image is $32 - 2 = 30$ cm behind the mirror.

Method 3. Shifting of mirror: By the principle of reversibility of light, we can say if light rays are coming from the mirror and passing through the slab, the mirror will shift 2 cm towards right for observer in front of the slab.

The position of the object from shifted mirror = 32 cm.



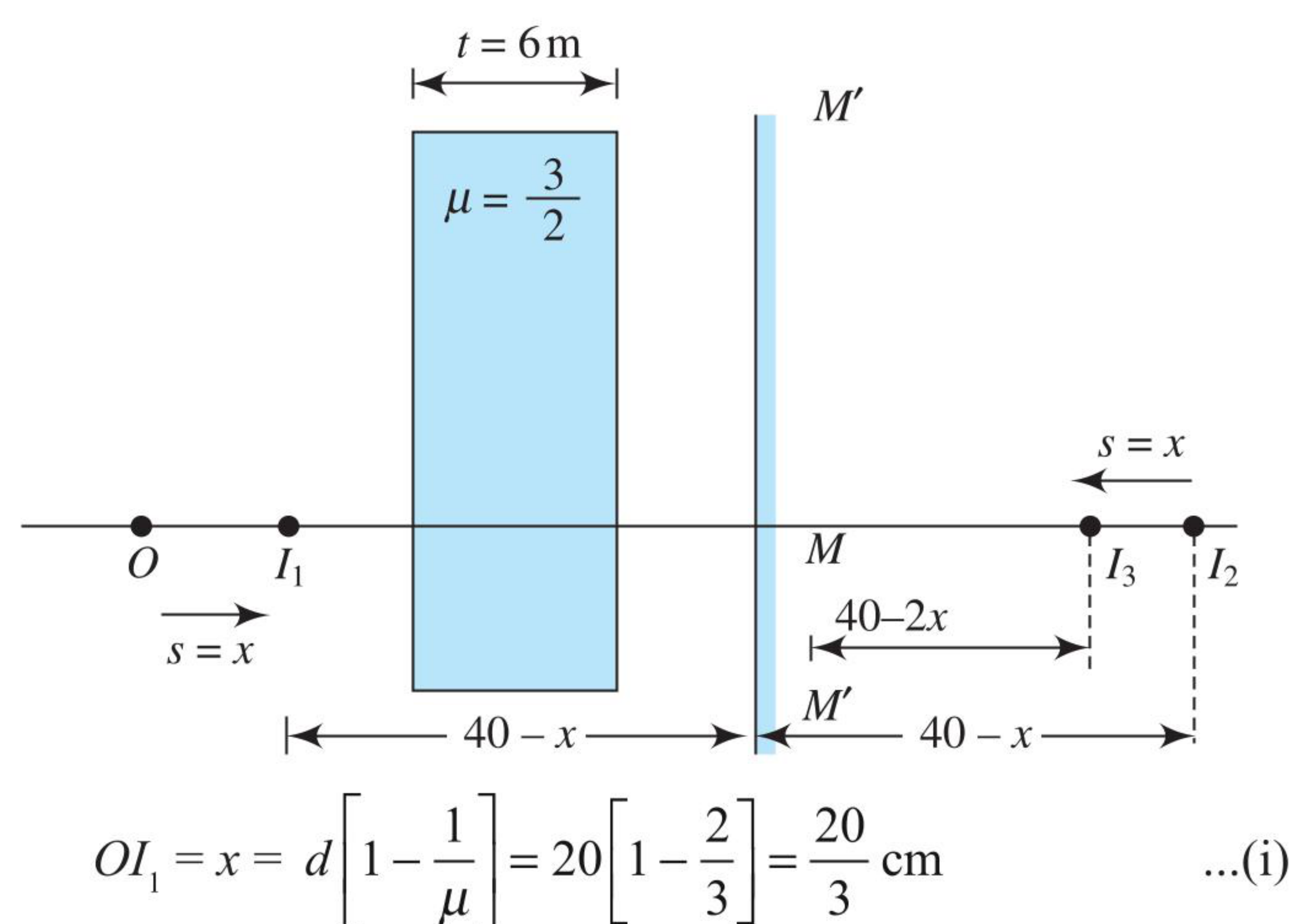
So, the position of the image formed by shifted mirror will be 32 cm behind it. Hence, the position of the image from surface 2 is 30 cm left to it and 36 cm left of surface 1.

Let us learn the combination of the slab and mirror through some more illustrations.

ILLUSTRATION 1.68

A 20 cm thick glass slab of refractive index 1.5 is kept in front of a plane mirror. Find the position of the image (relative to mirror) as seen by an observer through the glass slab when a point object is kept in air at a distance of 40 cm from the mirror.

Sol. The rays from O will first pass through slab and produce shifting towards right. The glass slab will form an image of O at I_1 such that



Now, the rays after passing through slab will (towards right)
So, the distance of I_1 from the mirror MM' ,

$$I_1M = (40 - x)$$

This image I_1 will act as an object for the mirror and the mirror will form an image I_2 , such that

$$MI_2 = MI_1 = 40 - x \quad [\text{with } x \text{ given by Eq. (i)}]$$

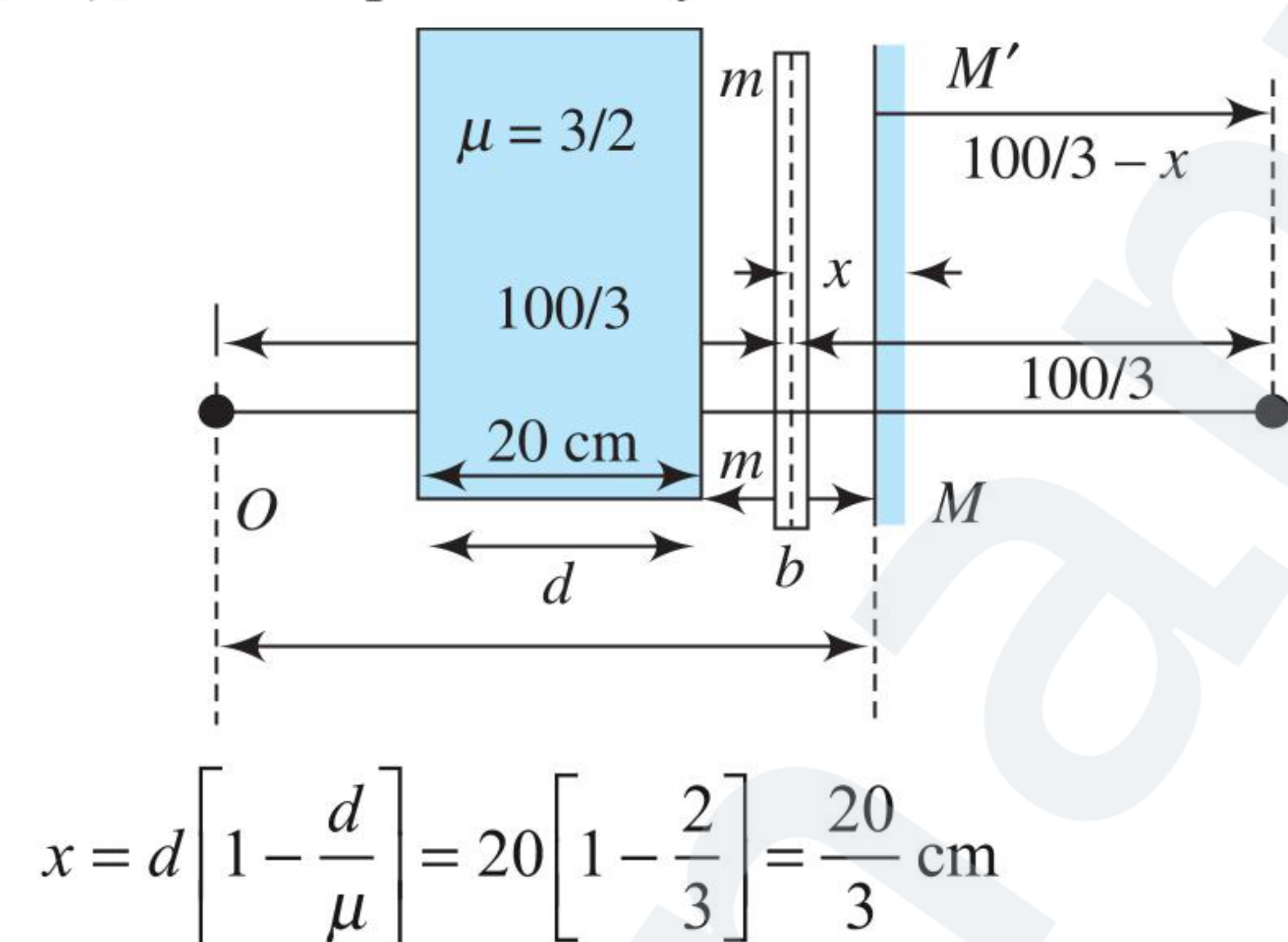
Now, image I_2 will act as an object again for the glass slab which by producing a shift of x forms the final image I_3 such that $I_2I_3 = x$. Hence, the distance of final image I_3 from the mirror will be

$$MI_3 = MI_2 - I_2I_3 = (40 - x) - x = 40 - 2x$$

$$MI_3 = 40 - 2 \times \left[\frac{20}{3} \right] = \left[\frac{80}{3} \right] \text{ cm}$$

[as from Eq. (i), $x = 20/3$ cm]

Alternative solution: As thickness of glass slab is 20 cm and $\mu = (3/2)$, so shift produced by it will be



So, the glass slab will shift the mirror from MM' to mm' as shown in.

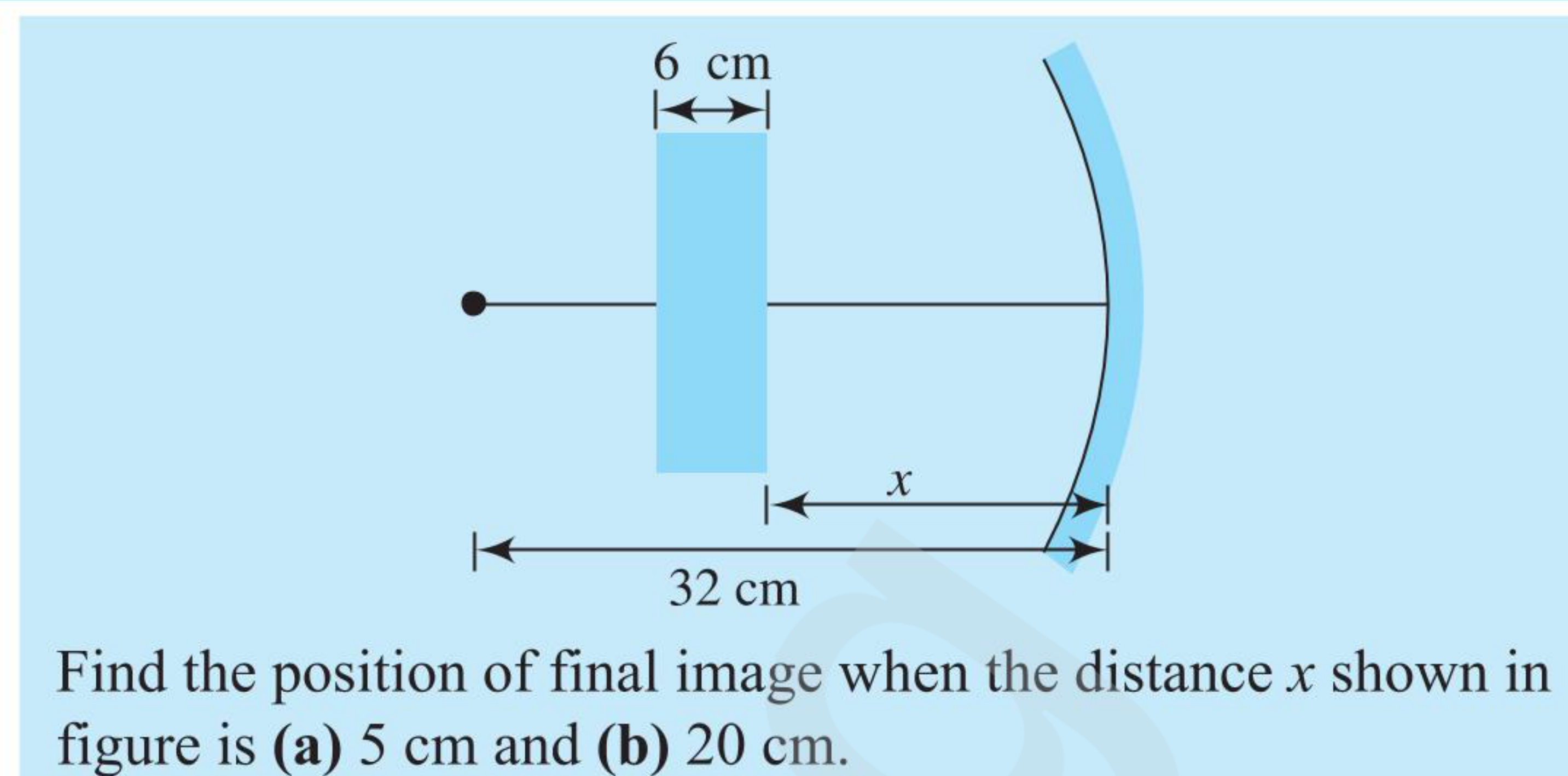
The distance of object from this virtual mirror will be

$$40 - x = 40 - (2/3) = (100/3) \text{ cm}$$

This virtual mirror will form the image of object O at a distance $(100/3)$ behind it and so the distance of image from actual mirror MM' will be $\frac{100}{3} - \frac{20}{3} = \frac{80}{3}$ cm [as mm' is $20/3$ cm in front of MM']

ILLUSTRATION 1.69

A point object O is placed in front of a concave mirror of focal length 10 cm. A glass slab of refractive index $\mu = 3/2$ and thickness 6 cm is inserted between the object and mirror.



Sol. The normal shift produced by a glass slab is

$$S = \left(1 - \frac{1}{\mu} \right) t = \left(1 - \frac{2}{3} \right) (6) = 2 \text{ cm}$$

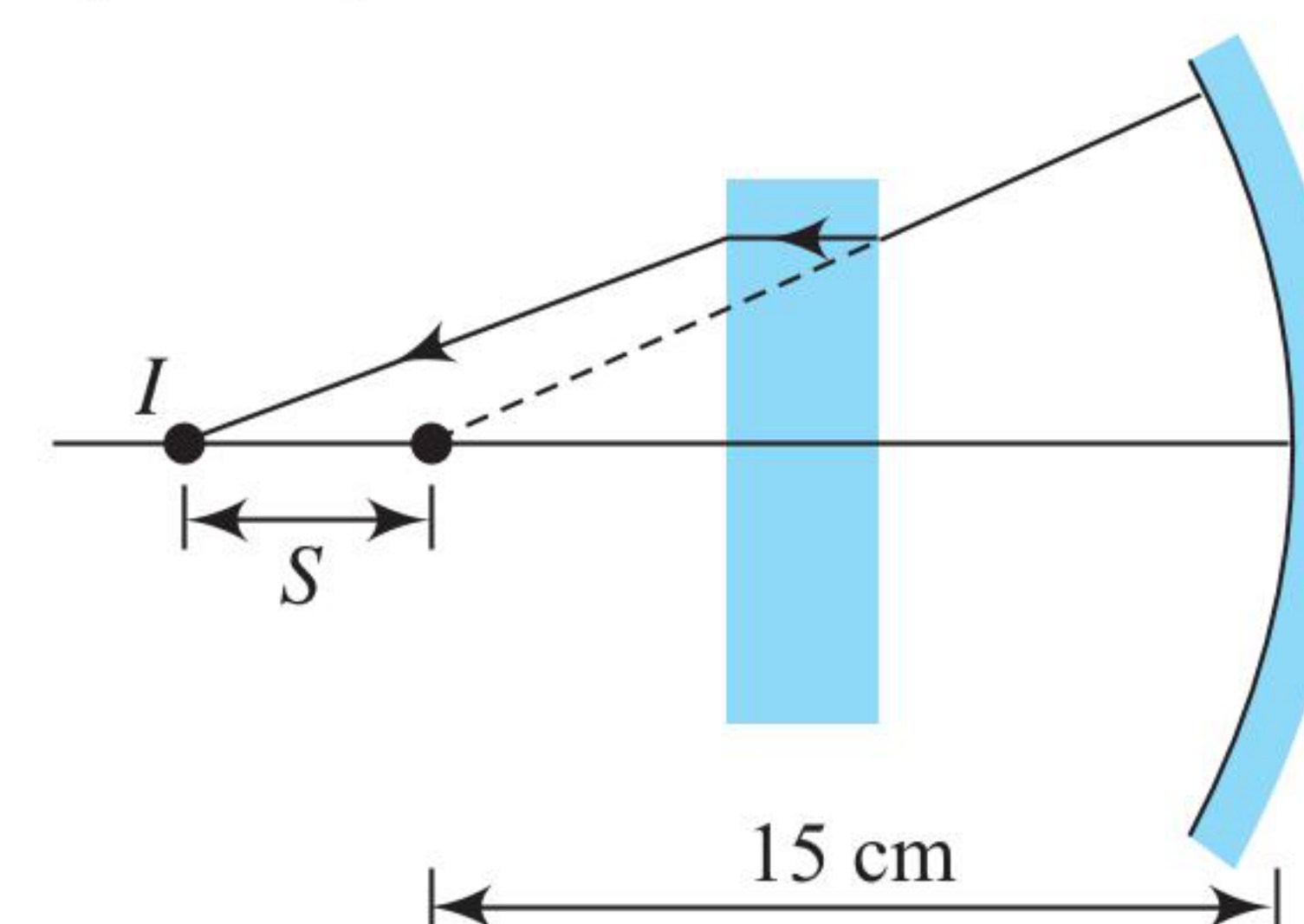
i.e., for the mirror the object is placed at a distance $(32 - S) = 30$ cm from it.

Applying mirror formula $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$$\text{or } \frac{1}{v} + \frac{1}{-30} = -\frac{1}{10}$$

$$\text{or } v = -15 \text{ cm}$$

(a) When $x = 5$ cm: The light falls on the slab on its return journey as shown. But the slab will again shift it by a distance $S = 2$ cm. Hence, the final real image is formed at a distance $(15 + 2) = 17$ cm from the mirror.



(b) When $x = 20$ cm: This time also the final image is at a distance 17 cm from the mirror but it is virtual as shown.

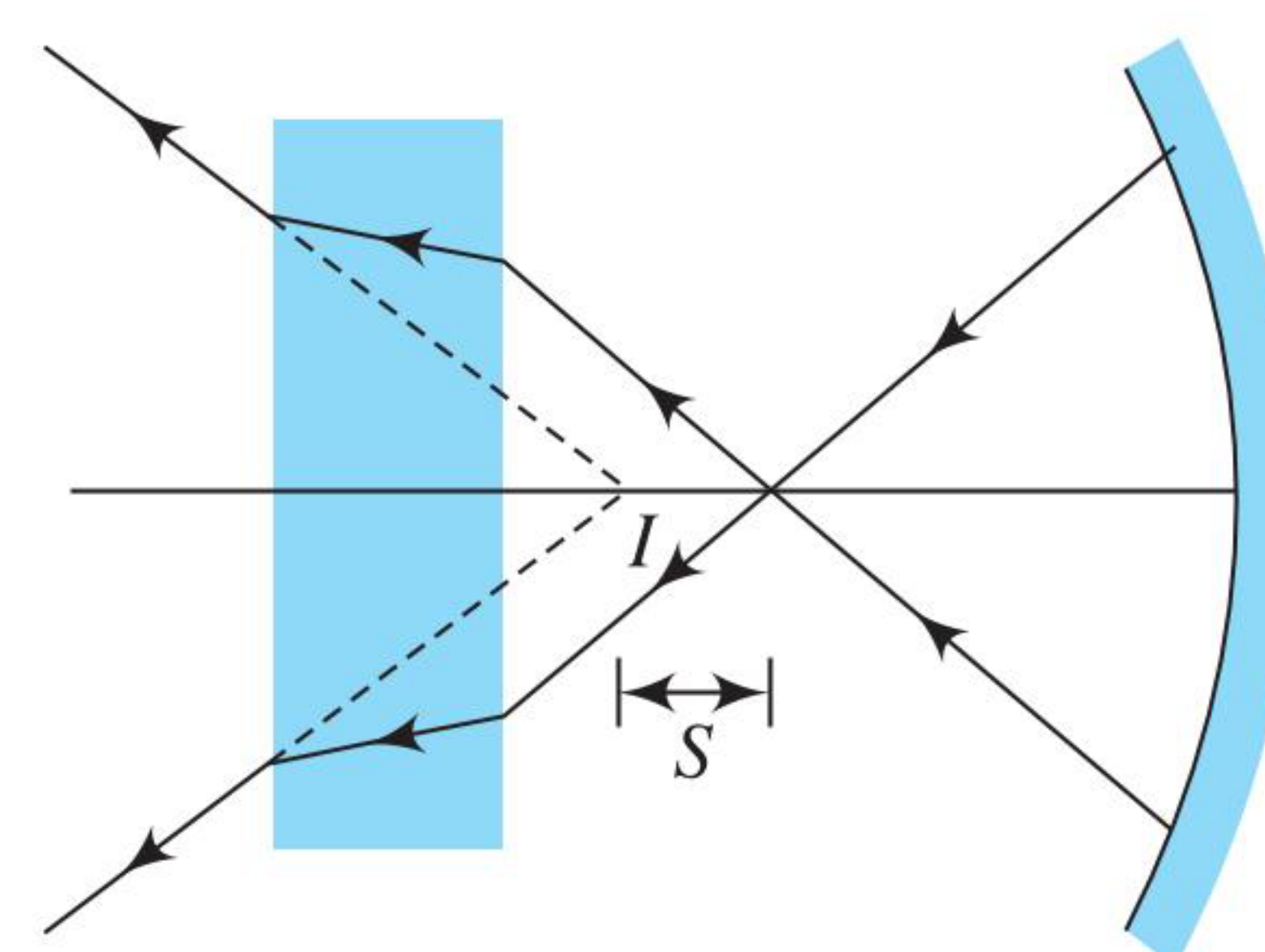
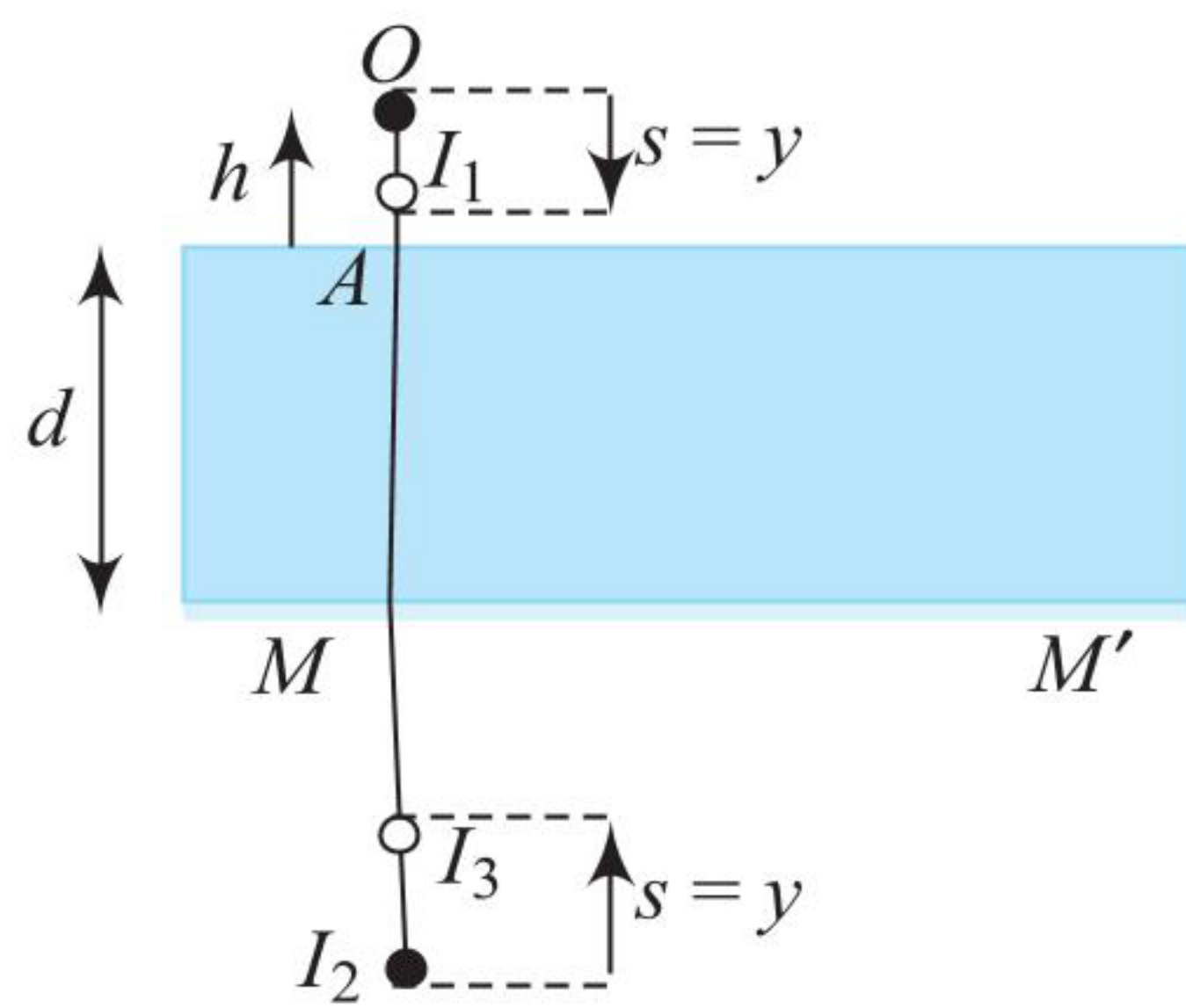


ILLUSTRATION 1.70

A vessel having perfectly reflecting plane bottom is filled with water ($\mu = 4/3$) to a depth d . A point source of light is placed at a height h above the surface of water. Find the distance of final image from water surface.

Sol. As shown in figure, water will form the image of object O at I_1 such that $OI_1 = y = d[1 - (1/\mu)]$, so that the distance of image I_1 from water surface will be $I_1A = h - y = h - d[1 - (1/\mu)]$. Hence, the distance of this image I_1 from mirror MM' ,

$$I_1M = I_1A + AM = \left[h - d \left(1 - \frac{1}{\mu} \right) \right] + d = h + \left[\frac{d}{\mu} \right]$$



Now, image I_1 will act as object for mirror MM' . As a plane mirror forms image at same distance behind the mirror as the object is in front of it, the image of I_1 formed by the mirror MM' will be I_2 such that $I_1M = I_2M = h + (d/\mu)$.

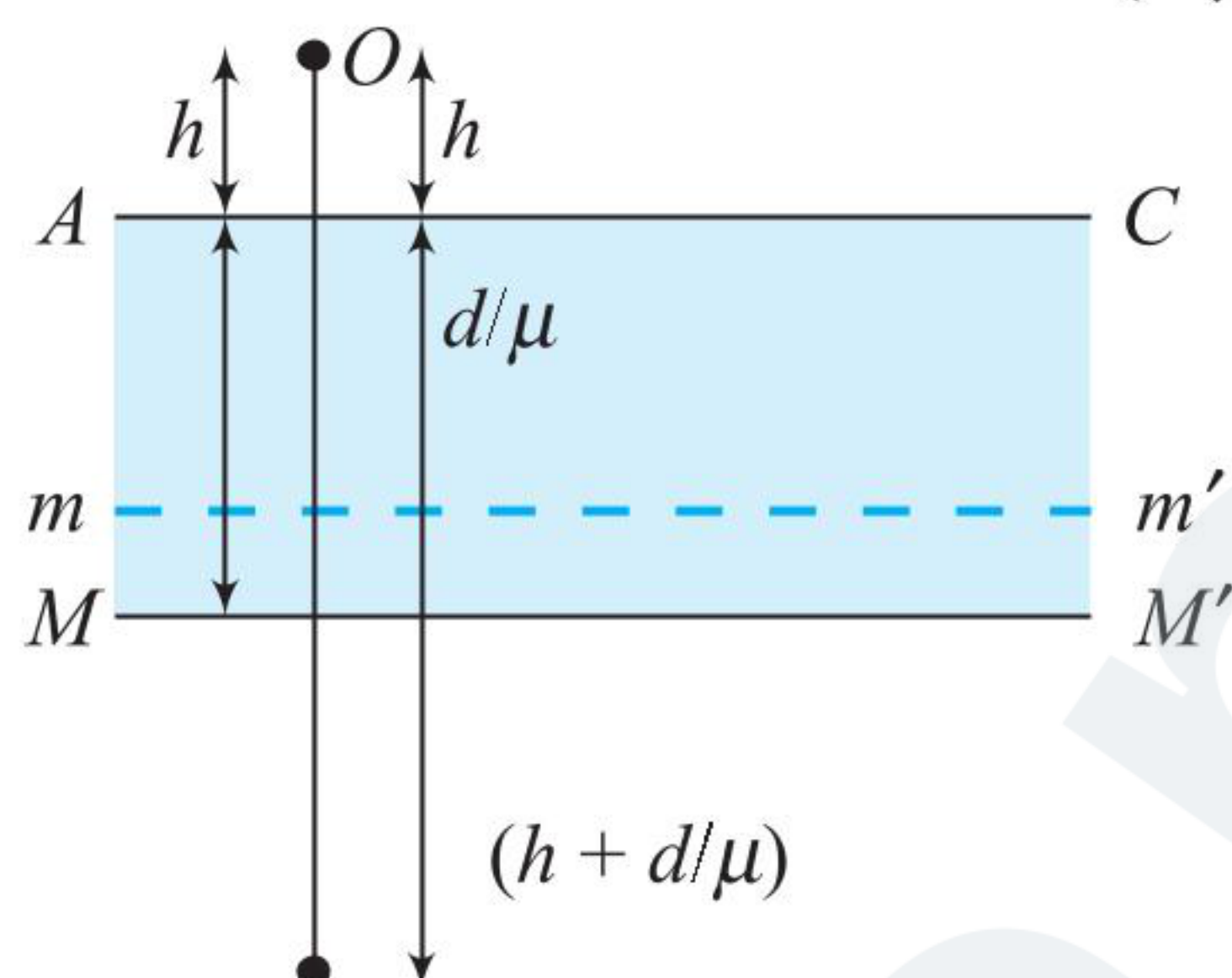
Now, this image I_2 will act as object for water again and water will produce image I_3 such that $I_2I_3 = y = d \left(1 - \frac{1}{\mu} \right)$. So, the distance of image I_3 from the surface of water AC will be

$$AI_3 = AM + MI_2 - I_2I_3 = d + \left[h + \frac{d}{\mu} \right] - d \left[1 - \frac{1}{\mu} \right]$$

$$AI_3 = h + 2 \frac{d}{\mu} = h + \frac{3}{2} d$$

Alternative solution:

As shown in figure, water will form the image of bottom, i.e., mirror MM' at a depth (d/μ) from its surface. So, the distance of object O from virtual mirror mm' will be $h + (d/\mu)$.

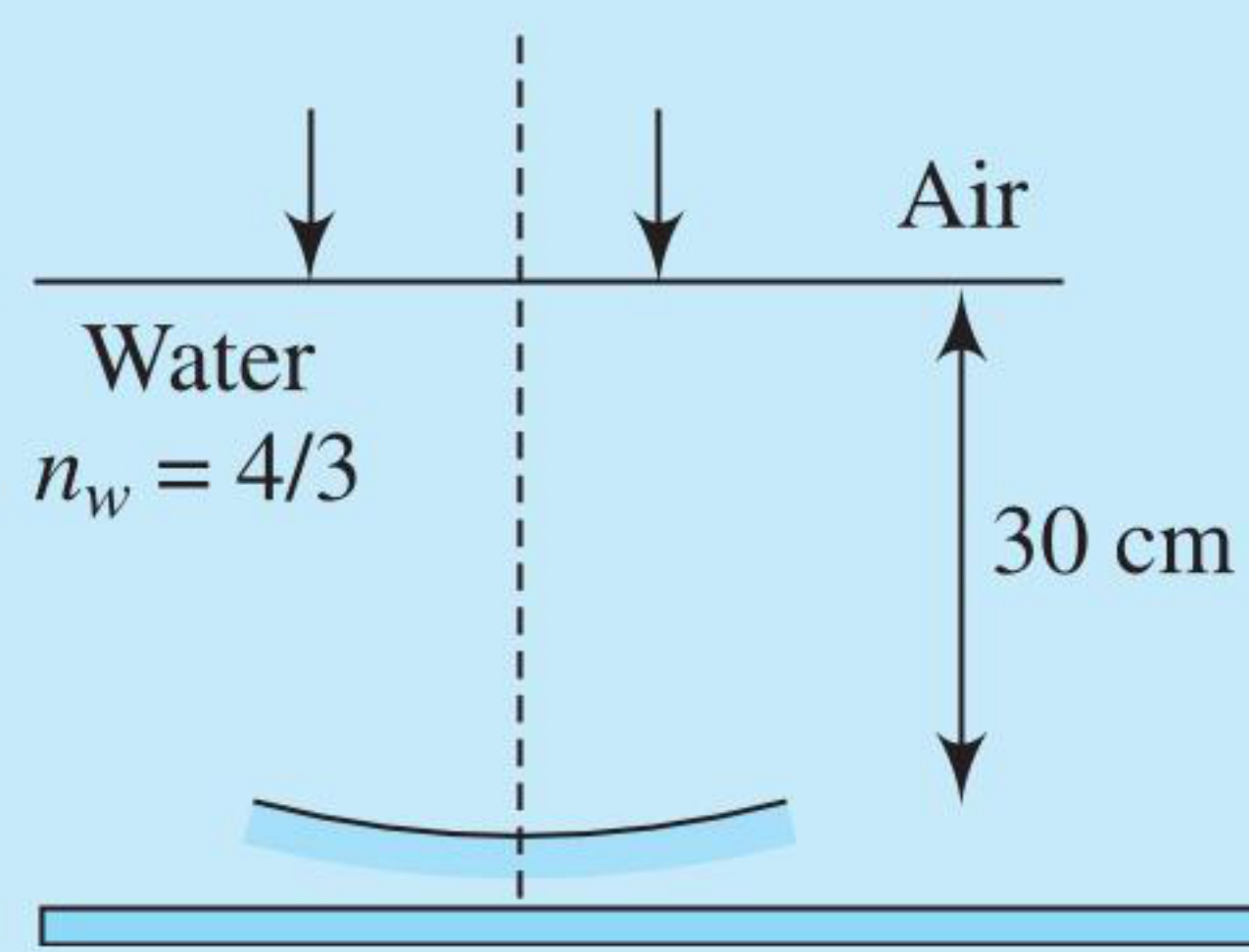


Now, as a plane mirror forms image behind the mirror at the same distance as the object is in front of it, the distance of image I from mm' will be $h + (d/\mu)$. Also, as the distance of virtual mirror from the surface of water is (d/μ) , the distance of image I from the surface of water will be

$$\left[h + \frac{d}{\mu} \right] + \frac{d}{\mu} = h + \frac{2d}{\mu} = h + \frac{3}{2} d \quad \left[\text{as } \mu = \frac{4}{3} \right]$$

ILLUSTRATION 1.71

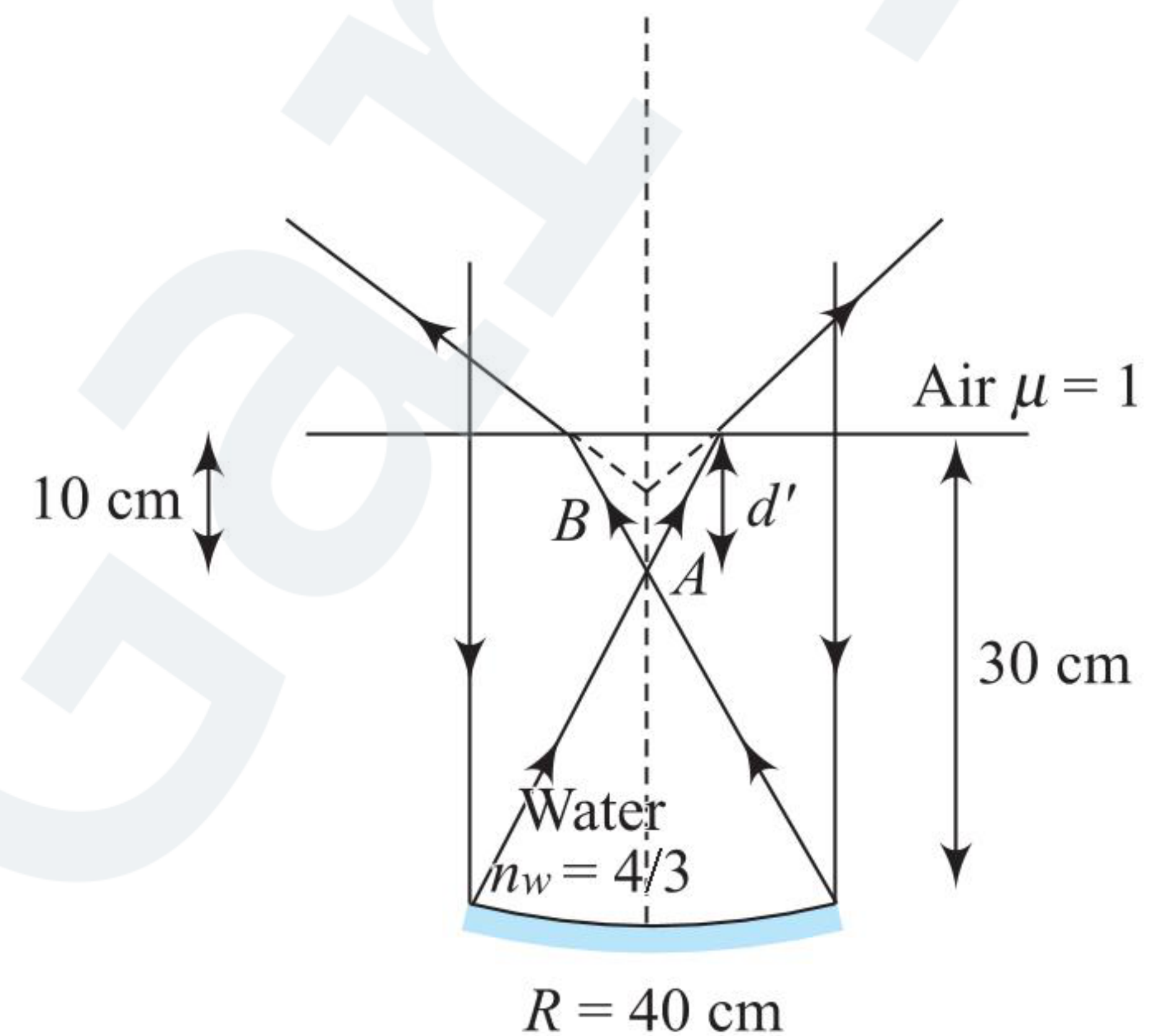
A concave mirror of focal length 20 cm is placed inside water with its shining surface upwards and principal axis vertical as shown in figure. Rays are incident parallel to the principal axis of concave mirror. Find the position of final image.



Sol. The incident rays will pass undeviated through the water surface and strike the mirror parallel to its principal axis. Therefore, for the mirror, object is at ∞ . Its image A (in figure) will be formed at focus which is 20 cm from the mirror. Now, for the interface between water and air. For observer at air, the image formed by mirror will act as an object for observer with real depth $d = 30 - 20 = 10$ cm.

Apparent depth observed by observer,

$$d' = \frac{d}{\left(\frac{n_w}{n_a} \right)} = \frac{10}{\left(\frac{4/3}{1} \right)} = 7.5 \text{ cm}$$



REFRACTION IN A MEDIUM WITH VARIABLE REFRACTIVE INDEX

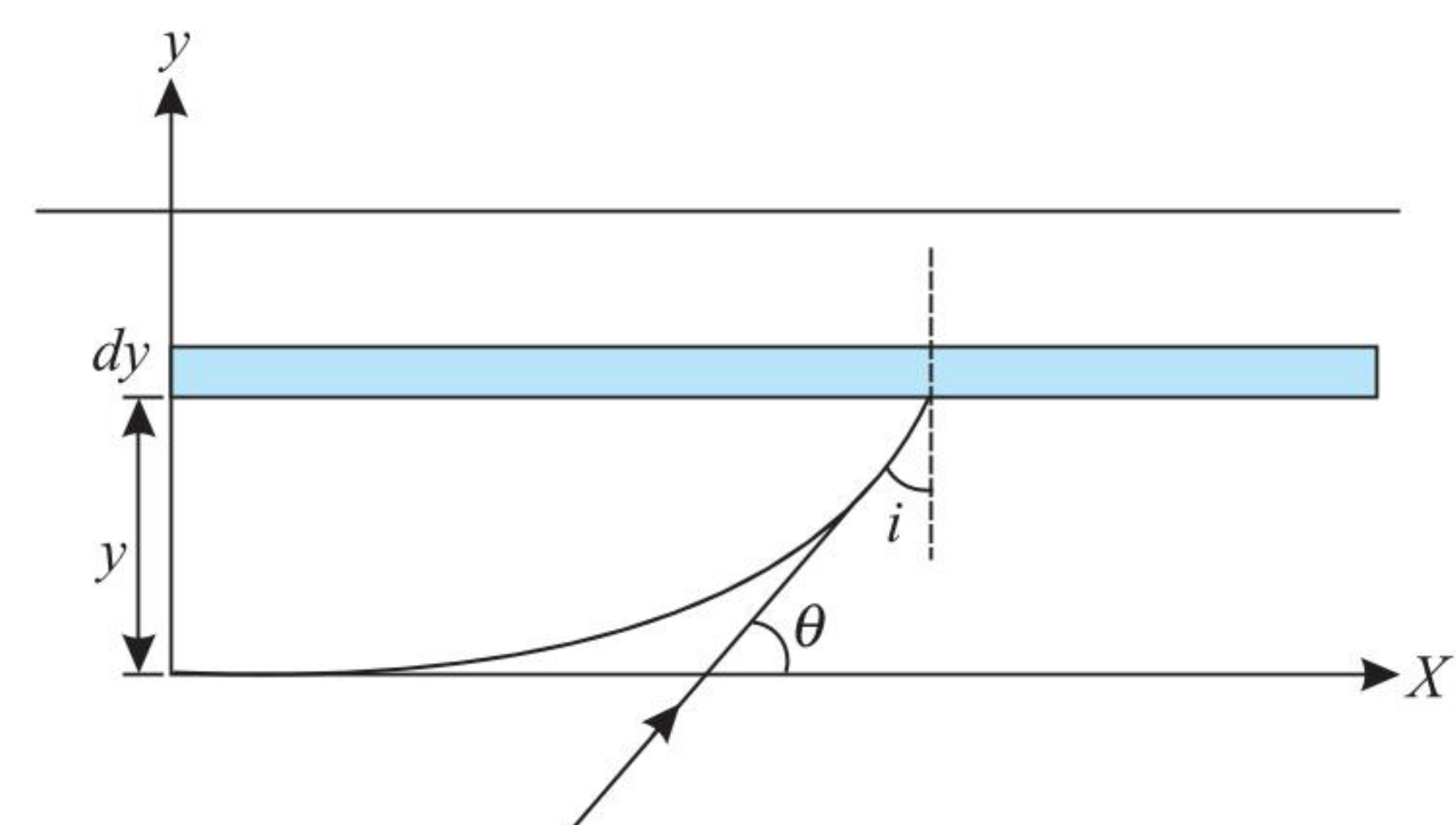
So far, we have assumed that the refractive index of the slab is a constant. This need not be necessarily true. An example of such a situation is the atmosphere. The atmosphere becomes thinner as we go up. Hence, the refractive index of air is highest close to surface of earth and decreases as we move upward. How do we analyze such a problem?

Divide the medium into different layers. This indicates that each layer has different refractive index having value according to a given expression. If μ is a function of x , then a differential layer will be a thin layer of thickness dx . If μ is a function of y , then a differential layer will be a thin layer of thickness dy .

At a given layer, let the angle of incidence be i . Relate this angle of incidence to the initial condition and the refractive index at that point using the relation

$$\mu \sin(i) = \text{constant} \quad \dots(i)$$

From the given ray diagram (figure), draw a tangent to the path taken by the ray of light. Geometrically, relate the slope of this tangent to the angle of incidence.



$$\tan \theta = \frac{dy}{dx} = \tan(90 - i) \quad \dots(ii)$$

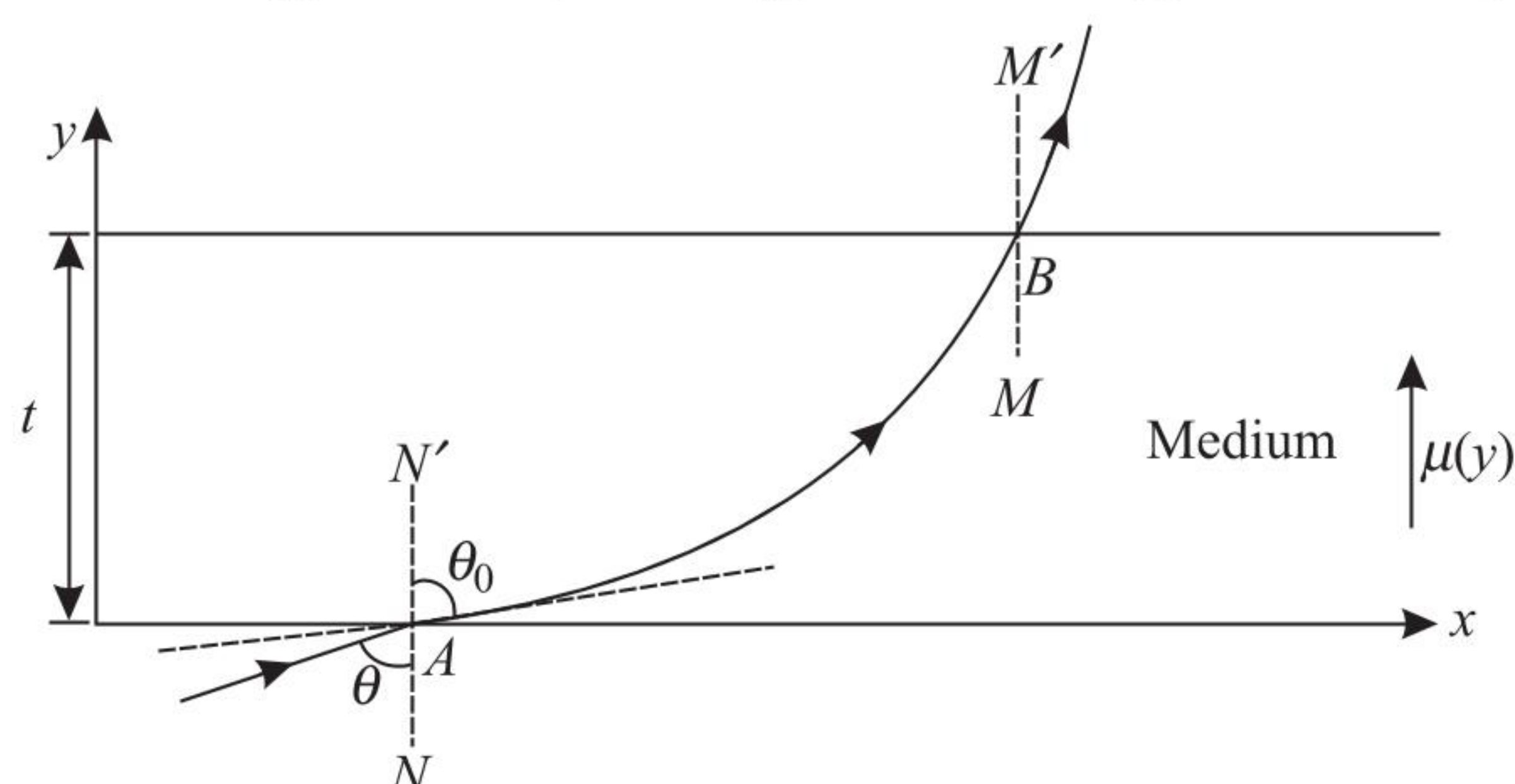
Substitute for i from Eq. (i) and determine dy/dx as a function of x and y .

Integrate and obtain an expression for y as a function of x .

Let us discuss a case where refractive index is changing in y -direction.

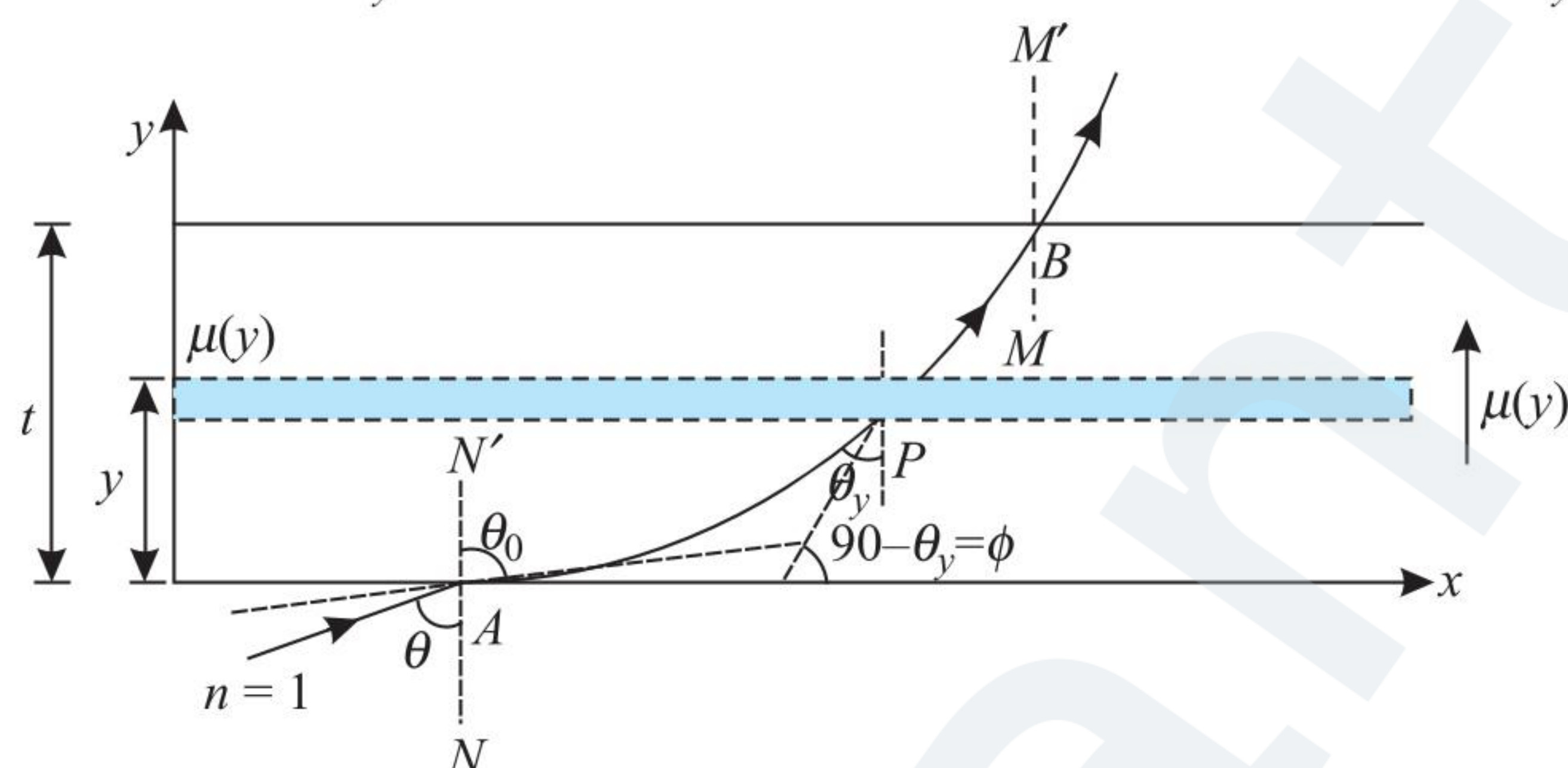
As 1: $\mu = f(y)$

Consider a slab with a medium where the refractive index varies from μ_1 to μ_2 in relation with y . Assume that the outside medium is air. Let light be incident on the glass slab at an angle θ . The light ray strikes the glass slab at the points. The normal at this point is NN' as shown in figure. Once the ray enters the medium, the refractive index varies continuously as if there are infinite number of successive glass slabs. Therefore, the angle of incidence also varies continuously and the path of the ray is curved. The ray finally reaches the second surface at the point B before exiting the slab, once again refracting at the exit point.



How can we derive a mathematical expression for the equation of the ray in the medium?

Consider refraction at some height y (figure). Here, the angle of incidence is θ_y and the refractive index of the medium is μ_y .



The product of the refractive index and the sine of the angle of incidence is constant. Therefore, at the height y we can say that

$$\mu_y \sin \theta_y = \text{constant} \quad \dots(i)$$

Furthermore, applying the law of refraction at points A which is the interface between air and the medium, we have

$$1 \times \sin(\theta) = \mu_y \sin \theta_y \quad \dots(ii)$$

$$\therefore \sin \theta_y = \frac{\sin \theta}{\mu_y}$$

$$\cos \theta_y = \sqrt{\frac{\mu_y^2 - \sin^2 \theta}{\mu_y^2}}$$

$$\text{Hence, } \cot \theta_y = \frac{\sqrt{\mu_y^2 - \sin^2 \theta}}{\sin \theta}$$

Now, the slope of the curve at point P is given by

$$\tan \phi = \frac{dy}{dx} \quad \dots(iii)$$

$$\text{But } \phi = 90 - \theta_y$$

$$\text{Therefore, } \frac{dy}{dx} = \cot(\theta_y) = \frac{\sqrt{\mu_y^2 - \sin^2 \theta}}{\sin \theta} \quad \dots(iv)$$

$$\frac{\sin \theta dy}{\sqrt{\mu_y^2 - \sin^2 \theta}} = dx$$

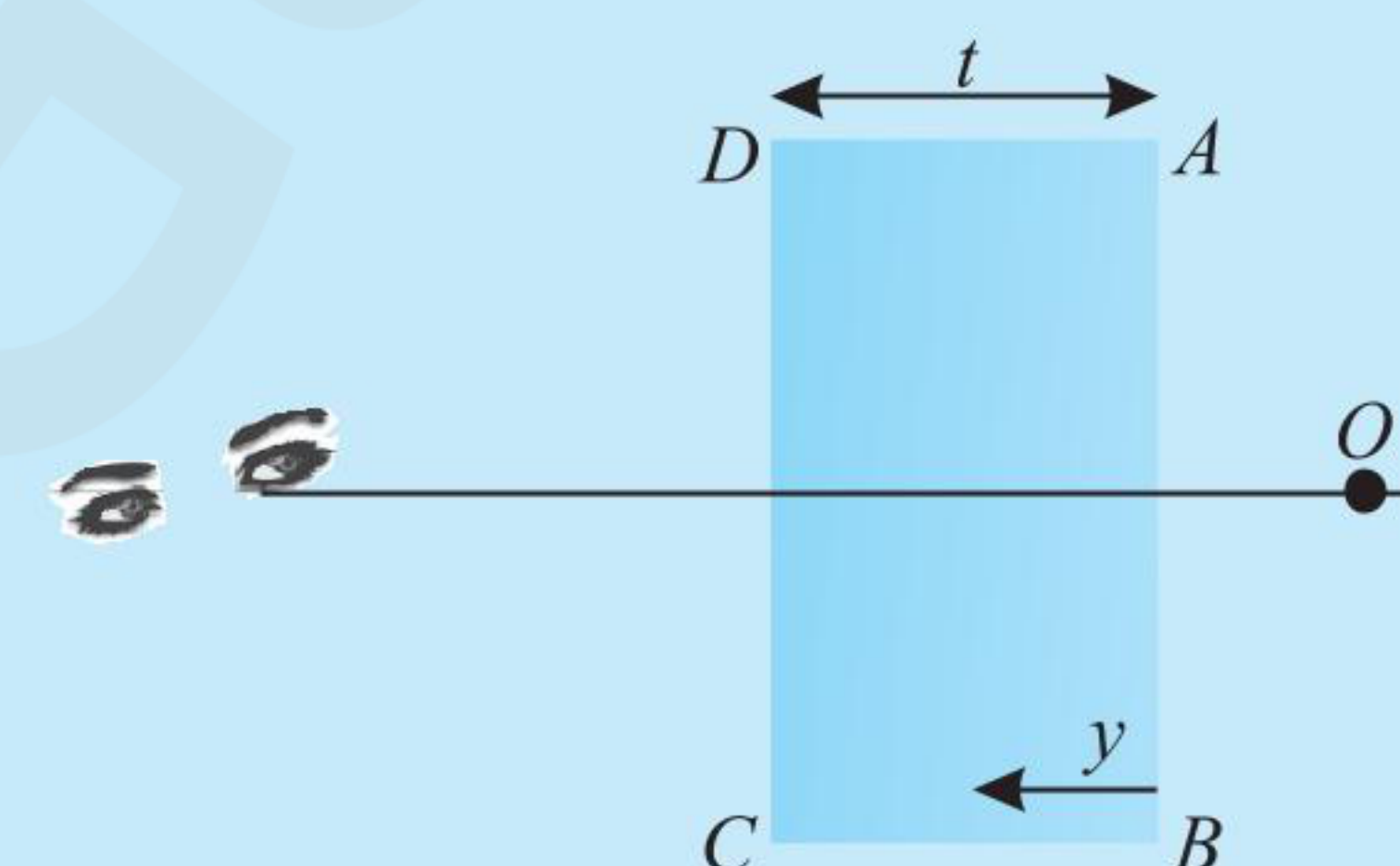
Integrating both side with proper limits.

If the functional dependence of " μ_y " is known, we can easily solve for the equation of the curve.

$$\sin \theta \int_{y_1}^{y_2} \frac{dy}{\sqrt{\mu_y^2 - \sin^2 \theta}} = \int_{x_1}^{x_2} dx$$

ILLUSTRATION 1.72

A glass slab is placed between an object (O) and an observer (E) with its refracting surfaces AB and CD perpendicular to the line OE .



The refractive index of the glass slab changes with distance (y) from the face AB as $\mu = \mu_0(1 + y)$. Thickness of the slab is t .

Find how much closer (compared to original distance) the object appears to the observer. Consider near normal incidence only.

Sol. Shift in position of the object caused due to thickness dy of the slab is

$$ds = dy \left(1 - \frac{1}{\mu} \right) = dy \left(1 - \frac{1}{\mu_0(1 + y)} \right)$$

Total shift due to all such layers is

$$s = \int ds = \int_0^t dy - \frac{1}{\mu_0} \int_0^t \frac{dy}{1 + y}$$

$$= t - \frac{1}{\mu_0} [\ln(1 + y)]_0^t$$

$$\Rightarrow s = t - \frac{\ln(1 + t)}{\mu_0}$$

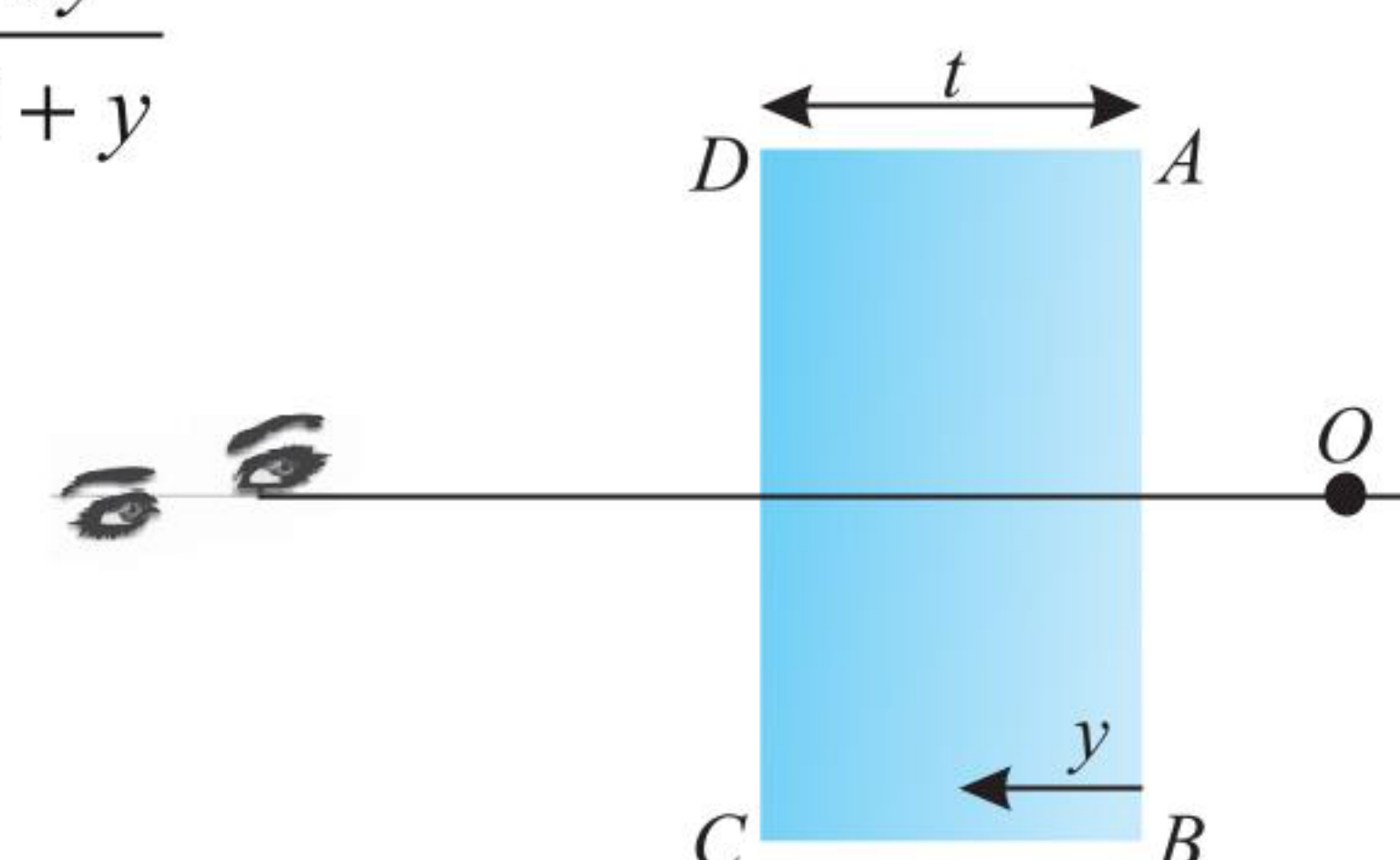
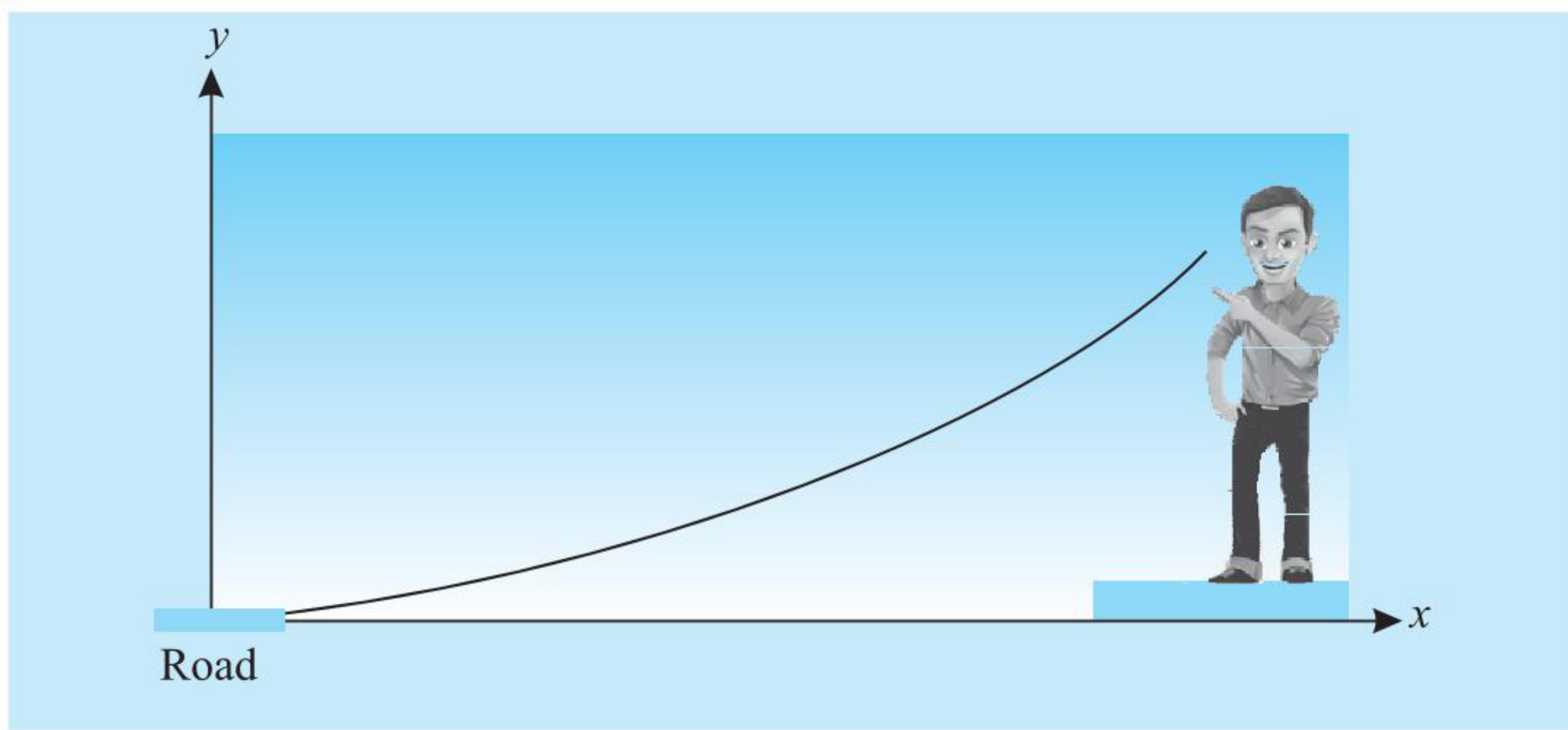
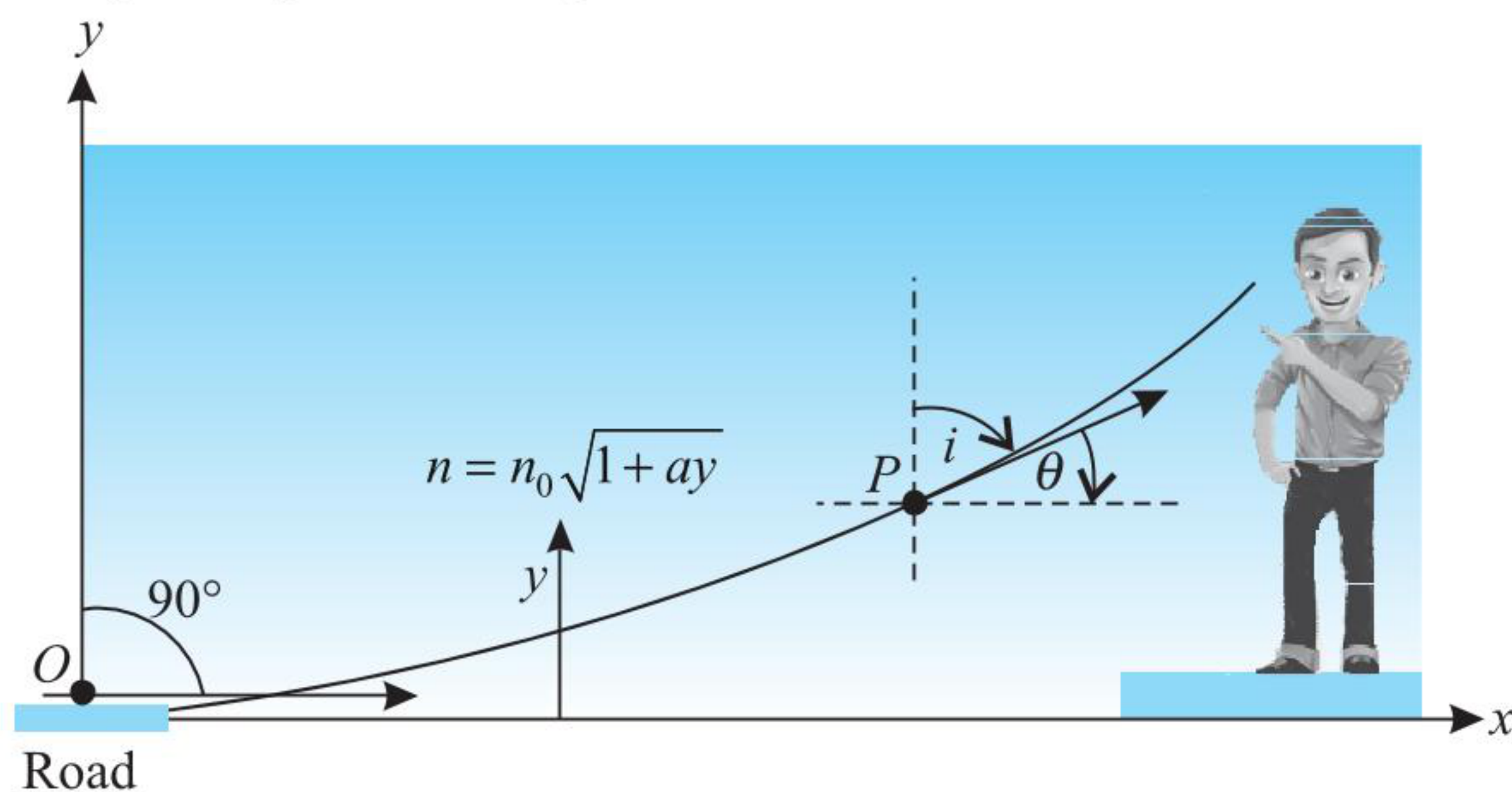


ILLUSTRATION 1.73

Due to a vertical temperature gradient in the atmosphere the index of refraction varies. Suppose index of refraction varies as $n = n_0 \sqrt{1 + ay}$ where n_0 is the index of refraction at the surface and $a = 2.0 \times 10^{-6} \text{ m}^{-1}$. A person of height $h = 2.0 \text{ m}$ stands on a level surface. Beyond what distance he cannot see the runway?



Sol. As refractive index is changing along y -direction, we can assume a number of thin layers of air placed parallel to x -axis. Let O be the distant object just visible to the man. Let P be a point on the trajectory of the ray.



From Snell's law, $n \cdot \sin i = \text{constant}$

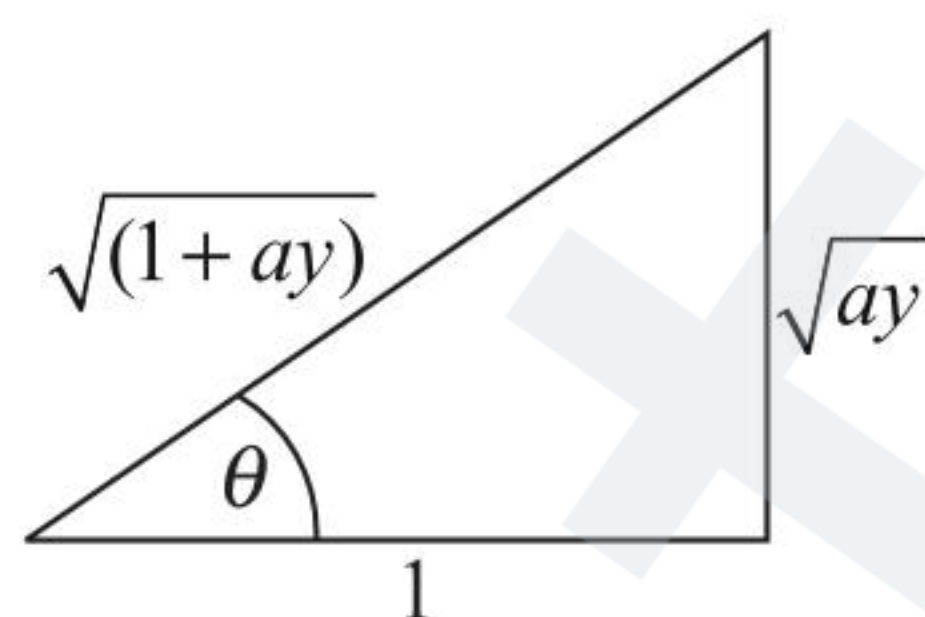
At the surface $n = n_0$ and $i = 90^\circ$

$$n_0 \sin 90^\circ = n \sin i = (n_0 \sqrt{1 + ay}) \sin (90^\circ - \theta)$$

$$\Rightarrow \cos \theta = \frac{1}{\sqrt{1 + ay}}$$

$$\Rightarrow \tan \theta = \sqrt{ay}$$

$$\tan \theta = \sqrt{ay} = \frac{dy}{dx} \Rightarrow dx = \frac{dy}{\sqrt{ay}}$$



$$\int_0^x dx = \int_0^y \frac{dy}{\sqrt{ay}} \Rightarrow x = \frac{1}{\sqrt{a}} \left[\frac{y^{\left(-\frac{1}{2}+1\right)}}{\left(-\frac{1}{2}+1\right)} \right]_0^y = 2 \frac{y^{\frac{1}{2}}}{\sqrt{a}} = 2 \sqrt{\frac{y}{a}}$$

Hence equation of trajectory $x = 2 \sqrt{\frac{y}{a}}$ or $y = \frac{ax^2}{4}$

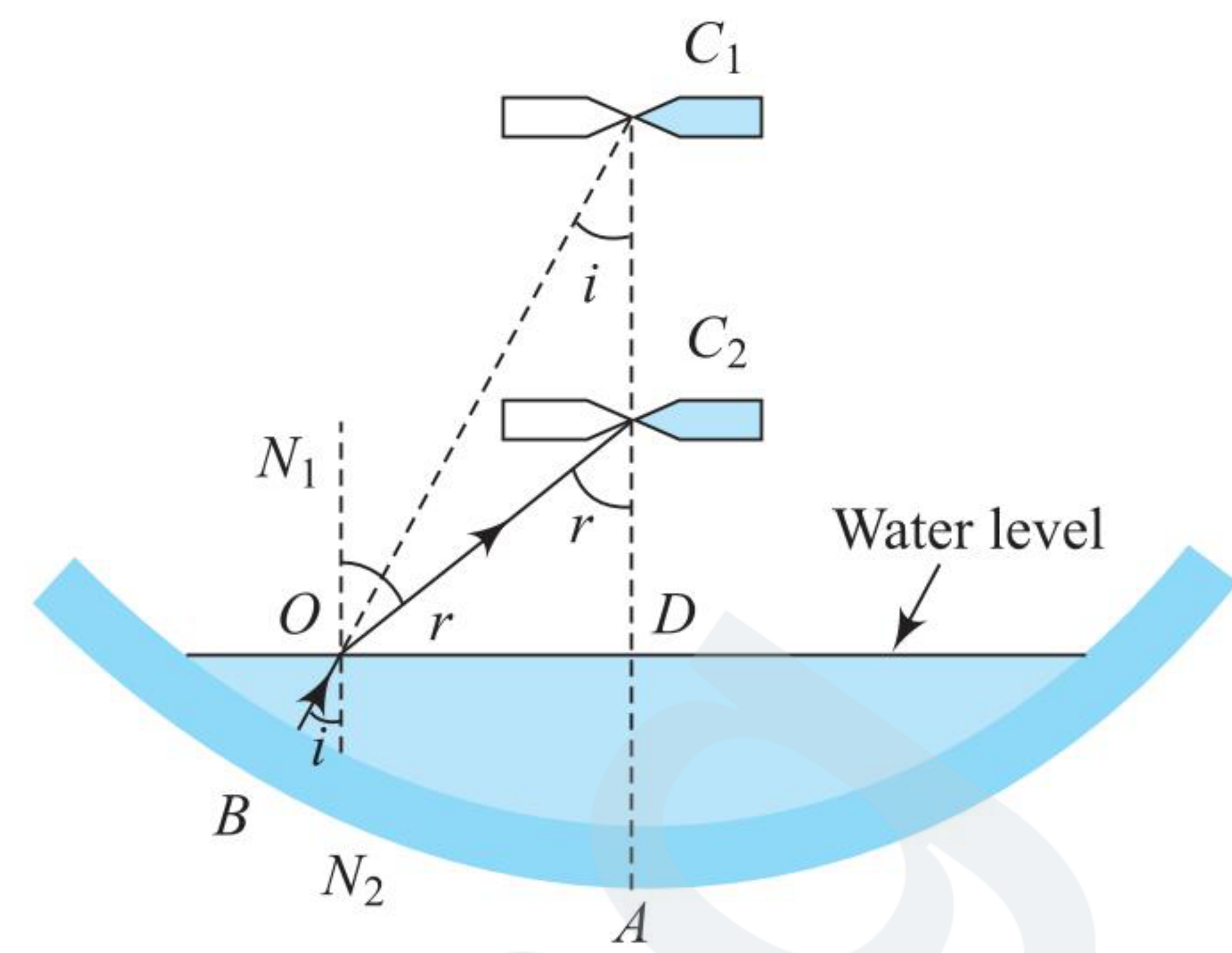
On substituting $y = 2.0$ m and $a_0 = 2 \times 10^{-6} \text{ m}^{-1}$,

$$x_{\max} = 2 \sqrt{\frac{2}{2 \times 10^{-6}}} = 2000 \text{ m}$$

MEASUREMENT OF REFRACTIVE INDEX OF A LIQUID BY A CONCAVE MIRROR

A concave mirror of large radius of curvature is placed on a table with its principal axis vertical, as shown in figure. A horizontal pin is placed with its tip on the principal axis of the mirror. The pin is moved till there is no parallax between the tip of the pin and its image, when the pin lies at the centre of curvature of the mirror.

A small quantity of liquid whose refractive index is to be measured is poured into the mirror. The pin is moved down in order to remove the parallax between the tip of the pin and its image.



In the figure, C_1 is the position of the pin when its image coincides with itself without liquid, and C_2 is the position of the pin when its image coincides with itself after pouring the liquid into the concave mirror.

The ray BO that is normal to the mirror passes through C_1 before pouring the liquid. It is refracted away from the normal when the liquid is poured, now it passes through C_2 .

$$\text{In figure, } \sin i = \frac{OD}{OC_1}, \quad \sin r = \frac{OD}{OC_2}$$

From Snell's law, $\mu \sin i = 1 \sin r$

$$\mu = \frac{OD}{OC_2} \times \frac{OC_1}{OD} = \frac{OC_1}{OC_2}$$

Taking paraxial ray assumption, $OC_1 \approx DC_1$; $OC_2 \approx DC_2$

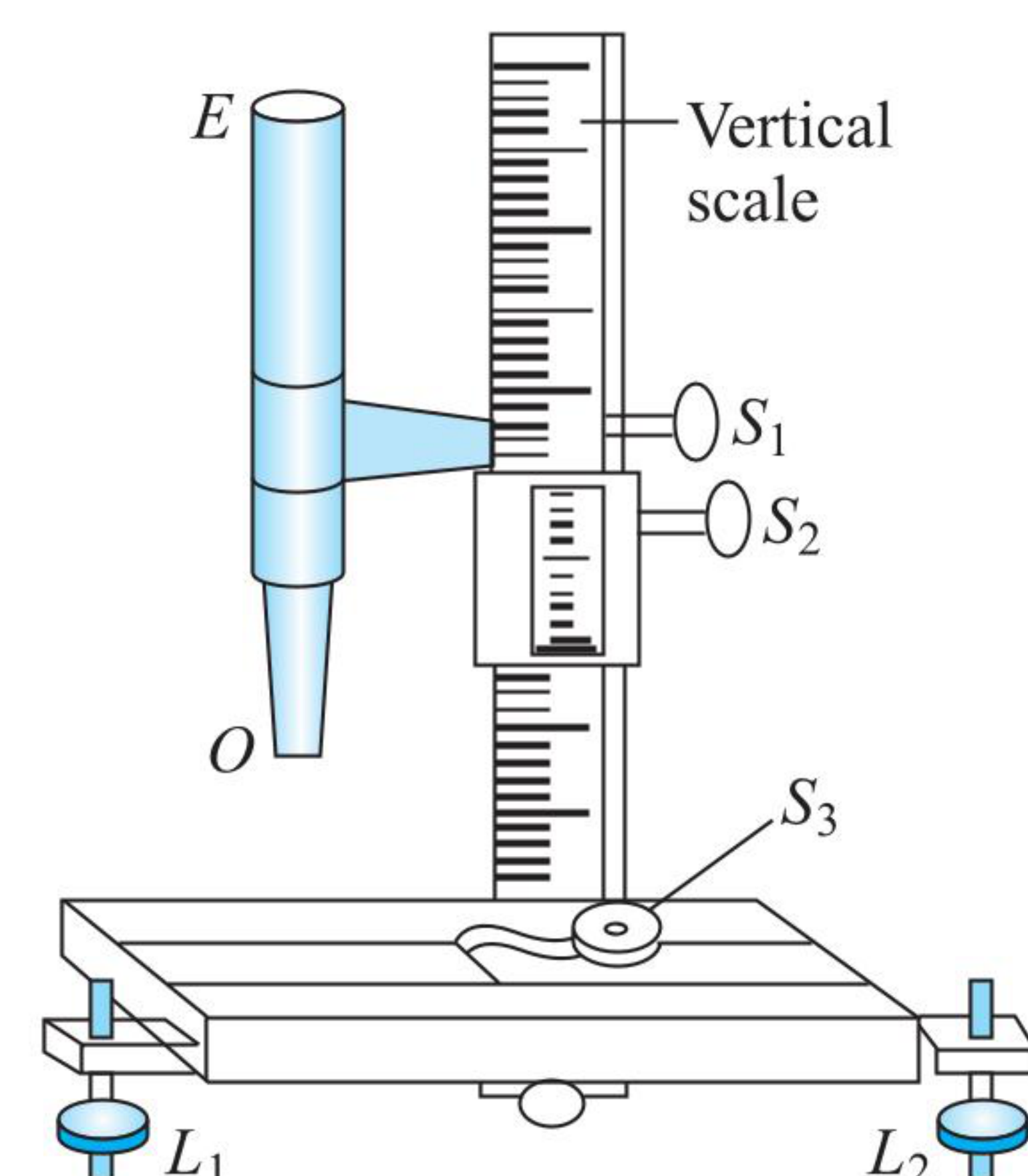
$$\text{Thus, } \mu = \frac{DC_1}{DC_2}$$

If we neglect the depth of the liquid, we have $\mu = \frac{AC_1}{AC_2}$

MEASUREMENT OF REFRACTIVE INDEX OF A LIQUID BY A TRAVELLING MICROSCOPE

Using the fact that $\mu = \frac{\text{Real depth}}{\text{Apparent depth}}$

We may determine the refractive index of a glass slab. (see figure)



The travelling microscope is first focused on a fine mark on a piece of paper and the reading on the scale is noted as Y_1 . Then, the glass slab is placed on the paper.

The microscope is now shifted up and is focused again on the mark so that it is distinctly visible. The reading of the microscope is noted as Y_2 .

Now, a little lycopodium powder is scattered on the upper surface of the glass block. The microscope is focused on the powder and the third reading of the scale is noted as Y_3 .

The real thickness of the glass slab is $Y_3 - Y_1$ and apparent thickness of the glass slab is $Y_3 - Y_2$. Thus, $\mu = \frac{Y_3 - Y_1}{Y_3 - Y_2}$.

The same method can also be used to find the refractive index of a liquid. Any mark on the bottom of a glass vessel is first focused and then liquid is poured into it and the same mark is again focused. Finally, a little lycopodium powder is sprinkled on the surface of the liquid and is focused as before.

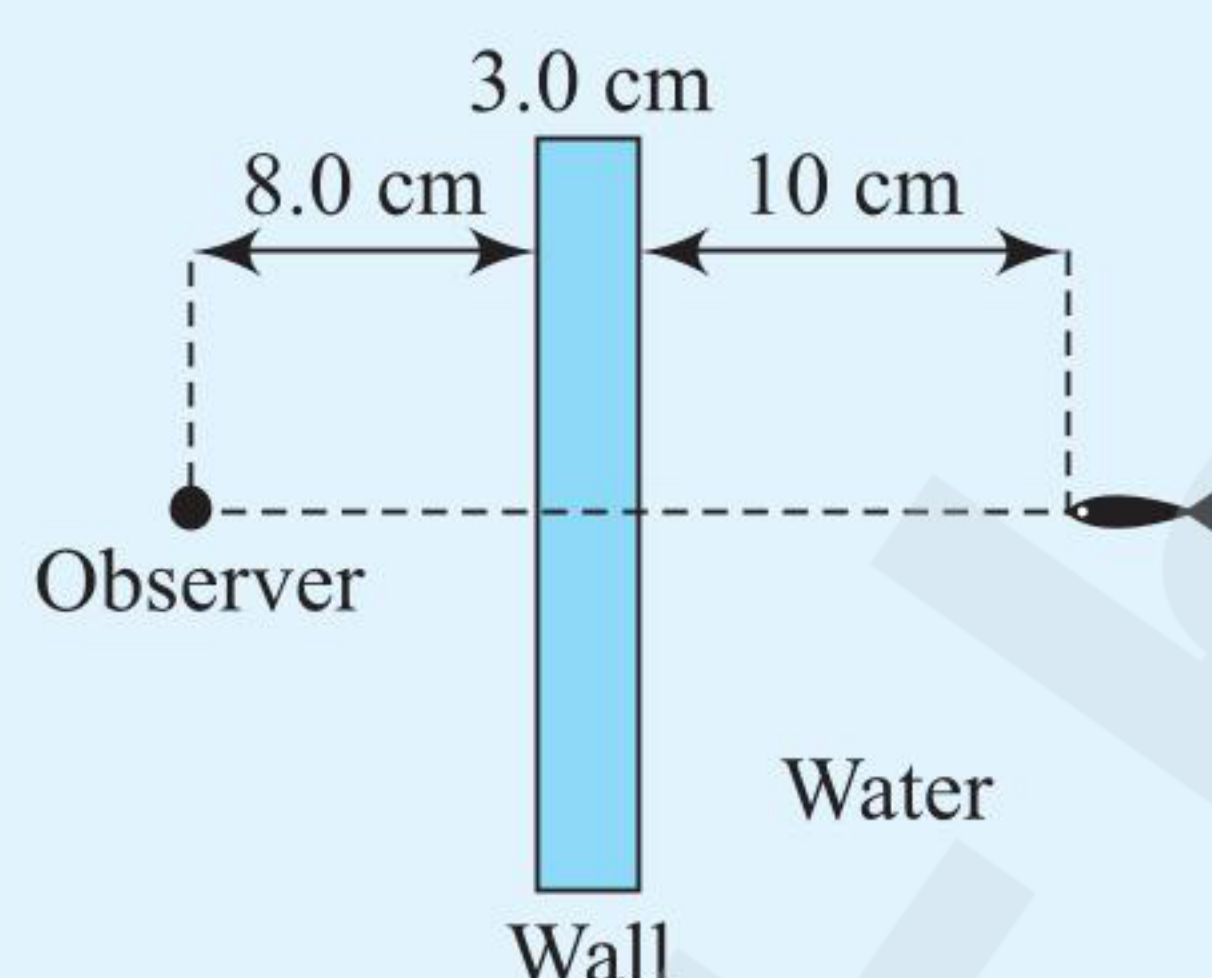
CONCEPT APPLICATION EXERCISE 1.5

1. Identify the True or False statements.

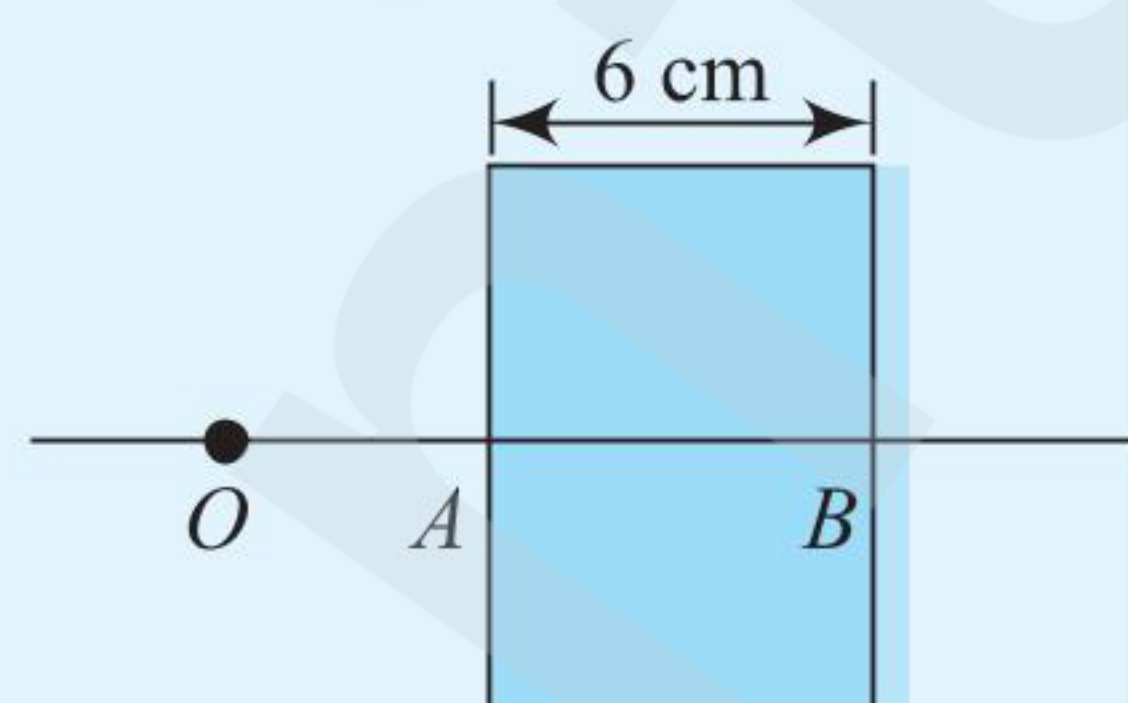
- A glass slab cannot deviate the light.
- A glass slab can produce lateral displacement.
- The shift produced by a slab depends on the converging and diverging nature of beam.
- Apparent shift in case of a slab always occurs in the direction of light ray travelling.
- The shift produced by a slab can never exceed its thickness.

2. In figure, a fish watcher watches a fish through a 3.0 cm thick glass wall of a fish tank. The watcher is in level with the fish; the index of refraction of the glass is $8/5$ and that of the water is $4/3$.

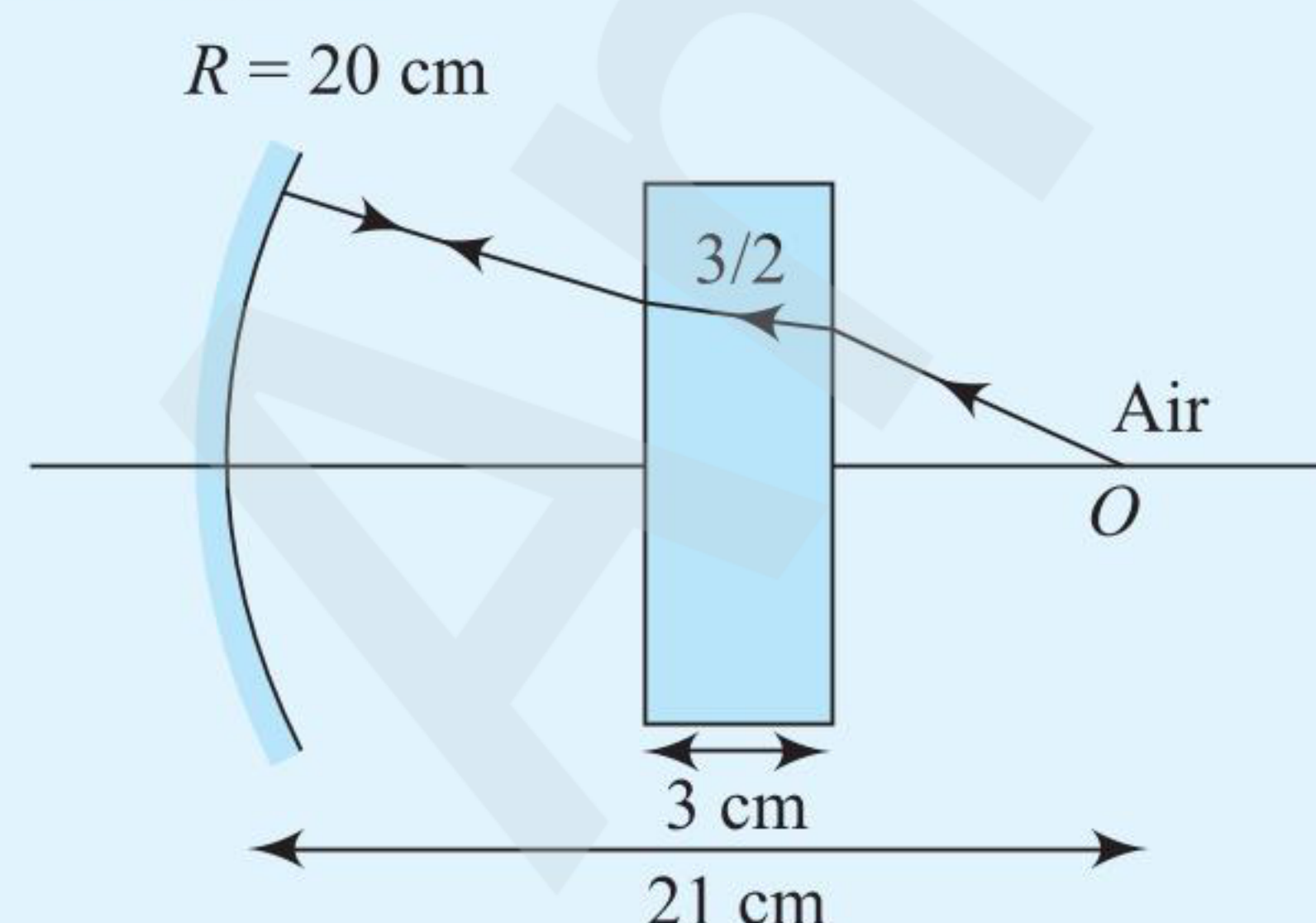
- To the fish, how far away does the watcher appear to be?
- To the watcher, how far away does the fish appear to be?



3. An object O is placed at 8 cm in front of a glass slab, whose one face is silvered as shown in figure. The thickness of the slab is 6 cm. If the image formed 10 cm behind the silvered face, find the refractive index of glass.

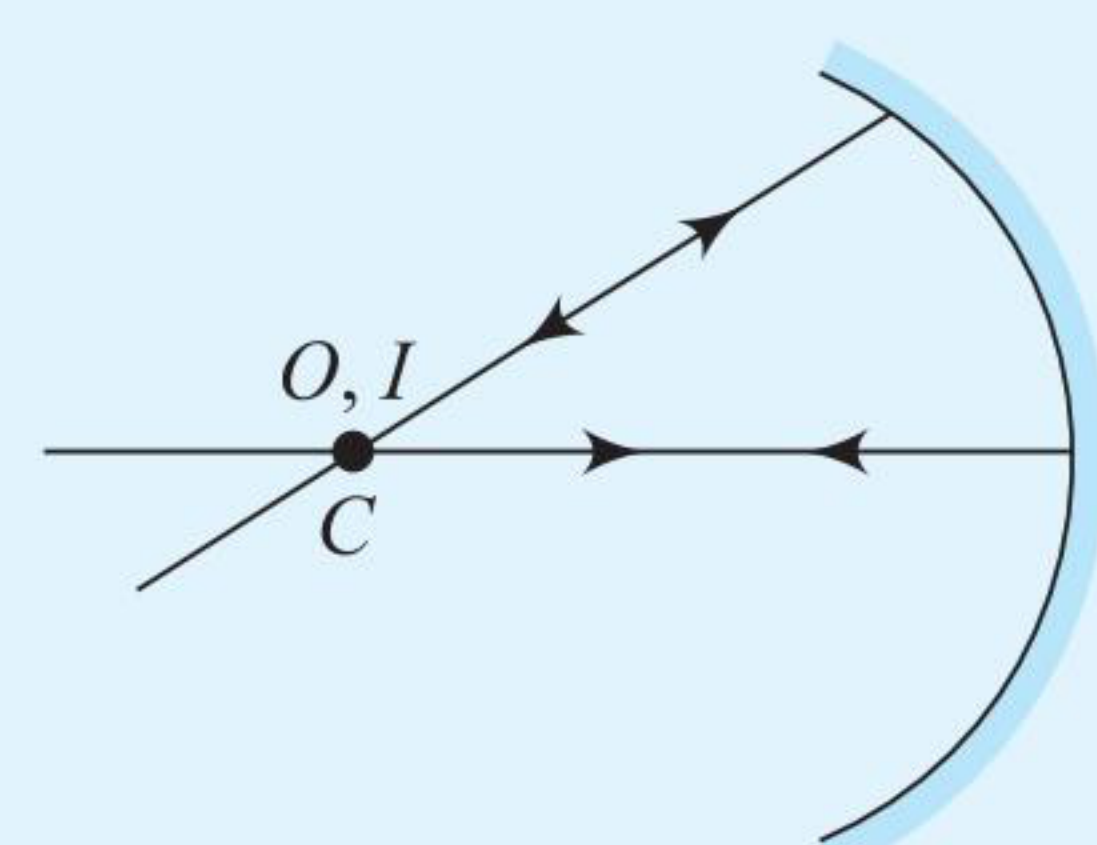


4. An object is placed on the principal axis of a concave mirror of focal length 10 cm at a distance of 21 cm from it. A glass slab is placed between the mirror and the object as shown in figure.



Find the distance of final image formed by the mirror.

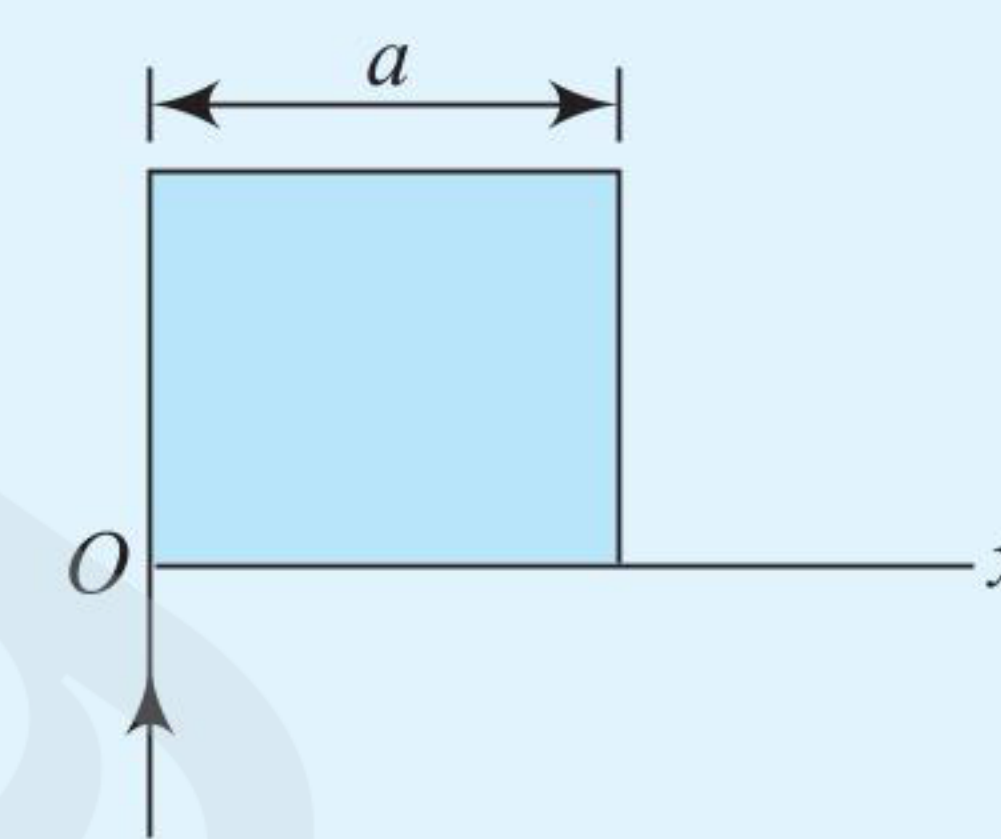
5. The image of an object kept at a distance of 30 cm in front of a concave mirror is found to coincide with itself. If a glass slab ($\mu = 1.5$) of thickness 3 cm is introduced between the mirror and the object, then



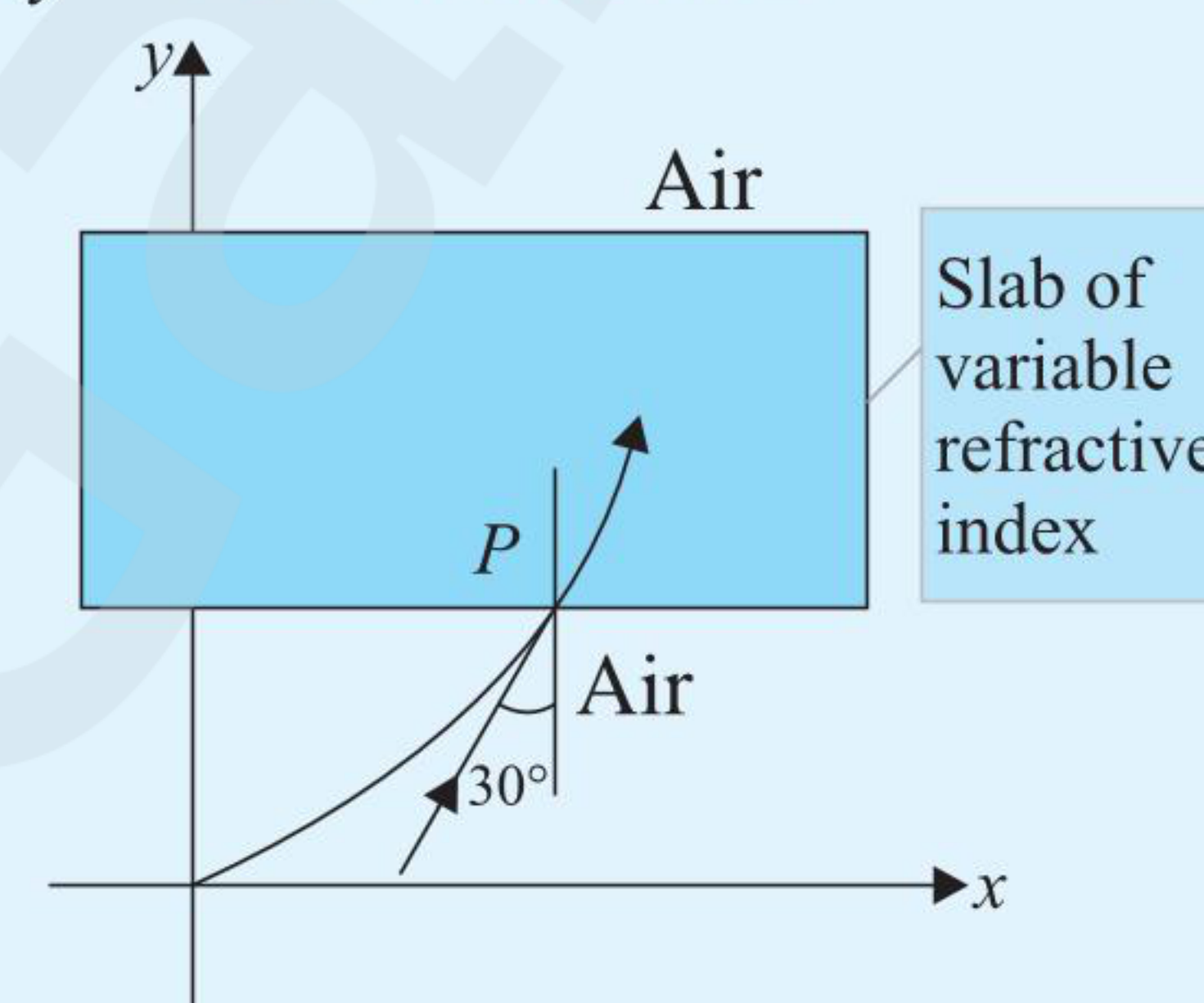
(a) Identify, in which direction the mirror should be displaced so that the final image may again coincide with the object itself.

(b) Find the magnitude of displacement.

6. The refractive index of an anisotropic medium varies as $\mu = \mu_0 \sqrt{(x+1)}$, where $0 \leq x \leq a$. A ray of light is incident at the origin just along y -axis (shown in figure). Find the equation of ray in the medium.

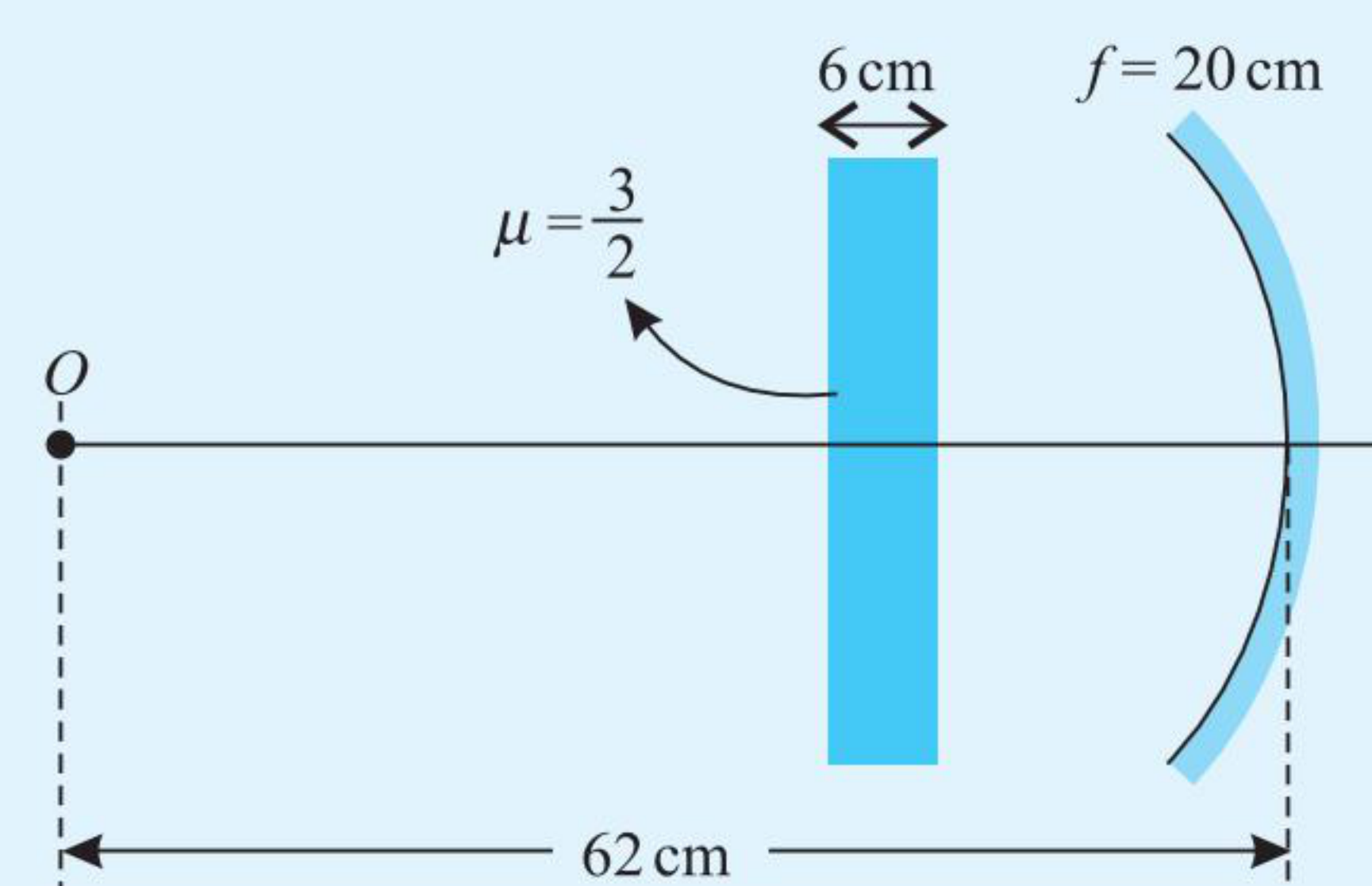


7. A ray of light travelling in air is incident at angle of incident 30° on one surface of slab in which refractive index varies with y . The light travels along the curve $y = 4x^2$ (y and x are in meter) in the slab. Find out the refractive index of the slab at $y = 1/2$ m in the slab.



8. A ray of light is incident on a glass slab at grazing incidence. The refractive index of the material of the slab is given by $\mu = \sqrt{(1+y)}$. If the thickness of the slab is $d = 2$ m, determine the equation of the trajectory of the ray inside the slab and the coordinates of the point where the ray exits from the slab. Take the origin to be at the point of entry of the ray.

9. A point object O is placed at a distance of 62 cm in front of a concave mirror of focal length $f = 20$ cm. A glass slab of refractive index $\mu = \frac{3}{2}$ and thickness 6 cm is inserted between the object and the mirror. Let's define final image as image formed after the light ray originating from O passes through the slab, gets reflected from the mirror and then again passes through the slab. Where final image will form?



ANSWERS

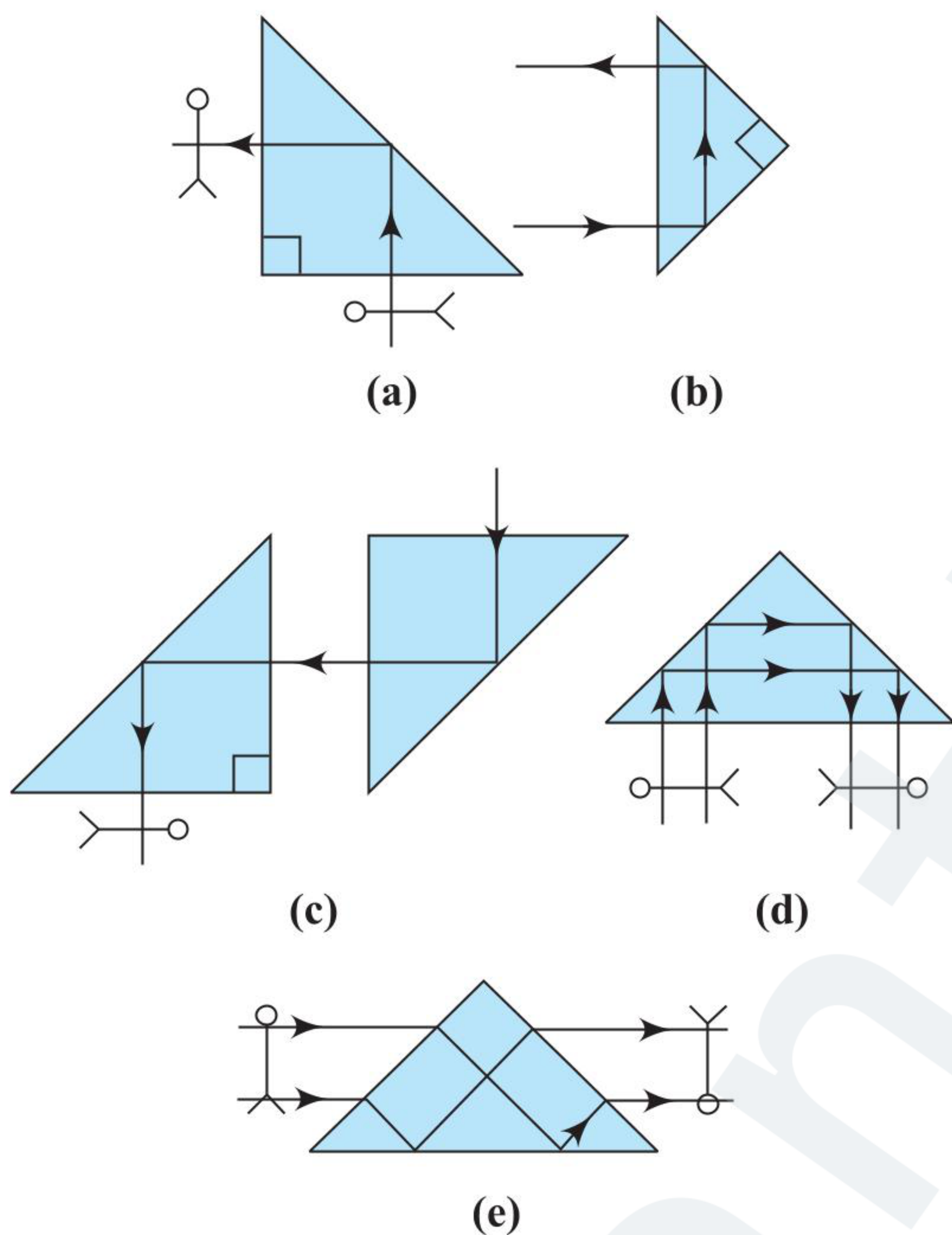
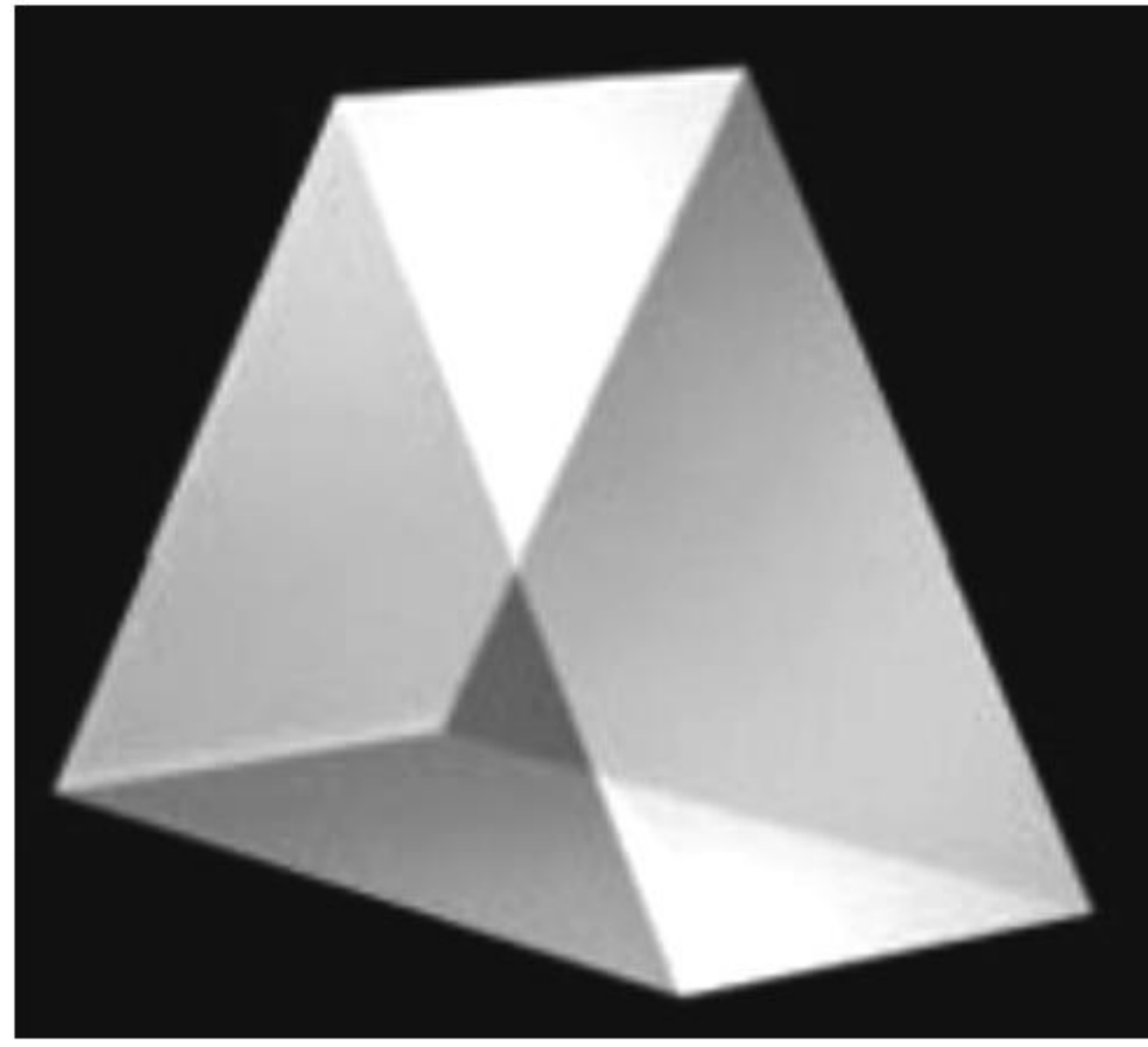
- (a) True; (b) True; (c) False; (d) False; (e) True
- (a) $\frac{139}{6}$ cm; (b) $\frac{139}{8}$ cm
- 1.5
- 21 cm
- (a) Rightwards; (b) 1 cm
- $y = 2\sqrt{x}$
- $\frac{3}{2}$
- $y = \frac{x^2}{4}$; $2\sqrt{2}$ m, 2 m
- 32 cm

PRISM

A prism is a transparent medium whose refracting surfaces are not parallel but are inclined to each other (see figure).

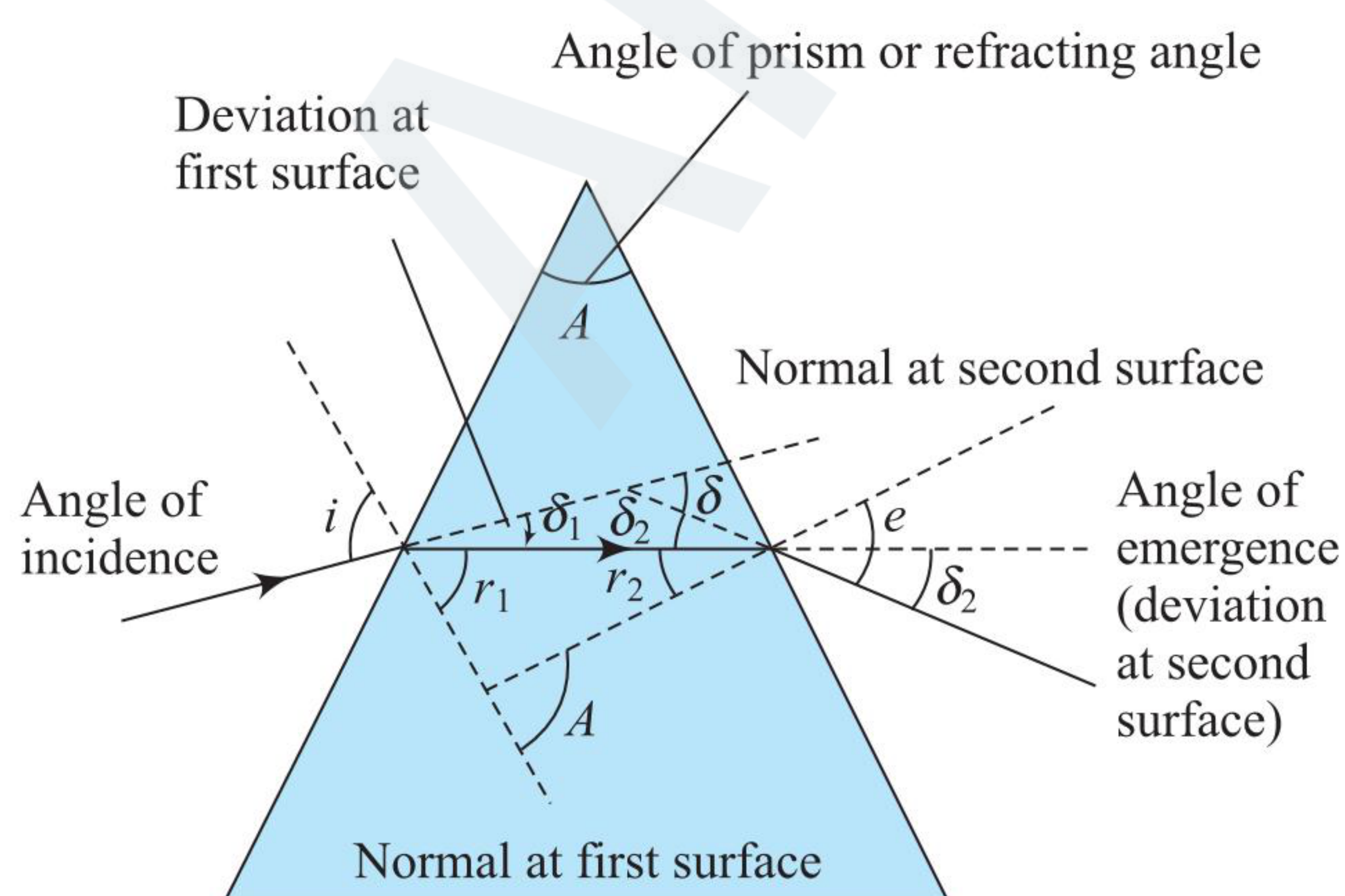
Prisms have the property to change the direction of light. Figures (a) to (e) present some of the useful applications.

The right-angled prism in Fig. (a) turns the light through 90° . A porro prism shown in Fig. (b) is a right-angled prism, it turns light through 180° . Figure (c) shows two right-angled prisms used in binoculars. Figures (d) and (e) illustrate image formation with or without deviation; in both the cases the image is laterally inverted.



Basic Terms

1. **Angle of prism or refracting angle (A):** The angle between the faces on which light is incident and from which it emerges (see figure).



2. **Angle of deviation (δ):** It is the angle between the emergent and the incident ray. In other words, it is the angle through which incident ray turns in passing through a prism.

$$\begin{aligned}\delta &= (i - r_1) + (e - r_2) \\ &= i + e - (r_1 + r_2) \\ &= i + e - A\end{aligned}$$

CONDITION OF NO EMERGENCE

A ray of light incident on a prism of angle A and refractive index μ will not emerge out of a prism (whatever may be the angle of incidence) if $A > 2\theta_c$, where θ_c is the critical angle, i.e., $\mu > 1 / [\sin (A/2)]$ (see figure).

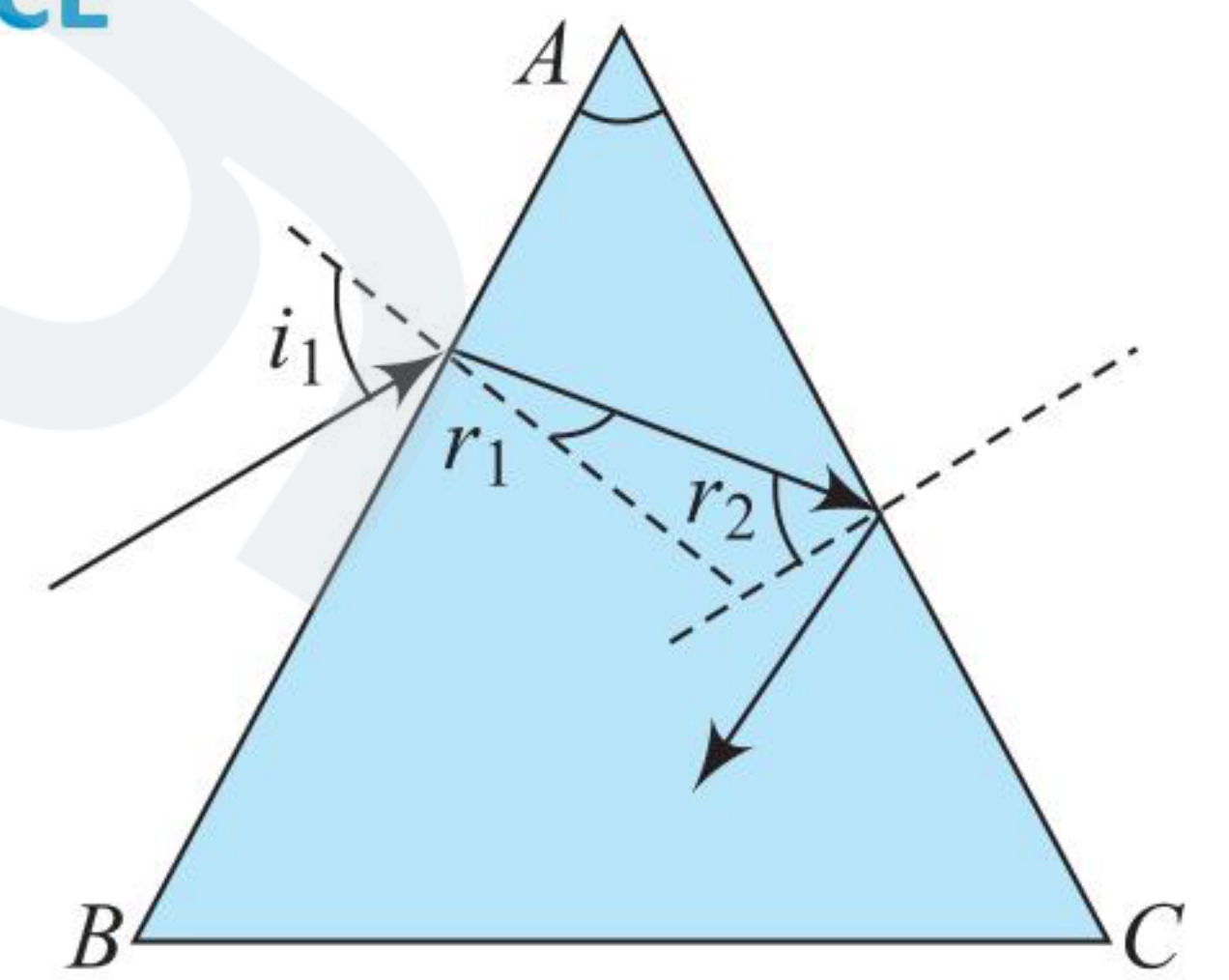
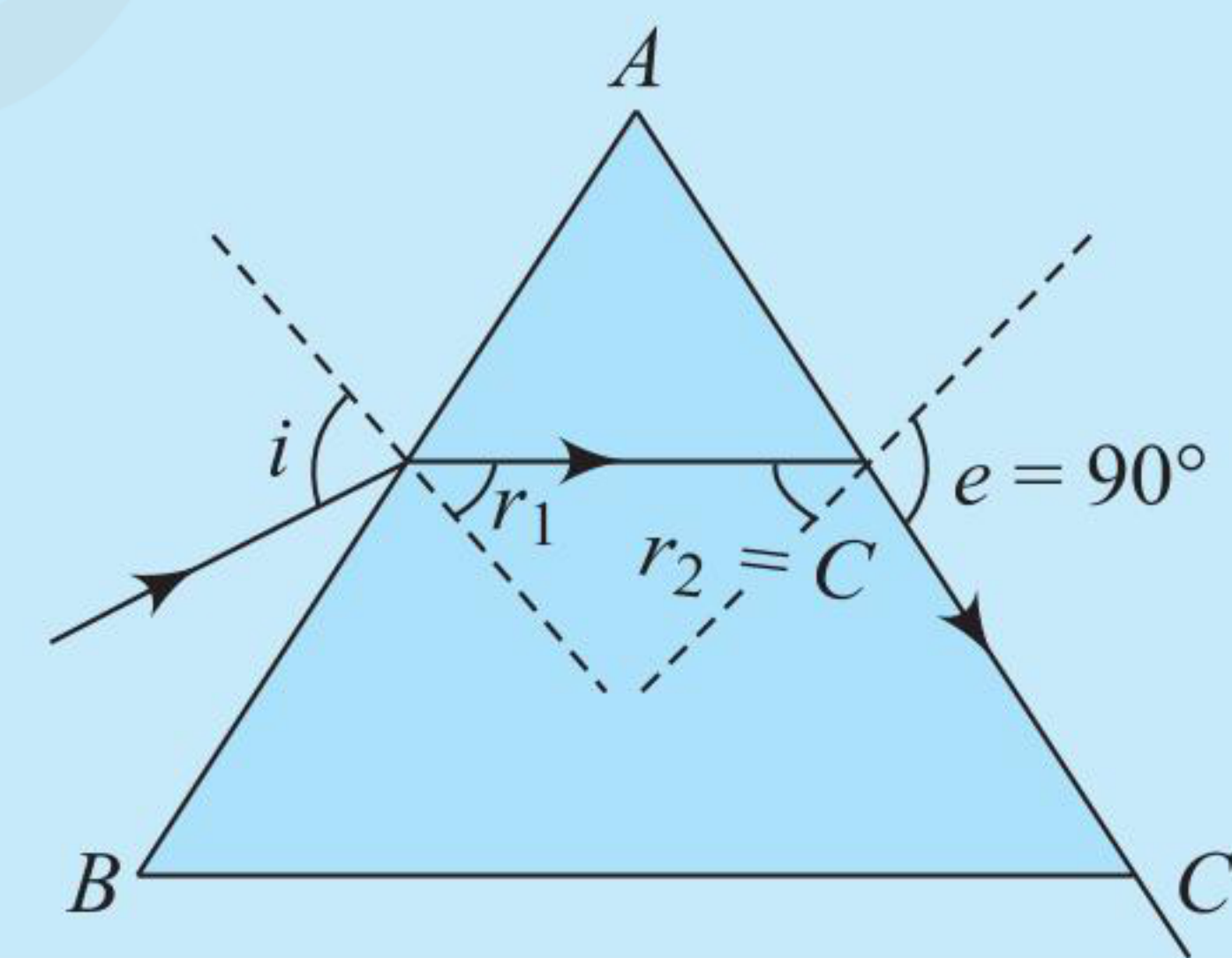


ILLUSTRATION 1.74

What should be the minimum value of refractive index of a prism, refracting angle A , so that there is no emergent ray irrespective of the angle of incidence?



Sol. If the ray just emerges from face AC ,

$$e = 90^\circ \quad \text{and} \quad r_2 = C \quad \dots(i)$$

From Snell's law at face AB , we have

$$1 \sin i = n \sin r_1 \quad \dots(ii)$$

$$A = r_1 + r_2 = r_1 + C \quad \dots(iii)$$

From Eq. (ii), n is minimum when r_1 is maximum, i.e., $r_1 = C$. In this case, $i = 90^\circ$

From Eq. (iii), $A = 2C$ or $C = A/2$

$$\text{As } \sin C = \frac{1}{n} \Rightarrow \sin \frac{A}{2} = \frac{1}{n}$$

$$n = \operatorname{cosec} \frac{A}{2}$$

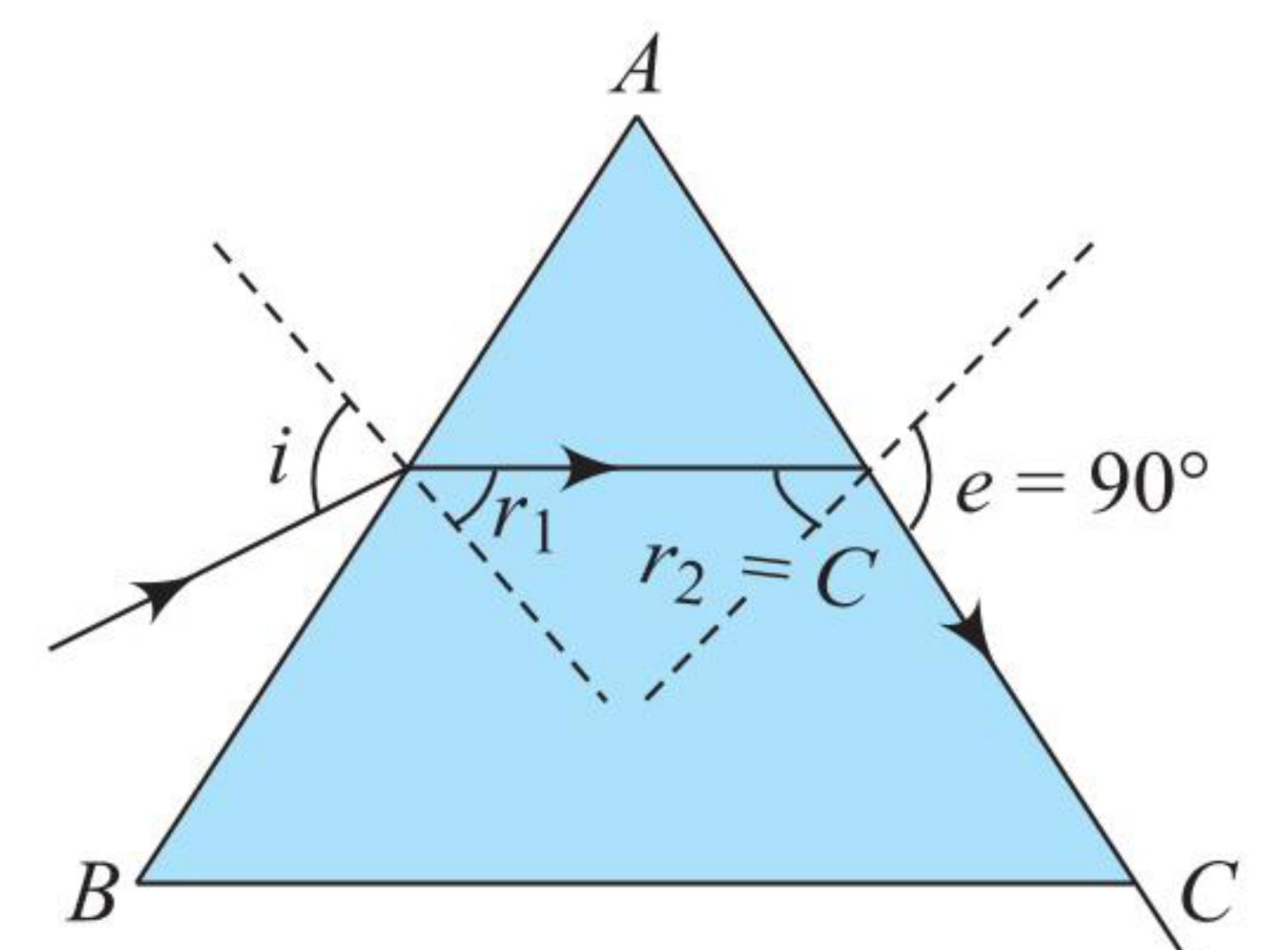
CONDITION OF GRAZING EMERGENCE

By the condition of grazing emergence, we mean the angle of incidence i at which the angle of emergence becomes 90° (see figure).

Consider a prism with refracting angle A and refractive index n .

The ray grazes face AC .

$$\text{So, } e = 90^\circ, r_2 = C \quad \dots(i)$$



$$\text{and } A = r_1 + r_2 = r_1 + C \quad \dots(ii)$$

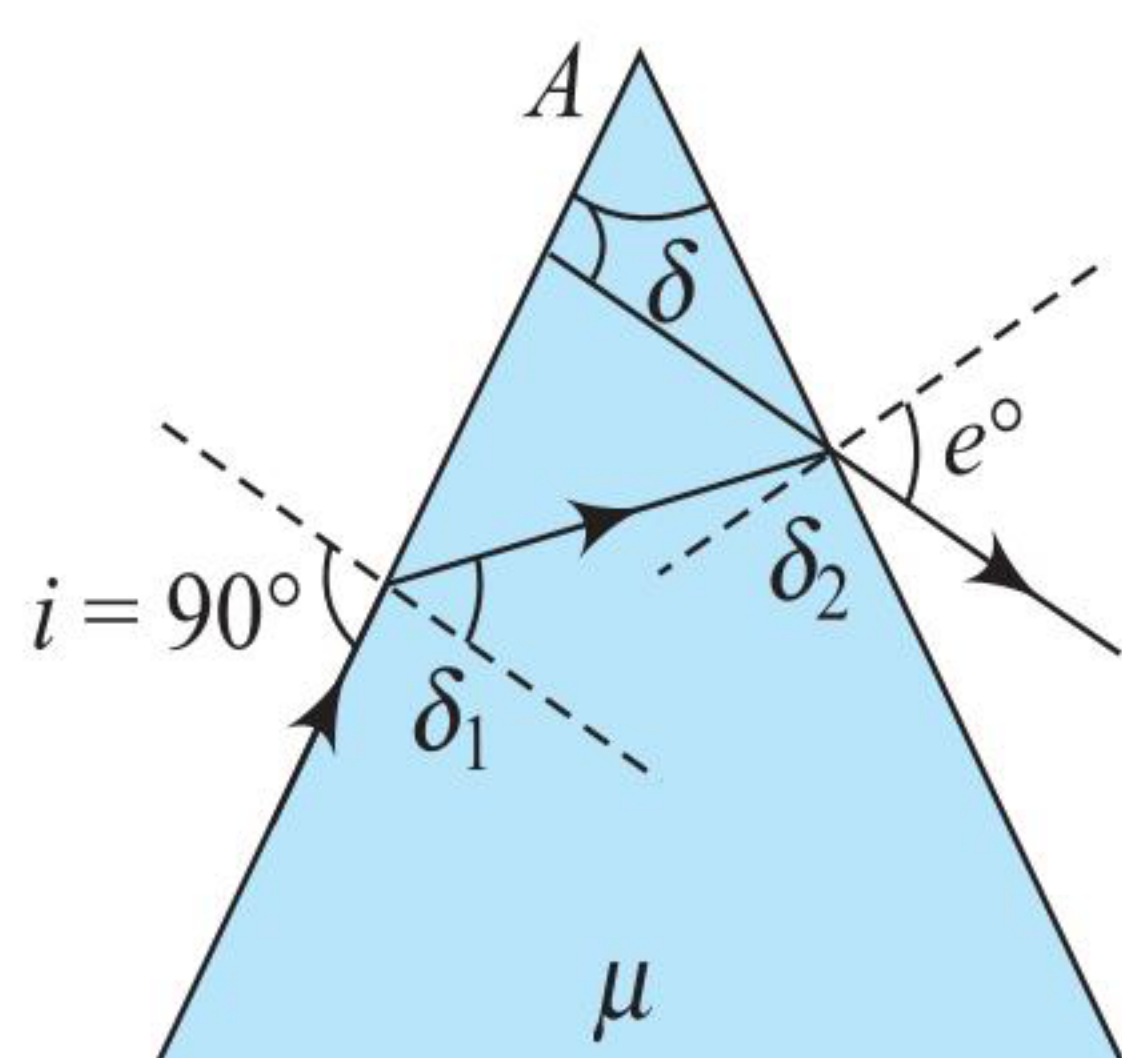
$$\text{Also, } \sin C = 1/n$$

From Snell's law at face AB , $1 \sin i = n \sin r_1$

$$\begin{aligned} \sin i &= n \sin (A - C) \\ &= n [\sin A \cos C - \cos A \sin C] \\ &= n \left[\sin A \sqrt{1 - \sin^2 C} - \cos A \sin C \right] \\ &= \sqrt{n^2 - 1} \sin A - \cos A \\ i &= \sin^{-1} \left[\sqrt{n^2 - 1} \sin A - \cos A \right] \end{aligned}$$

CONDITION OF MAXIMUM DEVIATION

Maximum deviation occurs when the angle of incidence is 90° (see figure).

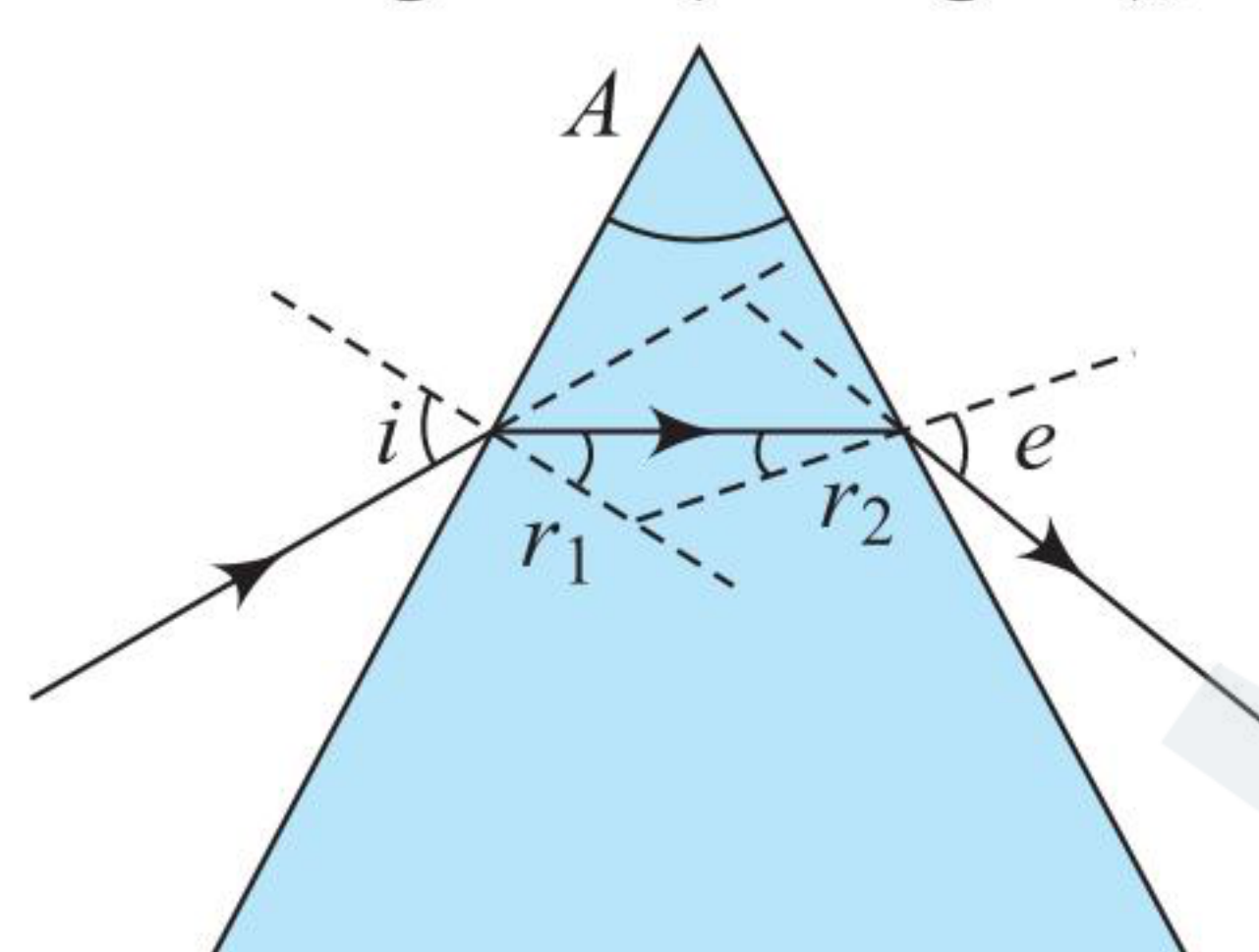


$$\delta_{\max} = 90^\circ + e - A$$

$$\text{where } e = \sin^{-1} [\mu \sin (A - \theta_c)]$$

CONDITION OF MINIMUM DEVIATION

The minimum deviation occurs when the angle of incidence is equal to the angle of emergence (see figure),

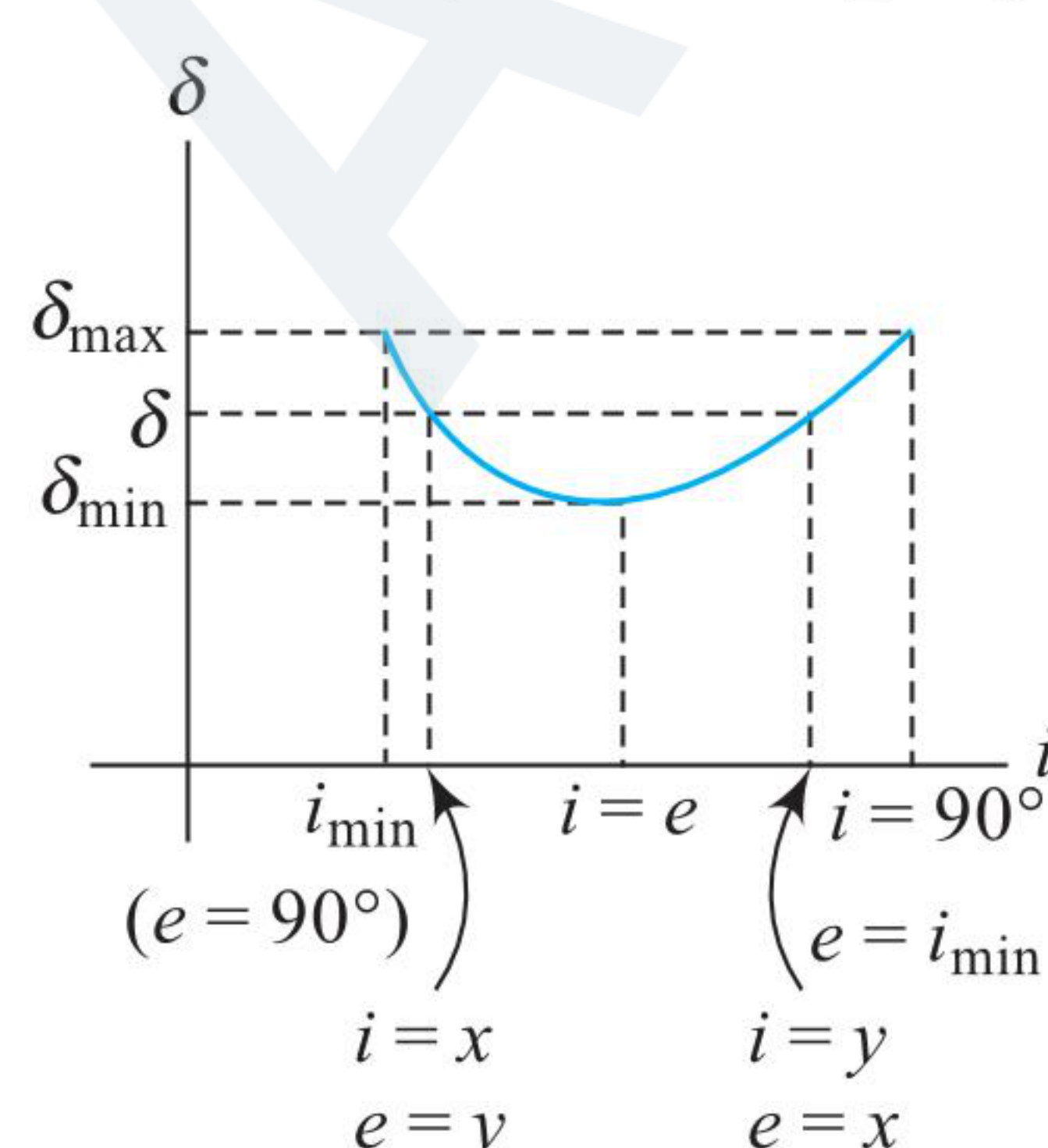


$$\text{i.e., } i = e; \delta_{\min} = 2i - A.$$

$$\text{Using Snell's law, we get } \mu = \frac{\sin [(\delta_{\min} + A)/2]}{\sin [A/2]}$$

Note that in the condition of minimum deviation the light ray passes through the prism symmetrically, i.e., the light ray in the prism becomes parallel to its base.

1. Variation of δ versus i (shown in figure).

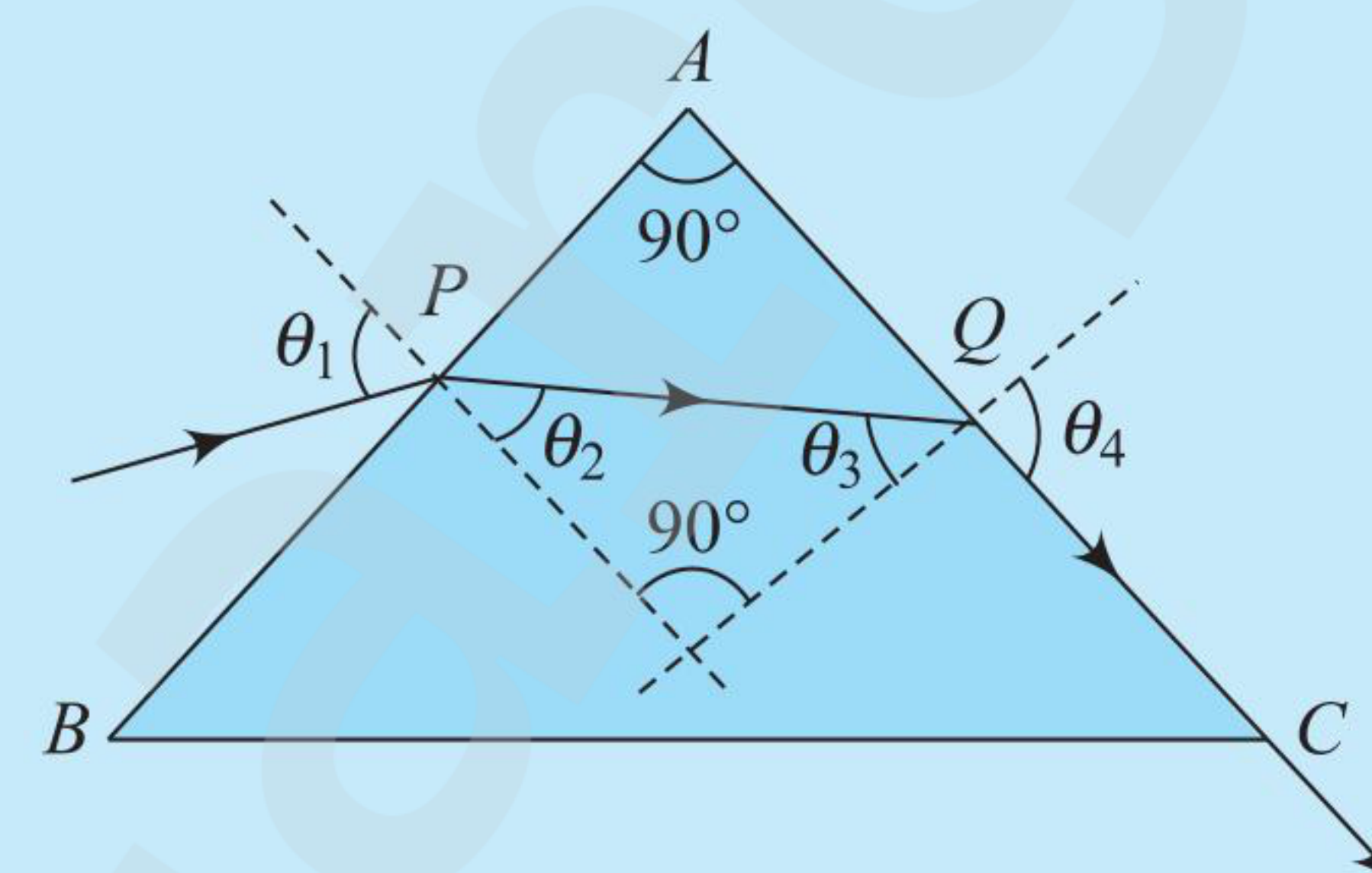


For each δ (except $\delta_{\min}$), there are two values of angle of incidence. If i and e are interchanged, then we get the same value of δ because of reversibility principle of light.

- There is one and only one angle of incidence for which the angle of deviation is minimum.

ILLUSTRATION 1.75

Figure shows a triangular prism of refracting angle 90° . A ray of light incident at face AB at an angle θ_1 refracts at point Q with an angle of refraction 90° .



- What is the refractive index of the prism in terms of θ_1 ?
- What is the maximum value that the refractive index can have?
- What happens to the light at Q if the incident angle at Q is (i) increased slightly, and (ii) decreased slightly?

Sol.

- Let the ray be incident at an angle θ_1 at the face AB . It refracts at an angle θ_2 and is incident at an angle θ_3 at face AC . Finally, the ray comes out at an angle $\theta_4 = 90^\circ$.

From the above figure, the normals at faces AB and AC make an angle of 90° with each other, $\theta_3 = 90^\circ - \theta_2$

$$\sin \theta_3 = \sin (90^\circ - \theta_2) = \cos \theta_2 = \sqrt{1 - \sin^2 \theta_2} \quad \dots(i)$$

From Snell's law at face AC , $n \sin \theta_3 = 1$

$$n \sqrt{1 - \sin^2 \theta_2} = 1 \quad \dots(ii)$$

From Snell's law at face AB , $1 \sin \theta_1 = n \sin \theta_2$

$$\sin \theta_2 = \frac{\sin \theta_1}{n} \quad \dots(iii)$$

From Eqs. (ii) and (iii), we have

$$n \sqrt{1 - \frac{\sin^2 \theta_1}{n^2}} = 1 \quad \dots(iv)$$

On squaring Eq. (iv) and solving for n , we get

$$n = \sqrt{1 + \sin^2 \theta_1}$$

- The greatest possible value of $\sin^2 \theta_1$ is 1; hence, the greatest possible value of n is $n_{\max} = \sqrt{2} = 1.41$.
- (i) For a given n , if θ_1 is increased the angle of refraction θ_2 increases. As $\theta_3 = 90^\circ - \theta_2$, the angle θ decreases, i.e., the angle of incidence at face AC is less than the critical angle for total reflection; hence light emerges into air.
(ii) If the angle of incidence is decreased, the angle of refraction θ_2 decreases. So, the angle θ_3 increases. The

angle of incidence at the second surface is greater than the critical angle; so light is reflected at Q .

Note: In the condition of minimum deviation, the light ray passes through the prism symmetrically, i.e., the light ray in the prism becomes parallel to its base.

ILLUSTRATION 1.76

A prism has refracting angle equal to $\pi/2$. It is given that γ is the angle of minimum deviation and β is the deviation of the ray entering at grazing incidence. Prove that $\sin \gamma = \sin^2 \beta$.

Sol. Applying condition of minimum deviation,

$$\mu = \frac{\sin \frac{(A + \gamma)}{2}}{\sin \frac{A}{2}} = \frac{\sin \frac{A}{2} \cos \frac{\gamma}{2} + \cos \frac{A}{2} \sin \frac{\gamma}{2}}{\sin \frac{A}{2}}$$

$$= \cos \frac{\gamma}{2} + \cot \frac{A}{2} \sin \frac{\gamma}{2}$$

Using $A = 90^\circ$, $\mu = \cos \frac{\gamma}{2} + \cot 45^\circ \sin \frac{\gamma}{2}$

$$\Rightarrow \cos \frac{\gamma}{2} + \sin \frac{\gamma}{2} = \mu$$

Squaring, $\cos^2 \frac{\gamma}{2} + \sin^2 \frac{\gamma}{2} + \sin \gamma = \mu^2$

$$\Rightarrow \sin \gamma = \mu^2 - 1 \quad \dots(i)$$

Deviation at grazing incidence,

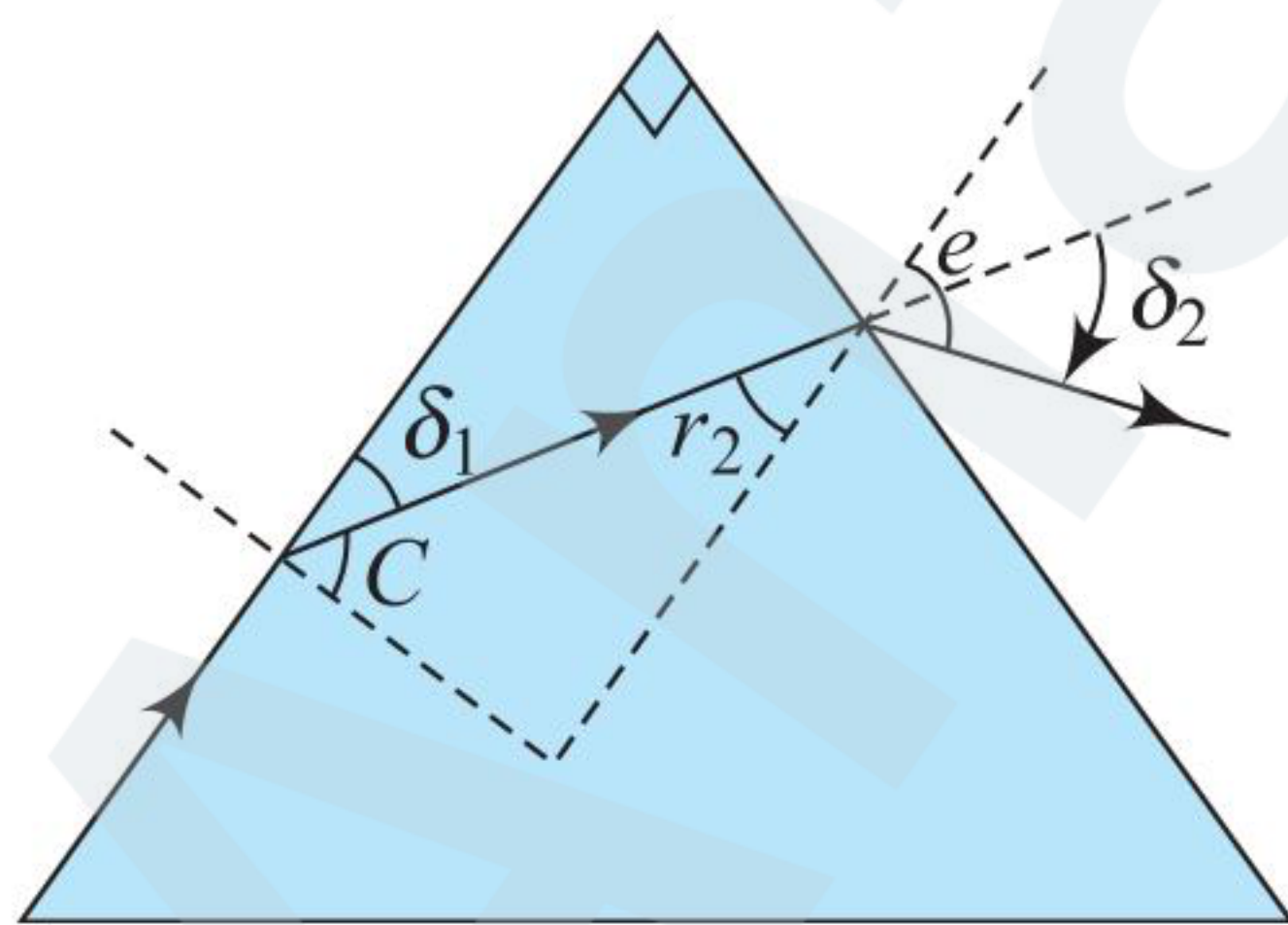
$$\beta = \delta_1 + \delta_2 = \left(\frac{\pi}{2} - C \right) + (e - r_2)$$

$$\Rightarrow \beta = \left(\frac{\pi}{2} - C \right) + \left[e - \left(\frac{\pi}{2} - C \right) \right]$$

$$\Rightarrow \beta = e$$

or $\sin \beta = \sin e = \mu \sin r_2 = \mu \sin \left(\frac{\pi}{2} - C \right)$

$$\Rightarrow \sin \beta = \mu \cos C \quad \dots(ii)$$



Squaring Eq. (ii),

$$\sin^2 \beta = \mu^2 \cos^2 C$$

$$\Rightarrow \sin^2 \beta = \mu^2 (1 - \sin^2 C)$$

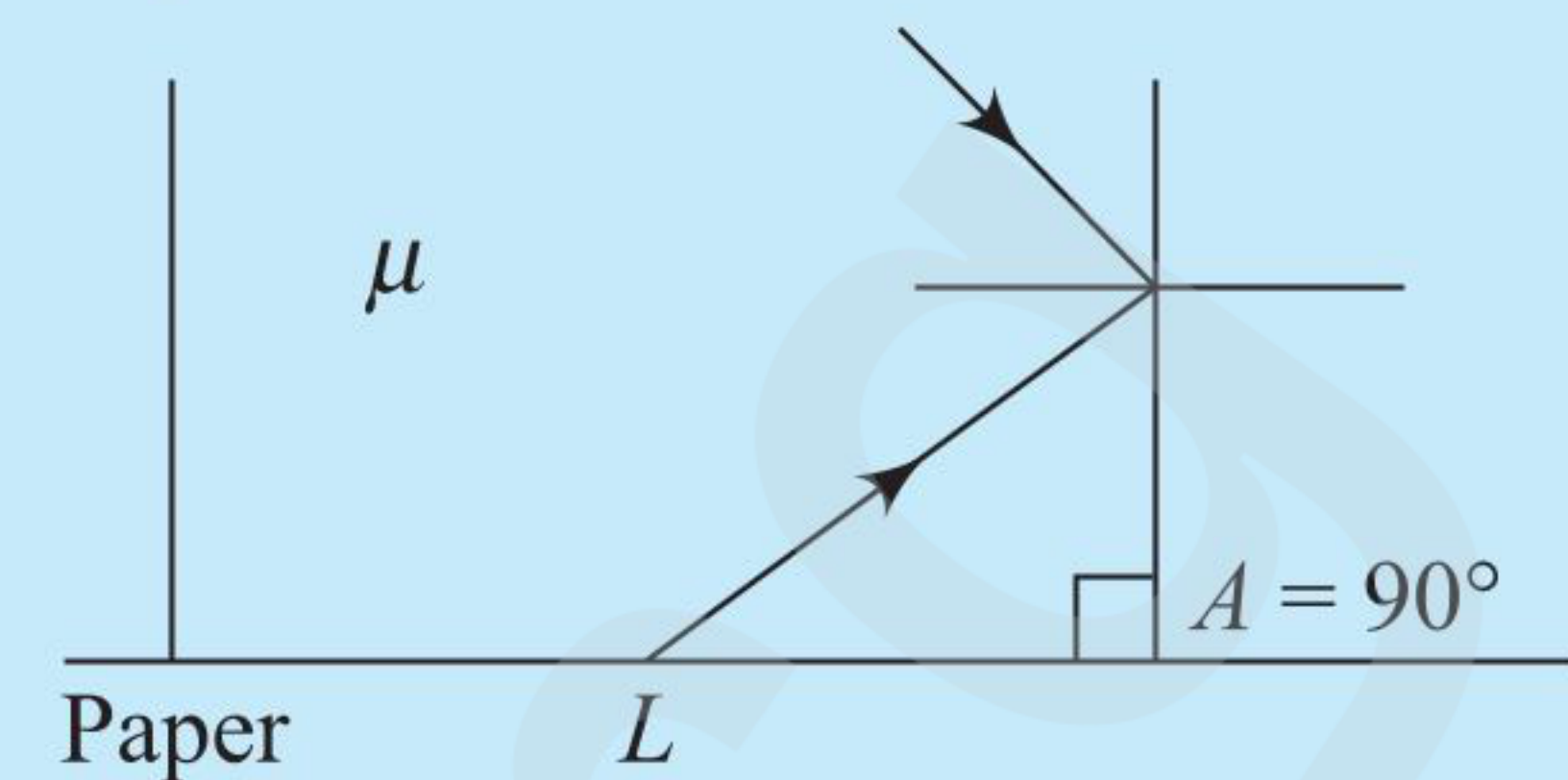
Using $\sin C = \frac{1}{\mu}$, $\sin^2 \beta = \mu^2 \left(1 - \frac{1}{\mu^2} \right)$

$$\Rightarrow \sin^2 \beta = \mu^2 - 1 \quad \dots(iii)$$

From Eqs. (i) and (iii), $\sin \gamma = \sin^2 \beta$

ILLUSTRATION 1.77

A rectangular block of refractive index μ is placed on a printed page lying on a horizontal surface as shown in figure. Find the minimum value of μ so that the letter L on the page is not visible from any of the vertical sides.



Sol. The letter L will not be visible from the vertical sides if the light ray does not enter through it.

We can apply the condition of no emergence for a prism of angle $A = 90^\circ$.

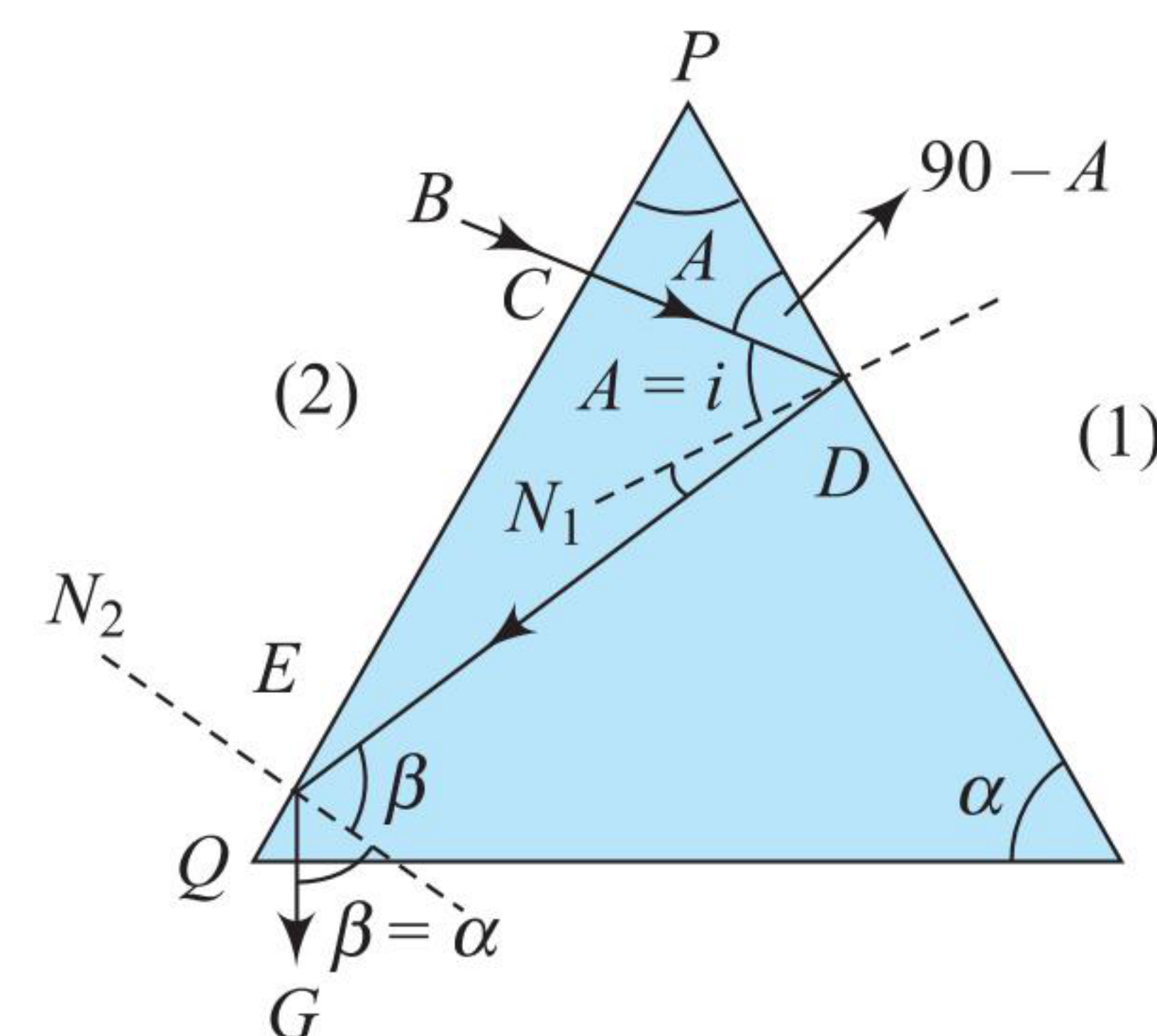
That is, $\mu > \frac{1}{\sin \frac{A}{2}}$

or $\mu > \frac{1}{\sin \frac{90^\circ}{2}}$ or $\mu > \sqrt{2}$

ILLUSTRATION 1.78

The cross section of a glass prism has the form of an isosceles triangle. One of the refracting faces is silvered. A ray of light falling normally on the other refracting face, being reflected twice, emerges through the base of the prism perpendicular to it. Find the angles of the prism.

Sol. The incident ray BC at normal incidence is reflected at silvered face, along DE and at E it again suffers reflection along EF . Since the ray emerges normally from the base, therefore the ray EF must fall normally on the base and emerges along EG .



We find $i = A$

Also, $\beta = \alpha$

Since $EN_2 \parallel CD$, $\beta = 2i$ (alternate angles)

$$\therefore \alpha = 2A \quad (\beta = \alpha, i = A) \quad \dots(i)$$

$$\text{Also, } 2\alpha + A = 180^\circ \quad \dots(ii)$$

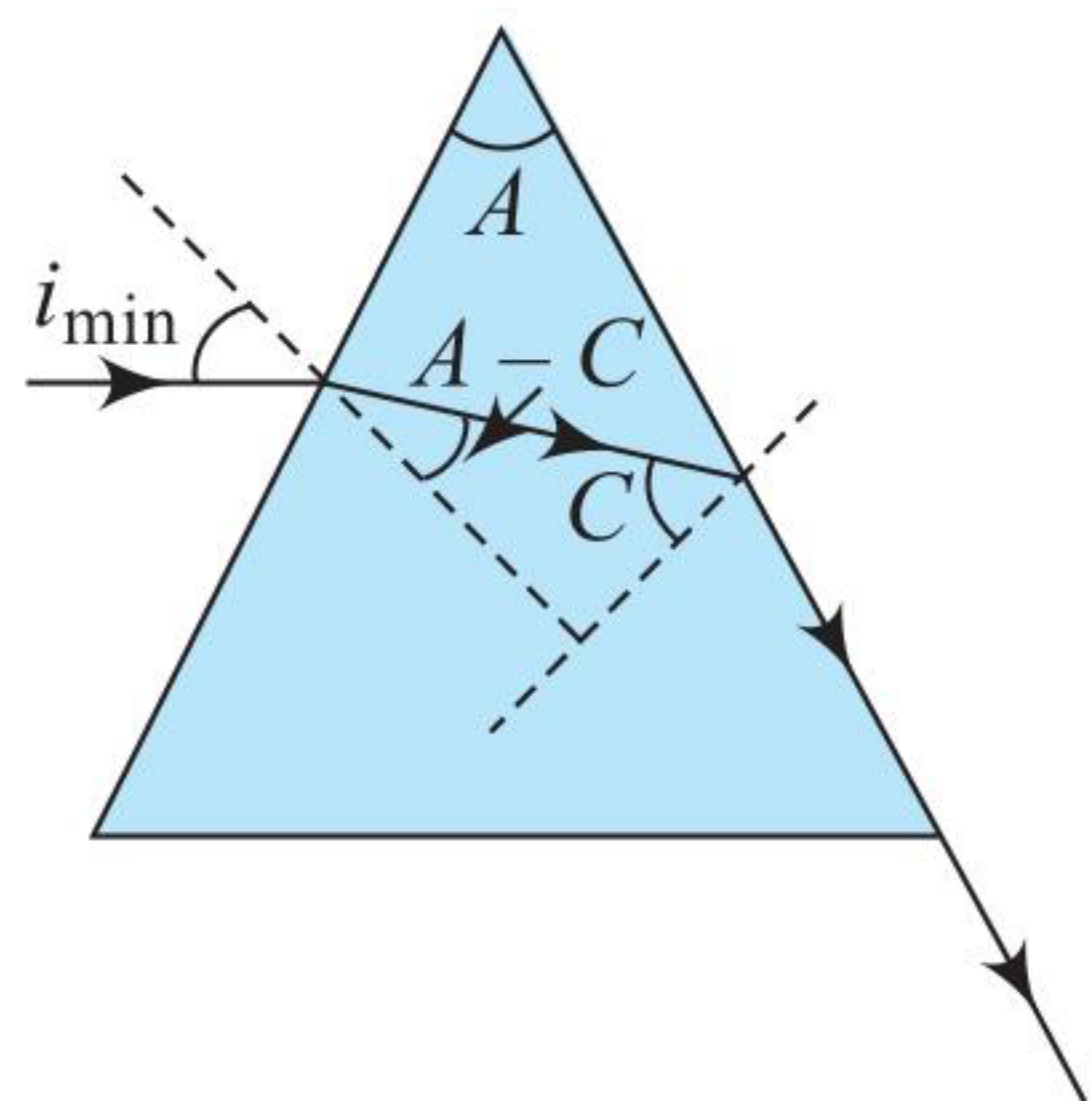
($\therefore$ Sum of angles of a triangle = 180°)

Solving Eqs. (i) and (ii), we get $A = 36^\circ$, $\alpha = 72^\circ$.

ILLUSTRATION 1.79

For a prism, $A = 60^\circ$, $n = \sqrt{7/3}$. Find the minimum possible angle of incidence, so that the light ray is refracted from the second surface. Also, find $\delta_{\max}$.

Sol. In minimum incidence case, the angles will be as shown in figure.



Applying Snell's law, we get

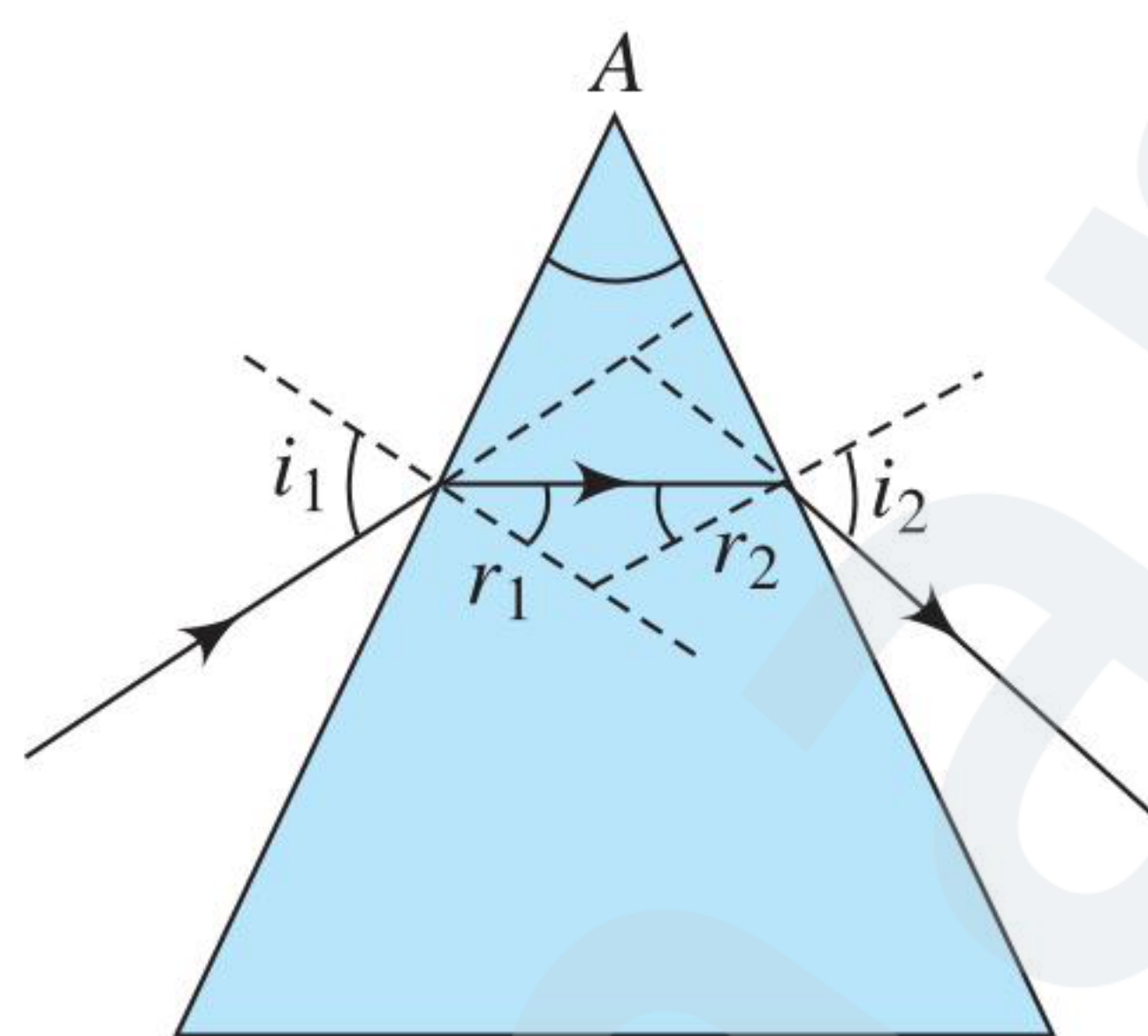
$$\begin{aligned} 1 \times \sin i_{\min} &= \sqrt{\frac{7}{3}} \sin(A - C) \\ &= \sqrt{\frac{7}{3}} (\sin A \cos C - \cos A \sin C) \\ &= \sqrt{\frac{7}{3}} \left(\sin 60^\circ \sqrt{1 - \frac{3}{7}} - \cos 60^\circ \sqrt{\frac{3}{7}} \right) = \frac{1}{2} \end{aligned}$$

$$\therefore i_{\min} = 30^\circ$$

$$\therefore \delta_{\max} = i_{\min} + 90^\circ - A = 30^\circ + 90^\circ - 60^\circ = 60^\circ$$

THIN PRISMS

In thin prisms, the distance between the refracting surfaces is negligible and the angle of prism (A) is very small. Since $A = r_1 + r_2$, therefore if A is small then both r_1 and r_2 are also small, and the same is true for i_1 and i_2 .



According to Snell's law, $\sin i_1 = \mu \sin r_1$

$$\text{or } i_1 = \mu r_1$$

$$\sin i_2 = \mu \sin r_2$$

$$\text{or } i_2 = \mu r_2$$

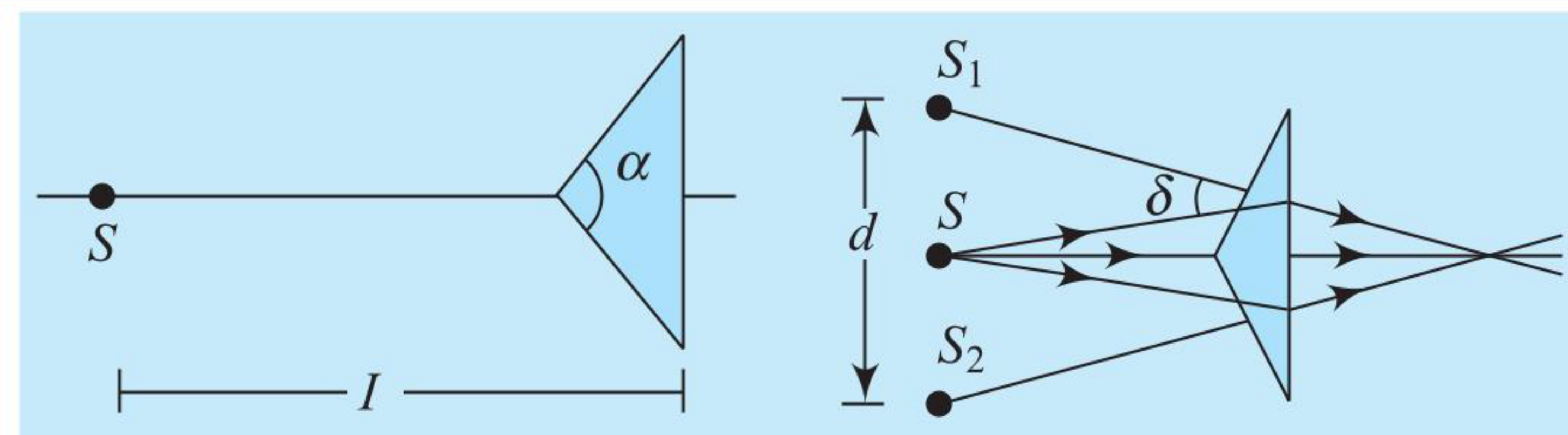
Therefore, deviation,

$$\delta = (i_1 - r_1) + (i_2 - r_2)$$

$$\Rightarrow \delta = (r_1 + r_2)(\mu - 1) \Rightarrow \delta = A(\mu - 1)$$

ILLUSTRATION 1.80

A thin biprism (see figure) of obtuse angle $\alpha = 178^\circ$ is placed at a distance $l = 20$ cm from a slit. How many images are formed and what is the separation between them? Refractive index of the material $\mu = 1.6$.



Sol. Two images are formed by the two thin prisms—one above the axis and the other below the axis by the same distance. The refracting angle of each thin prism is

$$\frac{\pi - \alpha}{2} = \frac{1}{2}(\pi - \alpha)$$

$$\text{Then, } \delta (\text{deviation of a ray}) = (\mu - 1) \frac{1}{2}(\pi - \alpha)$$

$$\frac{d}{2} = l\delta$$

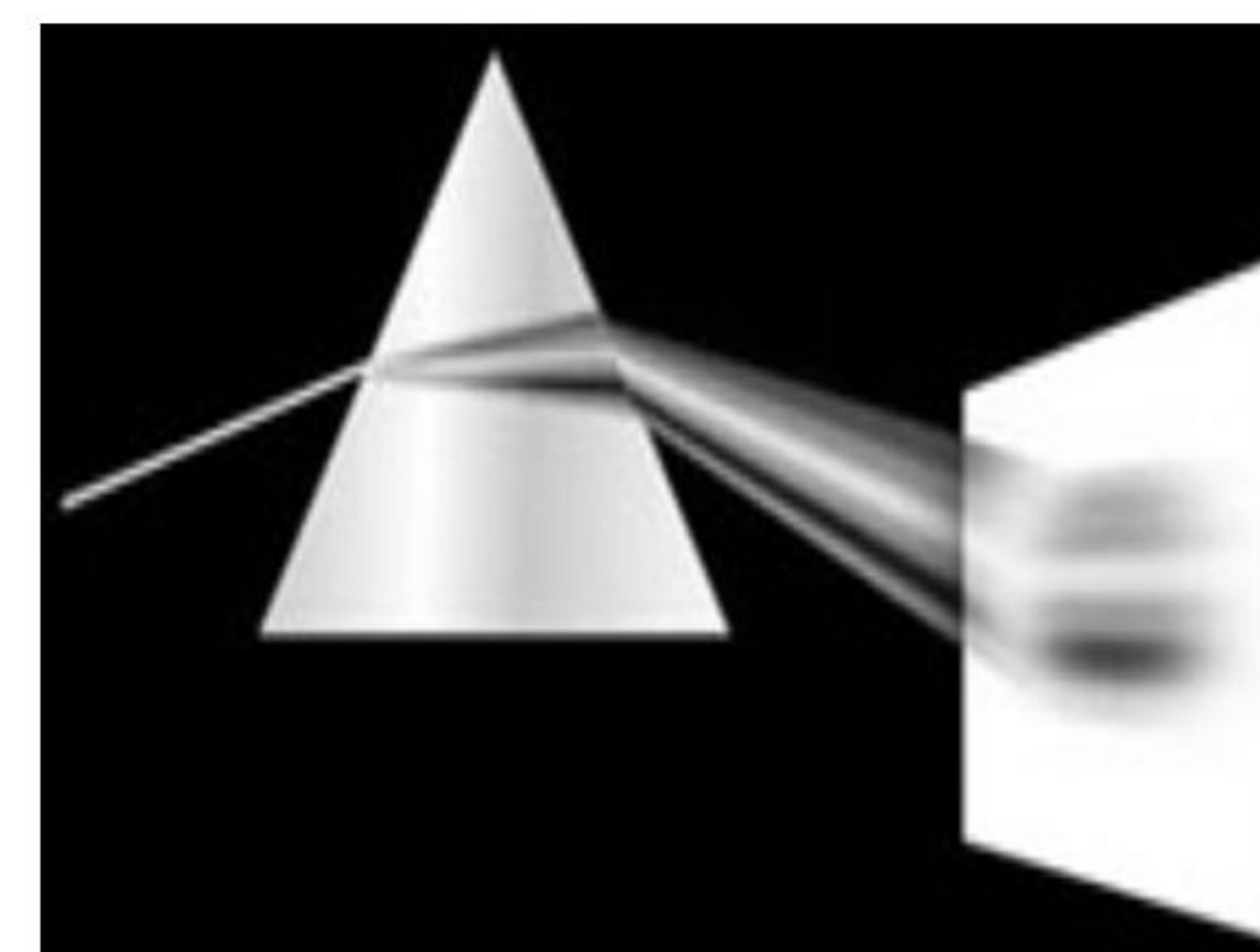
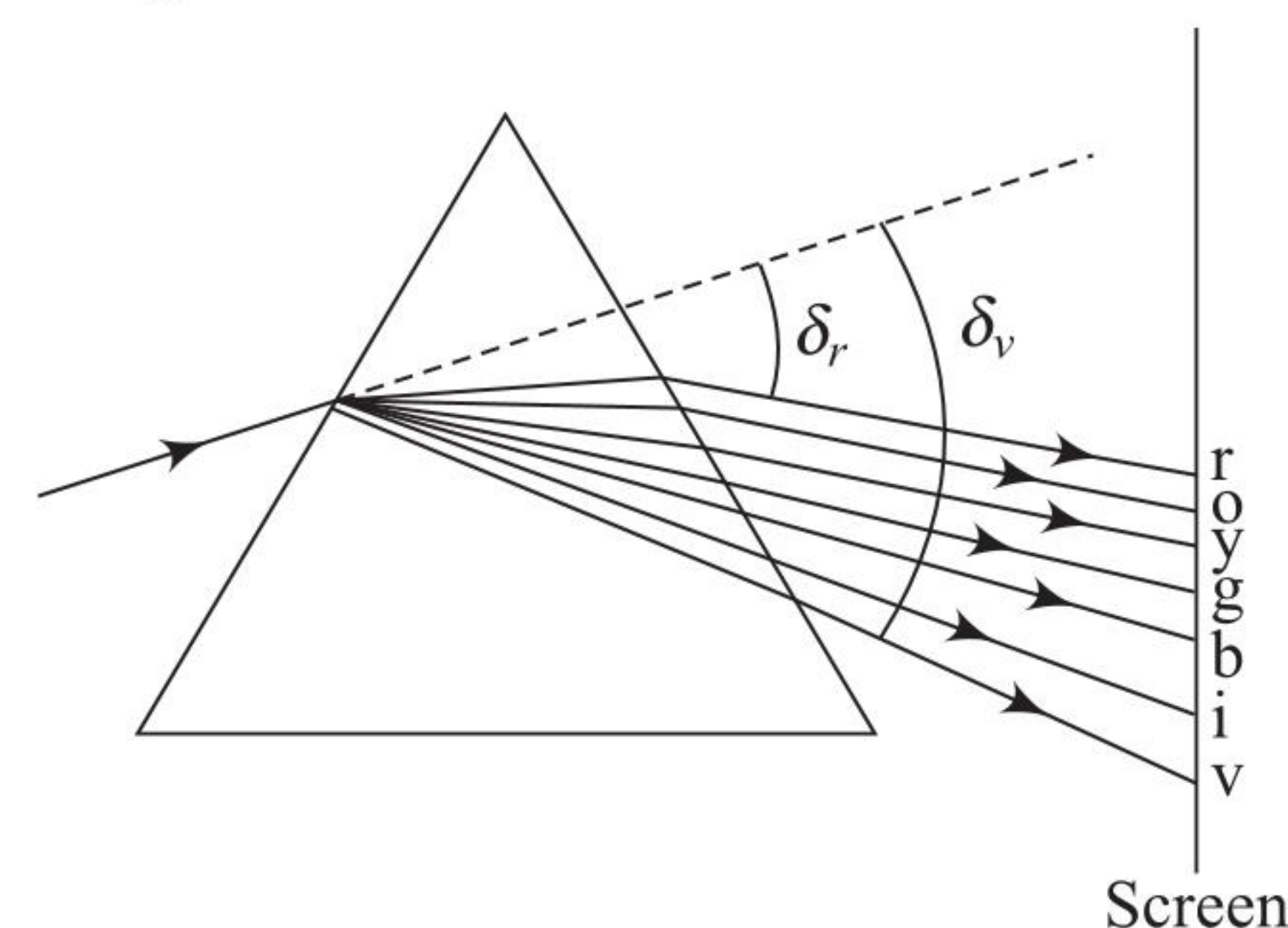
$$d = 2l(\mu - 1) \frac{1}{2}(\pi - \alpha)$$

$$d = (\mu - 1)l(\pi - \alpha)$$

$$\begin{aligned} d &= (1.6 - 1) \times 0.20 \left(\pi - 178^\circ \times \frac{\pi}{180} \right) \\ &= 0.6 \times 0.20 \times \pi \times \frac{1}{90} = 0.004 \text{ m} = 4 \text{ mm} \end{aligned}$$

DISPERSION OF LIGHT

When a ray of light passes through a prism, it splits up into rays of constituent colours or wavelengths. This phenomenon is called dispersion of light.



This phenomenon arises due to the fact that refractive index varies with wavelength. It has been observed for a prism that μ decreases with the increase of wavelength, i.e., $\mu_{\text{blue}} > \mu_{\text{red}}$.

Angular dispersion: $\theta = \delta_v - \delta_r$

Dispersive power: Ratio of angular dispersion to mean deviation.

$$\omega = \frac{\delta_v - \delta_r}{\delta}$$

where δ is deviation of mean ray (yellow)

As $\delta_v = (\mu_v - 1)A$, $\delta_r = (\mu_r - 1)A$

$$\Rightarrow \omega = \frac{\mu_v - \mu_r}{\mu_y - 1} \quad \text{where} \quad \mu_y = \frac{\mu_v + \mu_r}{2}$$

ILLUSTRATION 1.81

Calculate the dispersive power for crown glass from the given data $\mu_v = 1.523$ and $\mu_r = 1.5145$.

Sol. Here, $\mu_v = 1.523$ and $\mu_r = 1.5145$

Mean refractive index, $\mu = \frac{1.523 + 1.5145}{2} = 1.51875$

Dispersive power is given by

$$\omega = \frac{\mu_v - \mu_r}{(\mu - 1)} = \frac{1.523 - 1.5145}{(1.51875 - 1)} = 0.1639$$

ILLUSTRATION 1.82

Find the dispersion produced by a thin prism of 18° having refractive index for red light = 1.56 and for violet light = 1.68.

Sol. We know that dispersion produced by a thin prism,

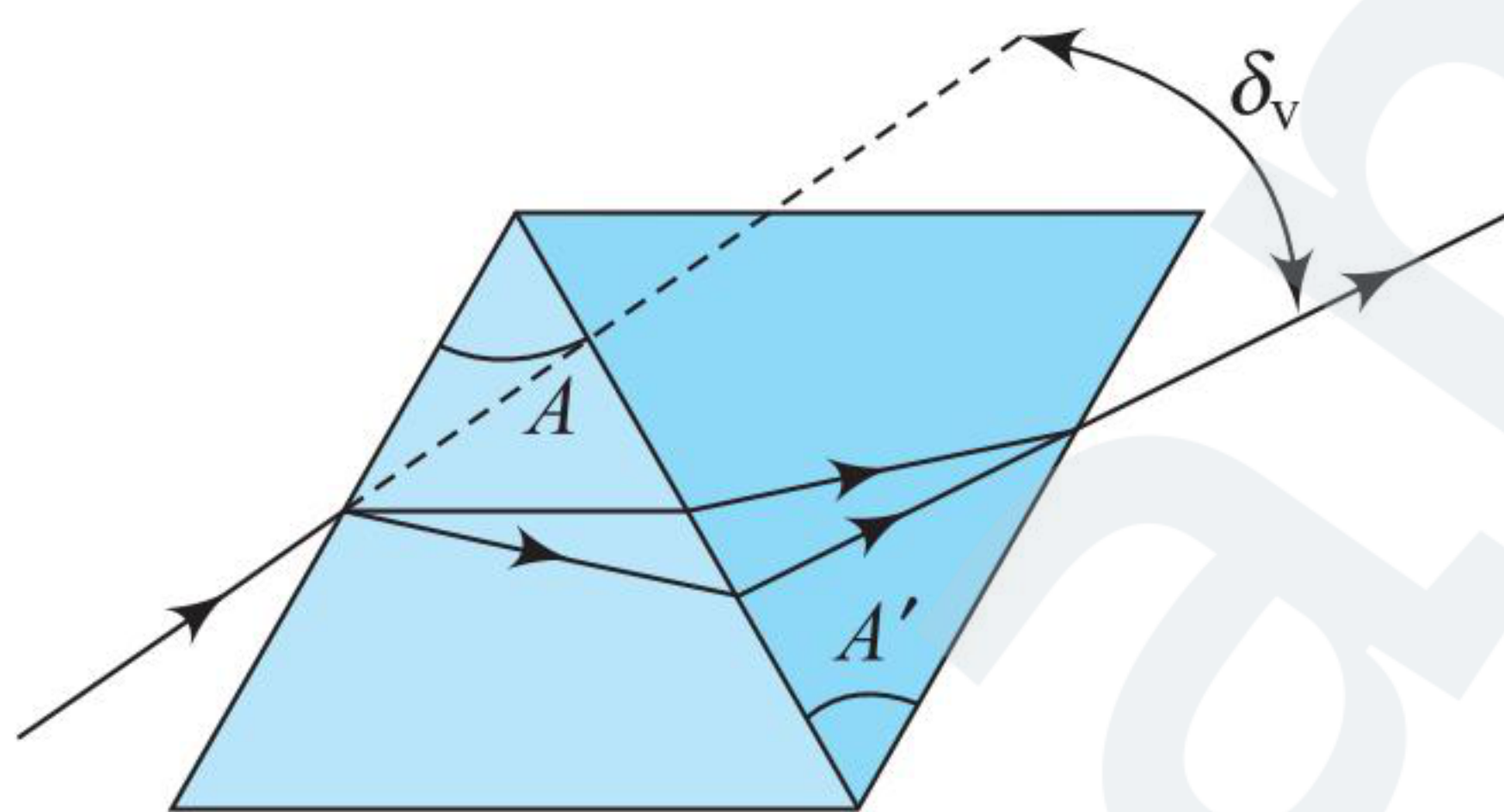
$$\theta = (\mu_v - \mu_r)A$$

Here, $\mu_v = 1.68$, $\mu_r = 1.56$ and $A = 18^\circ$

$$\Rightarrow \theta = (1.68 - 1.56) \times 18^\circ = 2.16^\circ$$

DEVIATION WITHOUT DISPERSION

This means an achromatic combination of two prisms in which net or resultant dispersion is zero and deviation is produced.



For the two prisms,

$$(\mu_v - \mu_1)A + (\mu_v - \mu_r)A' = 0$$

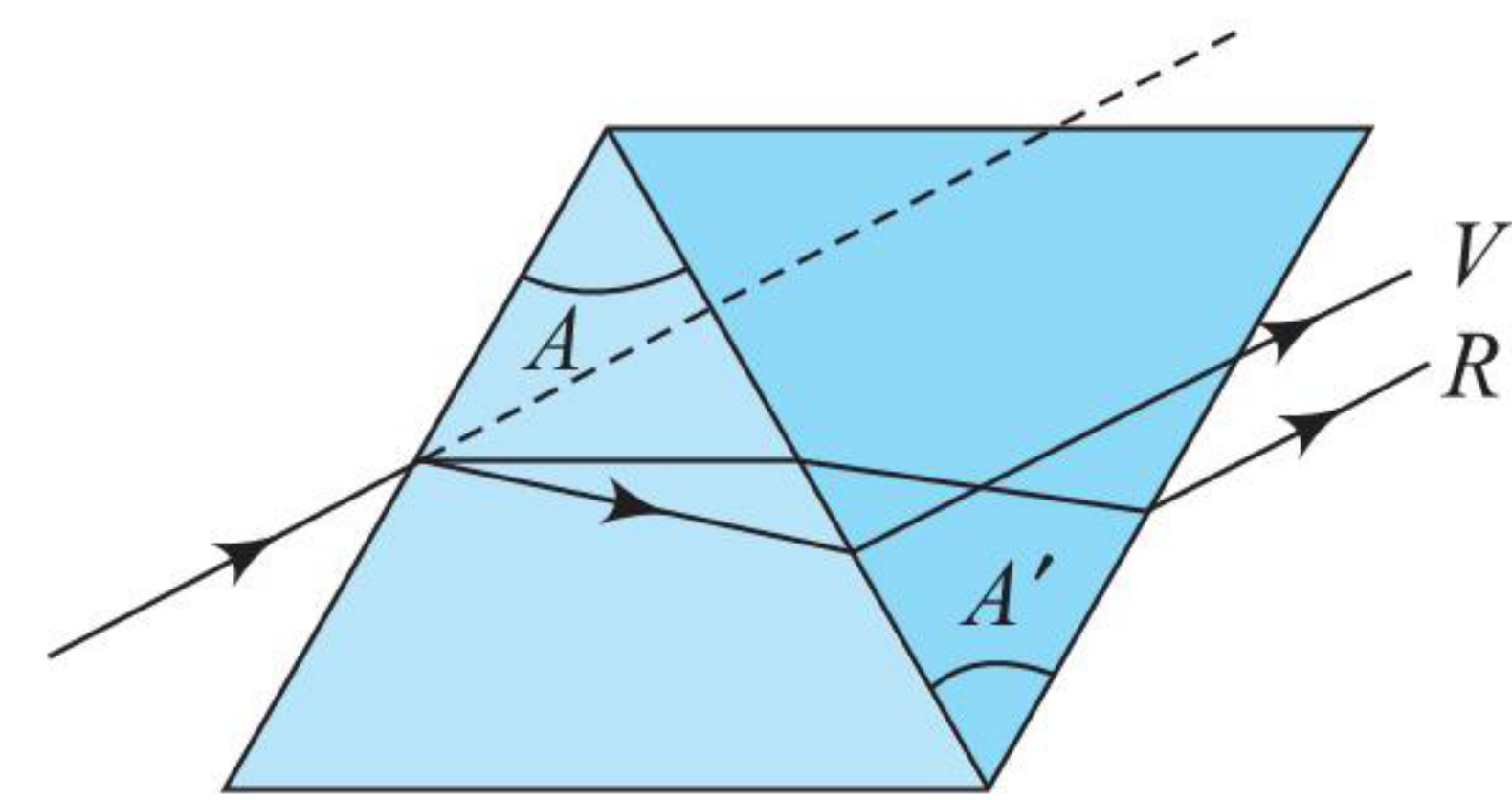
$$\Rightarrow A' = \frac{(\mu_v - \mu_r)A}{\mu_v - \mu_r} \quad \text{and} \quad \omega\delta + \omega'\delta' = 0$$

$$\delta = \delta_1 \left[1 - \frac{\omega}{\omega'} \right]$$

where ω and ω' are the dispersive powers of the two prisms and δ and δ' their mean deviations.

DISPERSION WITHOUT DEVIATION

A combination of two prisms in which deviation produced for the mean ray by the first prism is equal and opposite to that produced by the second prism is called a direct vision prism.



This combination produces dispersion without deviation.

For deviation to be zero, $(\delta + \delta') = 0$

$$\Rightarrow (\mu - 1)A + (\mu' - 1)A' = 0$$

$$\Rightarrow A' = \frac{(\mu - 1)A}{(\mu' - 1)}$$

(-ve sign $\Rightarrow$ prism A' has to be kept inverted)

$$\theta = \delta(\omega - \omega')$$

ILLUSTRATION 1.83

Two thin prisms are combined to form an achromatic combination. For prism I, $A = 4^\circ$, $\mu_r = 1.35$, $\mu_y = 1.40$, $\mu_v = 1.42$. For prism II, $\mu'_r = 1.7$, $\mu'_y = 1.8$ and $\mu'_v = 1.9$. Find the prism angle of prism II and the net mean deviation.

Sol. Condition for achromatic combination.

$$\theta = \theta'$$

$$(\mu_v - \mu_r)A = (\mu'_v - \mu'_r)A'$$

$$\Rightarrow A = \frac{(1.42 - 1.35)4^\circ}{1.9 - 1.7} = 1.4^\circ$$

$$\begin{aligned} \delta_{\text{Net}} &= \delta - \delta' = (\mu_y - 1)A - (\mu'_y - 1)A' \\ &= (1.40 - 1)4^\circ - (1.8 - 1)1.4^\circ = 0.48^\circ \end{aligned}$$

ILLUSTRATION 1.84

A crown glass prism of angle 5° is to be combined with a flint prism in such a way that the mean ray passes undeviated. Find (a) the angle of the flint glass prism needed and (b) the angular dispersion produced by the combination when white light goes through it. Refractive indices for red, yellow, and violet light are 1.514, 1.517, and 1.523, respectively, for crown glass and 1.613, 1.620, and 1.632 for flint glass.

Sol. The deviation produced by the crown prism is $\delta = (\mu - 1)A$ and by the flint prism is $\delta' = (\mu' - 1)A'$.

The prisms are placed with their angles inverted with respect to each other. The deviations are also in opposite directions. Thus, the net deviation is

$$\delta_{\text{net}} = \delta - \delta' = (\mu - 1)A - (\mu' - 1)A' \quad \dots(i)$$

(a) If the net deviation for the mean ray is zero,

$$(\mu - 1)A = (\mu' - 1)A'$$

$$\text{or} \quad A' = \frac{(\mu - 1)}{(\mu' - 1)}A = \frac{1.517 - 1}{1.620 - 1} \times 5^\circ \approx 4.2^\circ$$

(b) The angular dispersion produced by the crown prism is

$$\delta_v - \delta_r = (\mu_v - \mu_r)A$$

and that by the flint prism is $\delta'_v - \delta'_r = (\mu'_v - \mu'_r)A'$

The net angular dispersion is $(\mu_v - \mu_r)A - (\mu'_v - \mu'_r)A'$
 $= (1.523 - 1.514) \times 5^\circ - (1.632 - 1.613) \times 4.2^\circ$
 $= -0.0348^\circ$

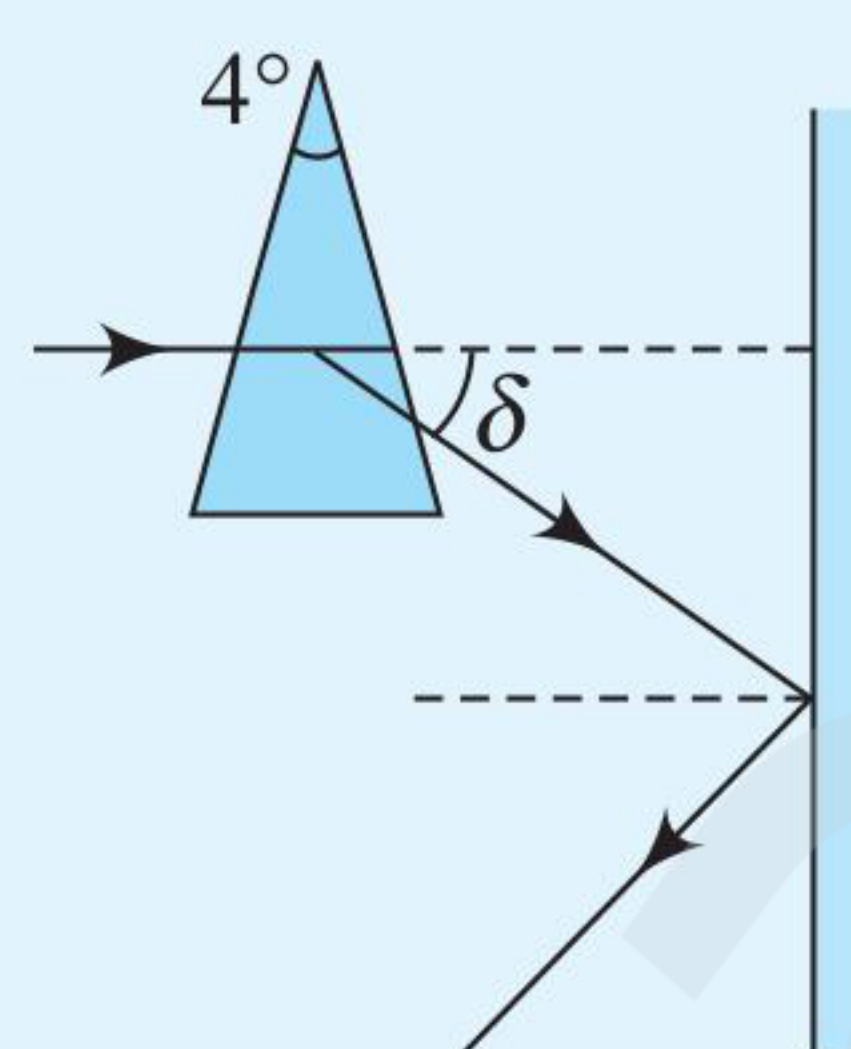
The angular dispersion has magnitude 0.0348° .

CONCEPT APPLICATION EXERCISE 1.6

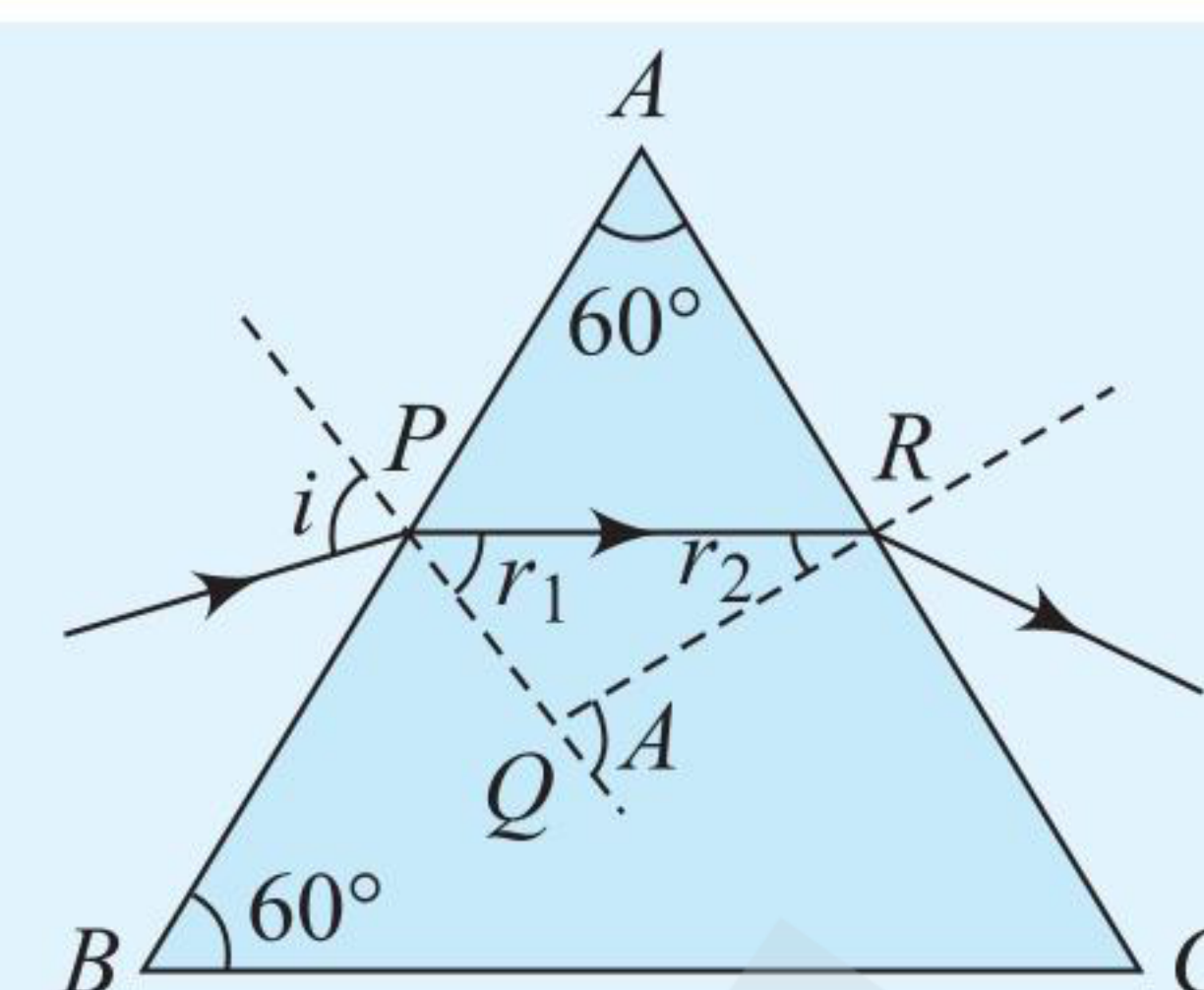
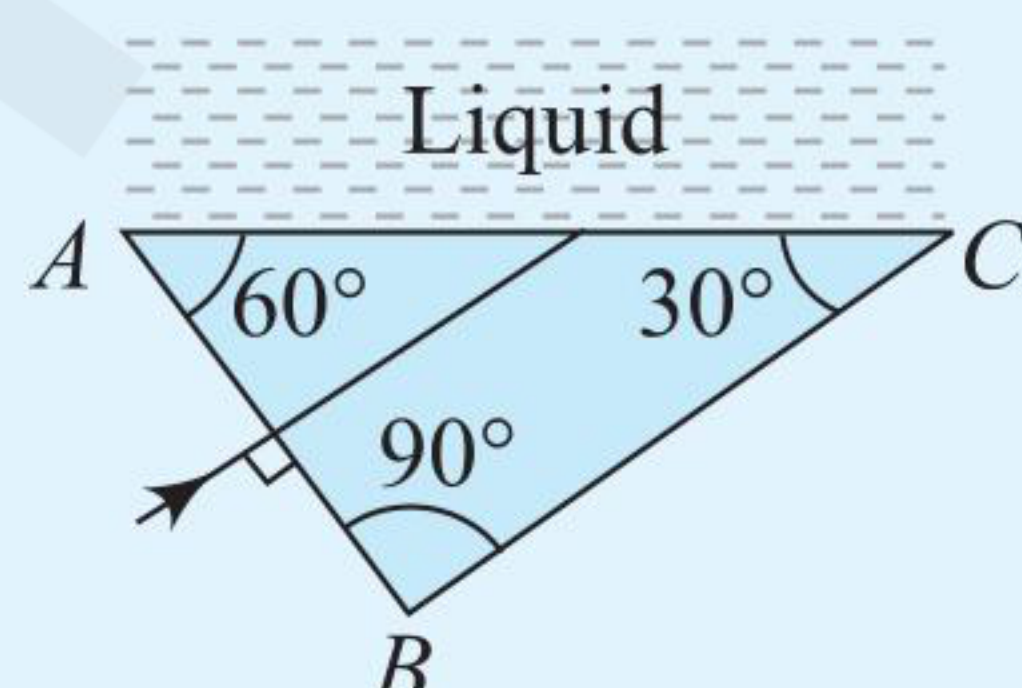
- The refractive indices of flint glass for red and violet lights are 1.613 and 1.632, respectively. Find the angular dispersion produced by a thin prism of flint glass having refracting angle 5° .
- Refractive index of glass for red and violet colours are 1.50 and 1.60, respectively. Find:
 - The refractive index for yellow colour, approximately.
 - Dispersive power of the medium.
- An equilateral prism is made of glass of refractive index 1.5. Calculate the angles of minimum and maximum deviations. Given:

$$A = 60^\circ, \mu = 1.5, \sin 48.6^\circ = \frac{3}{4}; \sin 41.8^\circ = \frac{3}{2}; \sin 27.9^\circ = \frac{3}{2} \sin 18.2^\circ$$

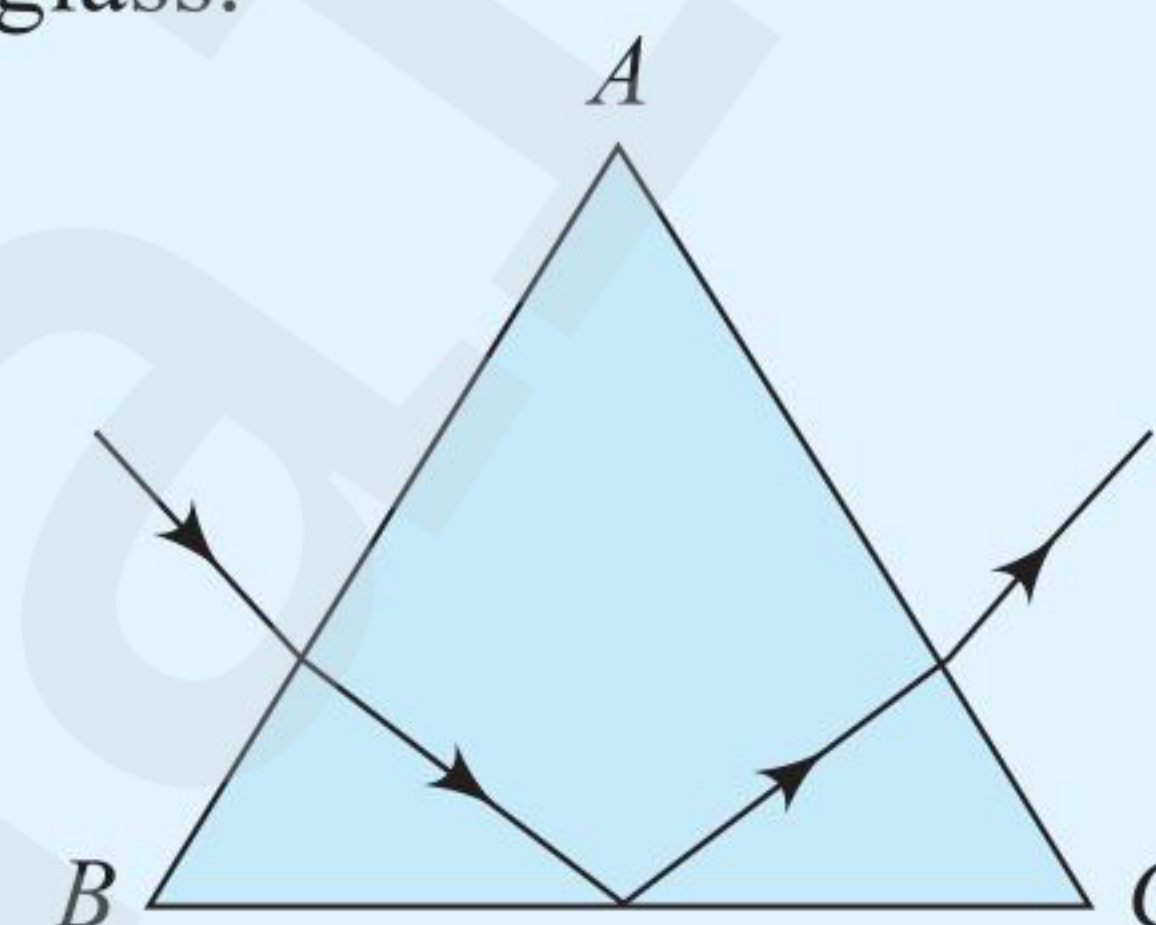
- A prism is made of glass of refractive index 1.5. If the angle of minimum deviation is equal to the refracting angle of the prism, calculate the angle of the prism.
- A horizontal ray of light passes through a prism whose apex angle is 4° and then strikes a vertical mirror M as shown in figure. For the ray to become horizontal after reflection, find the angle by which the mirror must be rotated.



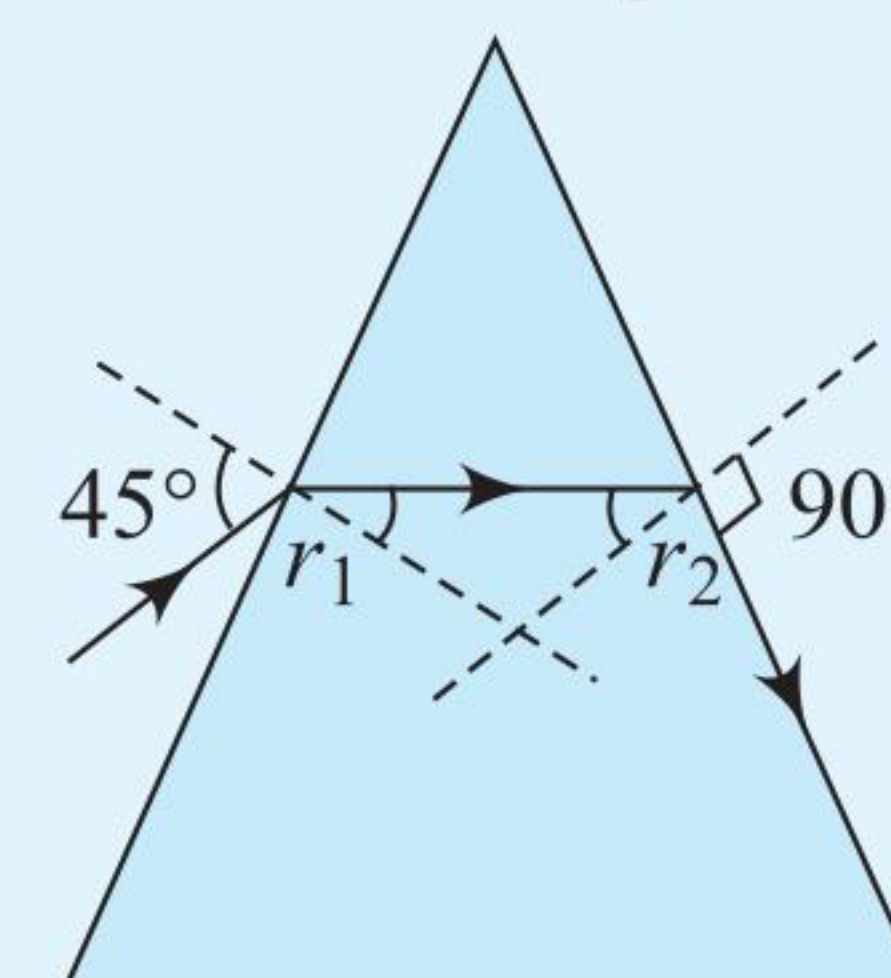
- Light is incident normally on face AB of a prism as shown in figure. A liquid of refractive index μ is placed on face AC of the prism. The prism is made of glass of refractive index $3/2$. Find the limits of μ for which total internal reflection takes place on the face AC .
- A ray of light suffers minimum deviation through a prism of refractive index $\sqrt{2}$. What is the angle of prism if the angle of incidence is double the angle of refraction within the prism?
- A ray of light undergoes a deviation of 30° when incident on an equilateral prism of refractive index $\sqrt{2}$. What is the angle subtended by the ray inside the prism with the base of the prism?



- The path of a ray of light passing through an equilateral glass prism ABC is shown in figure. The ray of light is incident on face BC at the critical angle for just total internal reflection. The total angle of deviation after the refraction at face AC is 108° . Calculate the refractive index of the glass.

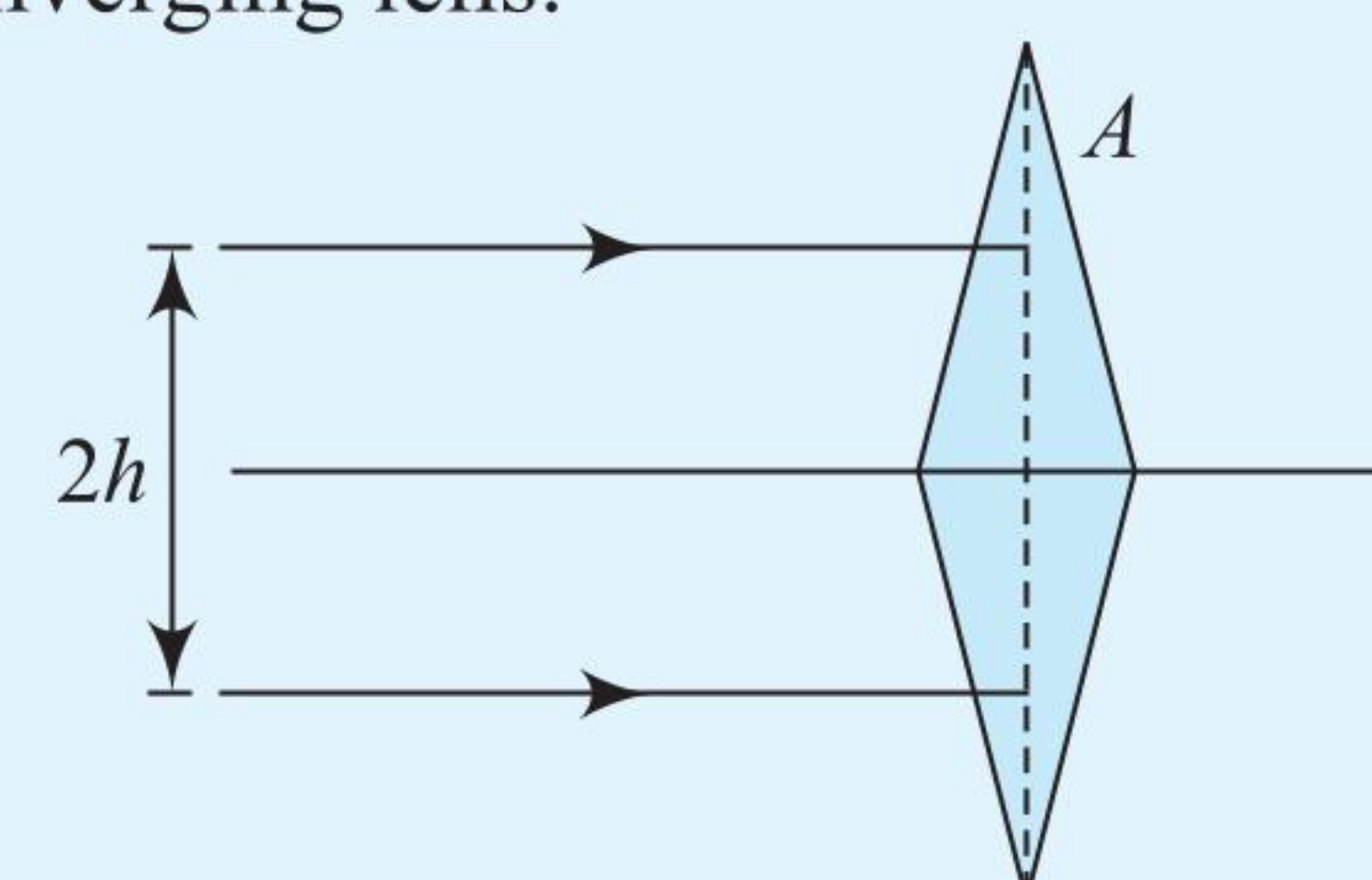


- In an isosceles prism of angle 45° , it is found that when the angle of incidence is same as the prism angle, the emergent ray grazes the emergent surface.



Find the refractive index of the material of the prism. For what angle of incidence, the angle of deviation will be minimum?

- Refracting angle of a prism $A = 60^\circ$ and its refractive index is $n = 3/2$. What is the angle of incidence i to get minimum deviation. Also, find the minimum deviation. Assume the surrounding medium to be air ($n = 1$).
- Two identical thin isosceles prisms of refracting angle A and refractive index μ are placed with their bases touching each other and this system can collectively act as a crude converging lens. A parallel beam of light is incident on this system as shown in figure. Find the focal length of this so called converging lens.



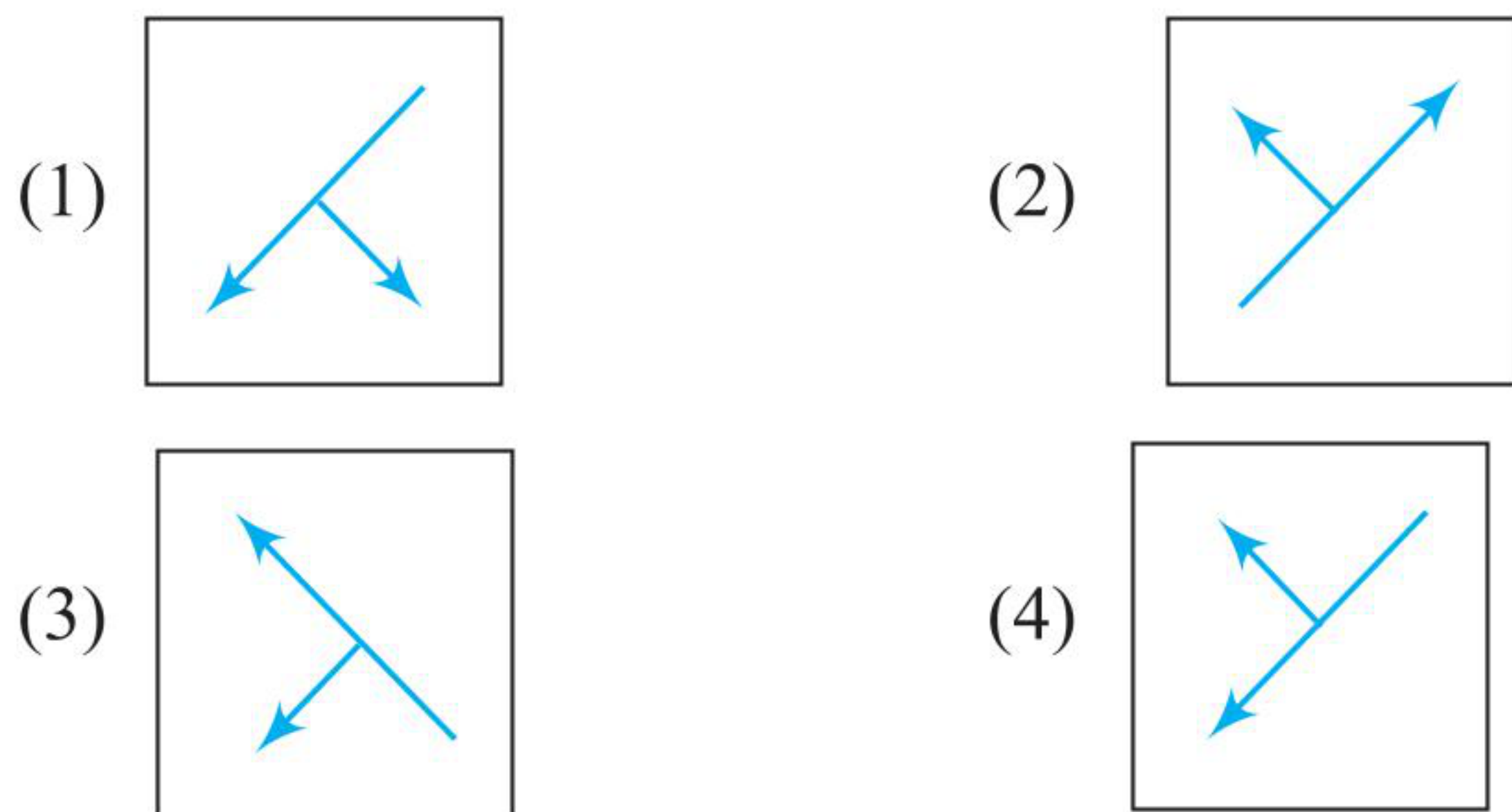
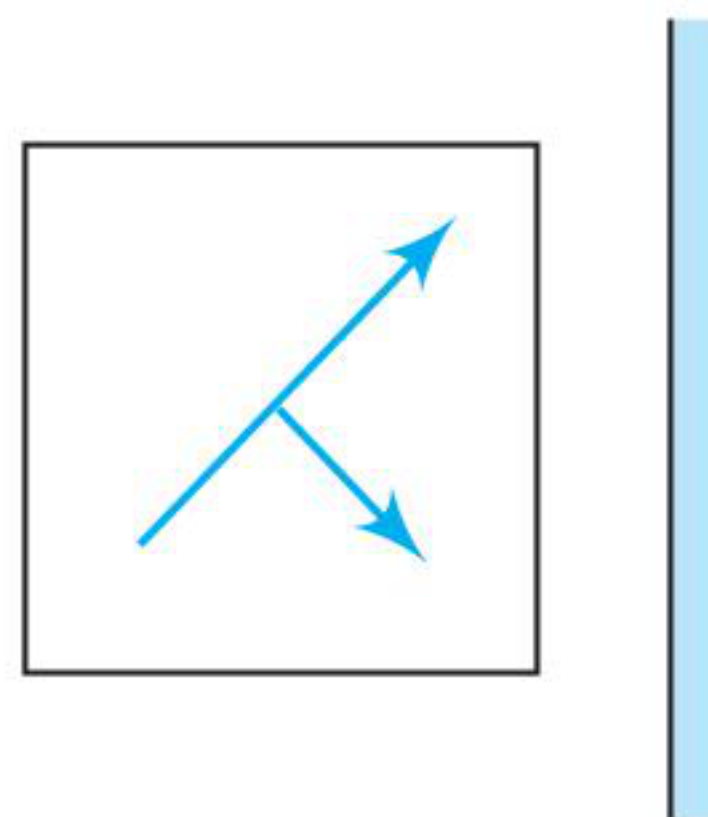
ANSWERS

1. 0.095° 2. (a) 1.55; (b) $2/11$ 3. 37.2° ; 57.9°
4. $2 \cos^{-1} \left(\frac{3}{4} \right)$ 5. 1° 6. $\mu > \sqrt{3}$ 7. 90°
8. 60° 9. 1.447 10. 58.8° 11. 48.5° 12. $h/(\mu-1)A$

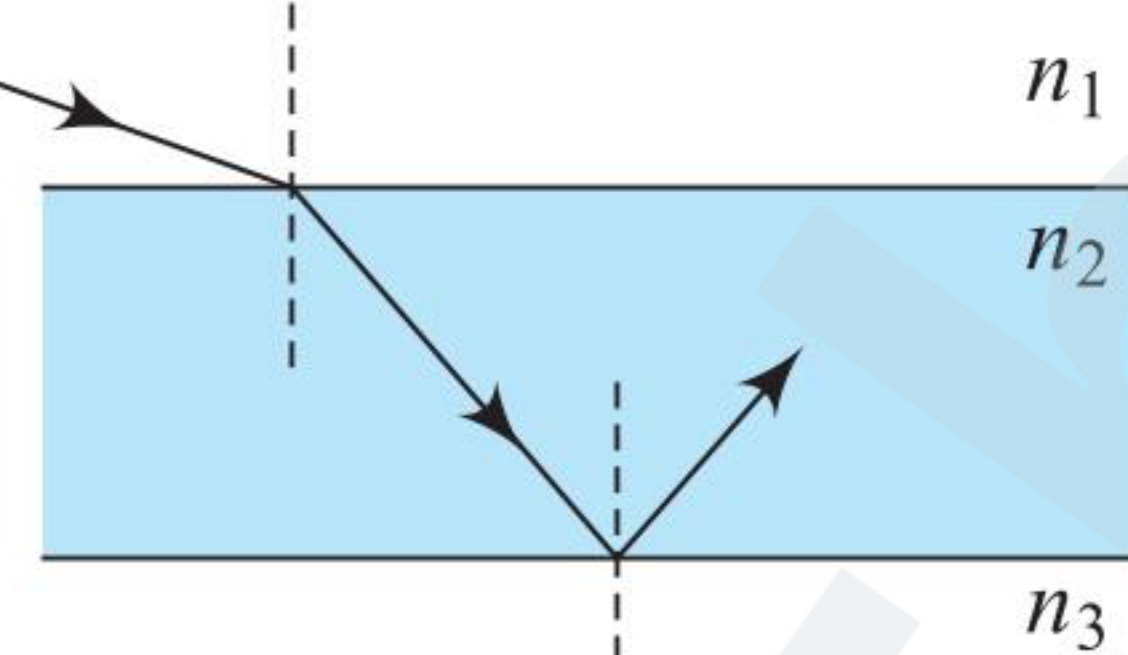
Exercises

Single Correct Answer Type

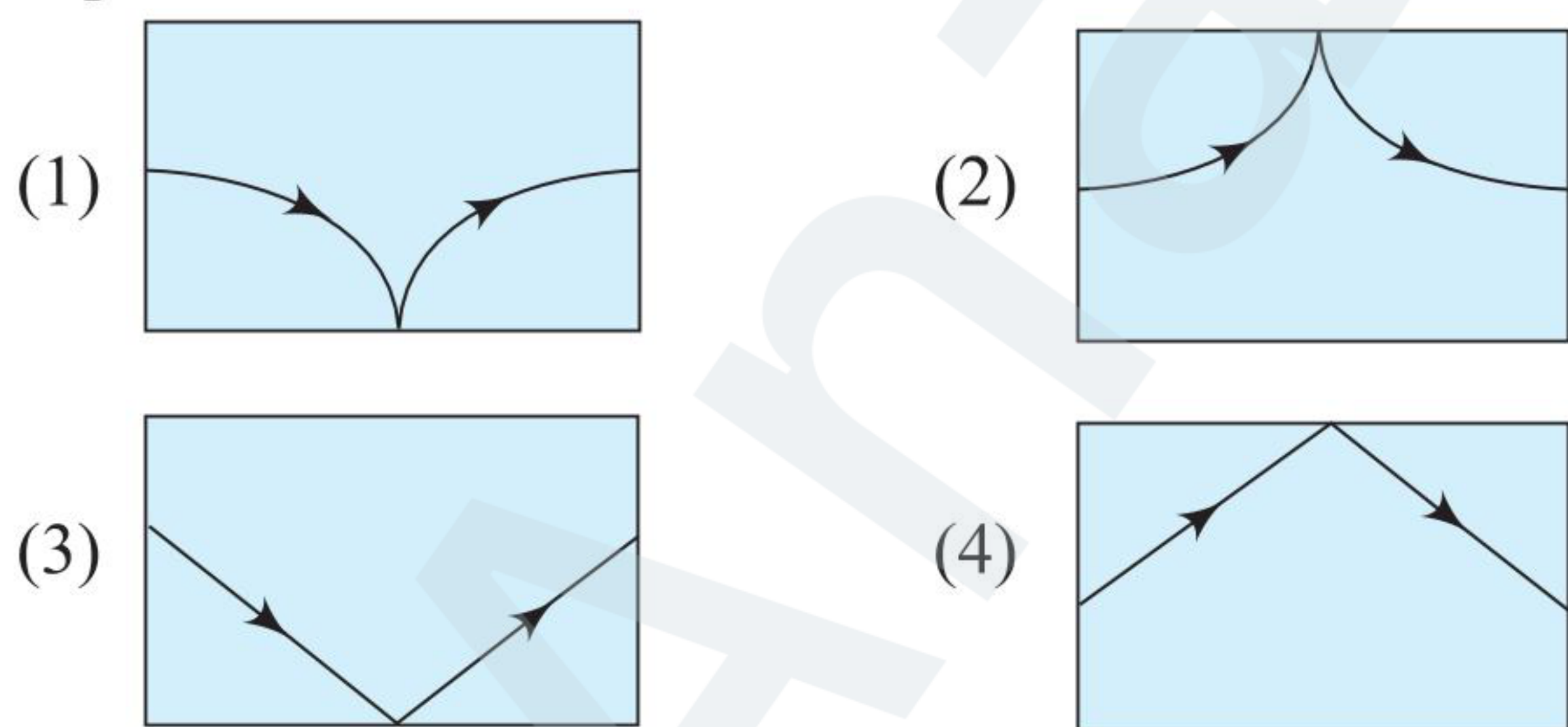
1. Choose the correct mirror image of object as shown in the following figure.



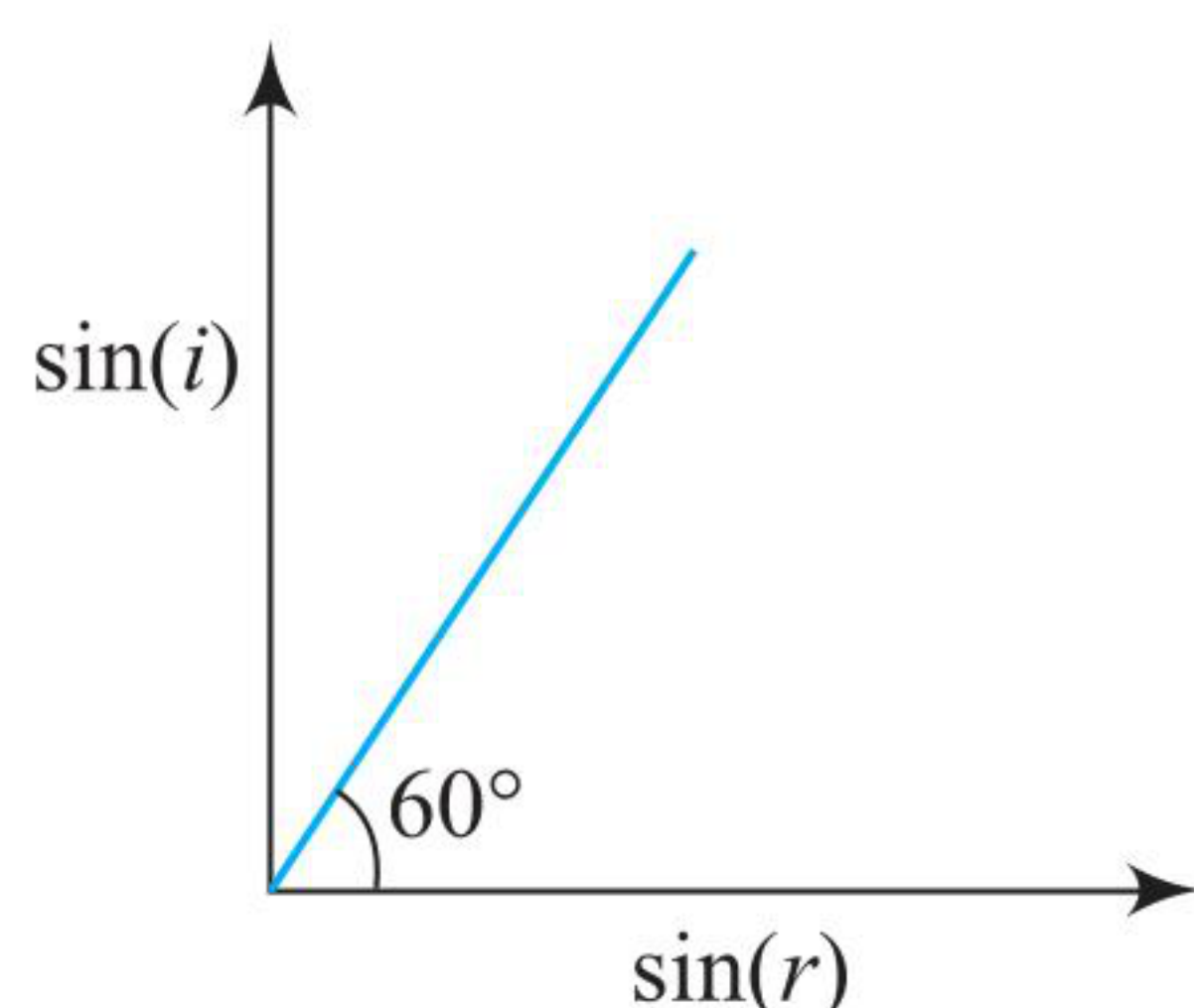
2. Light travelling through three transparent substances follows the path shown in figure. Arrange the indices of refraction in order from smallest to largest. Note that total internal reflection does occur on the bottom surface of medium 2.



- (1) $n_1 < n_2 < n_3$ (2) $n_2 < n_1 < n_3$
 (3) $n_1 < n_3 < n_2$ (4) $n_3 < n_1 < n_2$
3. A cubic container is filled with a liquid whose refractive index increases linearly from top to bottom. Which of the following represents the path of a ray of light inside the liquid?

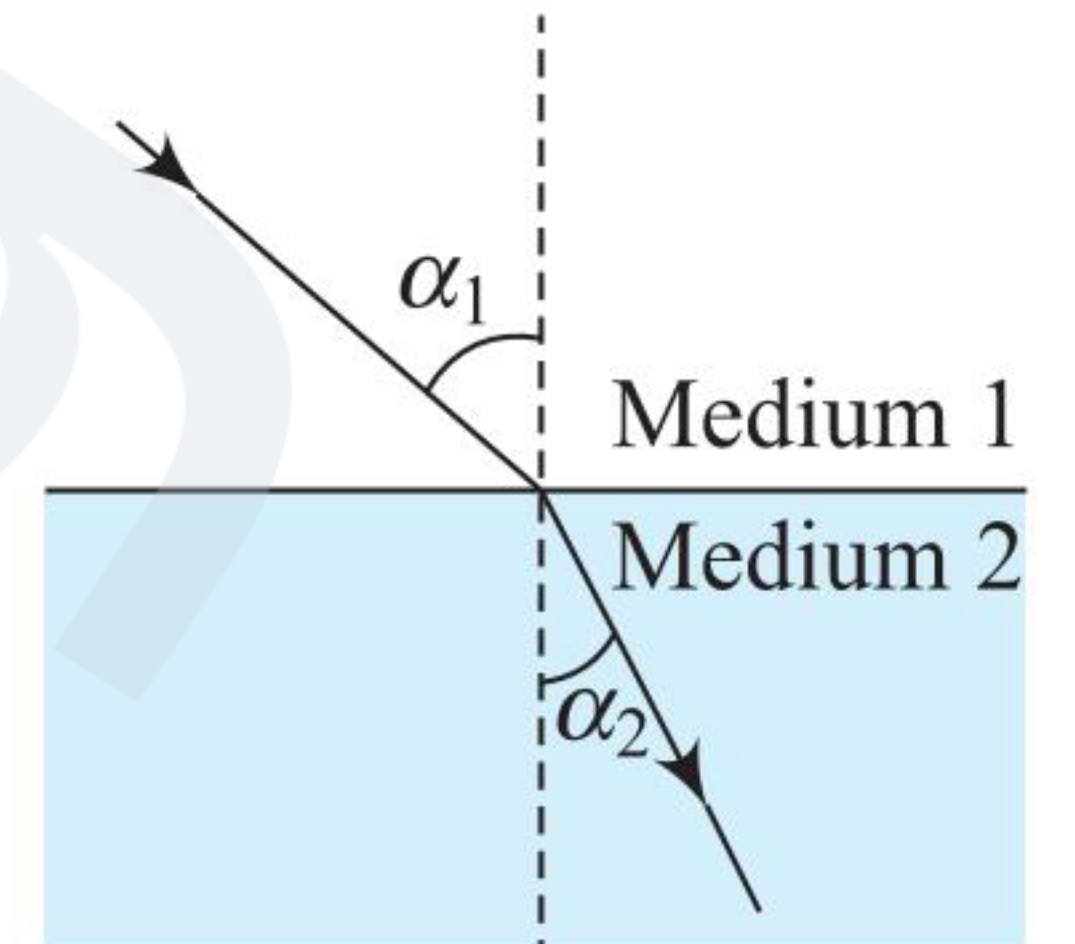


4. A ray of light is incident on a medium with angle of incidence i and refracted into a second medium with angle of refraction r . The graph of $\sin(i)$ vs. $\sin(r)$ is as shown in figure. Then, the velocity of light in the first medium is n times the velocity of light in the second medium. What should be the value of n ?



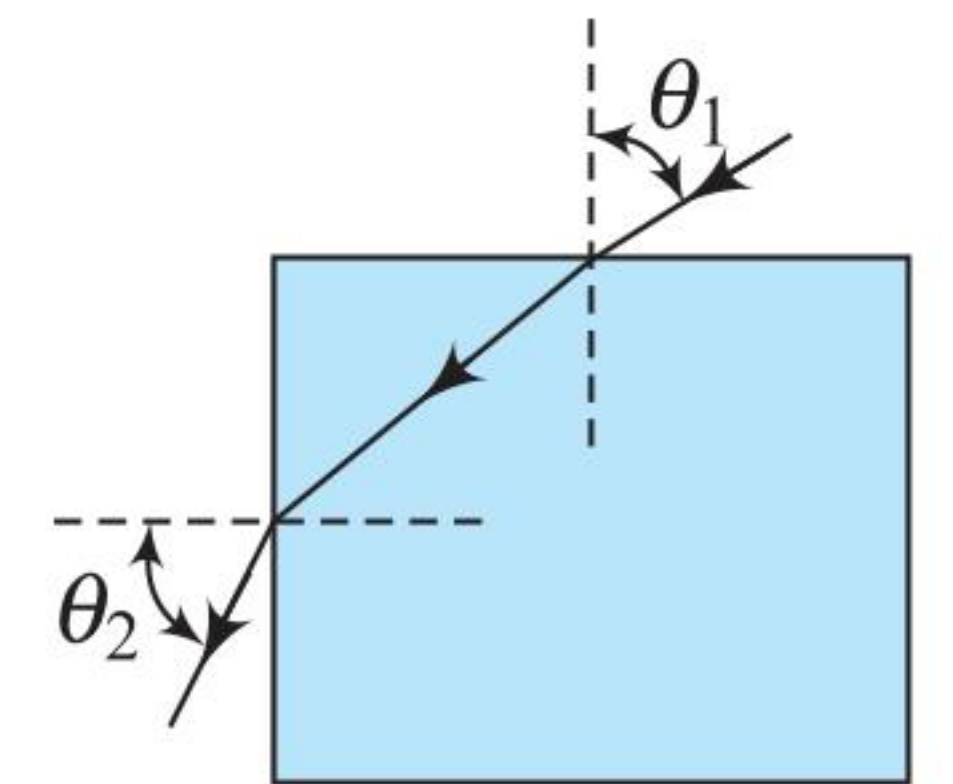
- (1) $\sqrt{3}$ (2) $1/\sqrt{3}$
 (3) $\sqrt{3}/2$ (4) $2/\sqrt{3}$

5. A beam of light propagates through a medium 1 and falls onto another medium 2, at an angle α_1 as shown in figure. After that, it propagates in medium 2 at an angle α_2 as shown. The light's wavelength in medium 1 is λ_1 . What is the wavelength of light in medium 2?



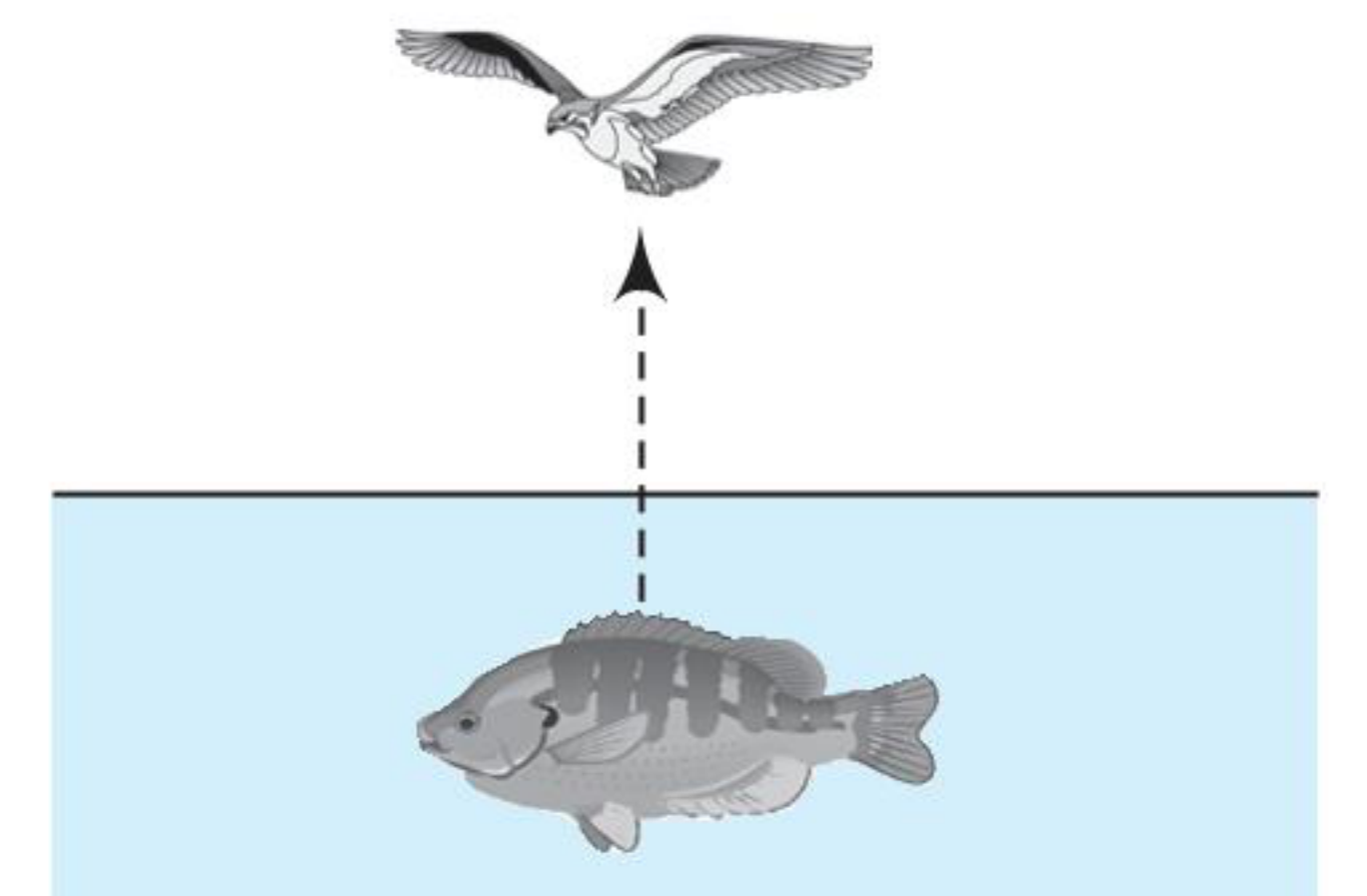
- (1) $\frac{\sin \alpha_1}{\sin \alpha_2} \lambda_1$ (2) $\frac{\sin \alpha_2}{\sin \alpha_1} \lambda_1$
 (3) $\frac{\cos \alpha_1}{\cos \alpha_2} \lambda_1$ (4) $\frac{\cos \alpha_2}{\cos \alpha_1} \lambda_1$

6. Light is incident on a glass block as shown in figure. If θ_1 is increased slightly, what happens to θ_2 ?



- (1) θ_2 also increases slightly
 (2) θ_2 is unchanged
 (3) θ_2 decreases slightly
 (4) θ_2 changes abruptly, since the ray experiences total internal reflection

7. A fish is vertically below a flying bird moving vertically down toward water surface. The bird will appear to the fish to be

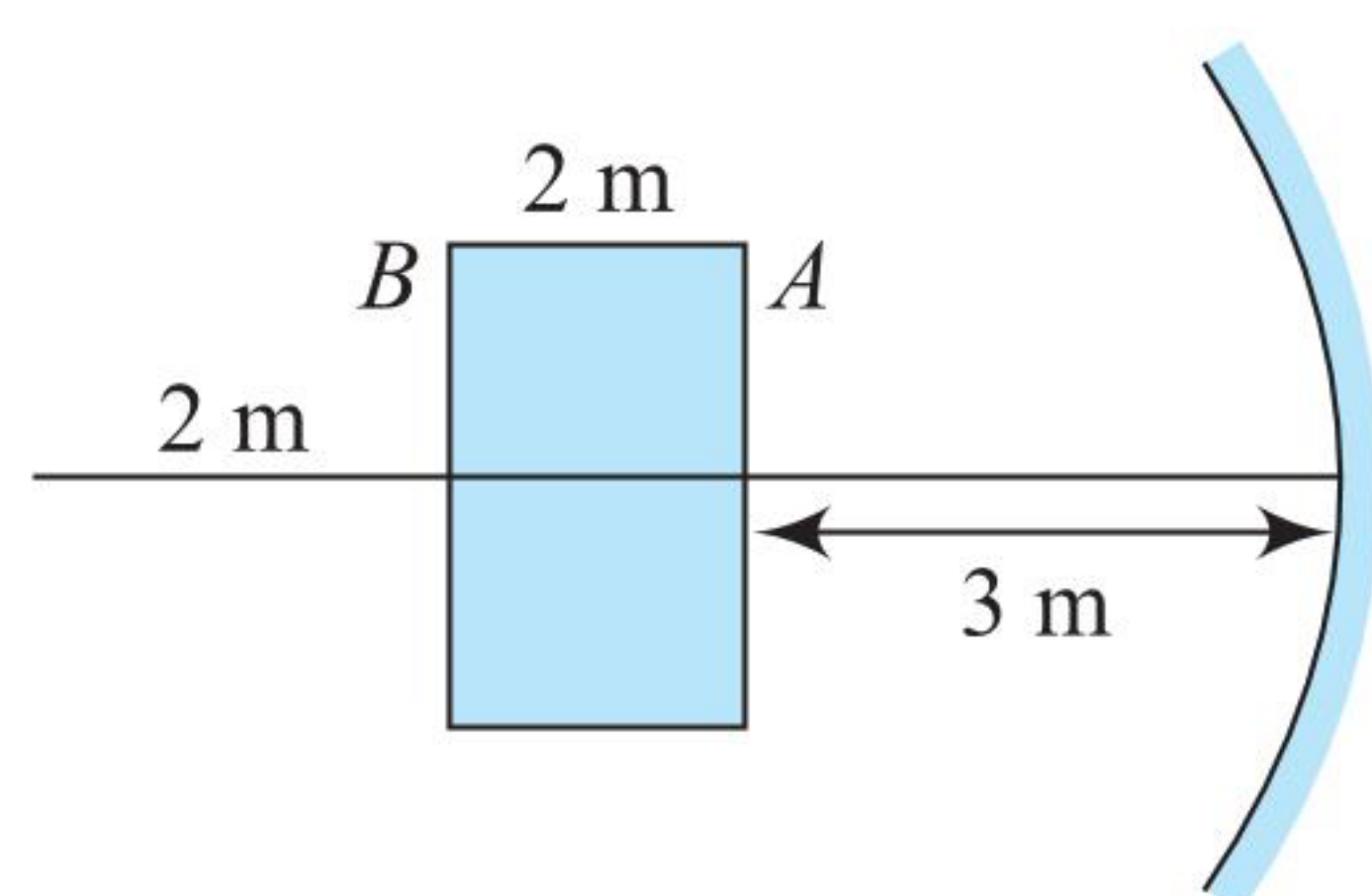


- (1) moving faster than its speed and also away from the real distance
 (2) moving faster than its real speed and nearer than its real distance
 (3) moving slower than its real speed and also nearer than its real distance
 (4) moving slower than its real speed and away from the real distance

8. What is the angle of incidence for an equilateral prism of refractive index $\sqrt{3}$ so that the ray is parallel to the base inside the prism?

- (1) 30° (2) 45°
 (3) 60° (4) Either 30° or 60°

9. A cube of side 2 m is placed in front of a concave mirror of focal length 1 m with its face A at a distance of 3 m and face B at a distance of 5 m from the mirror. The distance between the images of faces A and B and heights of images of A and B are, respectively,



- (1) 1 m, 0.5 m, 0.25 m
 (2) 0.5 m, 1 m, 0.25 m
 (3) 0.5 m, 0.25 m, 1 m
 (4) 0.25 m, 1 m, 0.5 m

10. A circular beam of light of diameter $d = 2$ cm falls on a plane surface of glass. The angle of incidence is 60° and refractive index of glass is $\mu = 3/2$. The diameter of the refracted beam is

- (1) 4.00 cm (2) 3.0 cm
 (3) 3.26 cm (4) 2.52 cm

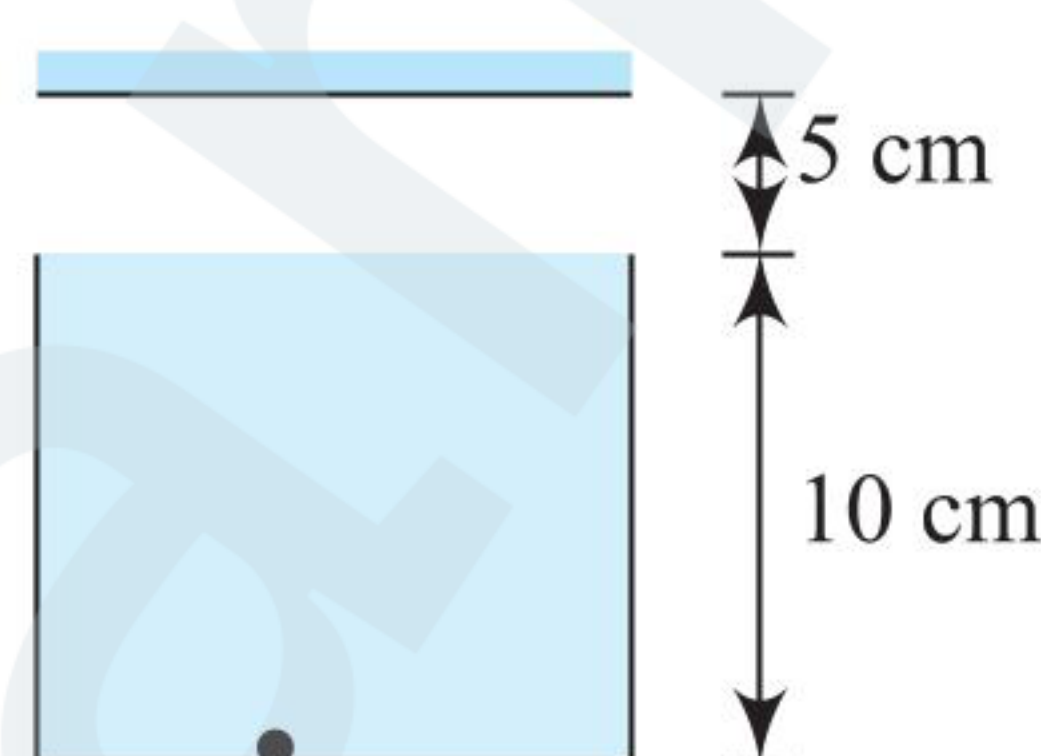
11. The velocity of light in a medium is half its velocity in air. If a ray of light emerges from such a medium into air, the angle of incidence, at which it will be totally internally reflected, is

- (1) 15° (2) 30°
 (3) 45° (4) 60°

12. A candle is placed 20 cm from the surface of a convex mirror and a plane mirror is also placed so that the virtual images in the two mirrors coincide. If the plane mirror is 12 cm away from the object, what is the focal length of the convex mirror?

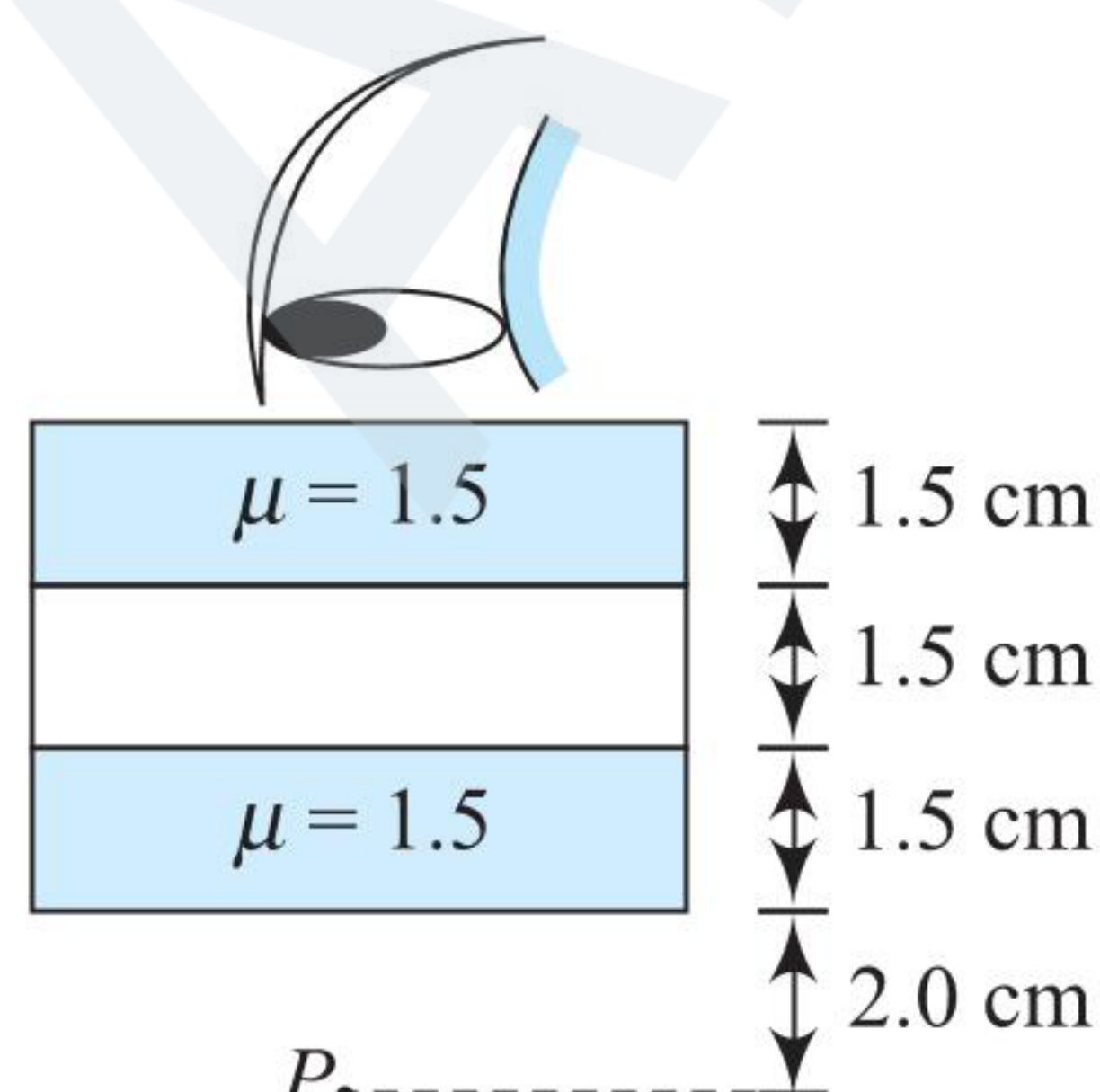
- (1) 20 cm (2) 15 cm
 (3) 10 cm (4) 5 cm

13. Consider the situation shown in figure. Water ($\mu = 4/3$) is filled in a beaker up to a height of 10 cm. A plane mirror is fixed at a height of 5 cm from the surface of water. Distance of image from the mirror after reflection from it of an object O at the bottom of the beaker is



- (1) 15 cm (2) 12.5 cm
 (3) 7.5 cm (4) 10 cm

14. The image of point P when viewed from top of the slabs will be



- (1) 2.0 cm above P (2) 1.5 cm above P
 (3) 2.0 cm below P (4) 1 cm above P

15. One of the refracting surfaces of a prism of angle 30° is silvered. A ray of light incident at angle of 60° retraces its path. The refractive index of the material of prism is

- (1) $\sqrt{2}$ (2) $\sqrt{3}$
 (3) $3/2$ (4) 2

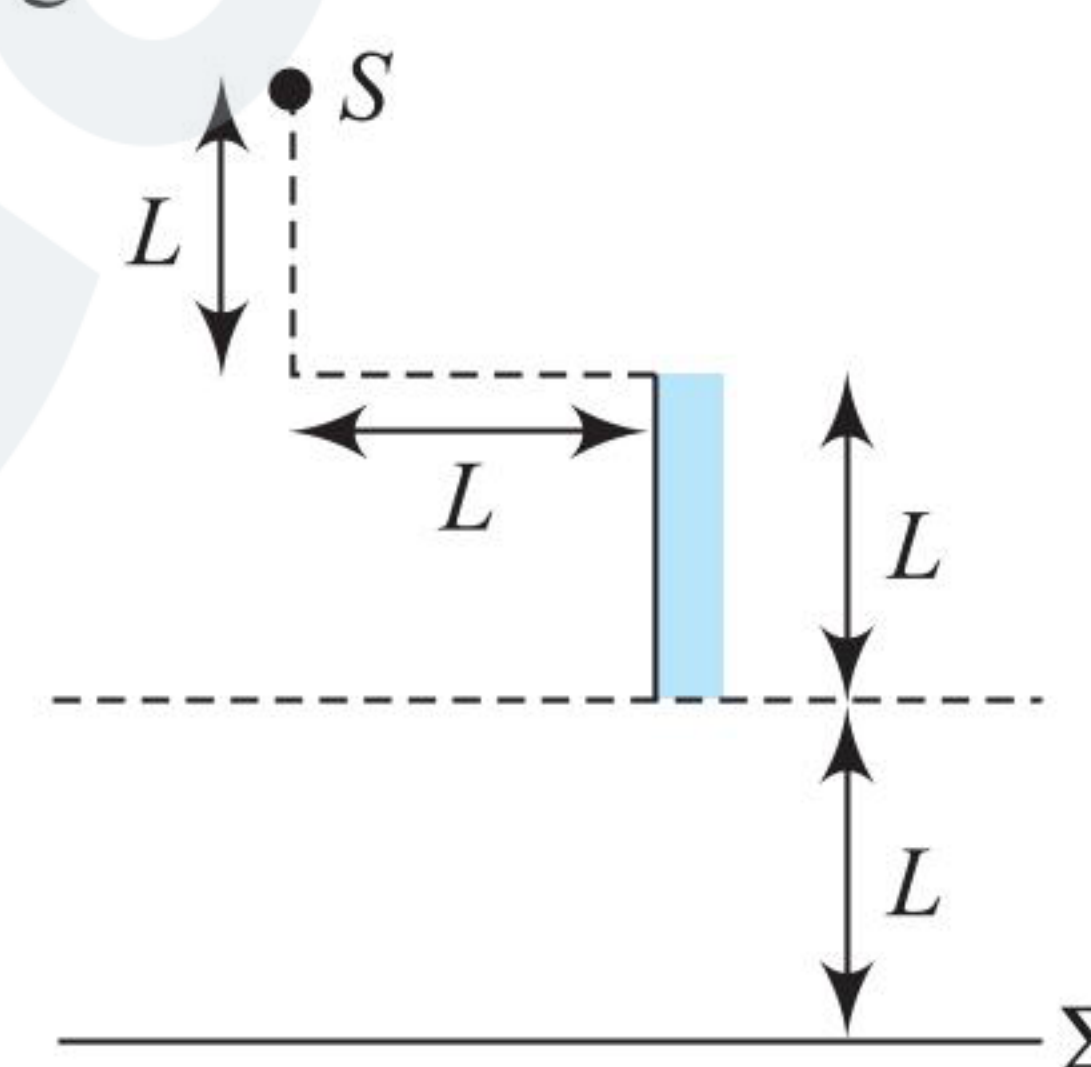
16. Angle of minimum deviation is equal to the angle of prism A of an equilateral glass prism. The angle of incidence at which minimum deviation will be obtained is

- (1) 60° (2) 30°
 (3) 45° (4) $\sin^{-1}(2/3)$

17. A spherical mirror forms an image of magnification 3. The object distance, if focal length of mirror is 24 cm, may be

- (1) 32 cm, 24 cm (2) 32 cm, 16 cm
 (3) 32 cm only (4) 16 cm only

18. A point source of light is placed in front of a plane mirror as shown in the figure.



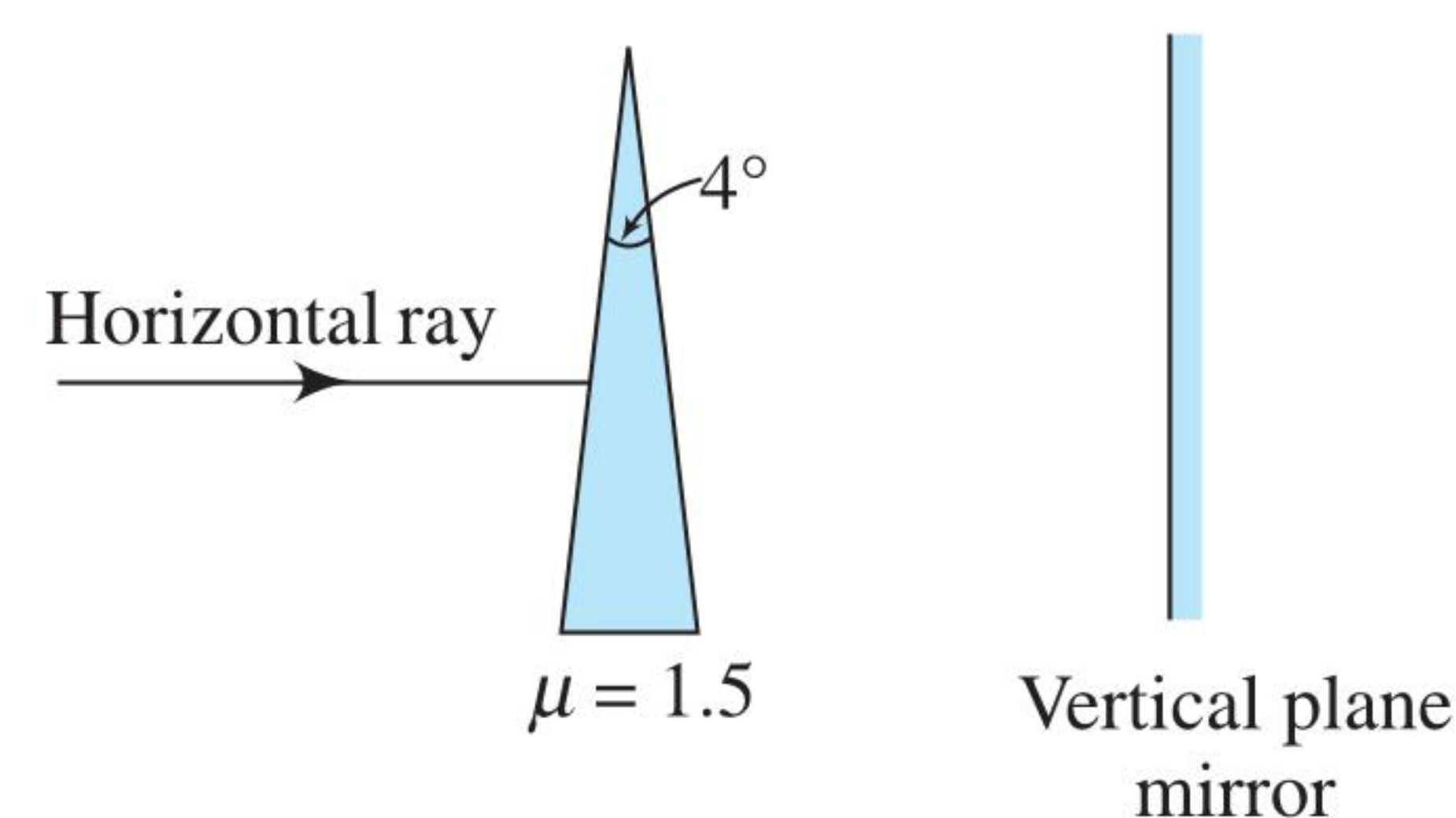
Determine the length of reflected path of light on the screen Σ .

- (1) L (2) $2L$
 (3) $3L/2$ (4) $L/2$

19. A fish looks upward at an unobstructed overcast sky. What total angle does the sky appear to subtend? (Take refractive index of water as $\sqrt{2}$.)

- (1) 180° (2) 90°
 (3) 75° (4) 60°

20. For the situations shown in figure, determine the angle by which the mirror should be rotated, so that the light ray will retrace its path after refraction through the prism and reflection from the mirror?



- (1) 1° ACW (2) 1° CW
 (3) 2° ACW (4) 2° CW

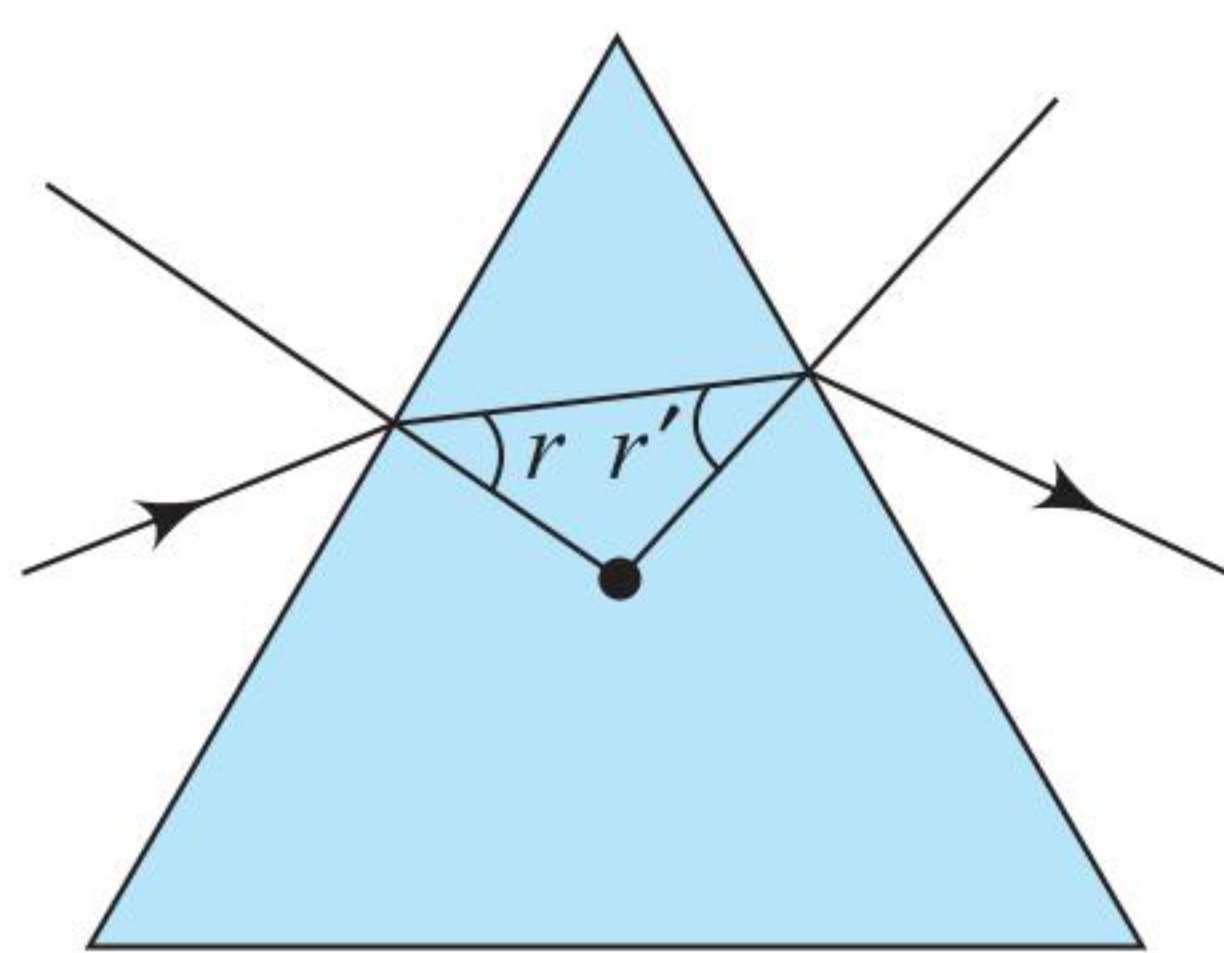
21. A real object is placed in front of a convex mirror (fixed). The object is moving toward the mirror. If v_0 is the speed of object and v_i is the speed of image, then

- (1) $v_i < v_0$ always
 (2) $v_i > v_0$ always

(3) $v_i > v_o$ initially and then $v_o > v_i$

(4) $v_i < v_o$ initially and then $v_i > v_o$

22. Let r and r' denote the angles inside an equilateral prism, as usual, in degrees. Consider that during some time interval from $t = 0$ to $t = t$, r' varies with time as $r' = 10 + t^2$. During this time, r will vary as (assume that r and r' are in degree)



- (1) $50 - t^2$ (2) $50 + t^2$
(3) $60 - t^2$ (4) $60 + t^2$

23. For a prism kept in air, it is found that for an angle of incidence 60° , the angle of refraction A , angle of deviation δ and angle of emergence e become equal. Then, the refractive index of the prism is

- (1) 1.73 (2) 1.15
(3) 1.5 (4) 1.33

24. The critical angle for light going from medium X into medium Y is θ . The speed of light in medium X is v . The speed of light in medium Y is

- (1) $v \cos \theta$ (2) $v / \cos \theta$
(3) $v \sin \theta$ (4) $v / \sin \theta$

25. A concave mirror is placed on a horizontal table with its axis directed vertically upward. Let O be the pole of the mirror C its centre of curvature and F is the focus. A point object is placed at C . It has a real image, also located at C . If the mirror is now filled with water, the image will be

- (1) real and will remain at C
(2) real and located at a point between C and ∞
(3) real and located at a point between C and O
(4) real and located at a point between C and F

26. A plane glass mirror of thickness 3 cm of material of $\mu = 3/2$ is silvered on the back surface. When a point object is placed 9 cm from the front surface of the mirror, then the position of the brightest image from the front surface is

- (1) 9 cm (2) 11 cm
(3) 12 cm (4) 13 cm

27. An object is placed at a distance of 25 cm from the pole of a convex mirror and a plane mirror is set at a distance 5 cm from convex mirror so that the virtual images formed by the two mirrors do not have any parallax. The focal length of the convex mirror is

- (1) 37.5 cm (2) -7.5 cm
(3) -37.5 cm (4) +7.5 cm

28. When an object is kept at a distance of 30 cm from a concave mirror, the image is formed at a distance of 10 cm. If the

object is moved with a speed of 9 cm s^{-1} the speed with which the image moves is

- (1) 0.1 m s^{-1} (2) 1 m s^{-1}
(3) 3 m s^{-1} (4) 9 m s^{-1}

29. A convex mirror and a concave mirror of radius 10 cm each are placed 15 cm apart facing each other. An object is placed midway between them. If the reflection first takes place in the concave mirror and then in convex mirror, the position of the final image is

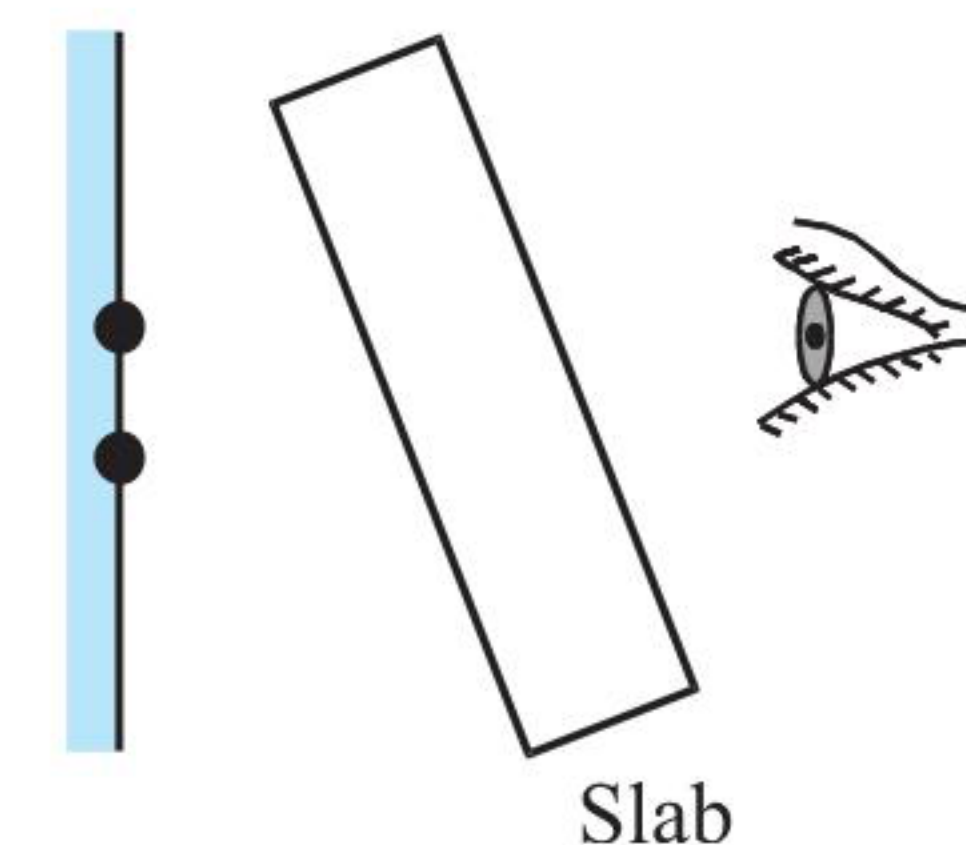
- (1) on the pole of the convex mirror
(2) on the pole of the concave mirror
(3) at a distance of 10 cm from the convex mirror
(4) at a distance of 5 cm from the concave mirror

30. A small piece of wire bent into an L shape, with upright and horizontal portions of equal lengths, is placed with the horizontal portion along the axis of the concave mirror whose radius of curvature is 10 cm. If the bend is 20 cm from the pole of the mirror, then the ratio of the lengths of the images of the upright and horizontal portions of the wire is

- (1) 1 : 2 (2) 3 : 1
(3) 1 : 3 (4) 2 : 1

31. The observer at O views two closely spaced spots on a vertical wall through an angled glass slab as shown in figure. As seen by observer, the spots appear

- (1) shifted upward, such that distance between them remains same
(2) shifted downward, such that distance between them remains same



- (3) spaced farther apart
(4) spaced closer together

32. The image of an object placed on the principal axis of a concave mirror of focal length 12 cm is formed at a point which is 10 cm more distance from the mirror than the object. The magnification of the image is

- (1) $8/3$ (2) 2.5
(3) 2 (4) -1.5

33. An object is approaching a fixed plane mirror with velocity 5 m s^{-1} making an angle of 45° with the normal. The speed of image w.r.t. the mirror is

- (1) 5 m s^{-1} (2) $5/\sqrt{2} \text{ m s}^{-1}$
(3) $5\sqrt{2} \text{ m s}^{-1}$ (4) 10 m s^{-1}

34. A clear transparent glass sphere ($\mu = 1.5$) of radius R is immersed in a liquid of refractive index 1.25. A parallel beam of light incident on it will converge to a point. The distance of this point from the centre will be

- (1) $-3R$ (2) $+3R$
(3) $-R$ (4) $+R$

35. A ray of light from a denser medium strikes a rarer medium at an angle of incidence i . The reflected and refracted rays

make an angle of $\pi/2$ with each other. If the angles of reflection and refraction are r and r' , then the critical angle will be

- (1) $\tan^{-1}(\sin i)$ (2) $\sin^{-1}(\sin r)$
 (3) $\sin^{-1}(\tan i)$ (4) $\sin^{-1}(\tan r)$

36. A ray of light travelling in glass ($\mu = 3/2$) is incident on a horizontal glass–air surface at the critical angle θ_c . If a thin layer of water ($\mu = 4/3$) is now poured on the glass–air surface, the angle at which the ray emerges into air at the water–air surface is

- (1) 60° (2) 45°
 (3) 90° (4) 180°

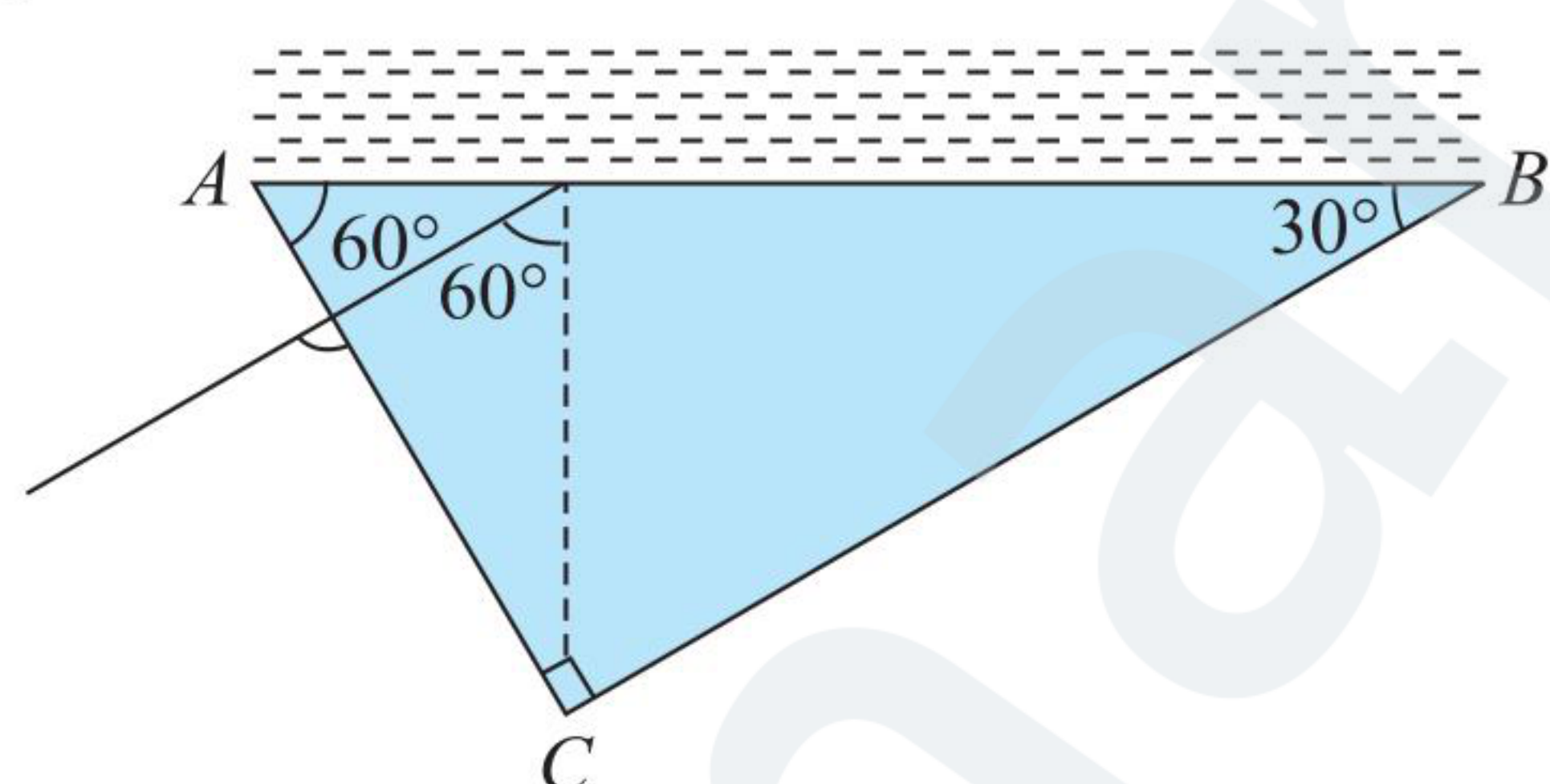
37. A ray of light enters a rectangular glass slab of refractive index $\sqrt{3}$ at an angle of incidence 60° . It travels a distance of 5 cm inside the slab and emerges out of the slab. The perpendicular distance between the incident and the emergent rays is

- (1) $5\sqrt{3}$ cm (2) $\frac{5}{2}$ cm
 (3) $5\sqrt{3/2}$ cm (4) 5 cm

38. A ray of monochromatic light is incident on the refracting face of a prism (angle 75°). It passes through the prism and is incident on the other face at the critical angle. If the refractive index of the prism is $\sqrt{2}$, then the angle of incidence on the first face of the prism is

- (1) 15° (2) 30°
 (3) 45° (4) 60°

39. ACB is right-angled prism with other angles as 60° and 30° . Refractive index of the prism is 1.5. AB has thin layer of liquid on it as shown. Light falls normally on the face AC . For total internal reflections, maximum refractive index of the liquid is

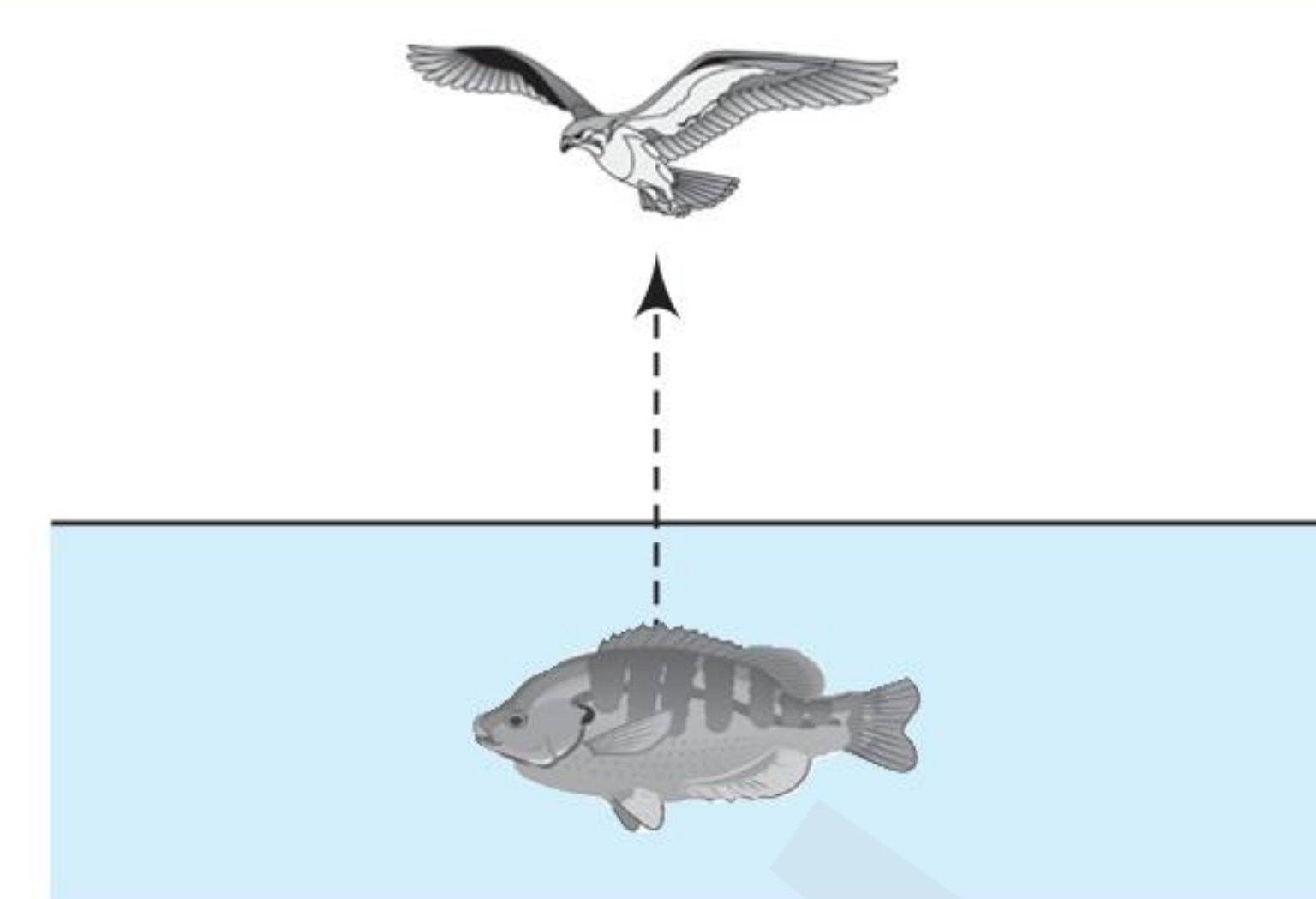


- (1) 1.4 (2) 1.3
 (3) 1.2 (4) 1.6

40. A glass prism has refractive index $\sqrt{2}$ and refracting angle 30° . One of the refracting surface of the prism is silvered. A beam of monochromatic light will retrace its path if its angle of incidence on the unsilvered refracting surface of the prism is

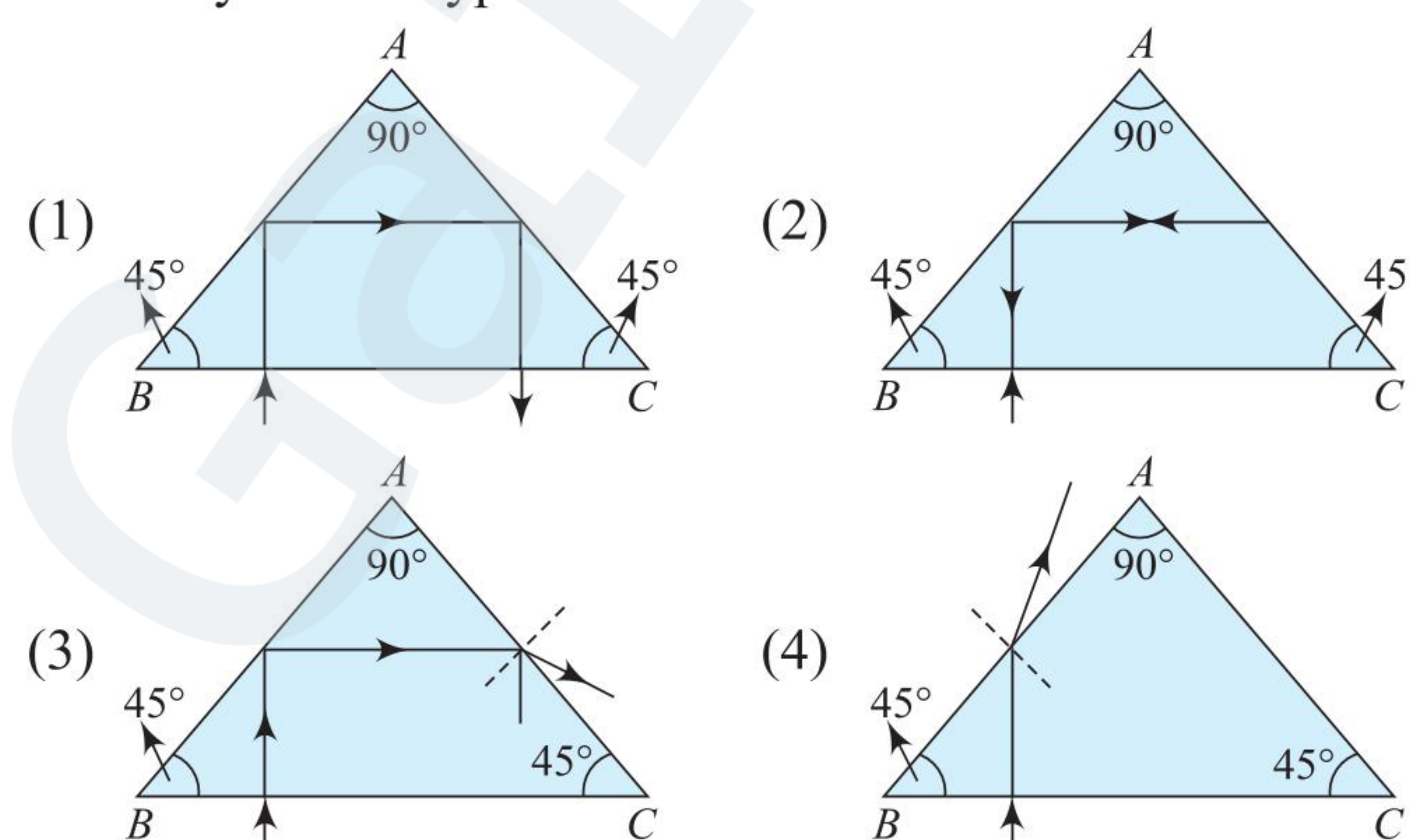
- (1) 0 (2) $\pi/6$
 (3) $\pi/4$ (4) $\pi/3$

41. A fish rising up vertically toward the surface of water with speed 3 ms^{-1} observes a bird diving down vertically towards it with speed 9 ms^{-1} . The actual velocity of bird is



- (1) 4.5 m s^{-1} (2) 5.4 m s^{-1}
 (3) 3.0 m s^{-1} (4) 3.4 m s^{-1}

42. The refractive index of material of a prism of angles 45° , -45° , and -90° is 1.5. The path of the ray of light incident normally on the hypotenuse side is shown in



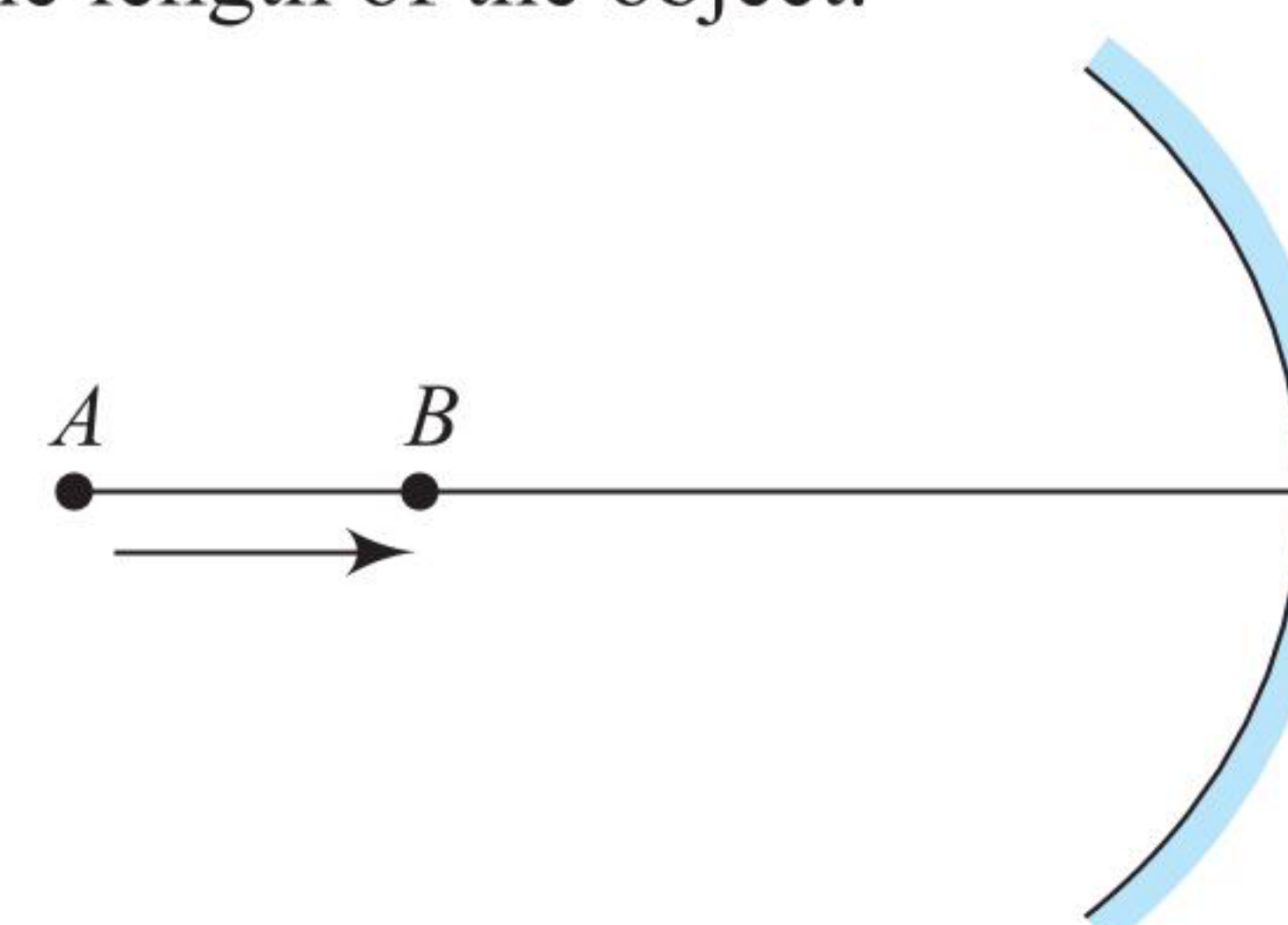
43. An object is placed in front of a convex mirror at a distance of 50 cm. A plane mirror is introduced covering the lower half of the convex mirror. If the distance between the object and the plane mirror is 30 cm, it is found that there is no parallax between the images formed by the two mirrors. What is the radius of curvature of the convex mirror?

- (1) 25 cm (2) 7 cm
 (3) 18 cm (4) 27 cm

44. The image produced by a concave mirror is one-quarter the size of object. If the object is moved 5 cm closer to the mirror, the image will only be half the size of the object. The focal length of mirror is

- (1) $f = 5.0 \text{ cm}$ (2) $f = 2.5 \text{ cm}$
 (3) $f = 7.5 \text{ cm}$ (4) $f = 10 \text{ cm}$

45. A linear object AB is placed along the axis of a concave mirror. The object is moving towards the mirror with speed U . The speed of the image of the point A is $4U$ and the speed of the image of B is also $4U$ but in opposite direction. If the centre of the line AB is at a distance L from the mirror then find out the length of the object.

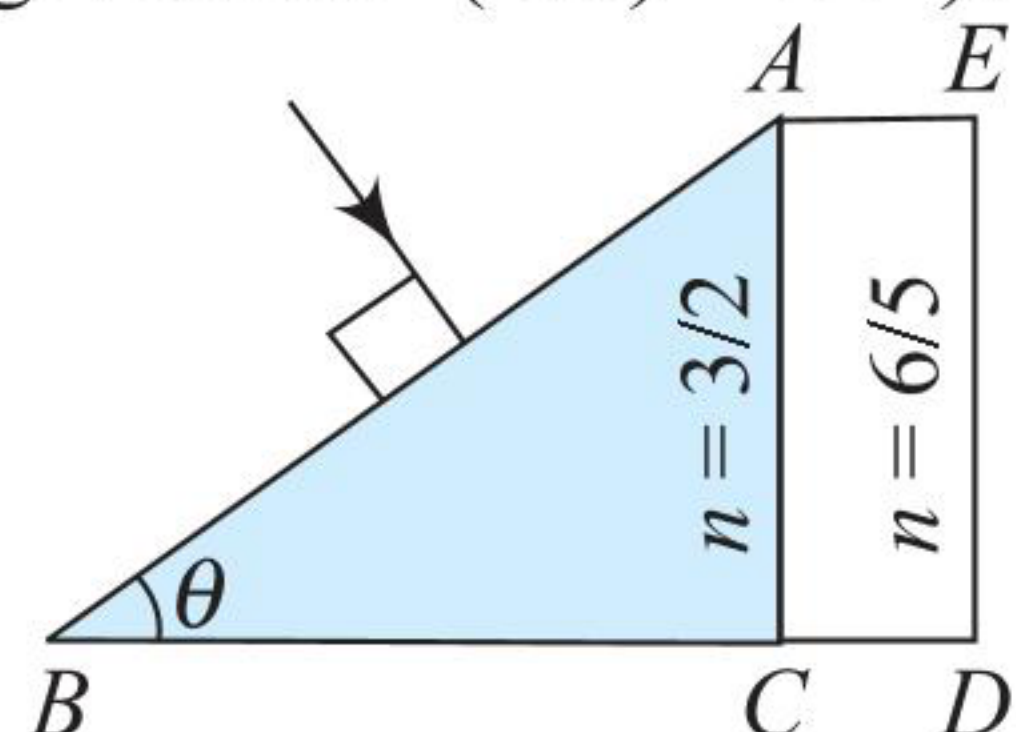


- (1) $3L/2$ (2) $5L/3$
 (3) L (4) None of these

46. A mango tree is at the bank of a river and one of the branch of tree extends over the river. A tortoise lives in the river. A mango falls just on the tortoise. The acceleration of the mango falling from tree as it appears to the tortoise is (refractive index of water is $4/3$ and the tortoise is stationary)

(1) g (2) $3g/4$
(3) $4g/3$ (4) none of these

47. In figure, ABC is the cross section of a right-angled prism and $ACDE$ is the cross section of a glass slab. The value of θ so that light incident normally on the face AB does not cross the face AC is (given $\sin^{-1}(3/5) = 37^\circ$).



(1) $\theta \leq 37^\circ$ (2) $\theta < 37^\circ$
(3) $\theta \leq 53^\circ$ (4) $\theta < 53^\circ$

48. A point object 'O' is at the centre of curvature of a concave mirror. The mirror starts to move at a speed u , in a direction perpendicular to the principal axis. Then, the initial velocity of the image is

(1) $2u$, in the direction opposite to that of mirror's velocity
(2) $2u$, in the direction same as that of mirror's velocity
(3) zero
(4) u , in the direction same as that of mirror's velocity

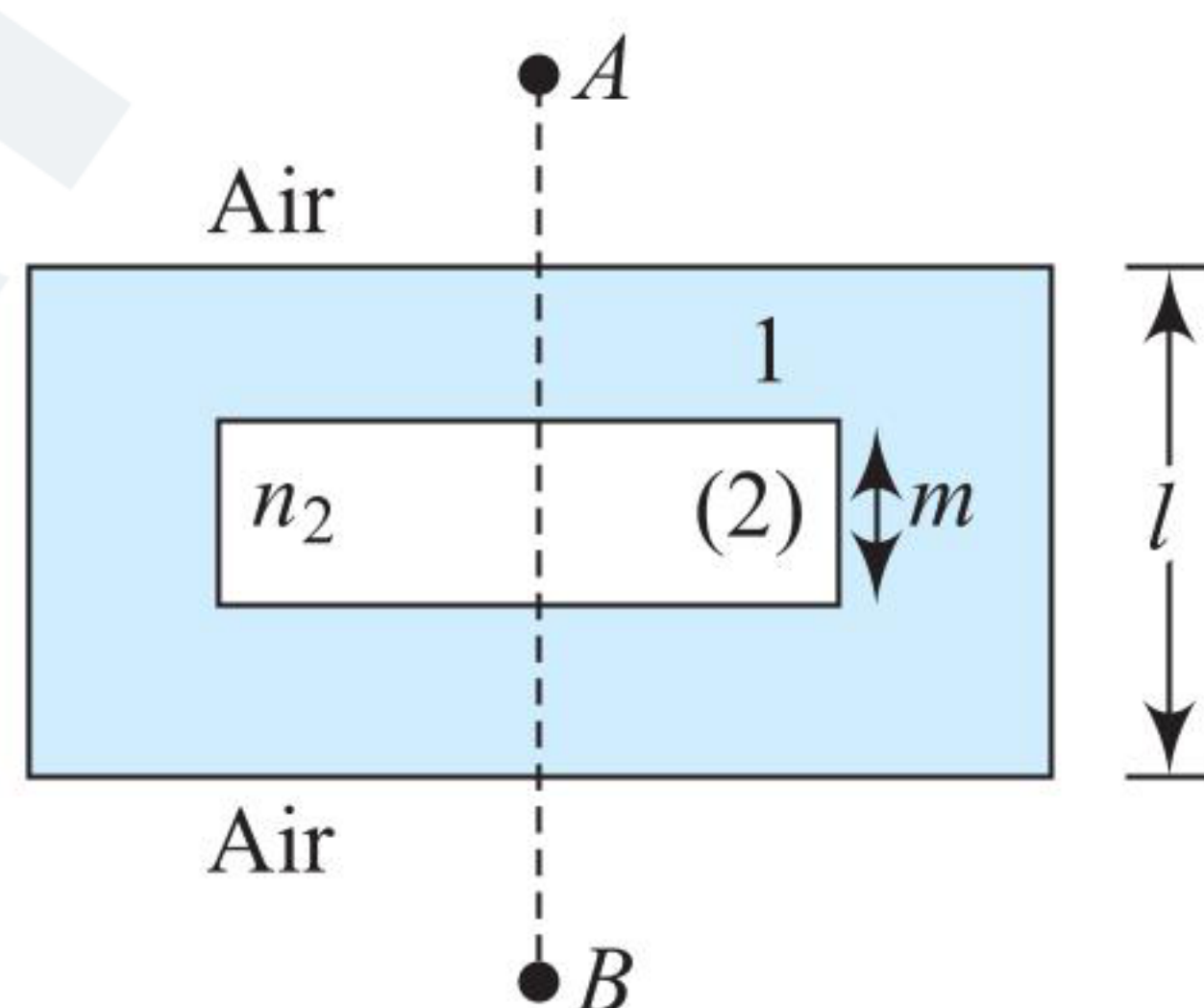
49. Refractive index of a prism is $\sqrt{7/3}$ and the angle of prism is 60° . The minimum angle of incidence of a ray that will be transmitted through the prism is

(1) 30° (2) 45°
(3) 15° (4) 50°

50. For a prism kept in air, it is found that for an angle of incidence 60° , the angle of refraction 'A', angle of deviation ' δ ', and angle of emergence 'e' become equal. The minimum angle of incidence of a ray that will be transmitted through the prism is

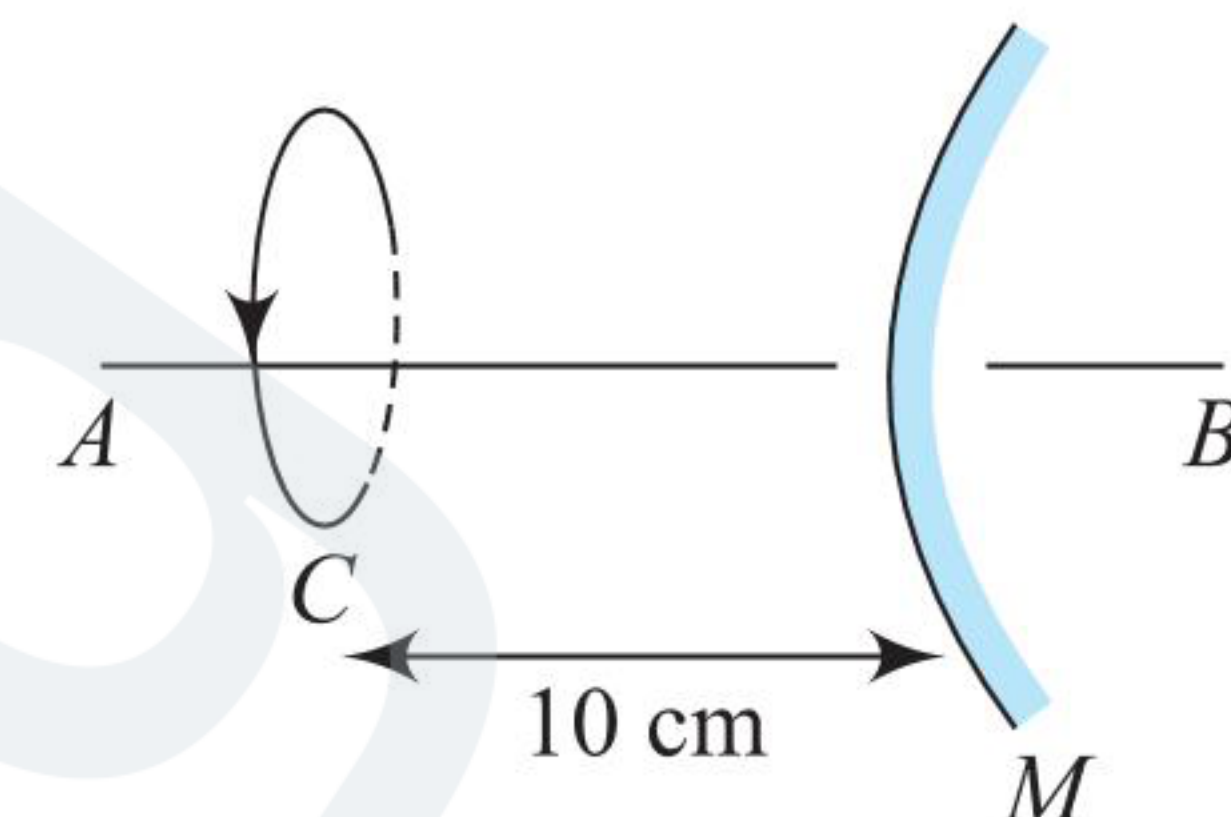
(1) 1.73 (2) 1.15
(3) 1.5 (4) 1.33

51. In a thick glass slab of thickness l , and refractive index n_1 , a cuboidal cavity of thickness ' m ' is carved as shown in figure and is filled with a liquid of RI n_2 ($n_1 > n_2$). The ratio l/m , so that shift produced by this slab is zero when an observer A observes an object B with paraxial rays is



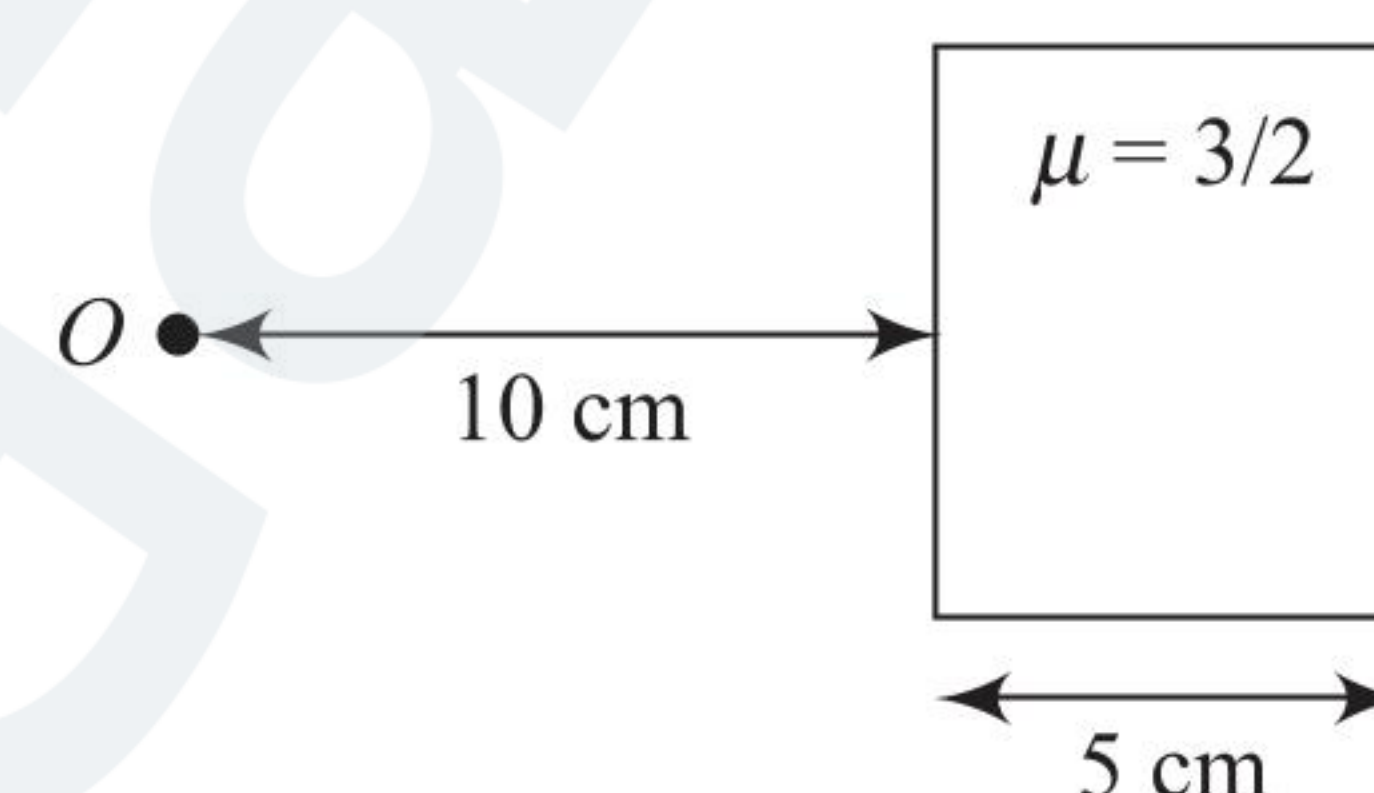
(1) $\frac{n_1 - n_2}{n_2 - 1}$ (2) $\frac{n_1 - n_2}{n_2(n_1 - 1)}$
(3) $\frac{n_1 - n_2}{n_1 - 1}$ (4) $\frac{n_1 - n_2}{n_1(n_2 - 1)}$

52. A particle revolves in clockwise direction (as seen from point A) in a circle C of radius 1 cm and completes one revolution in 2 sec. The axis of the circle and the principal axis of the mirror M coincide, call it AB. The radius of curvature of the mirror is 20 cm. Then, the direction of revolution (as seen from A) of the image of the particle and its speed is



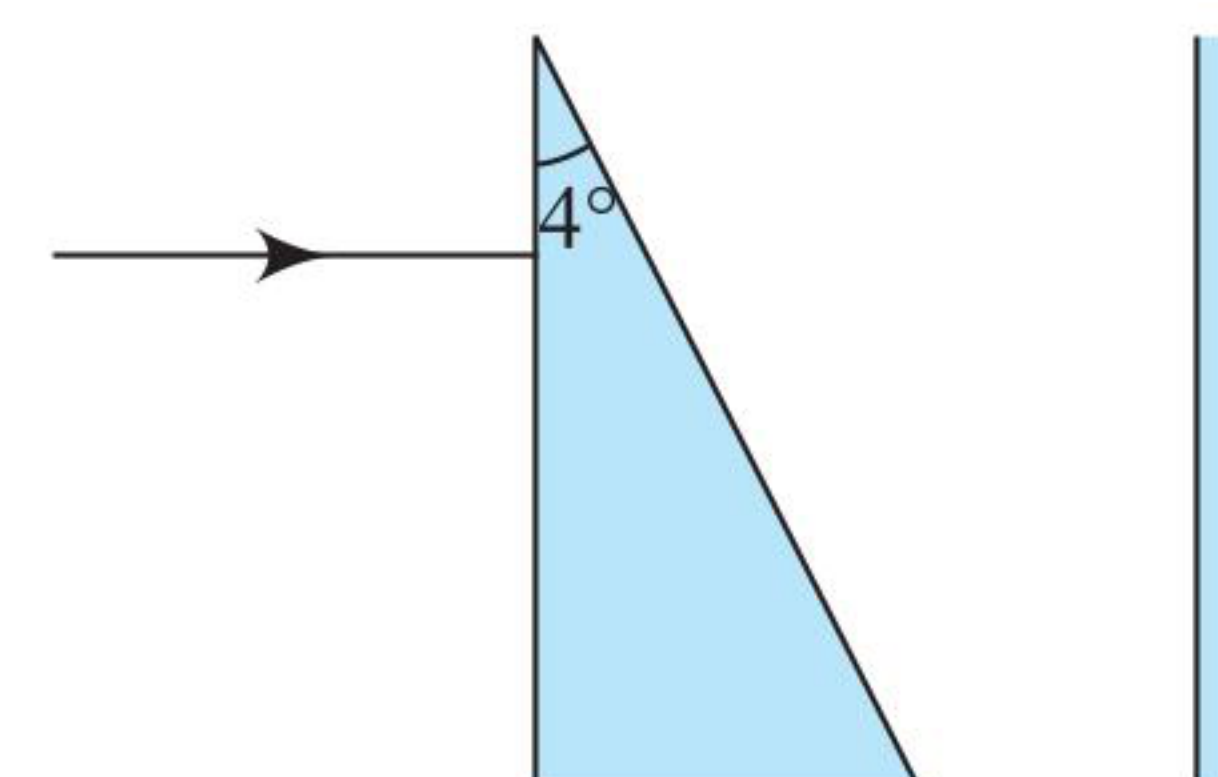
(1) clockwise, 1.57 cm s^{-1}
(2) clockwise, 3.14 cm s^{-1}
(3) anticlockwise, 1.57 cm s^{-1}
(4) anticlockwise, 3.14 cm s^{-1}

53. A point object is placed in front of a thick plane mirror as shown in figure. Find the location of final image w.r.t. object.



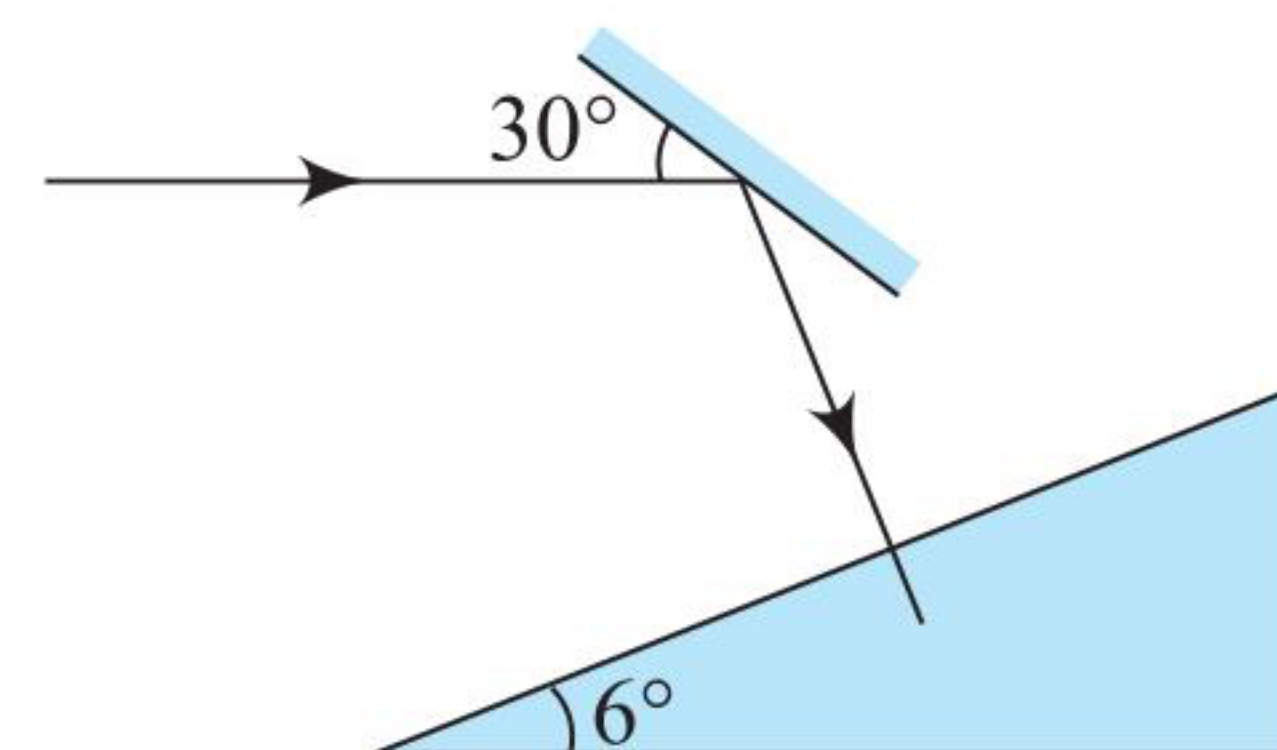
(1) $15/2 \text{ cm}$ (2) 15 cm
(3) $40/3 \text{ cm}$ (4) $80/3 \text{ cm}$

54. A right-angled prism of apex angle 4° and refractive index 1.5 is located in front of a vertical plane mirror as shown in figure. A horizontal ray of light is falling on the prism. Find the total deviation produced in the light ray as it emerges 2nd time from the prism.



(1) 8° CW (2) 6° CW
(3) 180° CW (4) 174° CW

55. Find the net deviation produced in the incident ray for the optical instrument shown in figure. (Take refractive index of the prism material as 2.)



(1) 66° CW
(2) 66° ACW
(3) 54° ACW
(4) 54° CW

56. The refracting angle of a prism is A and refractive index of the material of prism is $\cot(A/2)$. The angle of minimum deviation will be

(1) $180^\circ - 3A$ (2) $180^\circ + 3A$
(3) $90^\circ - 3A$ (4) $180^\circ - 2A$

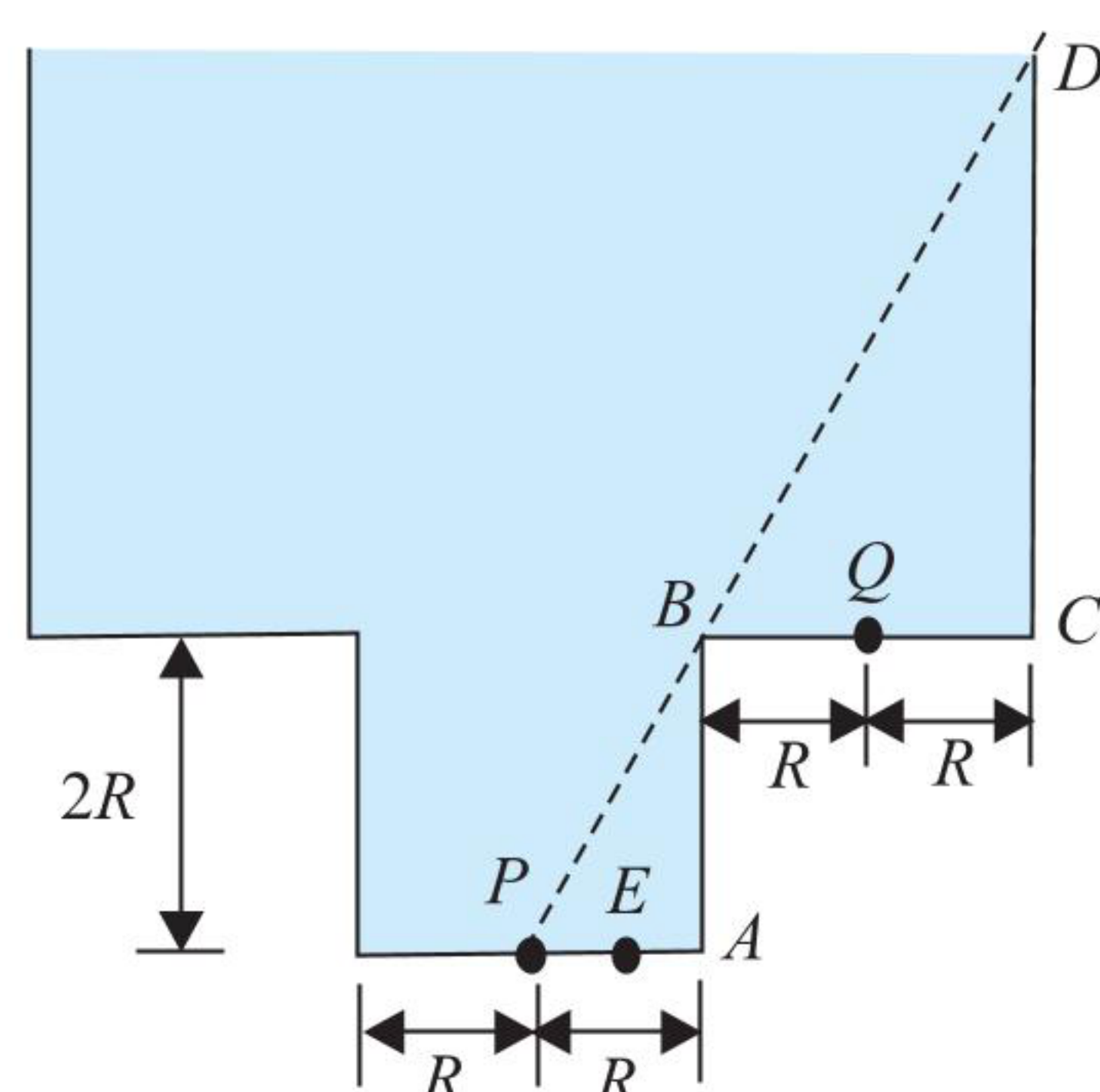
57. An object is placed 21 cm in front of a concave mirror of radius of curvature 10 cm. A glass slab of thickness 3 cm and refractive index 1.5 is then placed close to the mirror in the space between the object and the mirror. The distance of the near surface of the slab from the mirror is 1 cm. The final image from the mirror will be formed at

- (1) 4.67 cm (2) 6.67 cm
(3) 5.67 cm (4) 7.67 cm

58. A concave mirror of radius of curvature 40 cm forms an image of an object placed on the principal axis at a distance 45 cm in front of it. Now if the system (including object) is completely immersed in water ($\mu = 1.33$), then

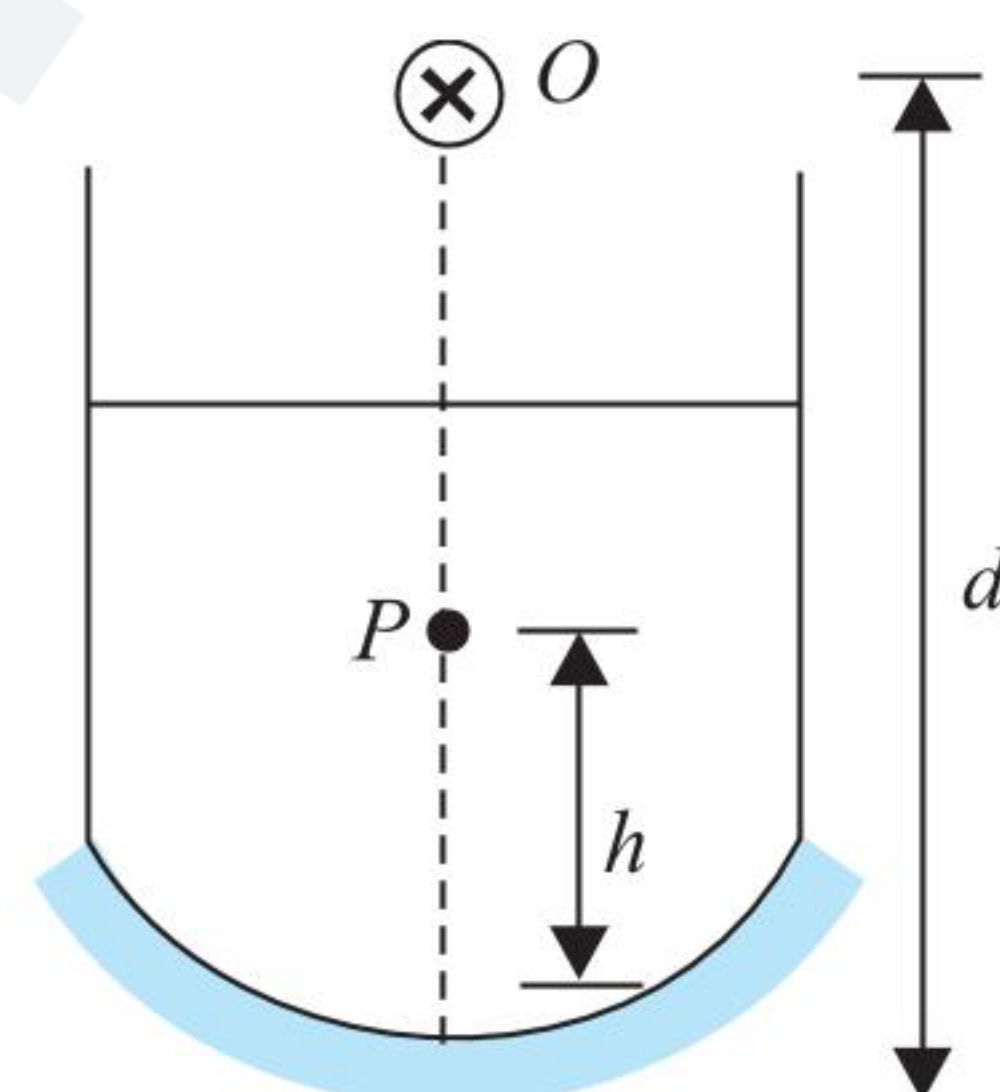
- (1) the image will shift towards the mirror
(2) the magnification will reduce
(3) the image will shift away from the mirror and magnification will increase.
(4) the position of the image and magnification will not change.

59. In a vessel, as shown in figure, point P is just visible when no liquid is filled in vessel through a telescope in the air. When liquid is filled in the vessel completely, point Q is visible without moving the vessel or telescope. Find the refractive index of the liquid.



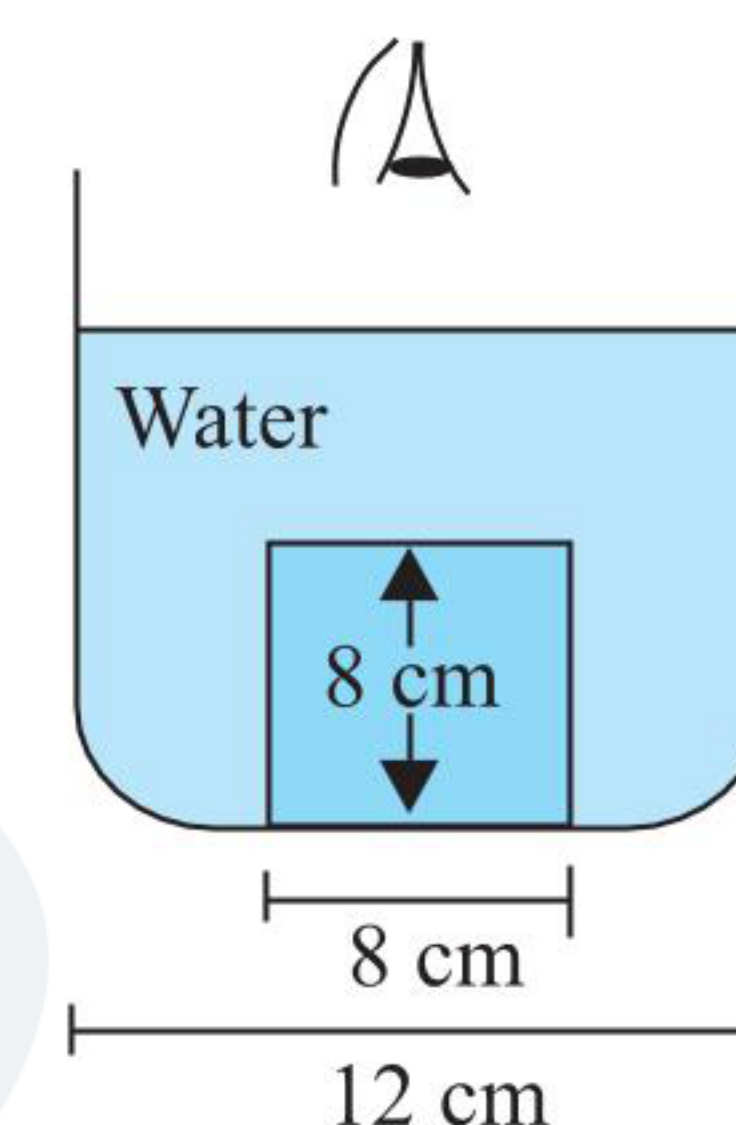
- (1) $\frac{\sqrt{14}}{3}$ (2) $\frac{\sqrt{85}}{5}$
(3) $\sqrt{2}$ (4) $\sqrt{3}$
60. In the previous problem, what will be refractive index of the liquid so that point E is visible ($PE = R/3$) through the telescope in air when the vessel is filled with liquid completely, without moving the telescope and vessel
- (1) 1.5 (2) $\sqrt{3}$
(3) $\sqrt{2}$ (4) none of these

61. A concave mirror of radius of curvature h is placed at the bottom of a tank containing a liquid of refractive index μ up to a depth d . An object P is placed at height h above the bottom of the mirror. Outside the liquid, an observer O views the object and its image in the mirror. The apparent distance between these two will be



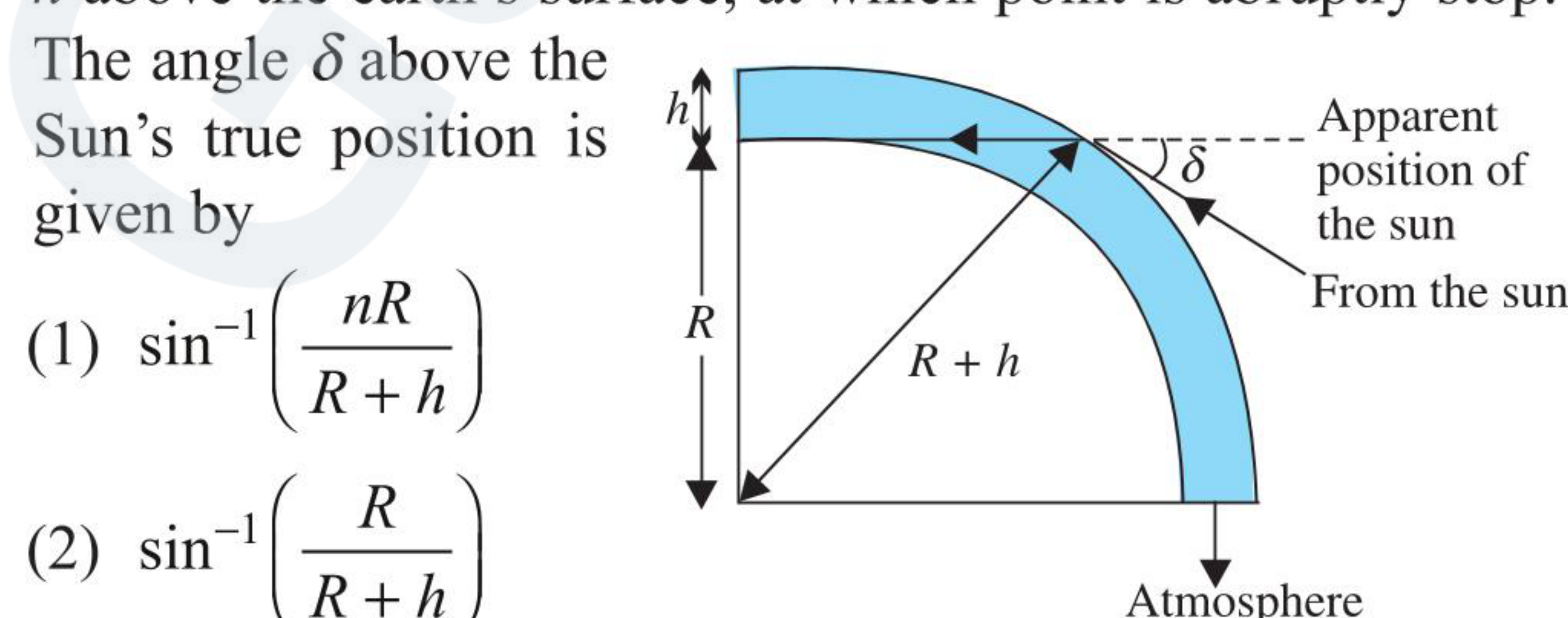
- (1) $h\left(1 - \frac{1}{\mu}\right)$ (2) $\frac{2h}{\mu - 1}$
(3) zero (4) $\frac{2h}{\mu}$

62. A cylindrical vessel of diameter 12 cm contains $800\pi\text{cm}^3$ of water. A cylindrical glass piece of diameter 8.0 cm and height 8.0 cm is placed in the vessel. If the bottom of the vessel under the glass piece is seen by the paraxial rays, locate its image. The index of refraction of glass is 1.50 and that of water is 1.33.



- (1) 9.2 cm above the bottom
(2) 12.5 cm above the bottom
(3) 7.1 cm above the bottom
(4) 7.1 cm below the bottom

63. When the sun is either rising or setting and appears to be just in the horizon, it is in fact below the horizon. The explanation for this seeming paradox is that light from the sun bends slightly when entering the earth's atmosphere as shown in figure. Assume that the atmosphere has uniform density and hence uniform index of refraction n and extends to a height h above the earth's surface, at which point is abruptly stop.

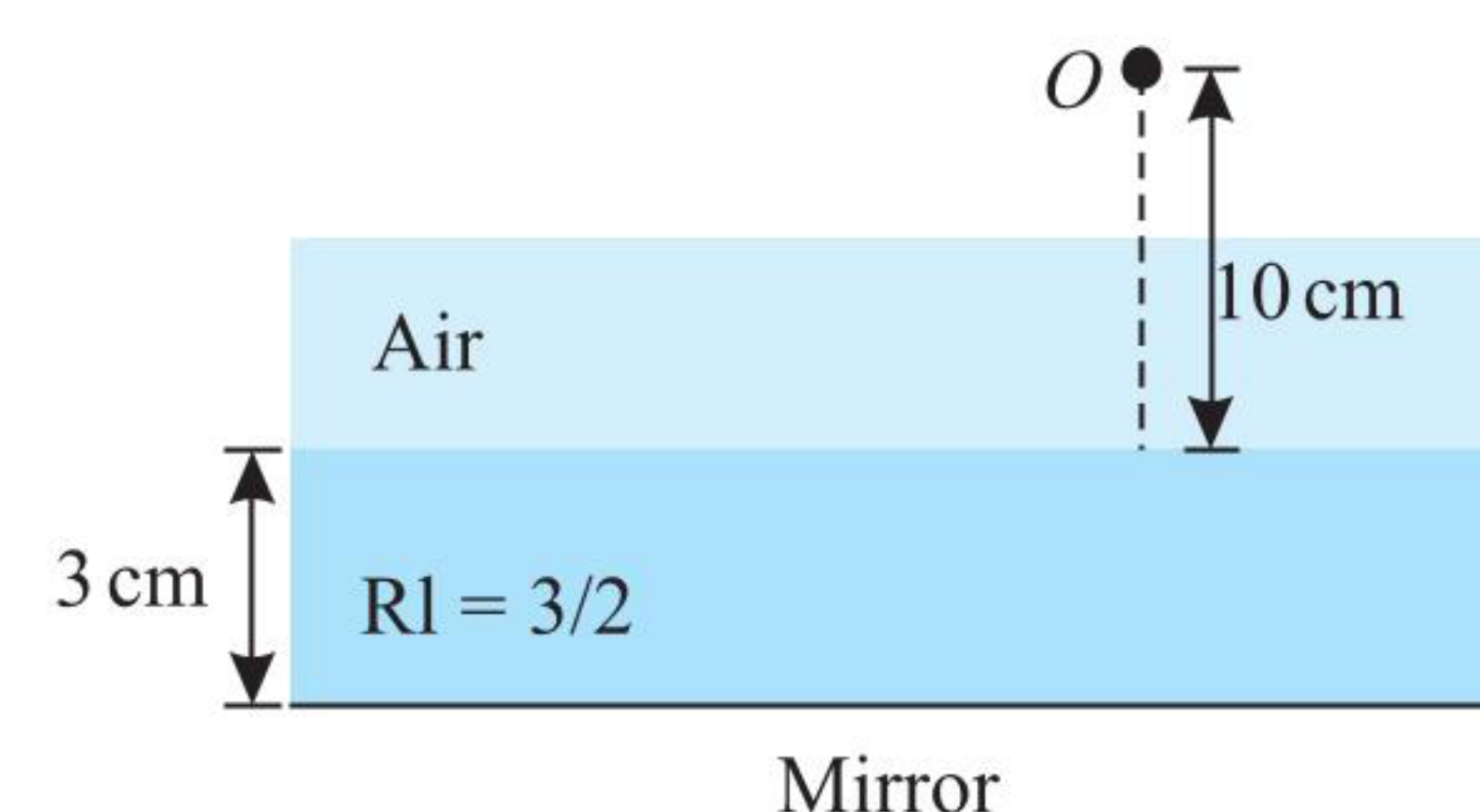


- The angle δ above the Sun's true position is given by
- (1) $\sin^{-1}\left(\frac{nR}{R+h}\right)$
(2) $\sin^{-1}\left(\frac{R}{R+h}\right)$
(3) $\sin^{-1}\left(\frac{nR}{R+h}\right) + \sin^{-1}\left(\frac{R}{R+h}\right)$
(4) $\sin^{-1}\left(\frac{nR}{R+h}\right) - \sin^{-1}\left(\frac{R}{R+h}\right)$

64. A gun of mass m_1 fires a bullet of mass m_2 with a horizontal speed v_0 . The gun is fitted with a concave mirror of focal length f facing toward a receding bullet. Find the speed of separation of the bullet and the image just after the gun was fired.

- (1) $\left(1 + \frac{m_2}{m_1}\right)v_0$ (2) $2\left(1 - \frac{m_2}{m_1}\right)v_0$
(3) $2\left(1 + \frac{2m_2}{m_1}\right)v_0$ (4) $2\left(1 + \frac{m_2}{m_1}\right)v_0$

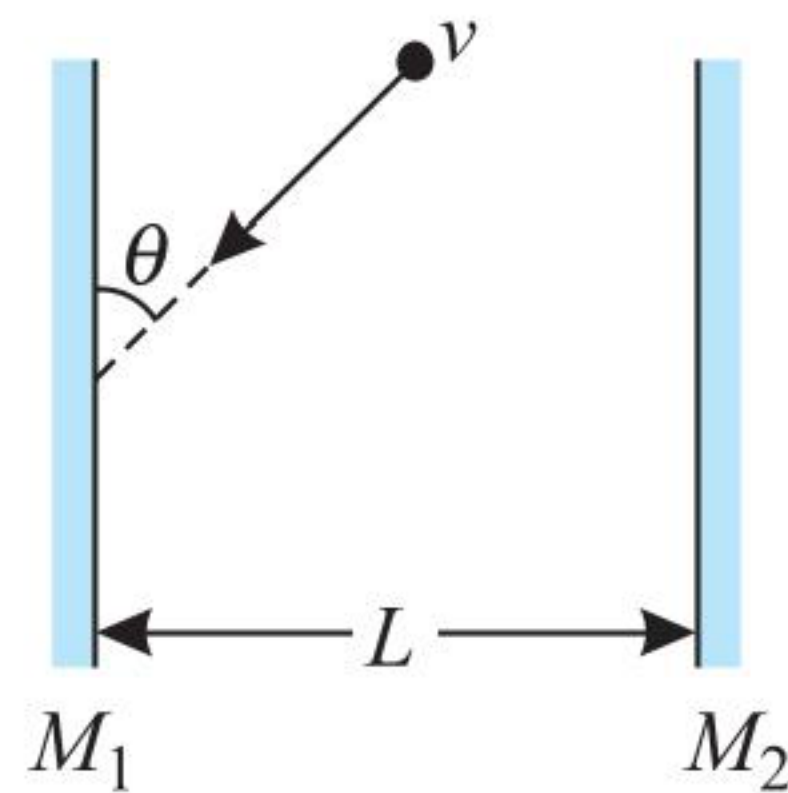
65. The position of final image of an object O as shown in figure is



- (1) 11 cm behind mirror (2) 12.33 cm behind mirror
(3) 18 cm behind mirror (4) 15 cm behind mirror

66. A bird is flying with a velocity v between two long vertical plane mirrors making an angle θ with mirror M_1 as shown. Then what will be the relative velocity of approach between

the images formed by the mirrors due to the 1st reflection in each of them



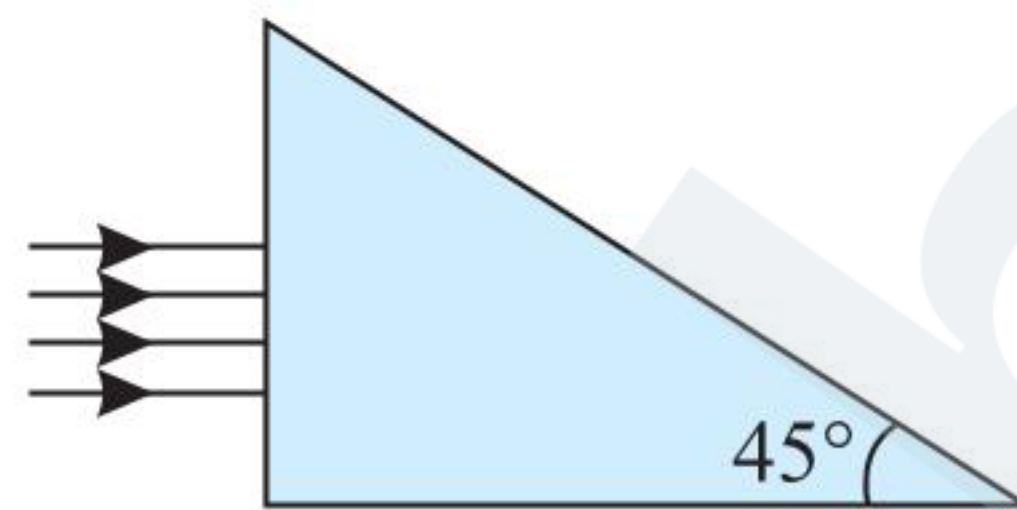
- (1) $v \sin \theta$ (2) $2v \sin \theta$
(3) $2(v \sin \theta + L)$ (4) zero

67. The x - z plane separates two media A and B of refractive indices $\mu_1 = 1.5$ and $\mu_2 = 2$. A ray of light travels from A to B . Its directions in the two media are given by unit vectors $\vec{u}_1 = a\hat{i} + b\hat{j}$ and $\vec{u}_2 = c\hat{i} + d\hat{j}$. Then

- (1) $\frac{a}{c} = \frac{4}{3}$ (2) $\frac{a}{c} = \frac{3}{4}$
(3) $\frac{b}{d} = \frac{4}{3}$ (4) $\frac{b}{d} = \frac{3}{4}$

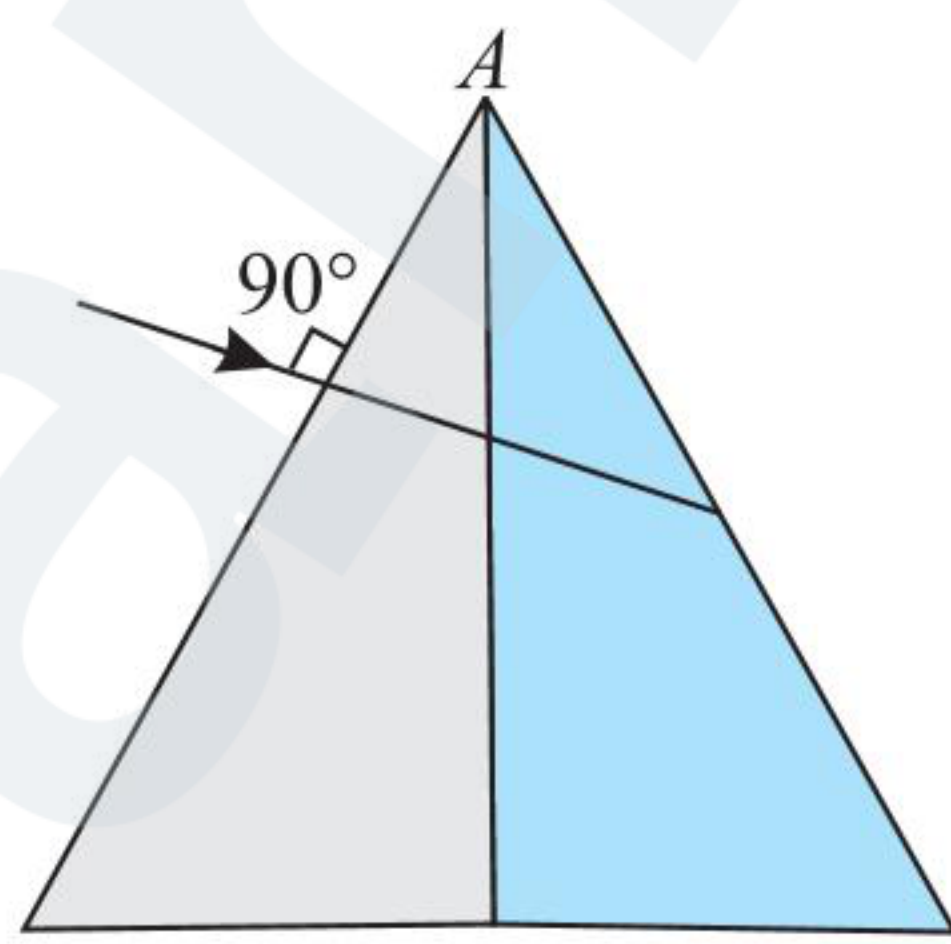
68. A beam of light consisting of red, green and blue and is incident on a right angled prism. The refractive index of the material of the prism for the above red, green and blue wavelengths are 1.39, 1.44 and 1.47 respectively. The prism will

- (1) separate part of the red color from the green and blue colors
(2) separate part of the blue color from the red and green colors
(3) separate all the three colors from the other two colors
(4) not separate even partially, any colors from the other two colors

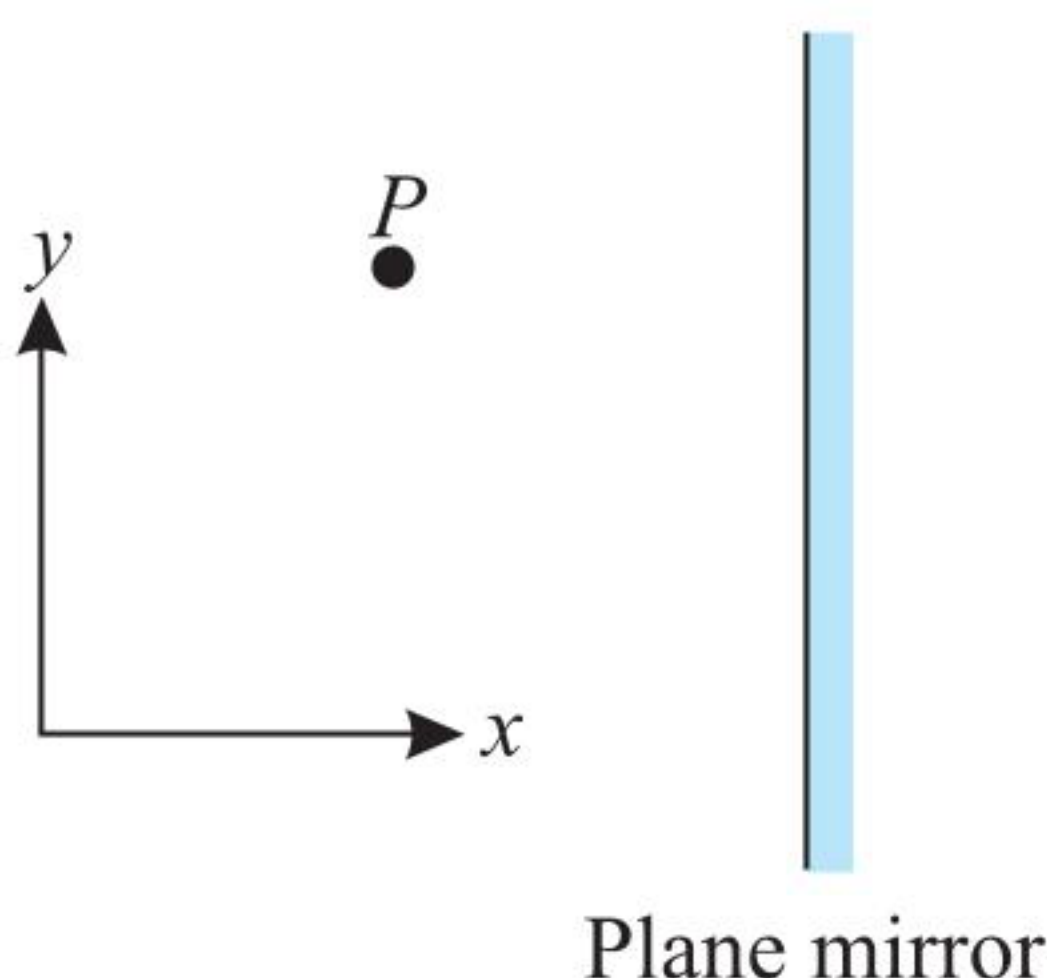


69. A ray of light is incident normally on the first refracting face of the prism of refracting angle A . The ray of light comes out at grazing emergence. If one half of the prism (shaded position) is knocked off, the same ray will

- (1) Emerge at an angle of emergence $\sin^{-1}\left(\frac{1}{2} \sec A/2\right)$
(2) Not emerge out of the prism
(3) Emerge at an angle of emergence $\sin^{-1}\left(\frac{1}{2} \sec A/4\right)$
(4) None of these



70. Shown plane mirror is lying in y - z plane. A particle P has velocity $\vec{u} = 5\hat{i} + 6\hat{j}$ at $t = 0$ and acceleration $\vec{a} = 2t\hat{j}$. Velocity of its image at $t = 2$ s relative to it is



- (1) $10\hat{i}$ (2) $-10\hat{i}$
(3) $20\hat{j}$ (4) $-20\hat{j}$

71. An infinitely long rod lies along the axis of a concave mirror of focal length f . The near end of the rod is at distance $u > f$ from the mirror. Its image will have a length

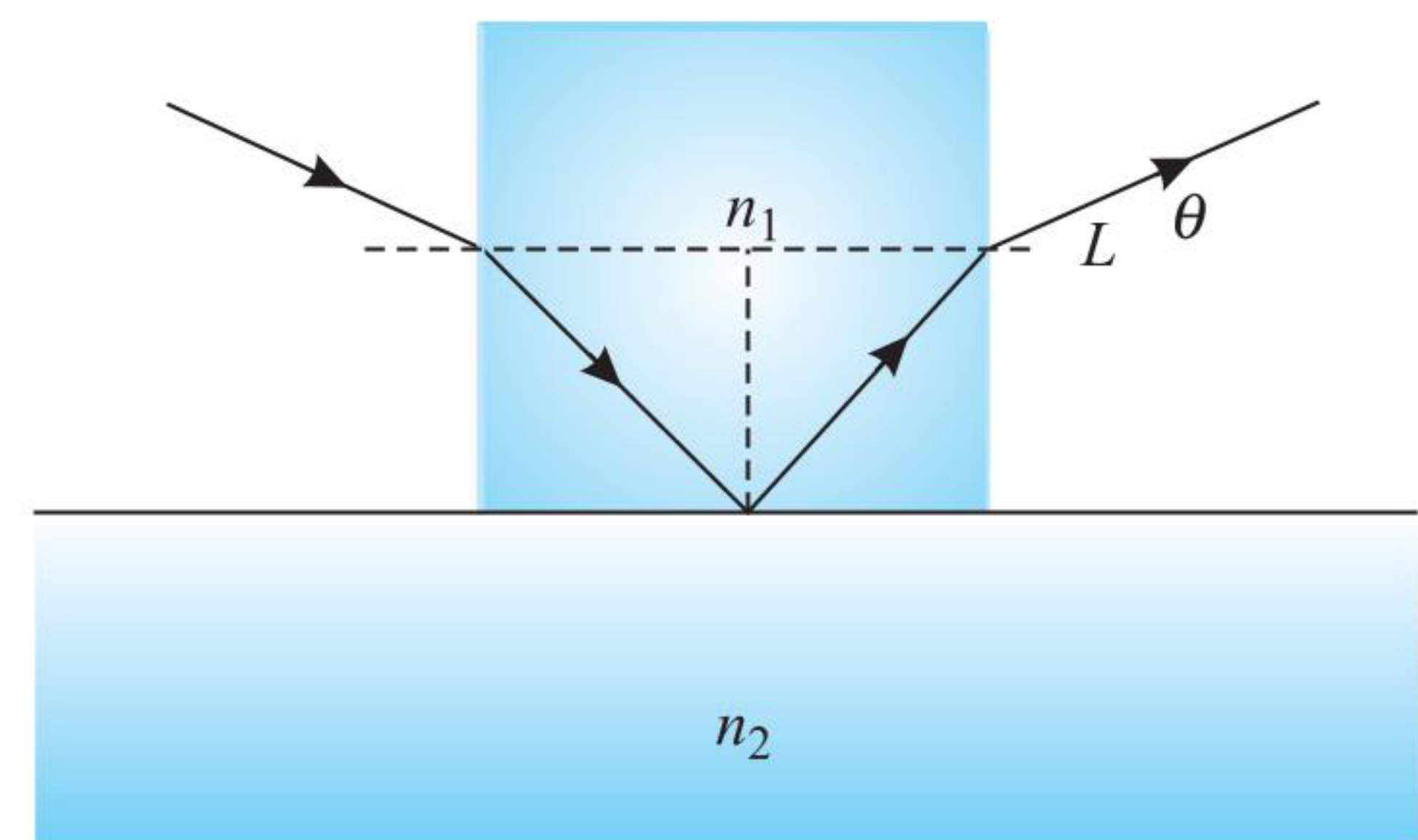
- (1) $\frac{uf}{u-f}$ (2) $\frac{uf}{u+f}$
(3) $\frac{f^2}{u+f}$ (4) $\frac{f^2}{u-f}$

72. What is the length of the image of the rod in mirror, according to the observer in air? (refractive index of the liquid is m)



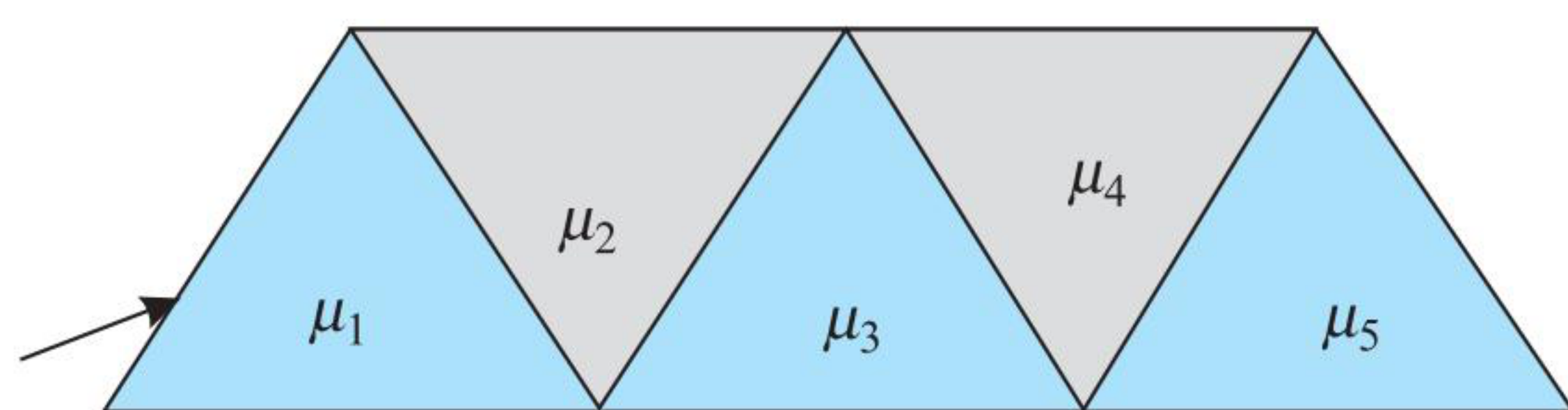
- (1) $\mu L + L$ (2) $L + \frac{L}{\mu}$
(3) $L\mu + \frac{L}{\mu}$ (4) None of these

73. A cubical block of glass of refractive index n_1 is in contact with the surface of water of refractive index n_2 . A beam of light is incident on vertical face of the block (see figure). After refraction, a total internal reflection at the base and refraction at the opposite vertical face, the ray emerges out at an angle θ . The value of θ is given by



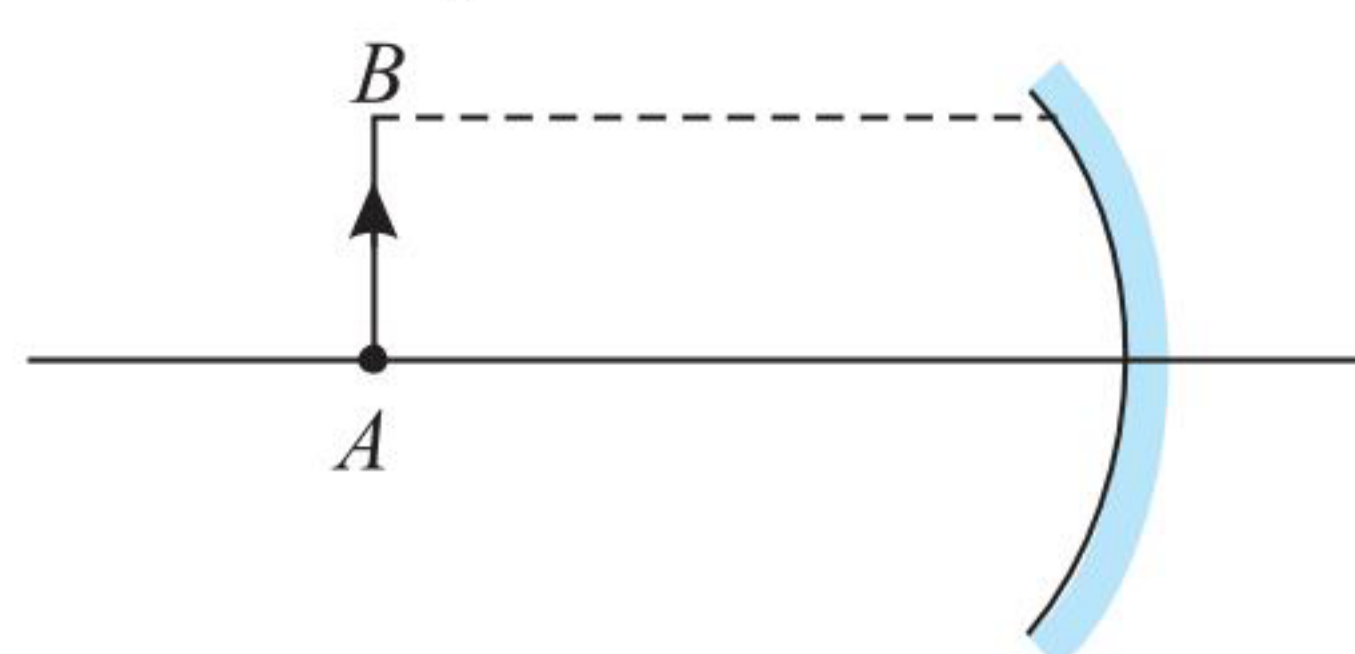
- (1) $\sin \theta < \sqrt{n_1^2 - n_2^2}$ (2) $\tan \theta < \sqrt{n_1^2 - n_2^2}$
(3) $\sin \theta < \frac{1}{\sqrt{n_1^2 - n_2^2}}$ (4) $\tan \theta < \frac{1}{\sqrt{n_1^2 - n_2^2}}$

74. The diagram shows five isosceles right angled prisms. A light ray incident at 90° at the first face emerges at same angle with the normal from the last face. Which of the following relations will hold regarding the refractive indices?



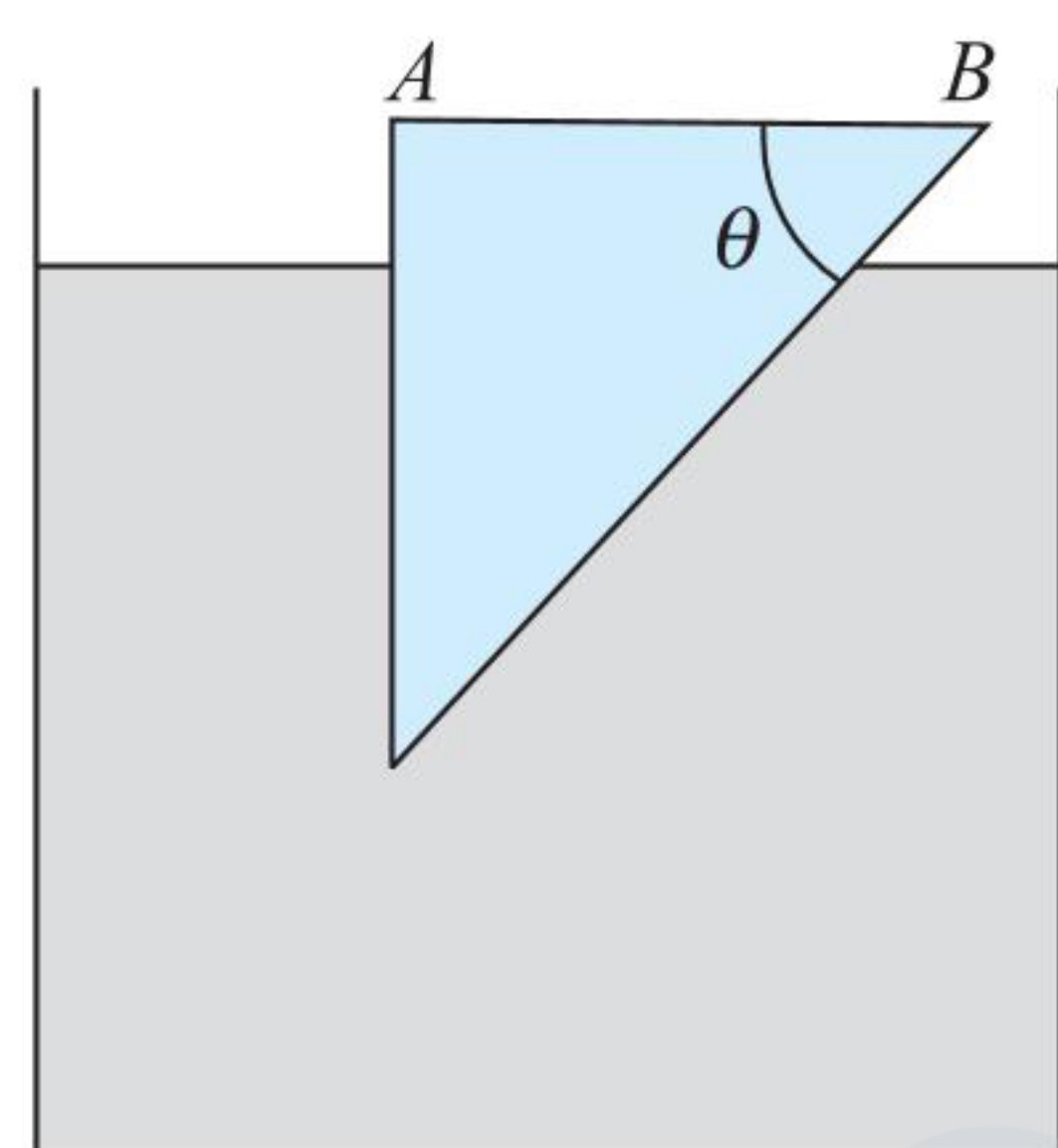
- (1) $\mu_1^2 + \mu_3^2 + \mu_5^2 = \mu_2^2 + \mu_4^2$
- (2) $\mu_1^2 + \mu_3^2 + \mu_5^2 = 1 + \mu_2^2 + \mu_4^2$
- (3) $\mu_1^2 + \mu_3^2 + \mu_5^2 = 2 + \mu_2^2 + \mu_4^2$
- (4) none of these

75. A converging mirror forms real image of object AB on screen. Now a hole is made on mirror just in front of point B , Select the correct option.



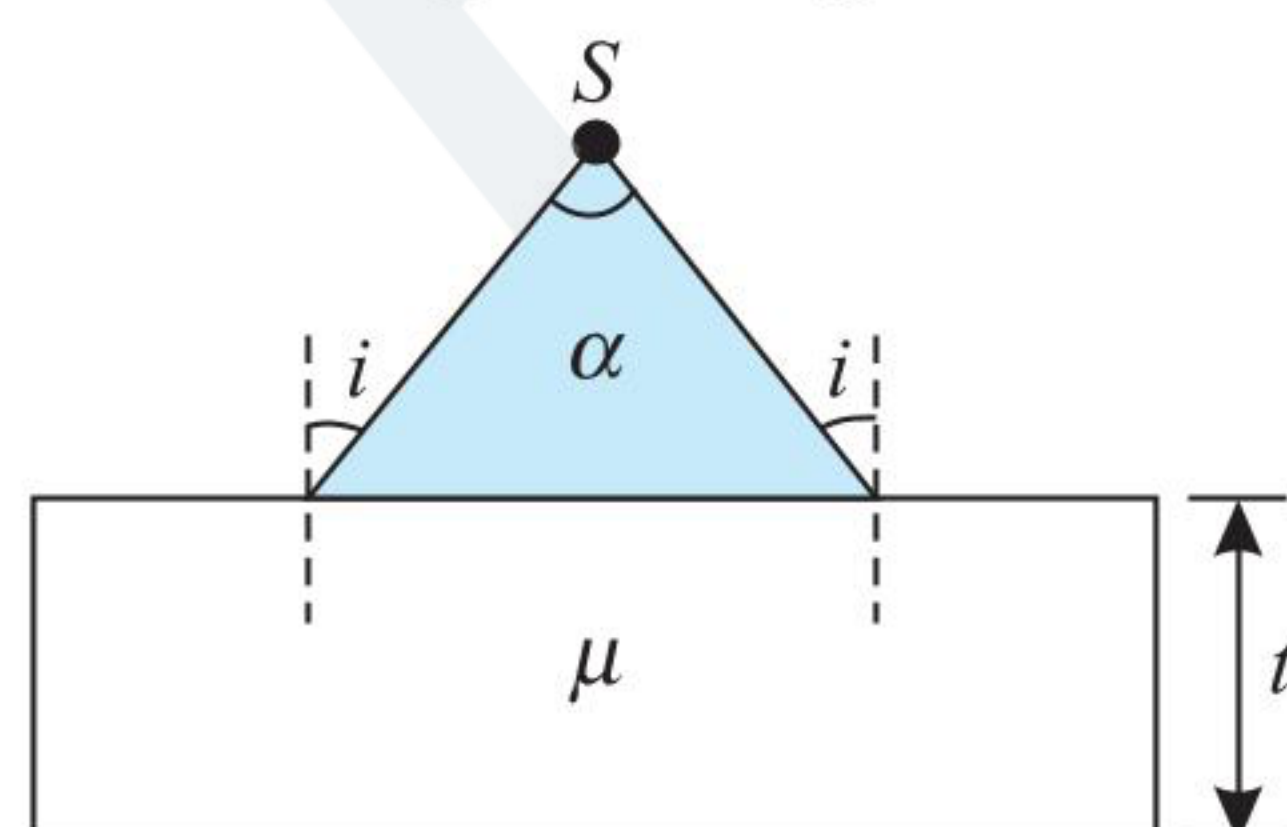
- (1) Image of point B will be slightly below the previous position in screen.
- (2) Image of point B will be at the same place where it was formed earlier.
- (3) Image of point B will be absent on screen.
- (4) Two images of point B will be formed.

76. A glass prism of refractive index 1.5 is immersed in water (refractive index $4/3$). A light beam normally on the face AB is totally reflected to reach on the face BC if



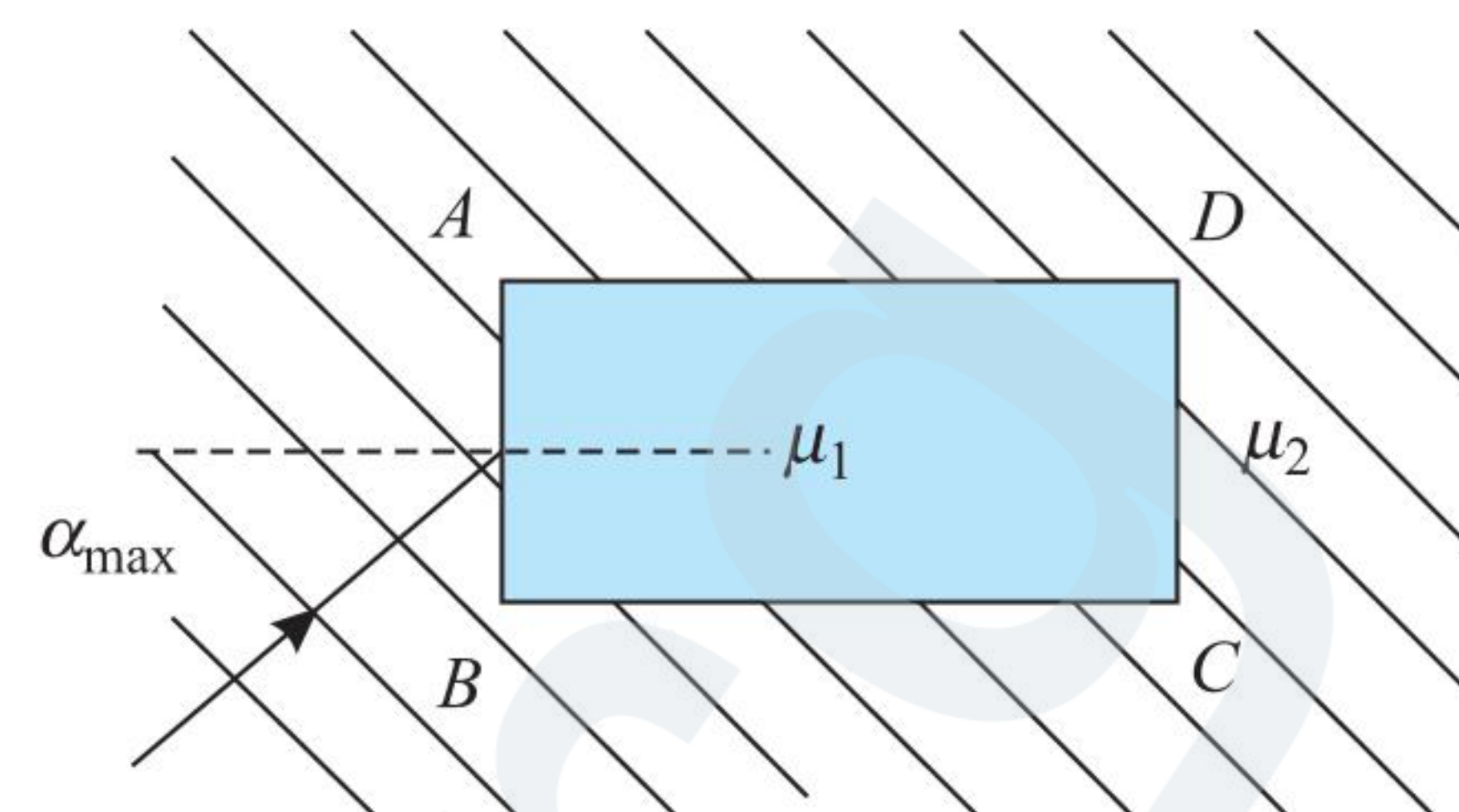
- (1) $\sin \theta \geq \frac{8}{9}$
- (2) $\frac{2}{3} < \sin \theta < \frac{8}{9}$
- (3) $\sin \theta \leq \frac{2}{3}$
- (4) $\sin \theta \leq \frac{8}{9}$

77. A diverging beam of light from a point source S having divergence angle α , falls symmetrically on a glass slab as shown. The angles of incidence of the two extreme rays are equal. If the thickness of the glass slab is t and the refractive index n , then the divergence angle of the emergent beam is



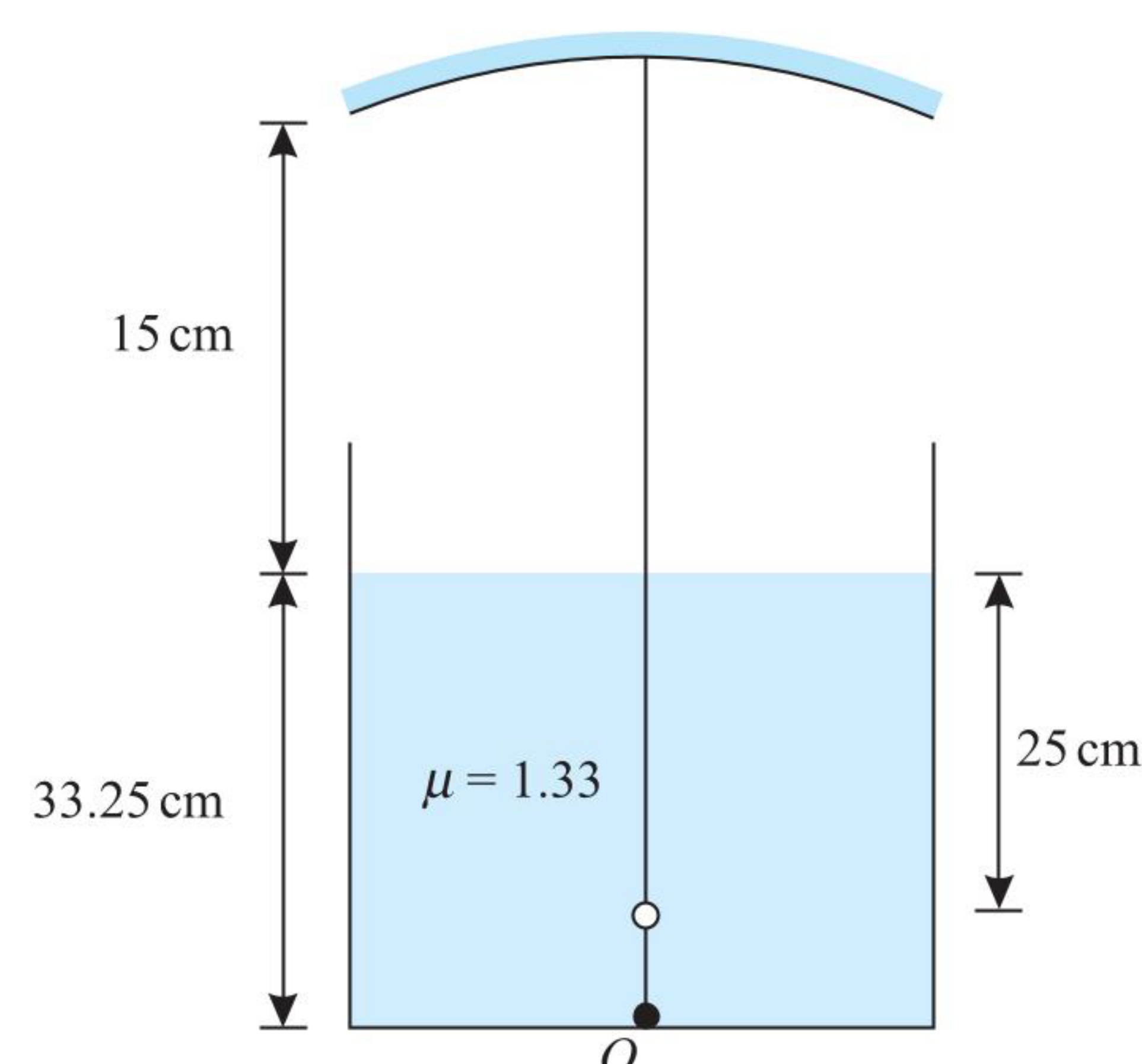
- (1) zero
- (2) α
- (3) $\sin^{-1}\left(\frac{1}{n}\right)$
- (4) $2\sin^{-1}\left(\frac{1}{n}\right)$

78. A rectangular glass slab $ABCD$ of refractive index n_1 is immersed in water of refractive index n_2 ($n_1 < n_2$). A ray of light is incident at the surface AB of the slab as shown. The maximum value of the angle of incidence $\alpha_{\max}$ such that the ray comes out from the other surface CD is given by



- (1) $\sin^{-1}\left[\frac{n_1}{n_2} \cos\left(\sin^{-1}\left(\frac{n_2}{n_1}\right)\right)\right]$
- (2) $\sin^{-1}\left[n_1 \cos\left(\sin^{-1}\left(\frac{1}{n_2}\right)\right)\right]$
- (3) $\sin^{-1}\left(\frac{n_1}{n_2}\right)$
- (4) $\sin^{-1}\left(\frac{n_2}{n_1}\right)$

79. A container is filled with water ($\mu = 1.33$) up to a height of 33.25 cm. A convex mirror is placed 15 cm above the water level and the image of an object placed at the bottom is formed 25 cm below the water level. Focal length of the mirror is



- (1) 15 cm
- (2) 20 cm
- (3) -18.31 cm
- (4) 10 cm

80. A light beam is travelling from Region I to Region IV (refer figure). The refractive indices in Regions I, II, III, and IV are n_0 , $n_0/2$, $n_0/6$, and $n_0/8$, respectively. The angle of incidence θ for which the beam just misses entering Region IV is

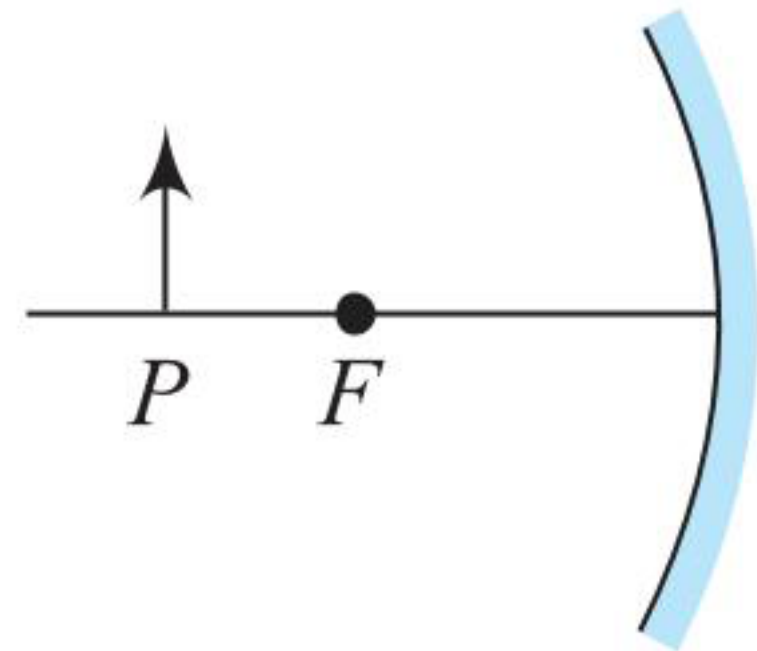
Region I	Region II	Region III	Region IV
n_0	$\frac{n_0}{2}$	$\frac{n_0}{6}$	$\frac{n_0}{8}$
0	0.2 m	0.6 m	

- (1) $\sin^{-1}(3/4)$
- (2) $\sin^{-1}(1/8)$
- (3) $\sin^{-1}(1/4)$
- (4) $\sin^{-1}(1/3)$

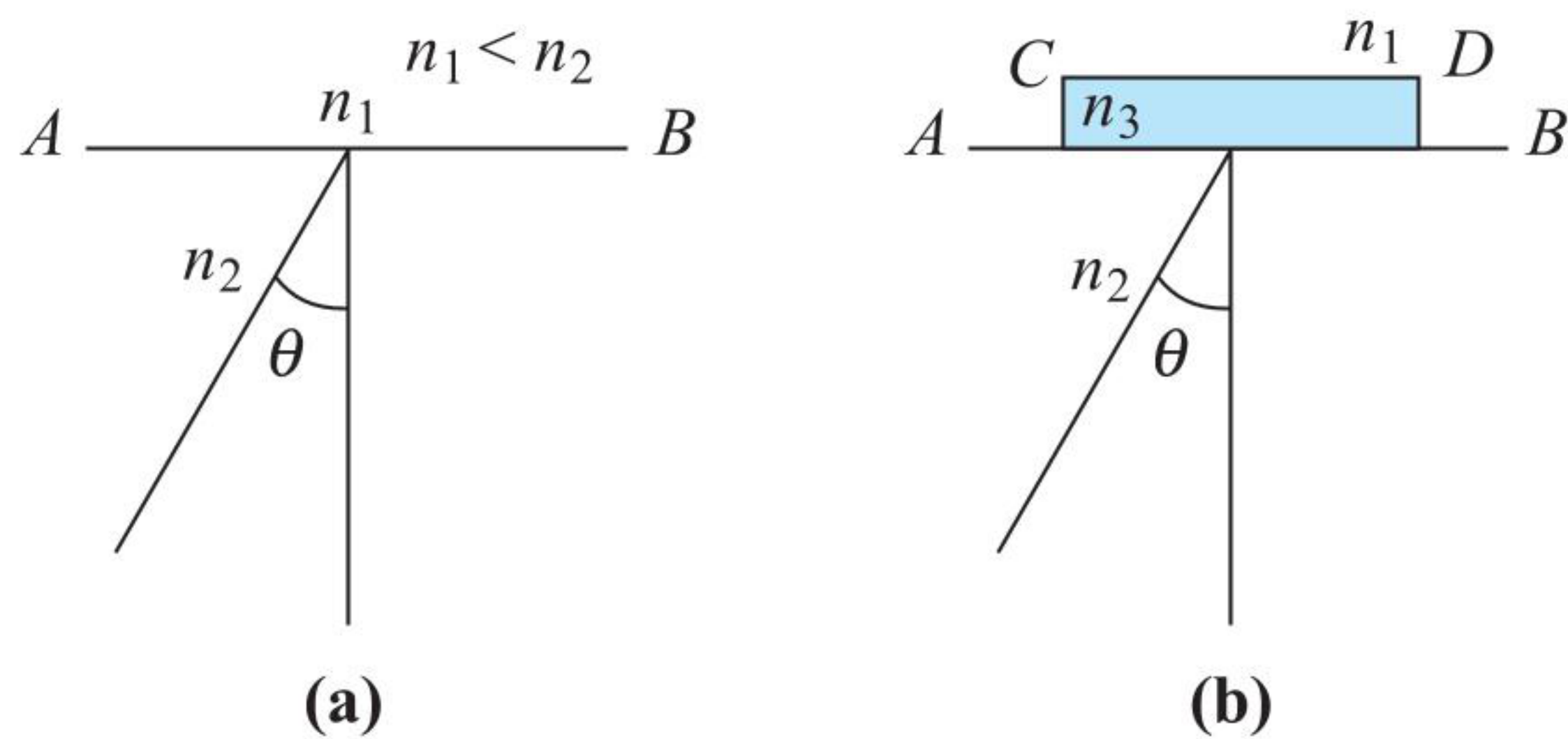
Multiple Correct Answers Type

1. Figure shows an object located at point P , 0.25 meter from concave spherical mirror with principal focus F . The focal length of the mirror is 0.10 m. How does the image change if the object is moved from point P toward point F ?

- (1) Its distance from the mirror decreases
- (2) The size of image decreases
- (3) Its distance from the mirror increases
- (4) The size of image increases

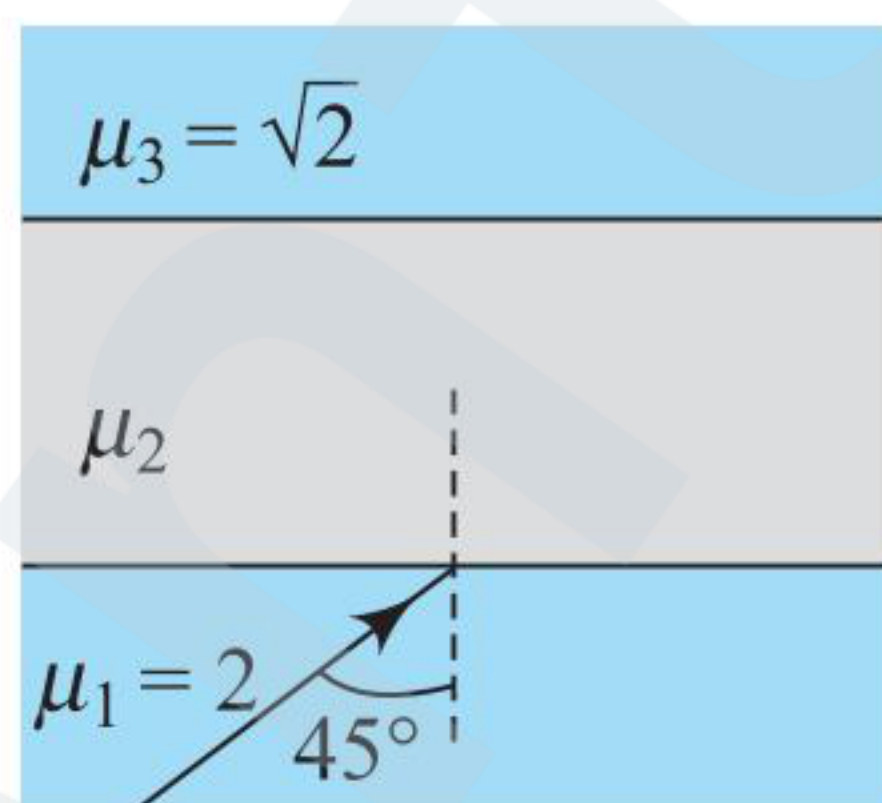


2. In figure, light is incident at an angle θ which is slightly greater than the critical angle. Now, keeping the incident angle fixed a parallel slab of refractive index n_3 is placed on surface AB . Which of the following statements are correct?



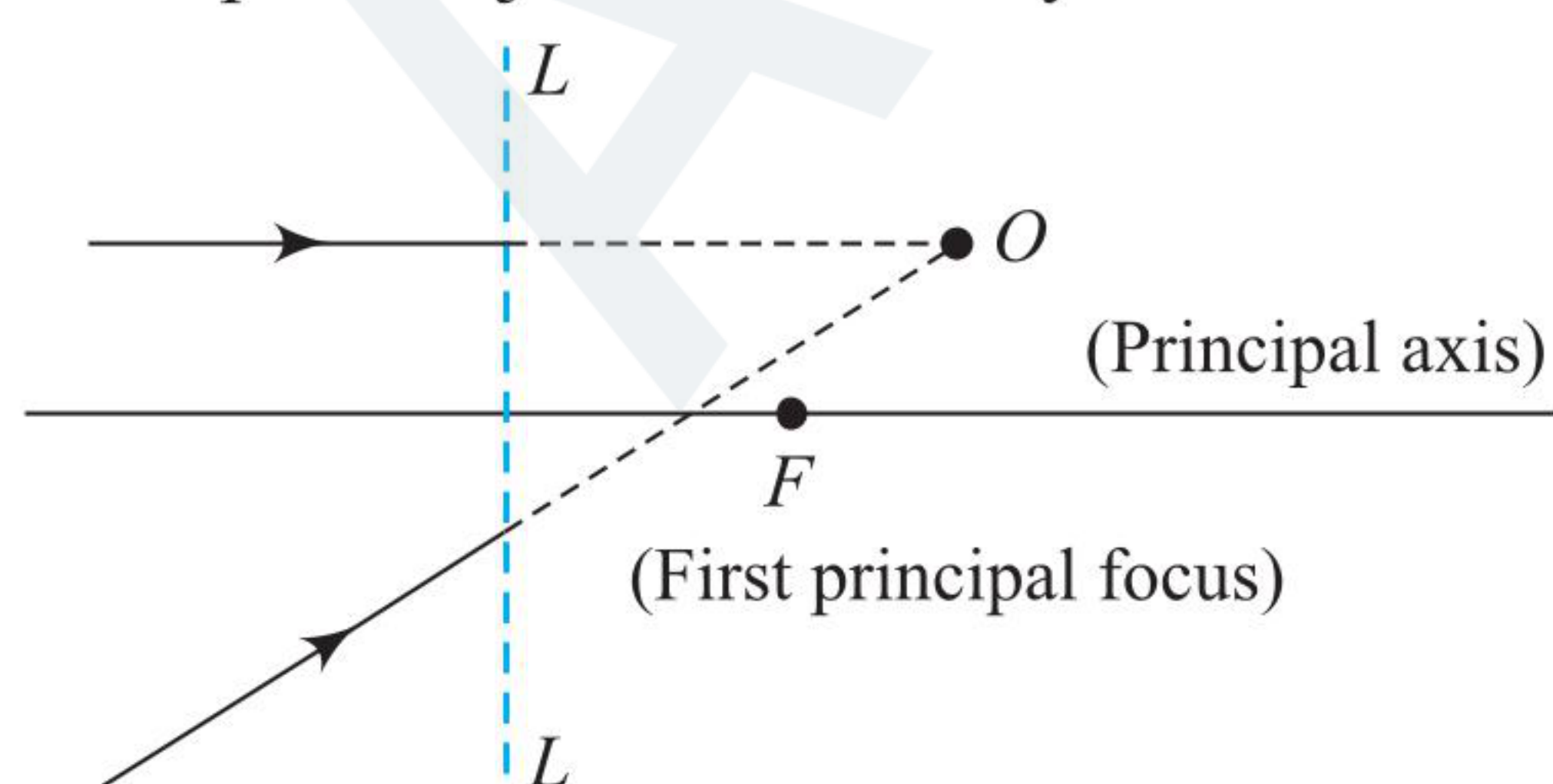
- (1) Total internal reflection occurs at AB for $n_3 < n_1$
- (2) Total internal reflection occurs at AB for $n_3 > n_1$
- (3) The ray will return back to the same medium for all values of n_3
- (4) Total internal reflection occurs at CD for $n_3 < n_1$

3. In figure, a light ray is incident on the lower medium boundary at an angle of 45° with the normal. Which of the following statements is/are true?



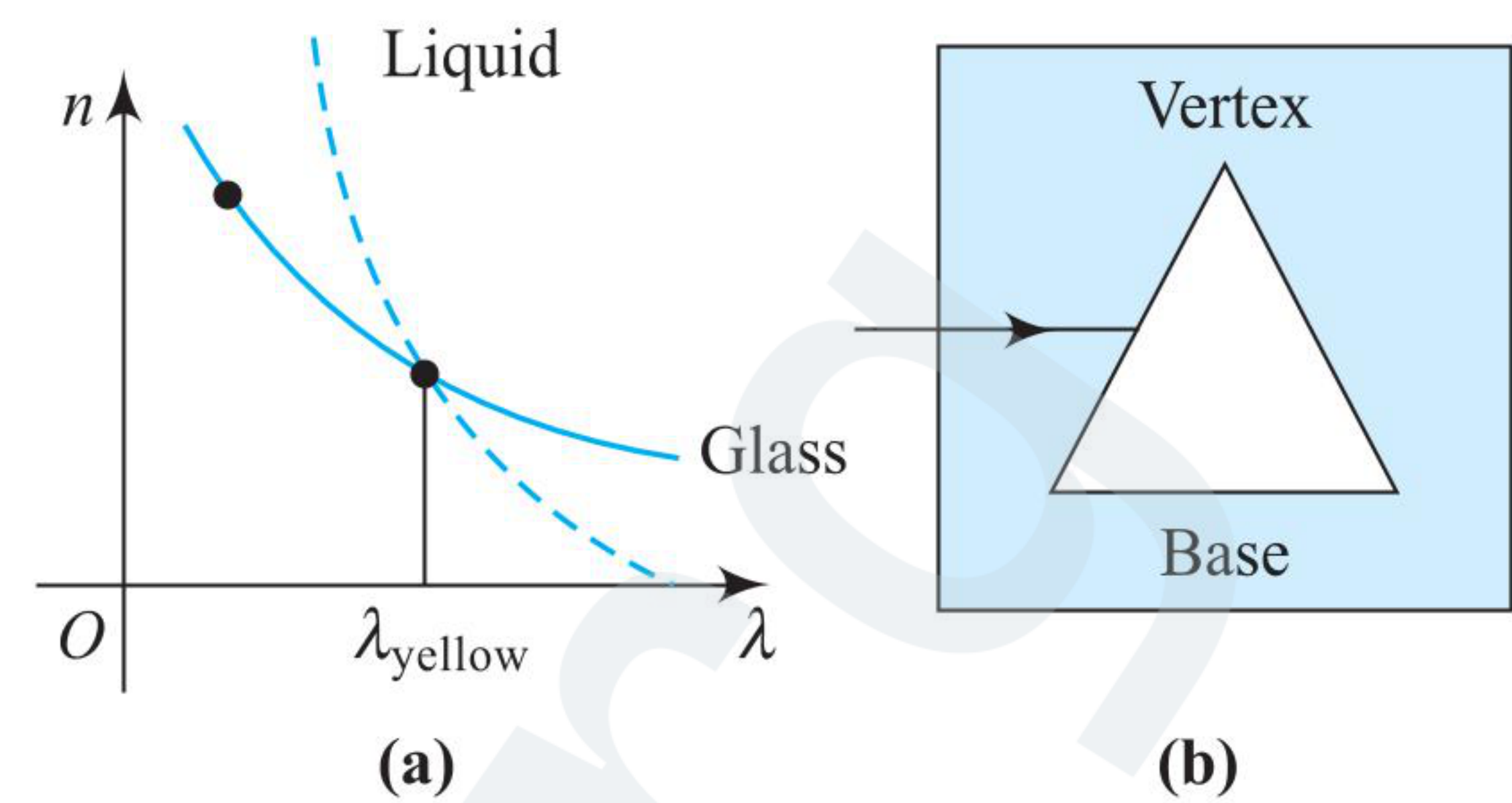
- (1) If $\mu_2 > \sqrt{2}$, then angle of deviation is 45° .
- (2) If $\mu_2 < \sqrt{2}$, then angle of deviation is 90° .
- (3) If $\mu_2 < \sqrt{2}$, then angle of deviation is 135° .
- (4) If $\mu_2 > \sqrt{2}$, then angle of deviation is 0° .

4. Consider the rays shown in figure as paraxial. The image of the virtual point object O formed by the lens LL is

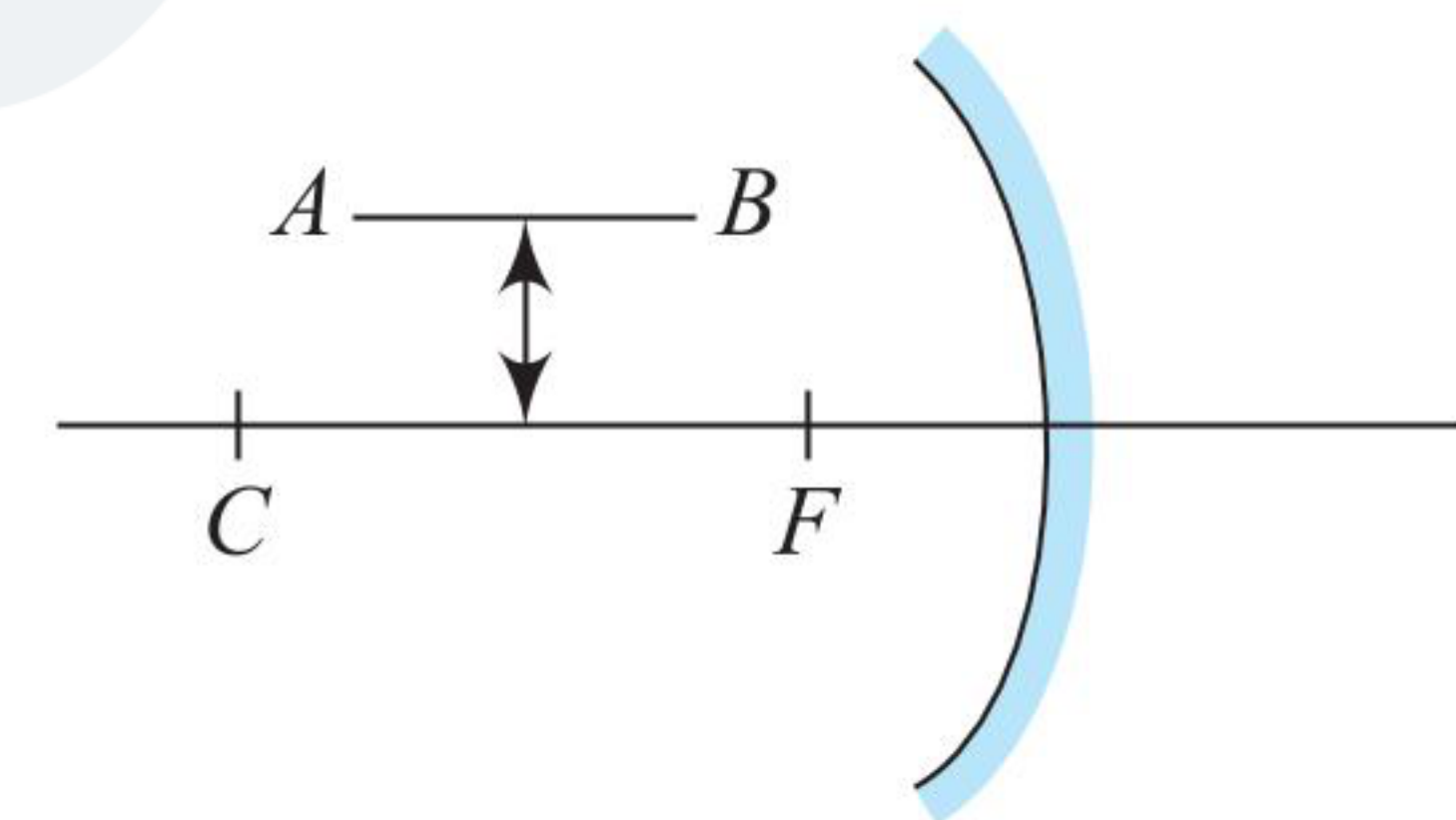


- (1) virtual
- (2) real
- (3) located below the principal axis
- (4) located to the left of the lens

5. A glass prism is immersed in a hypothetical liquid. The curves showing the refractive index n as a function of wavelength λ for glass and liquid are as shown in Figs. (a) and (b). When a ray of white light is incident on the prism parallel to the base

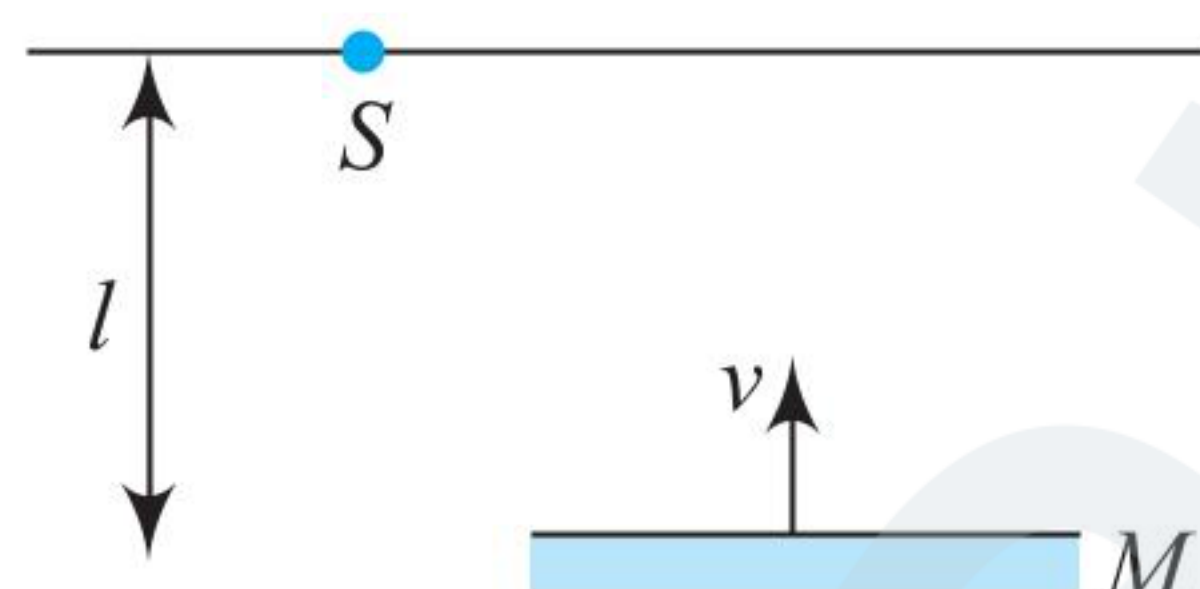


- (1) yellow ray travels without deviation
 - (2) blue ray is deviated toward the vertex
 - (3) red ray is deviated toward the base
 - (4) there is no dispersion
6. An object AB is placed parallel and close to the optical axis between focus F and centre of curvature C of a converging mirror of focal length f as shown in figure. Then,



- (1) Image of A will be closer than that of B from the mirror.
 - (2) Image of AB will be parallel to the optical axis.
 - (3) Image of AB will be a straight line inclined to the optical axis.
 - (4) Image of AB will not be a straight line.
7. Which of the following statements is/are correct about the refraction of light from a plane surface when light ray is incident in denser medium. [C is critical angle]
- (1) The maximum angle of deviation during refraction is $(\pi/2) - C$, it will be at angle of incidence C
 - (2) The maximum angle of deviation for all angles of incidence is $\pi - 2C$, when angle of incidence is slightly greater than C
 - (3) If angle of incidence is less than C , then deviation increases if angle of incidence is also increased
 - (4) If angle of incidence is greater than C , then angle of deviation decreases if angle of incidence is increased
8. Mark the correct statement(s) w.r.t. a concave spherical mirror.
- (1) For real extended object, it can form a diminished virtual image
 - (2) For real extended object, it can form a magnified virtual image
 - (3) For virtual extended object, it can form a diminished real image
 - (4) For virtual extended object, it can form a magnified real image

9. Mark the correct statement(s) from the following:
- Image formed by a convex mirror can be real
 - Image formed by a convex mirror can be virtual
 - Image formed by a convex mirror can be magnified
 - Image formed by a convex mirror can be inverted
10. A real object is moving toward a fixed spherical mirror. The image
- must move away from the mirror
 - may move away from the mirror
 - may move toward the mirror if the mirror is concave
 - must move toward the mirror if the mirror is convex
11. A real point source is 5 cm away from a plane mirror whose reflecting ability is 50%, while the eye of an observer (pupil diameter 5 mm) is 10 cm away from the mirror. Assume that both source and eye are on the same line perpendicular to the surface and refracted rays have no effect on intensity. Then,
- the area of the mirror used in observing the image of source is $(25\pi/36) \text{ mm}^2$
 - the area of the mirror used in observing the image of source is $25\pi \text{ mm}^2$
 - the ratio of the intensities of light as received by the observer in the presence to that in the absence of mirror is $(10/9)$
 - the ratio of the intensities of light as received in the presence to that in the absence of mirror is $19/18$
12. A plane mirror M is arranged parallel to a wall W at a distance l from it. The light produced by a point source S kept on the wall is reflected by the mirror and produces a patch of light on the wall. The mirror moves with velocity v towards the wall.

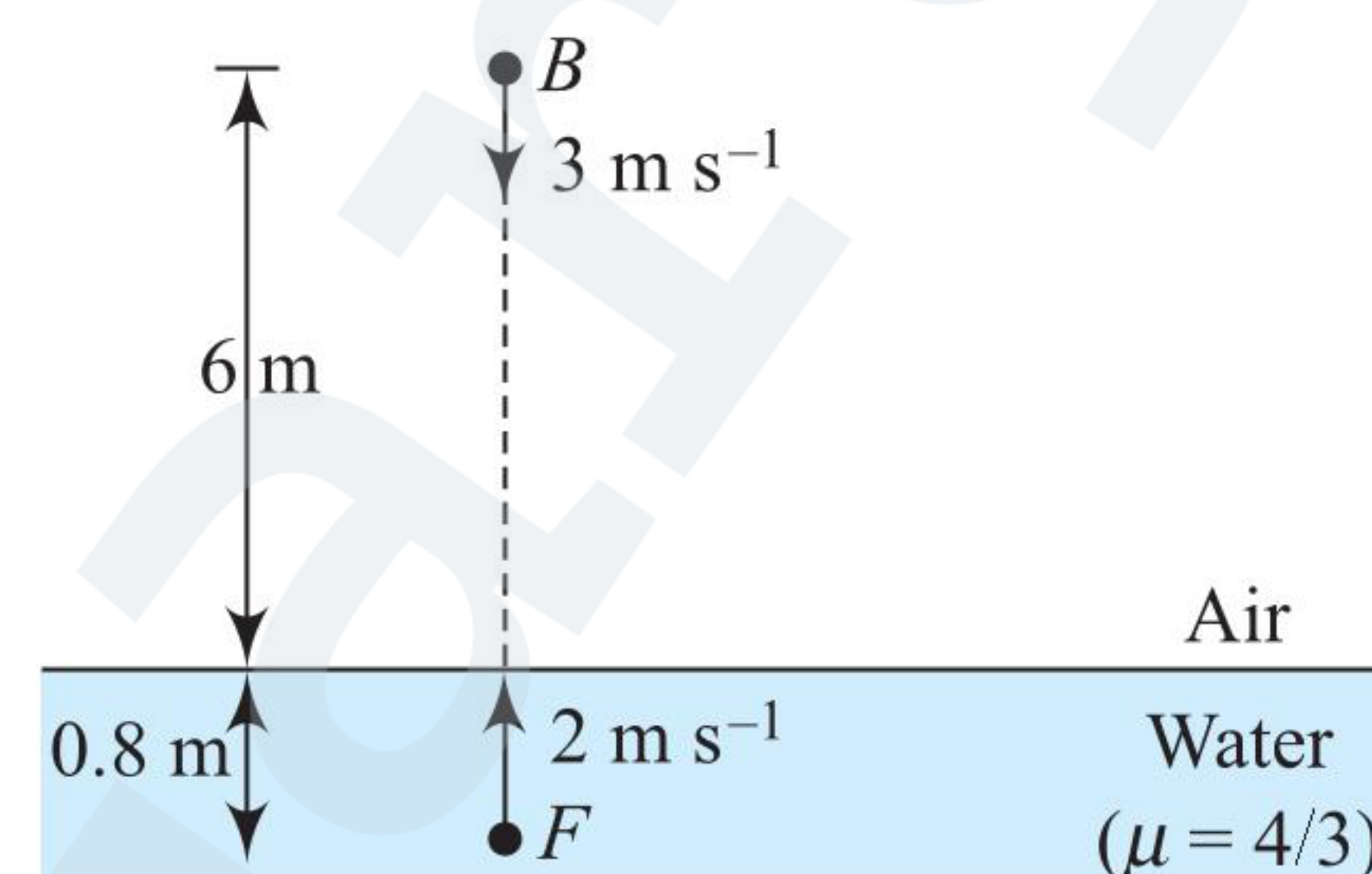


Which of the following statement(s) is/are correct?

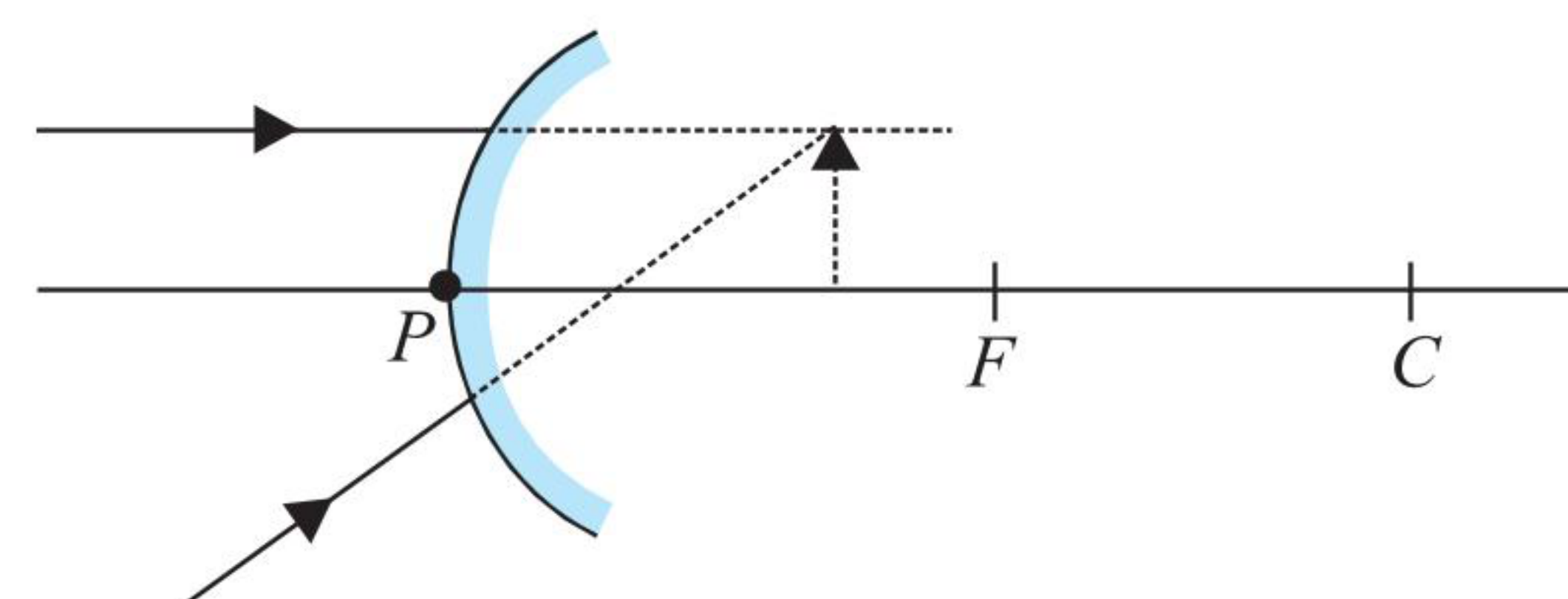
- The patch of light will move with speed v on the wall
 - The patch of light will not move on the wall
 - As the mirror comes closer, the patch of light will become larger and shift away from the wall with speed larger than v
 - The size of the patch of light on the wall remains the same
13. Which of the following statements are correct?
- A ray of light is incident on a plane mirror and gets reflected. If the mirror is rotated through an angle θ , then the reflected ray gets deviated through angle 2θ .
 - A ray of light gets reflected successively from two mirrors which are mutually inclined. Angular deviation suffered by the ray does not depend upon angle of incidence on first mirror.

- A plane mirror cannot form real image of a real object.
- If an object approaches toward a plane mirror with velocity v , then the image approaches the object with velocity $2v$.

14. A fish F , in the pond is at a depth of 0.8 m from the water surface and is moving vertically upward with velocity 2 ms^{-1} . At the same instant, a bird B is at a height of 6 m from the water surface and is moving downward with velocity 3 ms^{-1} . At this instant, both are on the same vertical line as shown in figure. Which of the following statements are correct?



- Height of B , observed by F (from itself), is equal to 5.30 m
 - Depth of F , observed by B (from itself), is equal to 6.60 m
 - Height of B , observed by F (from itself), is equal to 8.80 m
 - None of the above
15. In the previous question,
- velocity of B , observed by F (relative to itself), is equal to 4.25 m s^{-1}
 - velocity of B , observed by F (relative to itself), is equal to 6 m s^{-1}
 - velocity of B , observed by F (relative to itself), is equal to 5.50 m s^{-1}
 - velocity of F , observed by B (relative to itself), is equal to 4.50 m s^{-1}
16. Converging rays strike a spherical convex mirror such that they can form the image (in the absence of mirror) between pole and focus. Now what can you say about final image formed by mirror?



- real
 - virtual
 - erect
 - inverted
17. In the above question, if the rays were to converge between F and C of mirror, then find the nature of final image formed
- real
 - virtual
 - erect
 - inverted

18. In the above question, if the rays were to converge beyond C , then find the nature of final image formed.

- (1) real (2) virtual
(3) erect (4) inverted

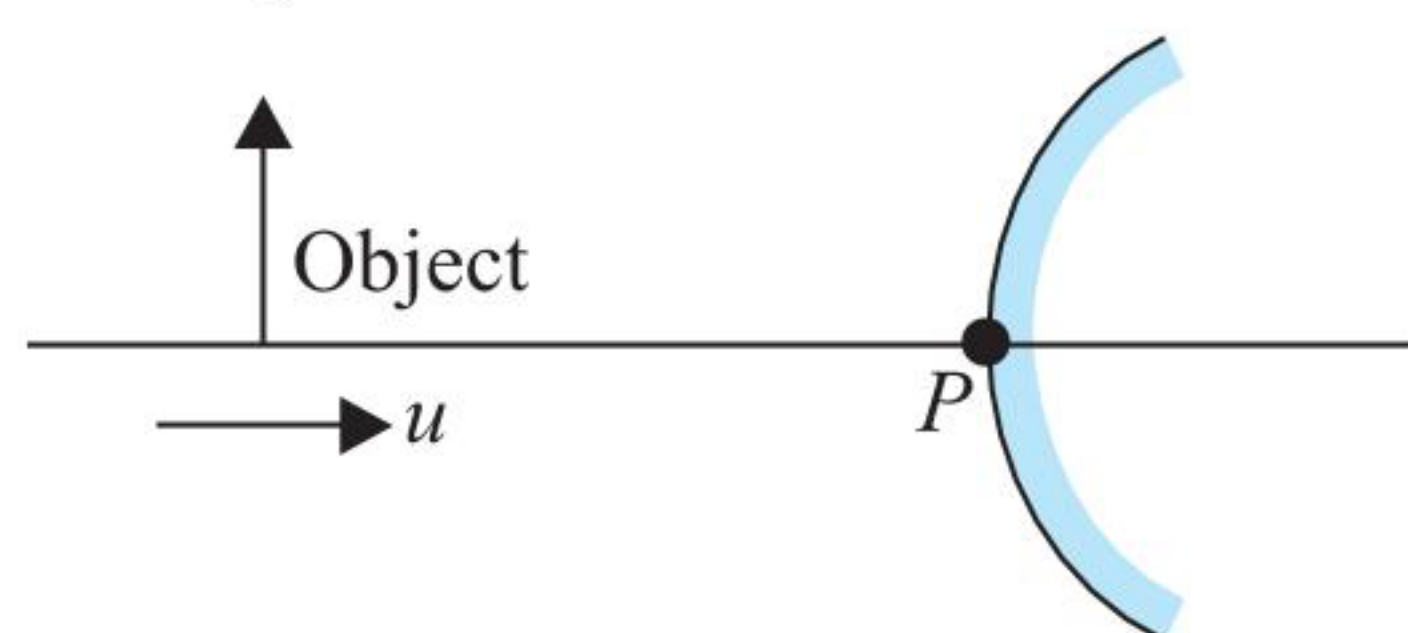
19. A concave mirror forms an image of the sun at a distance of 12 cm from it.

- (1) The radius of curvature of this mirror is 6 cm.
(2) To use it as a shaving mirror, it can be held at a distance of 8–10 cm from the face.
(3) If an object is kept at a distance of 24 cm from it, the image formed will be of the same size as the object.
(4) All the above alternatives are correct.

20. A real object is placed in front of a convex mirror (focal length f). It moves towards the mirror, the image also moves. If V_i = speed of image and V_o = speed of the object and u is the distance of object from mirror along principal axis, then

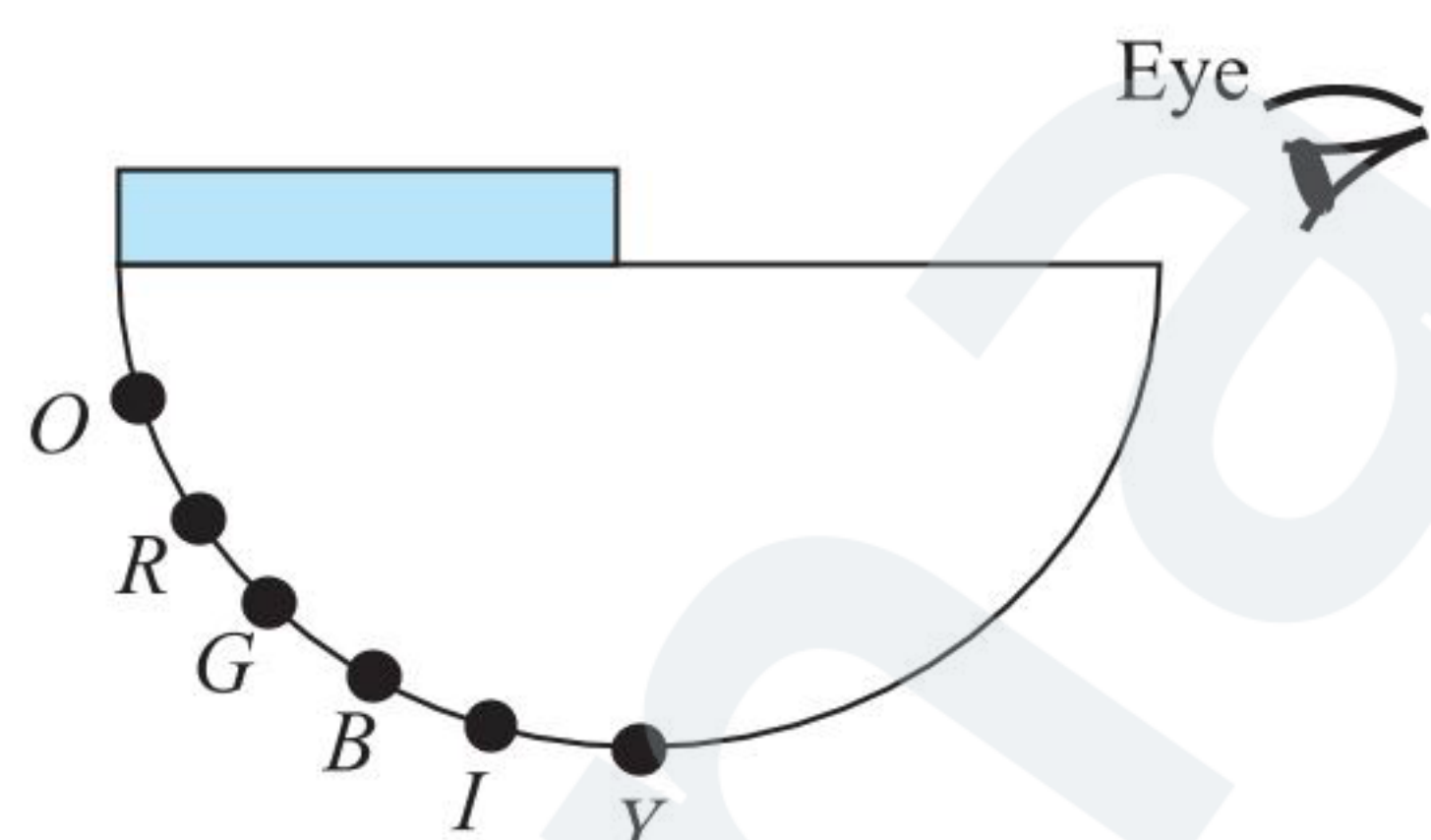
- (1) $V_i < V_o$ if $|u| < |f|$ (2) $V_i > V_o$ if $|u| > |f|$
(3) $V_i < V_o$ if $|u| > |f|$ (4) $V_i = V_o$ if $|u| = |f|$

21. An object is moving towards a convex mirror with a constant velocity. P is the pole of mirror.



- (1) Magnification of the image increases with time.
(2) Magnification of the image decreases with time.
(3) Velocity of the image increases with time.
(4) Velocity of the image decreases with time.

22. A person is looking at the flat surface of a transparent hemisphere of refractive index $\mu = \sqrt{2}$. Half of the flat surface is coloured black and half of curved surface is coloured in six equally spaced strips as shown in figure. Then



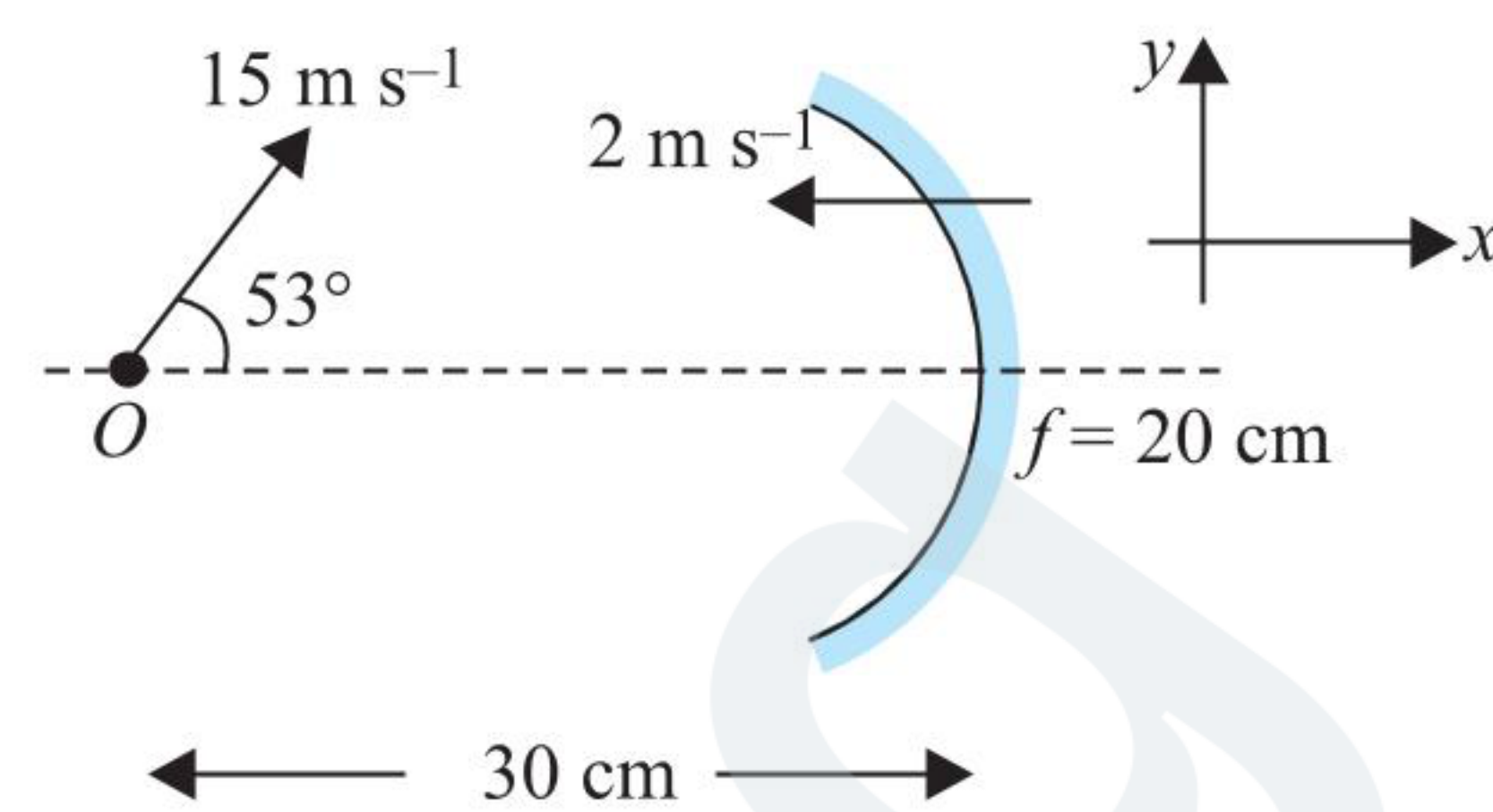
- (1) the person can see green, orange and red
(2) the person can see only yellow, indigo and blue
(3) the ray coming from red and orange strips will be totally reflected
(1) the ray coming from yellow, indigo and blue strips will be totally reflected

23. Refractive index of an equilateral prism is $\sqrt{2}$. Then

- (1) minimum deviation from this prism can be 30°
(2) minimum deviation from this prism can be 45°
(3) at angle of incidence 45° , deviation is minimum

(4) at angle of incidence 60° , deviation is minimum

24. In the situation as shown in figure $\left(\cos 53^\circ = \frac{3}{5}\right)$

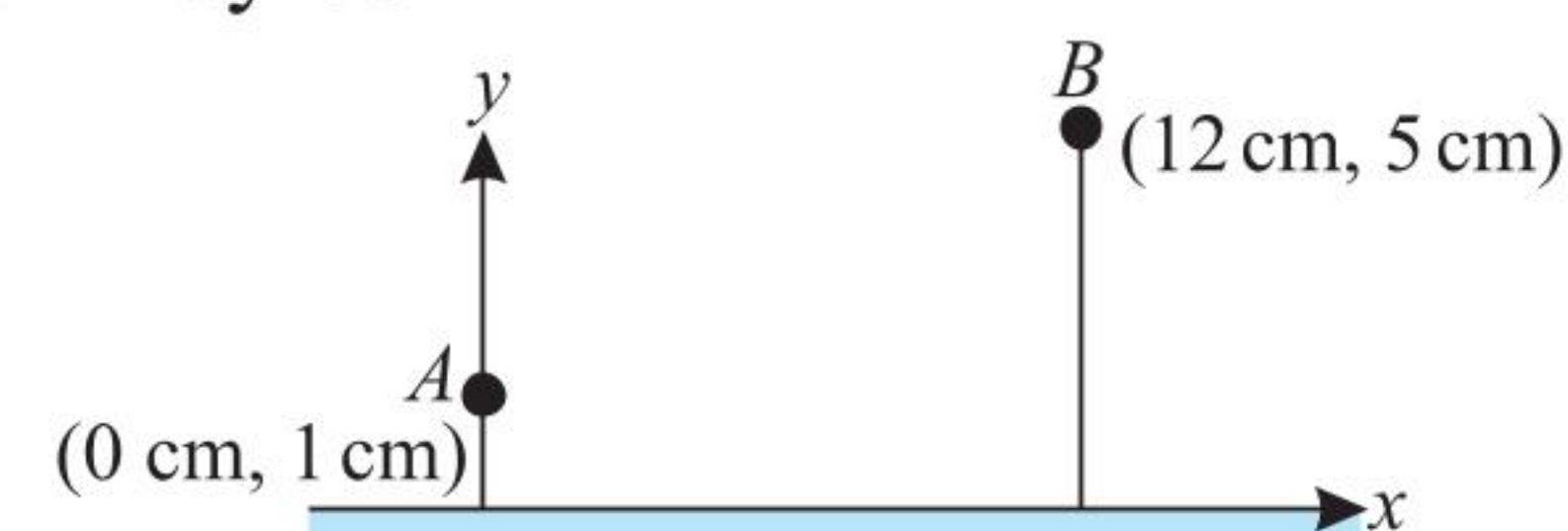


- (1) velocity of image w.r.t. mirror is $-22\hat{i} - 24\hat{j}$
(2) velocity of image w.r.t. mirror is $-44\hat{i} - 24\hat{j}$
(3) velocity of image w.r.t. ground is $-46\hat{i} - 24\hat{j}$
(4) velocity of image w.r.t. ground is $-24\hat{i} - 24\hat{j}$

25. For a concave mirror of focal length f , image is 2 times larger. Then the object distance from the mirror can be

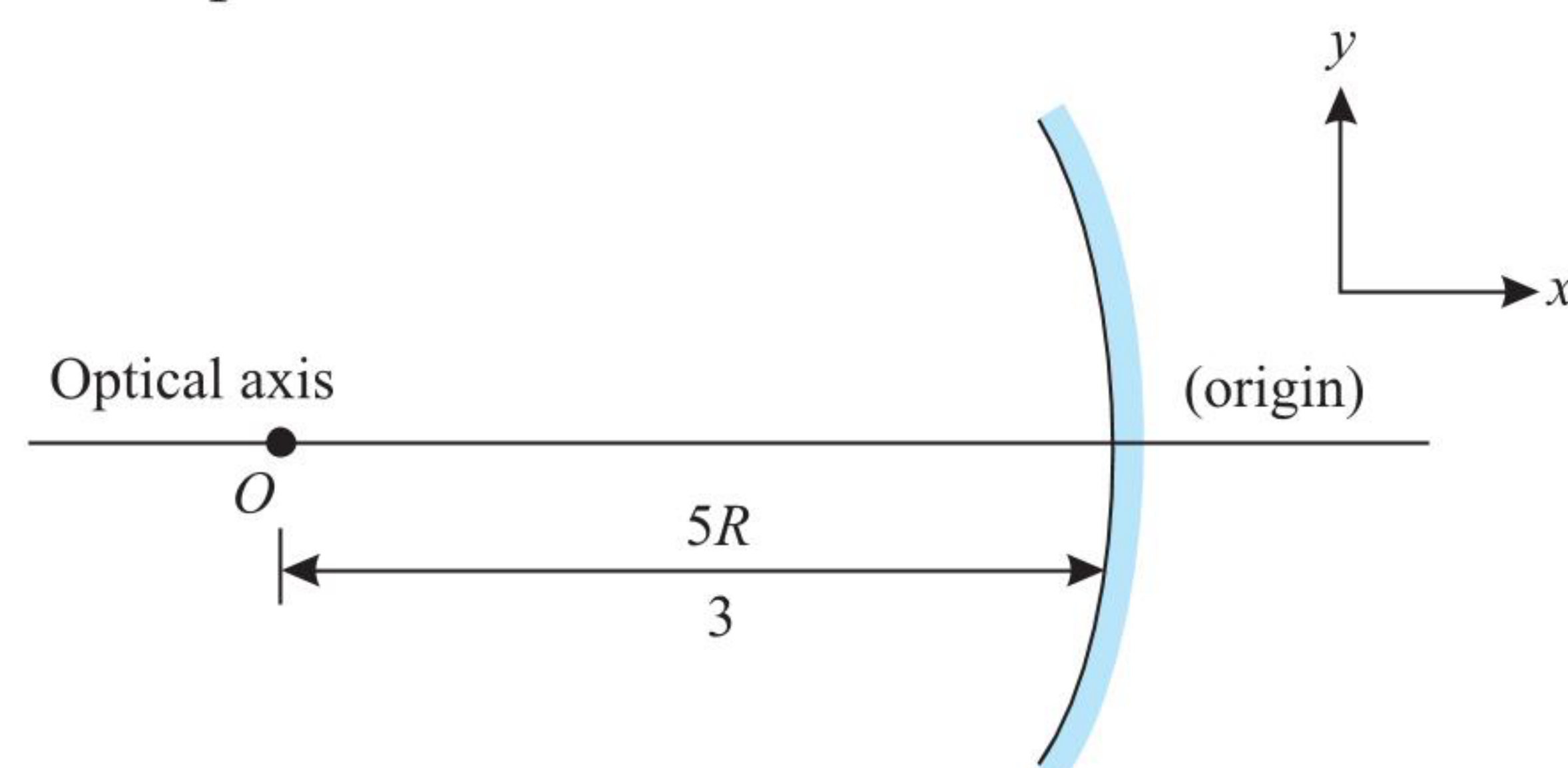
- (1) $\frac{f}{2}$ (2) $\frac{3f}{2}$
(3) $\frac{f}{4}$ (4) $\frac{4f}{3}$

26. Points A (0 cm, 1 cm) and B (12 cm, 5 cm) are object-image pair (one of the point acts as object and the other point as image), x -axis is the principal axis of the mirror. This object image pair may be



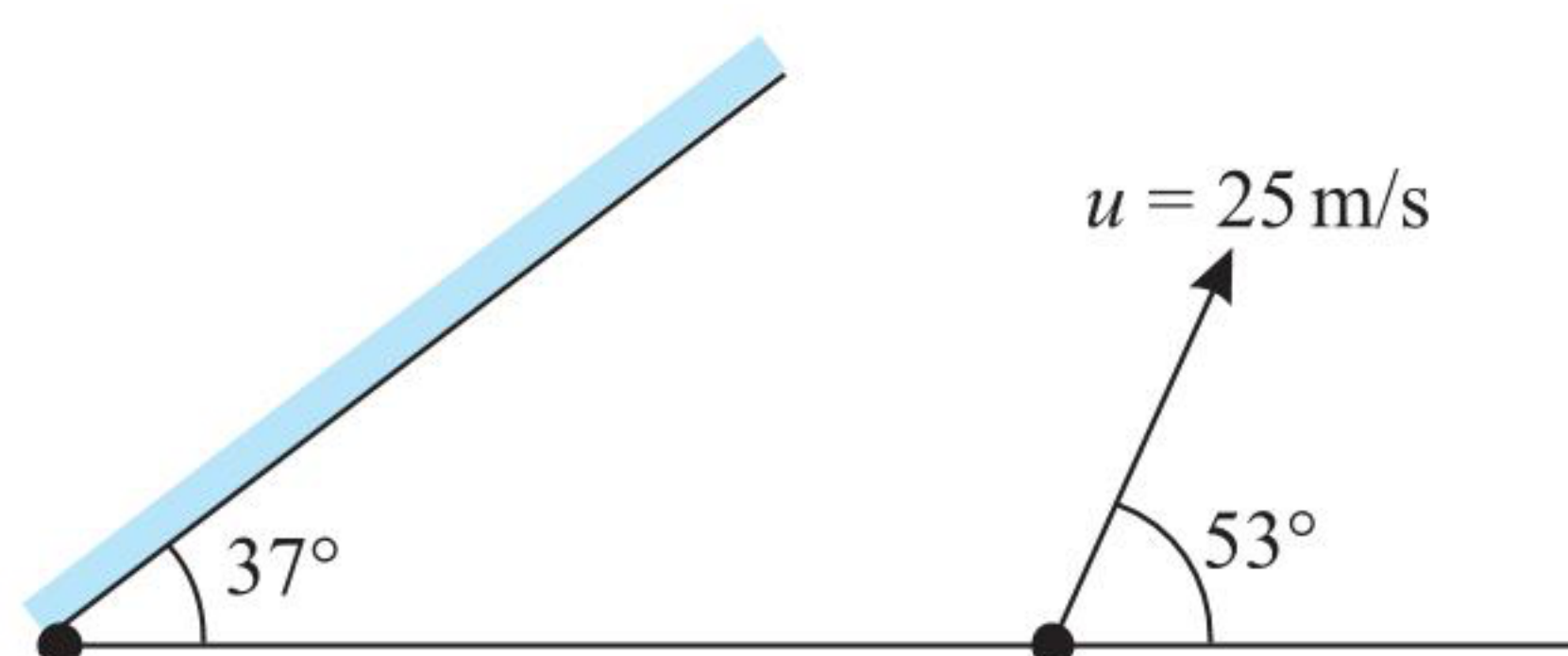
- (1) due to a convex mirror of focal length 2.5 cm
(2) due to a concave mirror having its pole at (2, 0)
(3) real virtual pair
(4) due to a concave mirror of focal length 5 cm

27. A point object is placed at a distance of $5R/3$ from the pole of concave mirror of small aperture and radius of curvature R . Point object oscillates with amplitude 1 mm perpendicular to the optical axis. Then

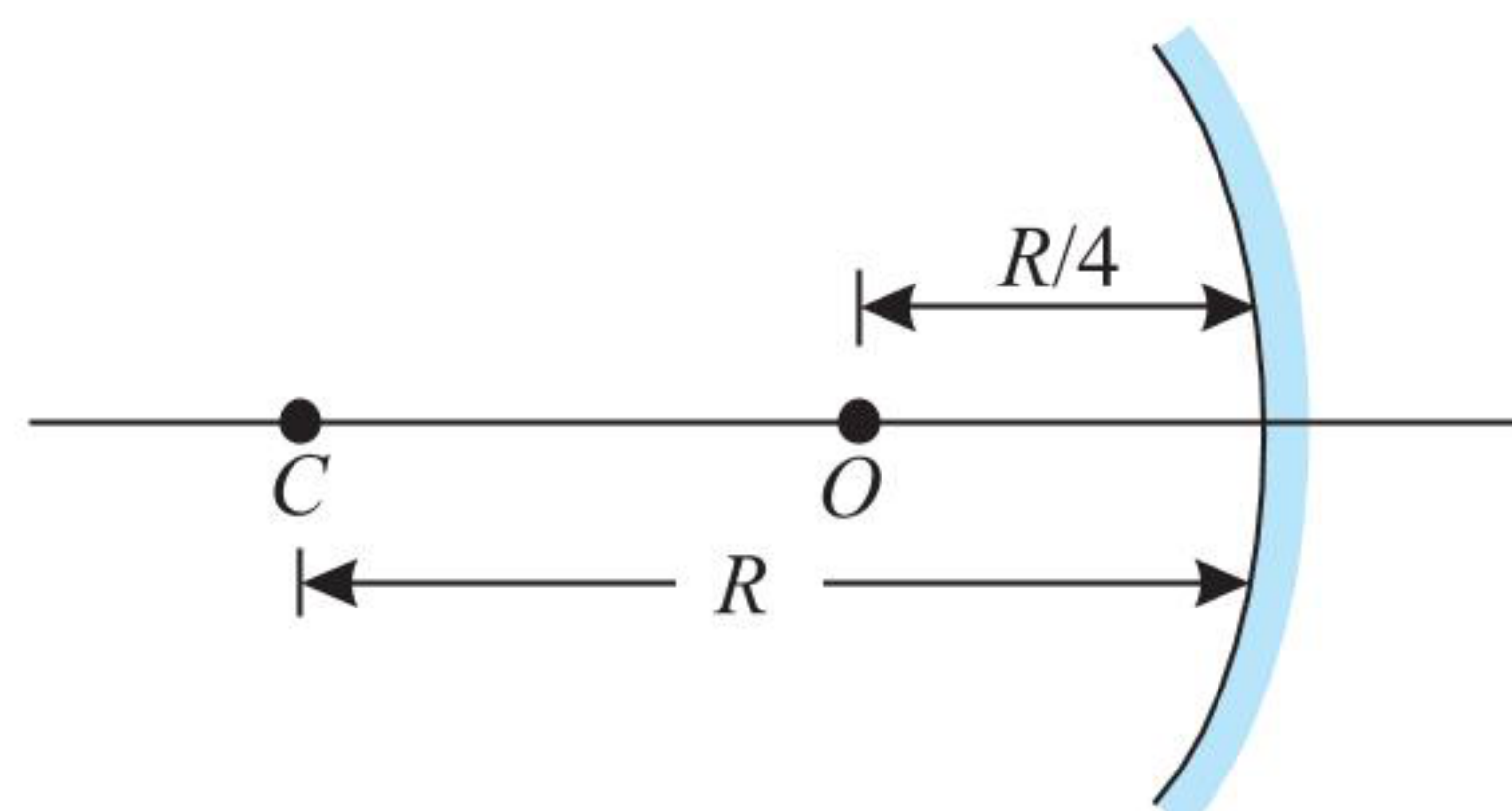


- (1) position of image of object from pole is $\left(-\frac{5R}{7}, 0\right)$
(2) image of object is real
(3) amplitude of image is $\frac{3}{7}$ mm
(4) phase difference between motion of object and its image when object crosses optical axis is π

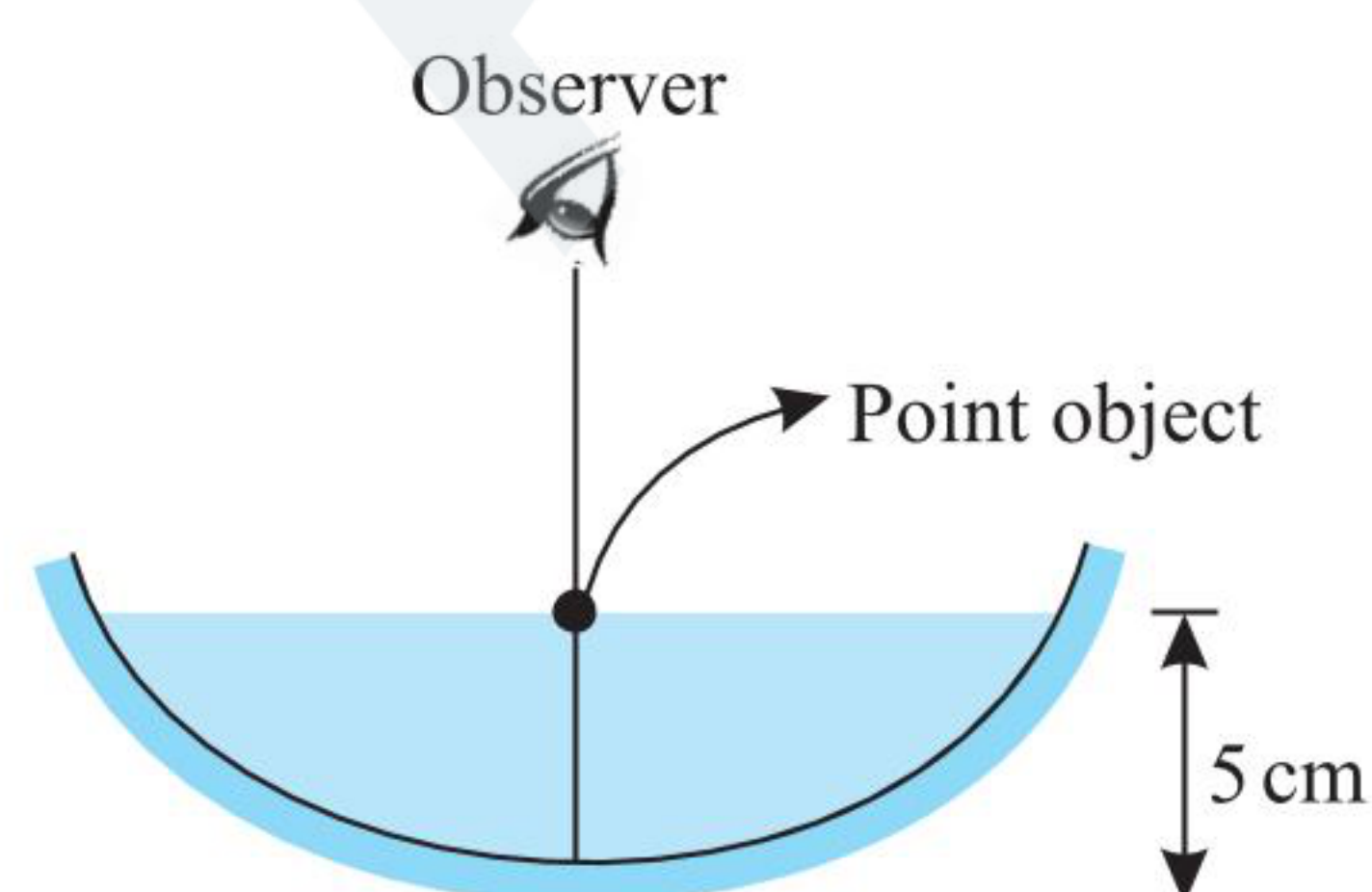
28. A particle is projected with speed 25 m/s at angle 53° from horizontal in front of an inclined plane mirror as shown in figure. Choose the correct options: ($g = 10 \text{ m/s}^2$)



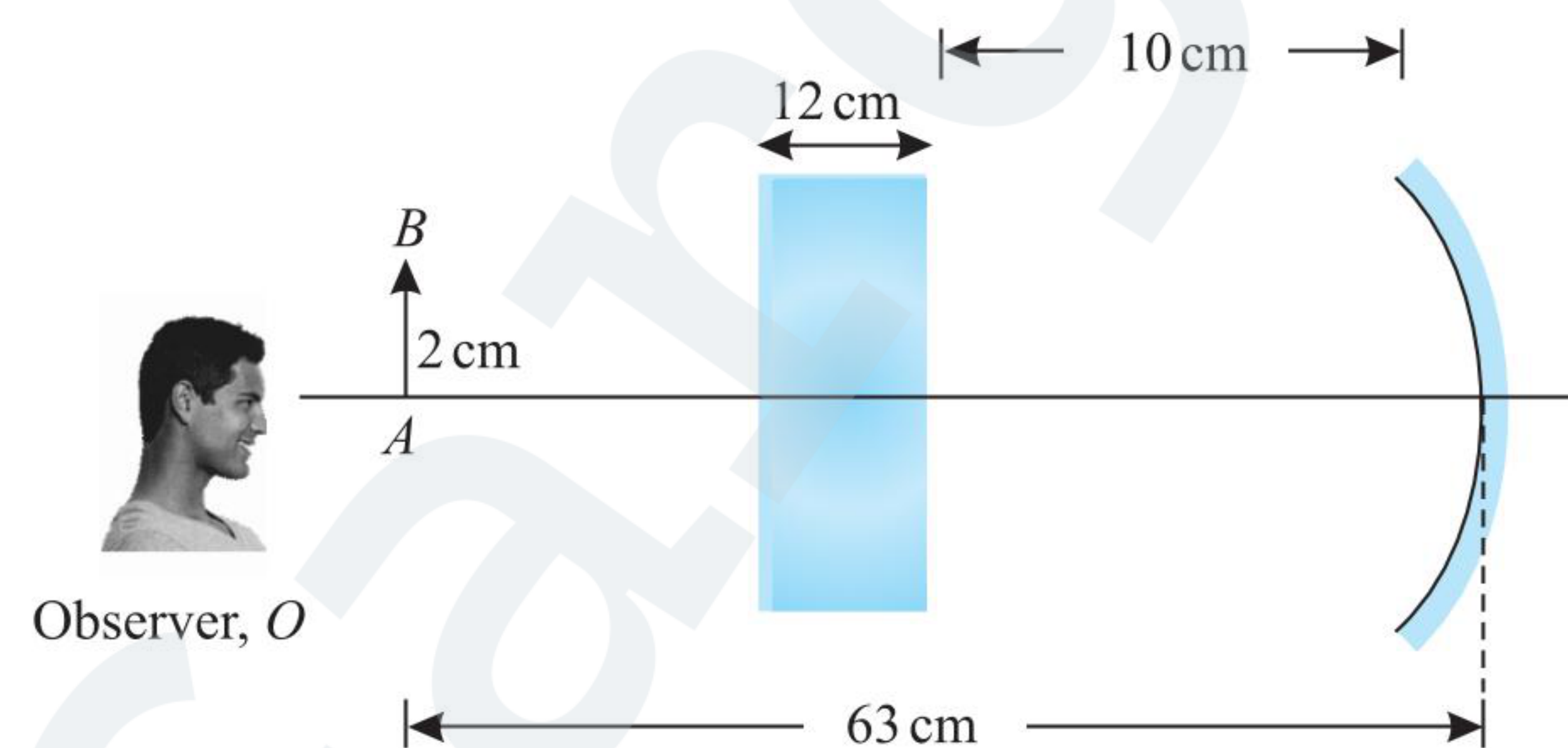
- (1) Speed of image w.r.t. object will be maximum after time 4 sec of projection
 (2) Minimum speed of image w.r.t. object is zero
 (3) Speed of image w.r.t. object will be minimum after time $7/8$ sec of projection
 (4) Maximum speed of image w.r.t. object is 50 m/s
29. The figure below shows an object O placed at a distance $R/4$ to the left of a concave spherical mirror that has radius of curvature R . Point C is the centre of curvature of the mirror. Size of the object is much smaller than the radius of curvature of the mirror. The image formed by the mirror is



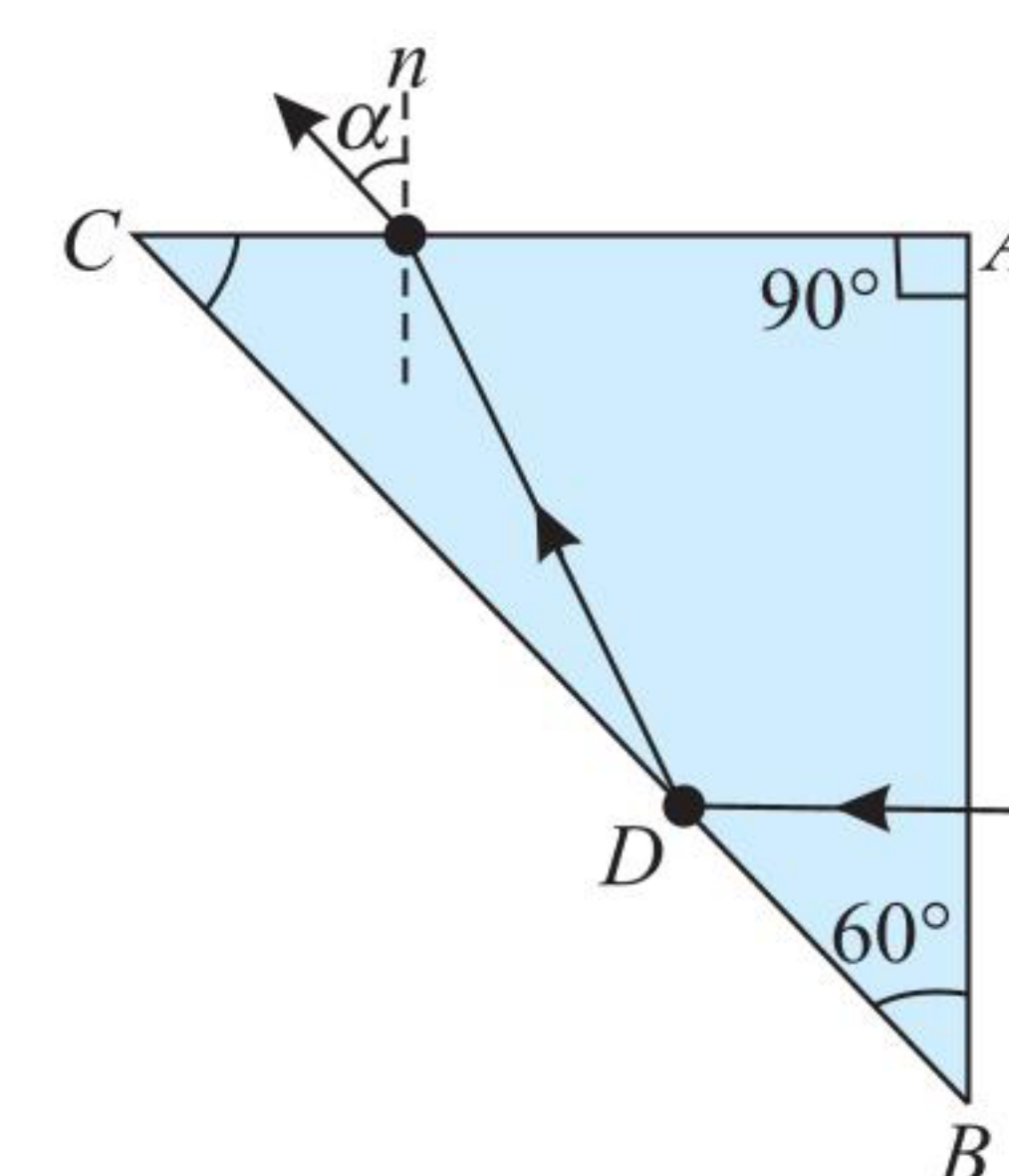
- (1) larger in size and inverted
 (2) larger in size and upright
 (3) at a distance $R/2$ to the right of the mirror
 (4) at a distance $R/2$ to the left of the mirror
30. A ray of light moving along vector $(3\sqrt{2}\hat{i} - 3\hat{j} - 3\hat{k})$ under goes refraction at an interface of two media, which is $y-z$ plane. The refractive index for $x \leq 0$ is one while for $x \geq 0$ it is $\sqrt{2}$. Then,
- (1) refracted ray bend towards y -axis
 (2) refracted ray bend towards x -axis
 (3) the unit vector along the refracted ray is $\frac{\sqrt{6}\hat{i} - \hat{j} - \hat{k}}{2}$
 (d) the unit vector along the refracted ray is $\frac{\sqrt{6}\hat{i} - \hat{j} - \hat{k}}{\sqrt{8}}$
31. A concave mirror of radius 40 cm is filled with water (refractive index, $\mu = 4/3$) up to height 5 cm. A point object P is floating on the surface of water as shown in the figure.



- (2) The image of point object P observed by observer is real
 (3) The image of point object P is 8.75 cm below the surface of water
 (4) The image of point object is 10 cm above the surface of water
32. A pencil AB of height 2 cm is placed at distance 63 cm from a concave mirror of radius of curvature 40 cm. A slab of refractive index $4/3$ and thickness 12 cm is placed between the pencil and the mirror.



- (1) Final image is real, inverted and 1 cm high
 (2) Final image is virtual, inverted and 1 cm high
 (3) Final image is 33 cm from the pole of mirror
 (4) Final image is 27 cm from the pole of mirror
33. A ray of light is incident normally on face AB of a prism of refractive index $5/3$ immersed in water of refractive index $4/3$ as shown in the figure.

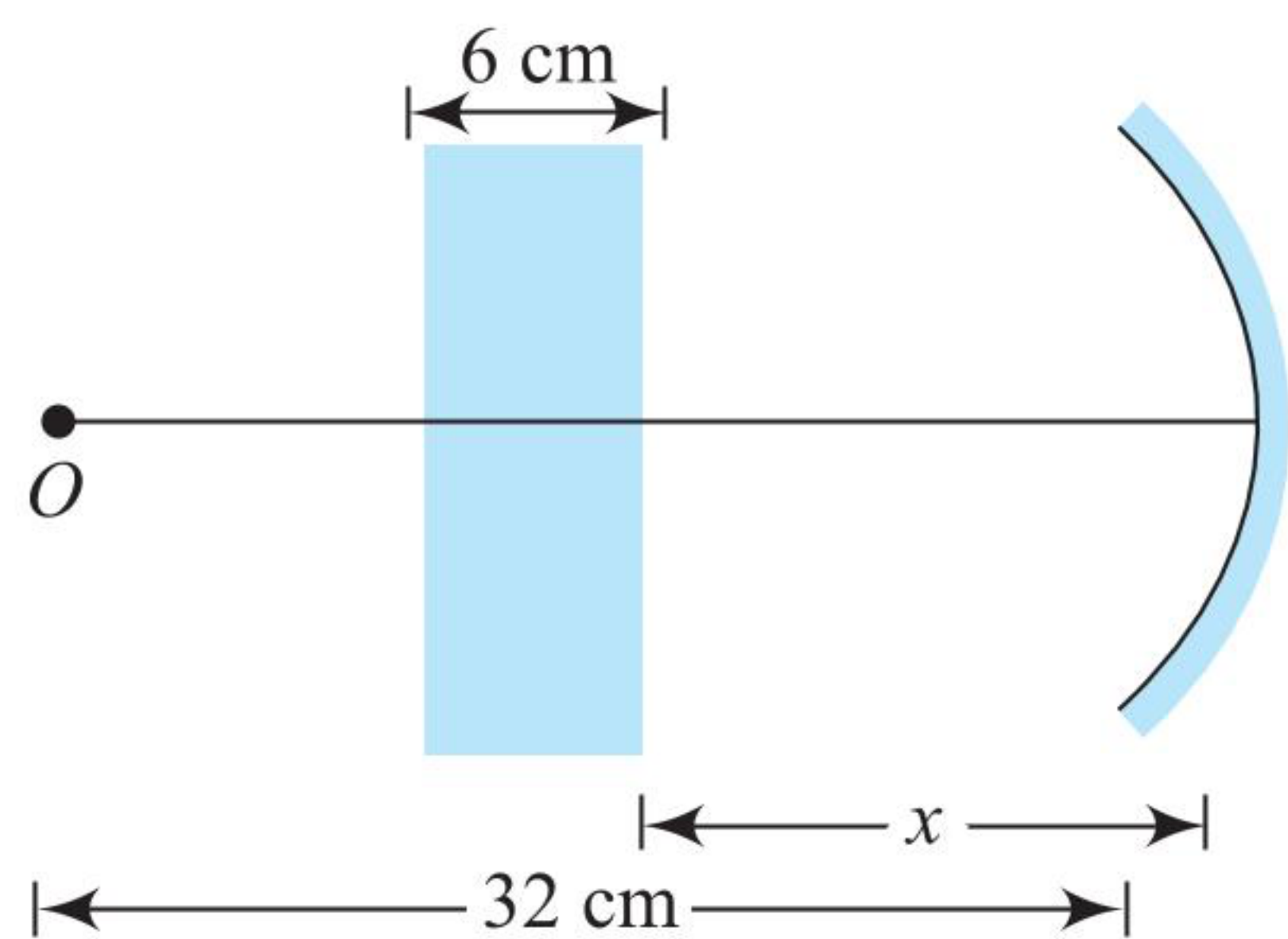


- (1) The exit angle α , of the ray is $\sin^{-1}\left(\frac{5}{8}\right)$
 (2) The exit angle α , of the ray is $\sin^{-1}\left(\frac{5}{4\sqrt{3}}\right)$
 (3) Total internal reflection does not take place at point D , if water is replaced by a medium of refractive index $\frac{5}{2\sqrt{3}}$
 (4) Total internal reflection does not take place at point D , if water is replaced by a medium whose refractive index $\frac{6}{5}$
34. A ray of light travelling in a transparent medium falls on a surface separating the medium from air at an angle of incidence of 45° . The ray undergoes total internal reflection. If n is the refractive index of the medium with respect to air, select the possible value(s) of n from the following:
- (1) 1.3 (2) 1.4
 (3) 1.5 (4) 1.6

Linked Comprehension Type

For Problems 1–2

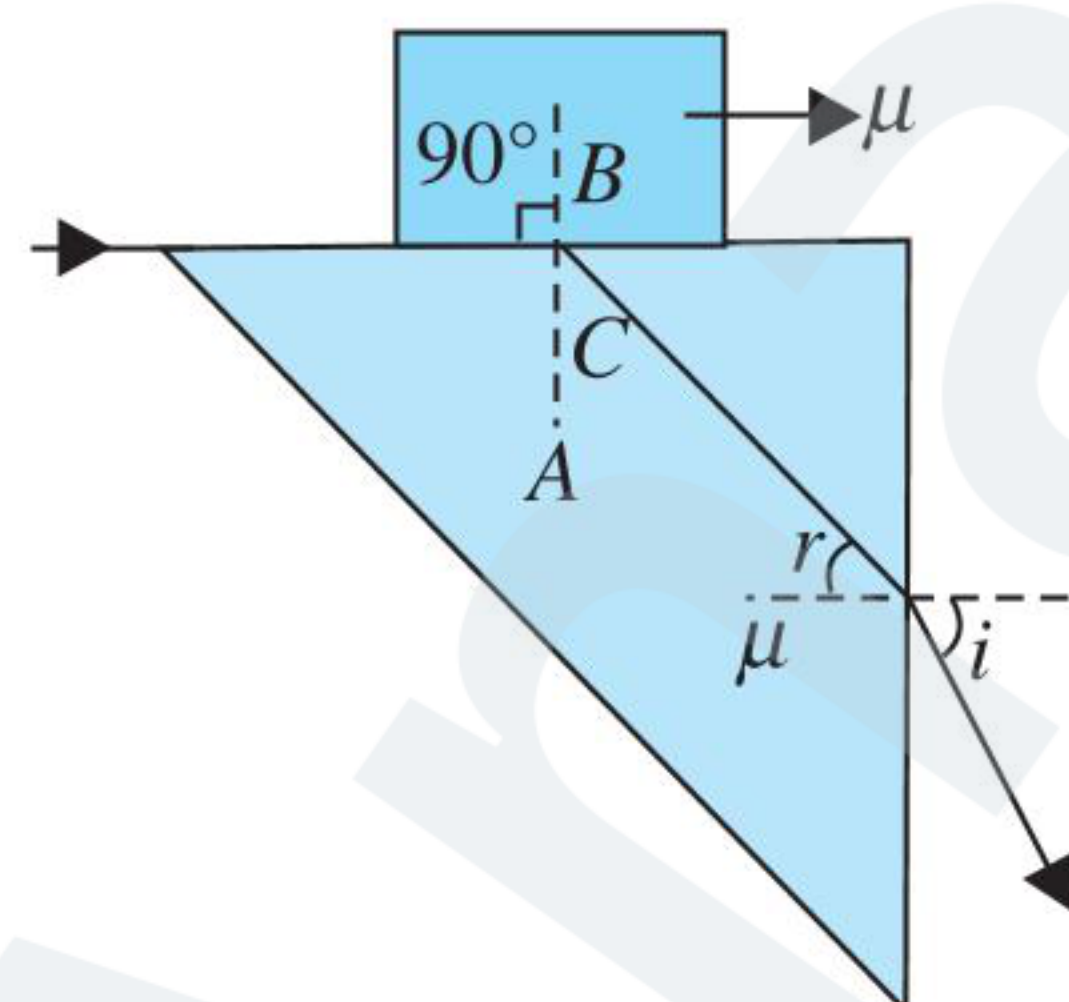
A point object O is placed in front of a concave mirror of focal length 10 cm. A glass slab of refractive index $\mu = 3/2$ and thickness 6 cm is inserted between the object and mirror.



- Find the position and nature of the final image when the distance x shown in figure is 5 cm.
 - 11 cm, virtual
 - 17 cm, real
 - 14 cm, real
 - 20 cm, virtual
- Find the position and nature of the final image when the distance x shown in figure is 20 cm.
 - 17 cm, virtual
 - 17 cm, real
 - 12 cm, virtual
 - 15 cm, virtual

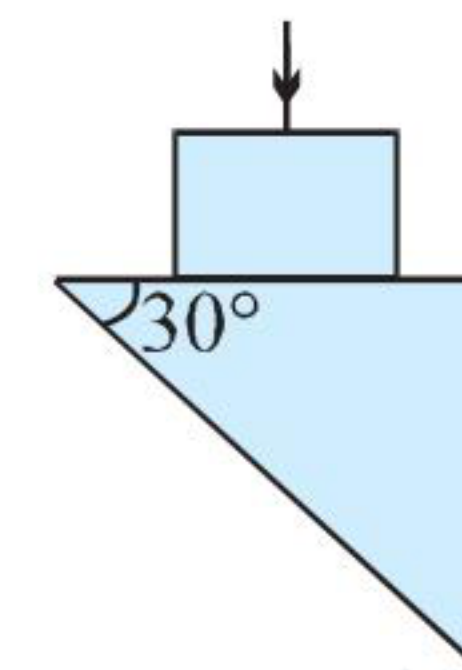
For Problems 3–5

Pulfrich refractometer is used to measure the refractive index of solids and liquid. It consists of a right-angled prism A having its two faces perfectly plane. One of the faces is horizontal and the other is vertical as shown in figure. The solid B whose refractive index is to be determined is taken having two faces cut perpendicular to one another. Light is incident in a direction parallel to the horizontal surface so that the light entering the prism A is at critical angle C . Finally, it emerges from the prism at an angle i . Let the refractive index of the solid be μ and that of the prism A be μ_0 (which is known). Here $\mu_0 > \mu$ and by measuring i , μ can be determined.



- Refractive index of the solid (μ) in terms of μ_0 and i is
 - $\sqrt{\mu_0^2 + \sin^2 i}$
 - $\mu_0 + \sin^2 i$
 - $\sqrt{\mu_0^2 - 2 \sin^2 i}$
 - $\sqrt{\mu_0^2 - \sin^2 i}$
- If $\mu_0 = \sqrt{2}$ and the ray just fails to emerge from the prism, refractive index μ of the solid will be
 - 1.21
 - $\sqrt{2}$
 - 1
 - $\sqrt{3}/2$
- A ray of light incident normally on the horizontal face of the slab and just fails to emerge from the diagonal face of the

prism. If prism angle 30° refractive index of the prism can be

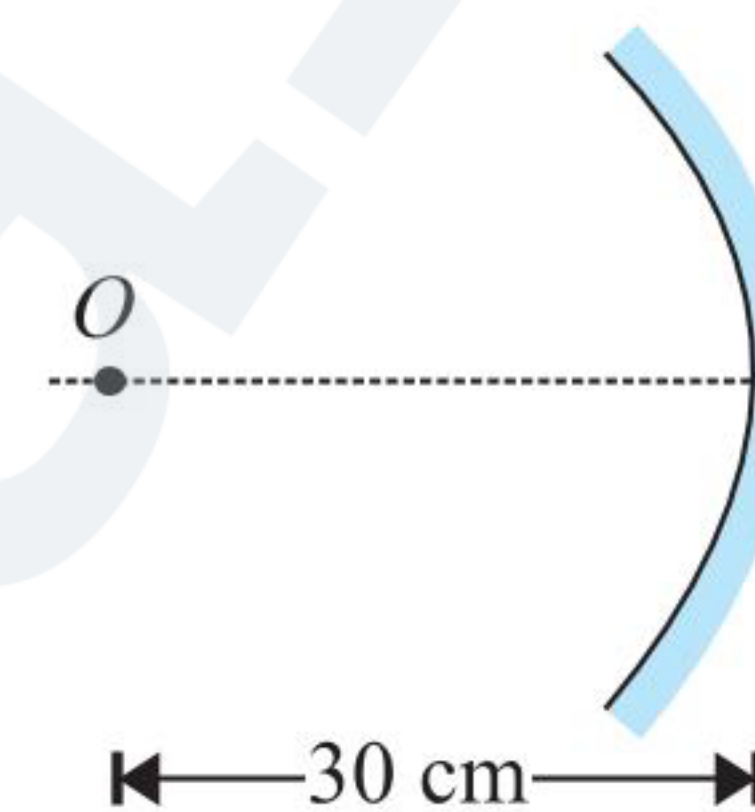


- $\sqrt{2}$
- $\sqrt{3}$
- slightly greater than 2
- slightly less than 2

For Problems 6–8

An object is present on the principal axis of a concave mirror at a distance 30 cm from it. Focal length of mirror is 20 cm.

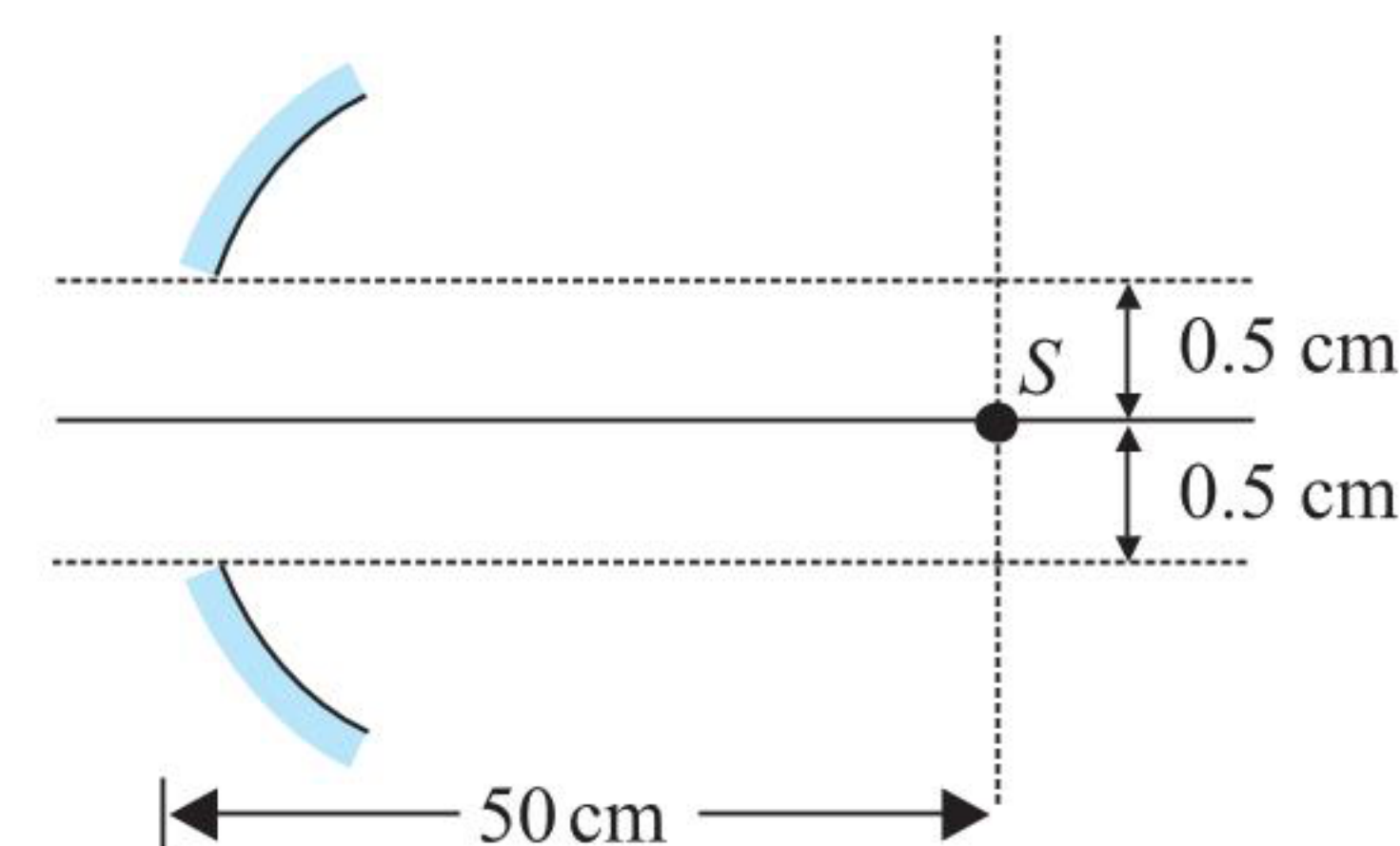
6. Image formed by mirror is



- Image formed by mirror is
 - at a distance 60 cm in front of mirror
 - at a distance 60 cm behind the mirror
 - at a distance 12 cm in front of mirror
 - at a distance 12 cm behind the mirror.
- If object starts moving with 2 cm s^{-1} along principal axis towards the mirror then,
 - image starts moving with 4 cm s^{-1} away from the mirror
 - image starts moving with 4 cm s^{-1} towards the mirror
 - image starts moving with 8 cm s^{-1} towards the mirror
 - image starts moving with 8 cm s^{-1} away from the mirror
- If object starts moving with 2 cm s^{-1} perpendicular to principal axis above the principal axis then,
 - image moves with velocity 4 cm s^{-1} below the principal axis
 - image moves with velocity 4 cm s^{-1} above the principal axis
 - image moves with velocity 8 cm s^{-1} below the principal axis
 - image moves with velocity 8 cm s^{-1} above the principal axis

For Problems 9–11

A concave mirror forms a real image of a point source lying on the optical axis at a distance of 50 cm from the mirror. The focal length of the mirror is 25 cm. The mirror is cut in two, and its halves are drawn at a distance of 1 cm apart in a direction perpendicular to the optical axis.



9. The distance between the two images formed by the two halves of the mirror is

(1) 0.5 cm (2) 1 cm
(3) 1.5 cm (4) 2 cm

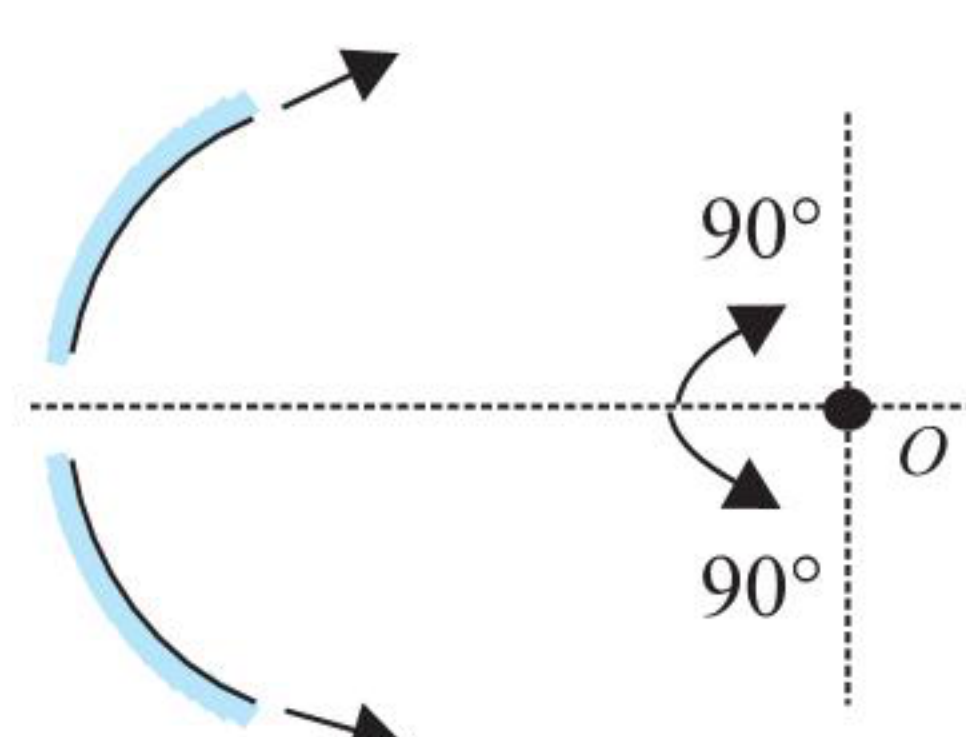
10. Two statements are provided. Choose which is true (T) and which is false (F).

Statement I: Image formed by half cut mirror has just half the intensity as that of the intensity of image formed by complete mirror.

Statement II: More the lateral separation between the two halves of the mirror, less will be the separation between the two images, as show in the given situation.

(1) T, T (2) T, F
(3) F, T (4) F, F

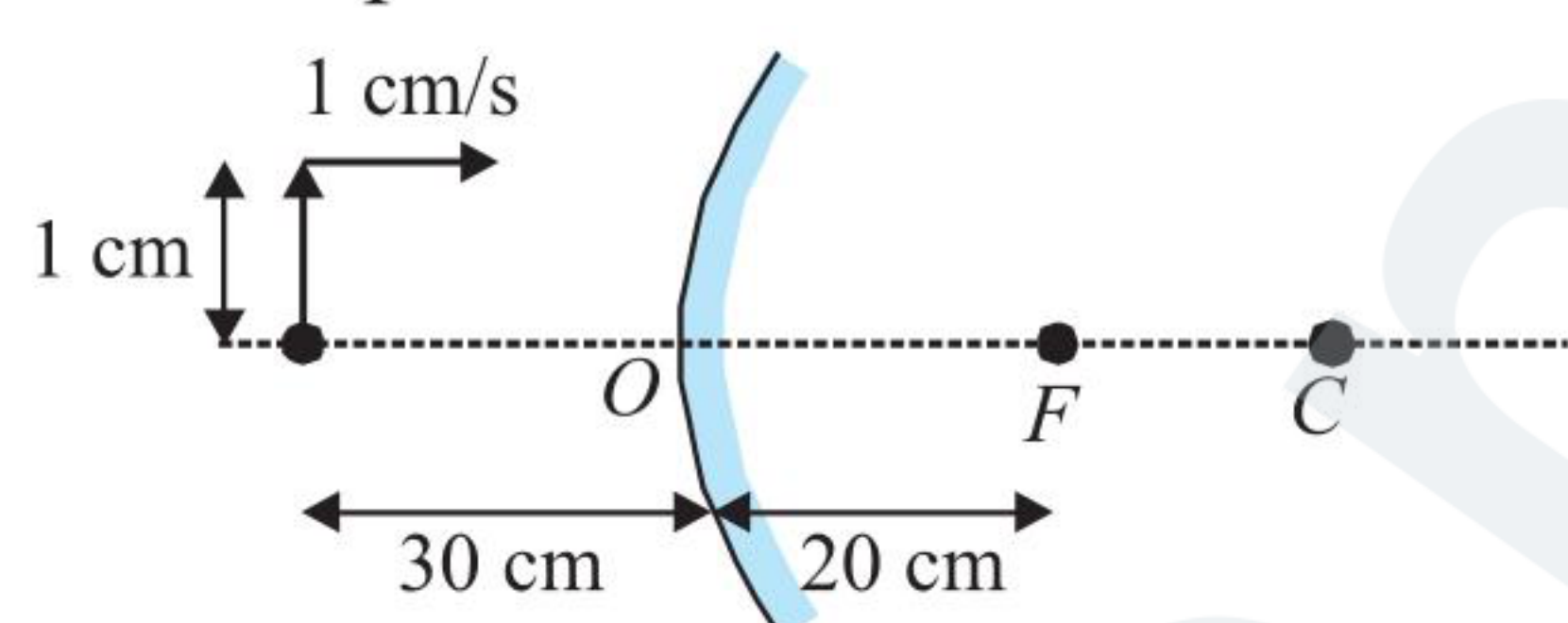
11. In the given situation, if the two halves are each rotated (before being separated) about S by 90° , with reference to original position, then the distance between the two images will be



(1) 0.5 cm (2) 2 cm
(3) 1 cm (4) zero

For Problems 12–13

An object AB of height 1 cm lying on the axis of convex mirror of focal length 20 cm, at a distance of 30 cm from its pole as shown in figure. Now the object starts moving with a constant speed 1 cm/sec towards the pole. Calculate



12. Magnitude of velocity of image at the given instant.

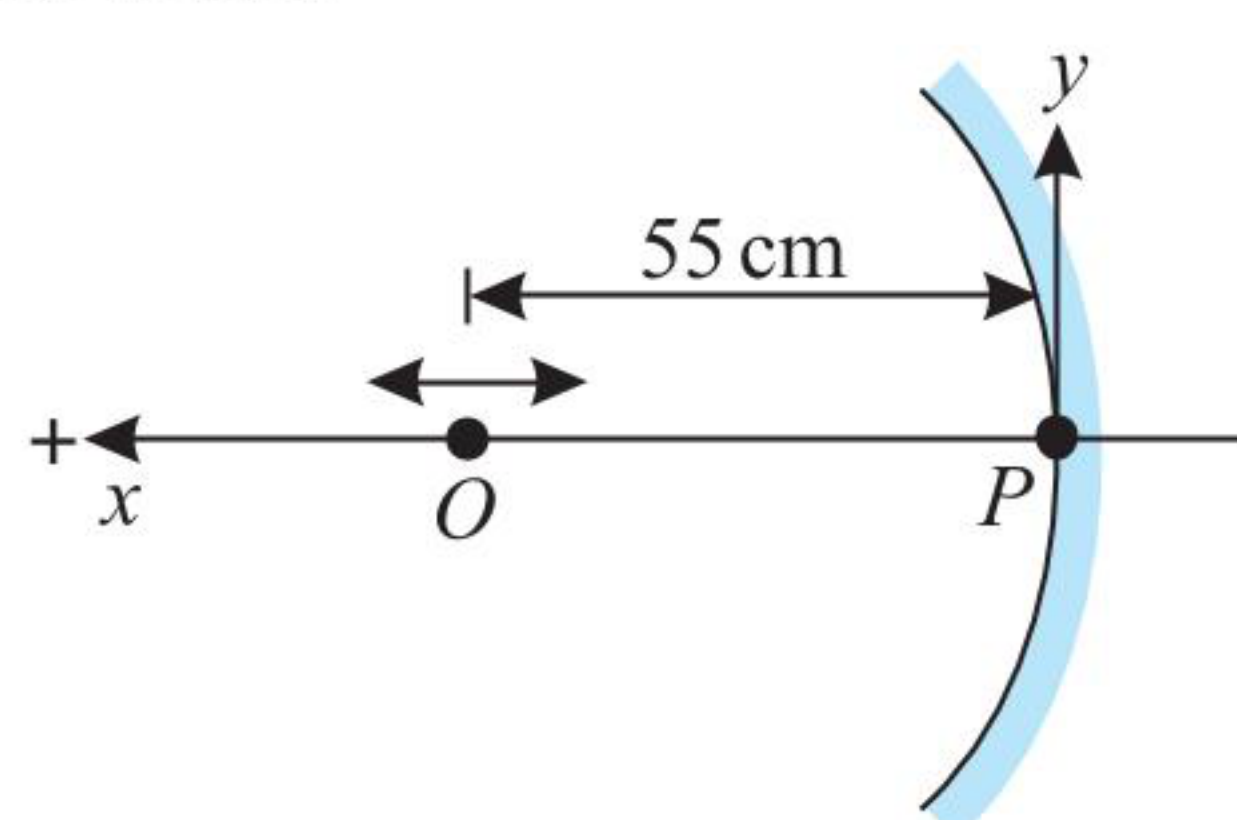
(1) 0.08 cm s^{-1} (2) 0.64 cm s^{-1}
(3) 0.16 cm s^{-1} (4) 0.32 cm s^{-1}

13. Rate of change of height of image.

(1) 0.16 cm s^{-1} (2) 0.004 cm s^{-1}
(3) 0.016 cm s^{-1} (4) 0.008 cm s^{-1}

For Problems 14–16

A particle performs simple harmonic motion, about $x = 55 \text{ cm}$ (mean position). The pole of the concave mirror is at origin of the coordinate system given in figure. The focal length of the mirror is 20 cm and the position of oscillating particle from its mean position is $s = 5 \cos 10\pi t$.



14. Find the range of vibration of image.

(1) 10 cm (2) 5 cm
(3) 3.33 cm (4) 20 cm

15. Find the maximum distance of image from pole of mirror,

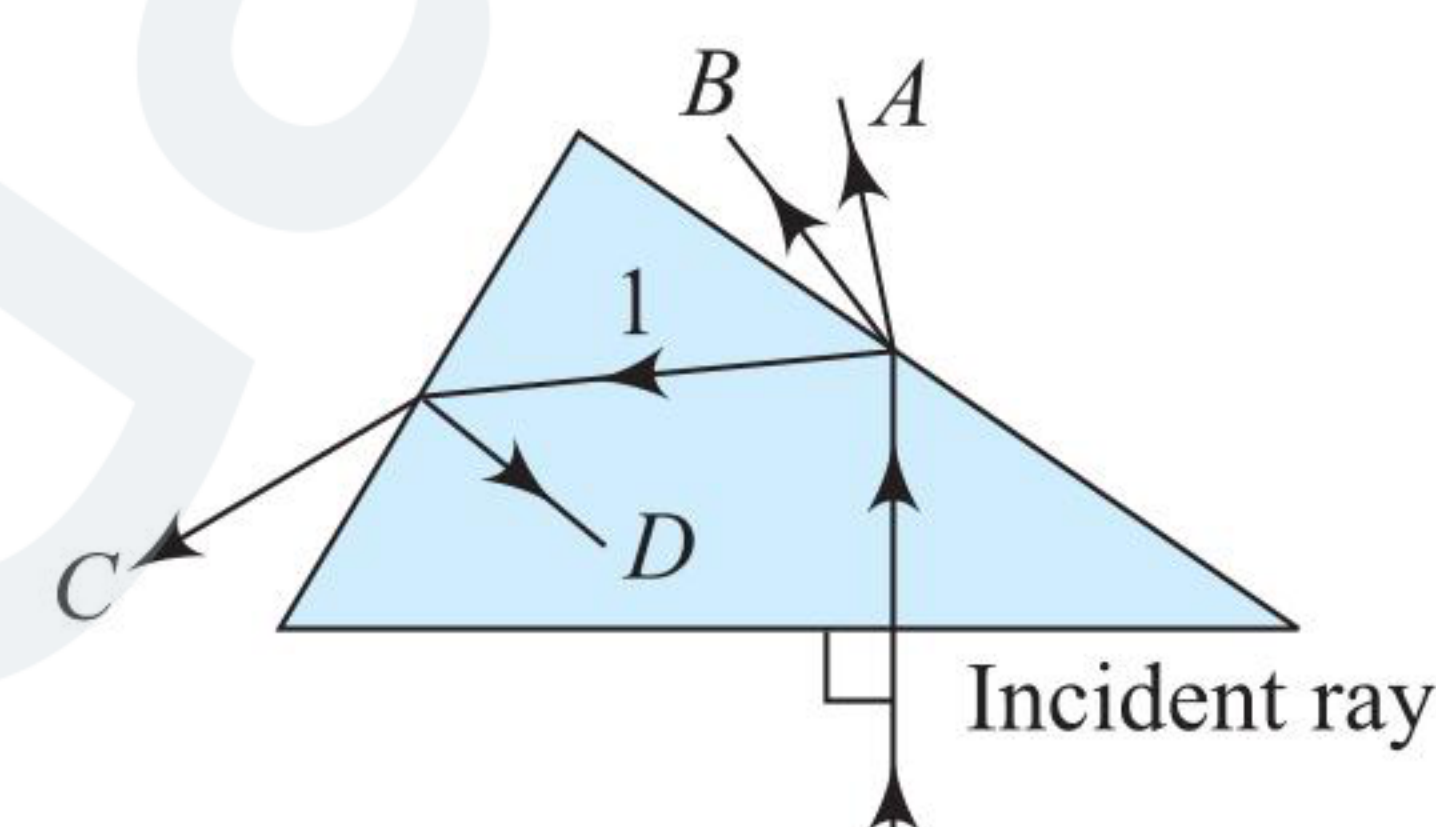
(1) 10 cm (2) 30 cm
(3) 25 cm (4) 33.33 cm

16. Find the time period of oscillation of image.

(1) 0.1 s (2) 0.2 s
(3) 0.4 s (4) 0.6 s

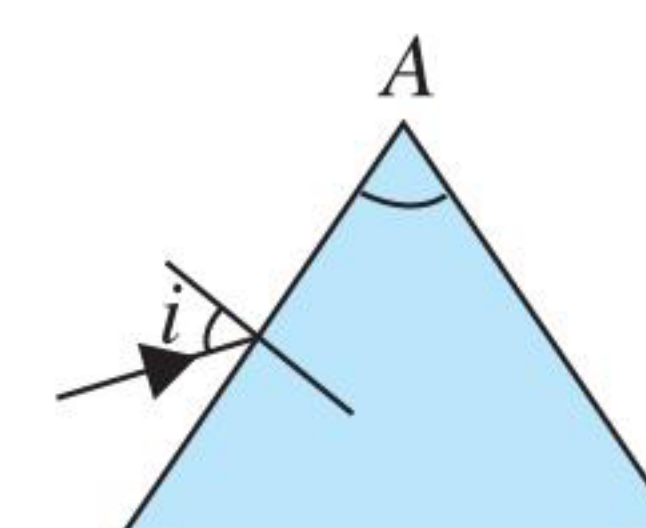
Matrix Match Type

1. A white light ray is incident on a glass prism, and it creates four refracted rays A , B , C , and D . Match the refracted rays with the colours given (1 and D are rays due to total internal reflection):



Column I (Ray)	Column II (Colour)
i. A	a. red
ii. B	b. green
iii. C	c. yellow
iv. D	d. blue

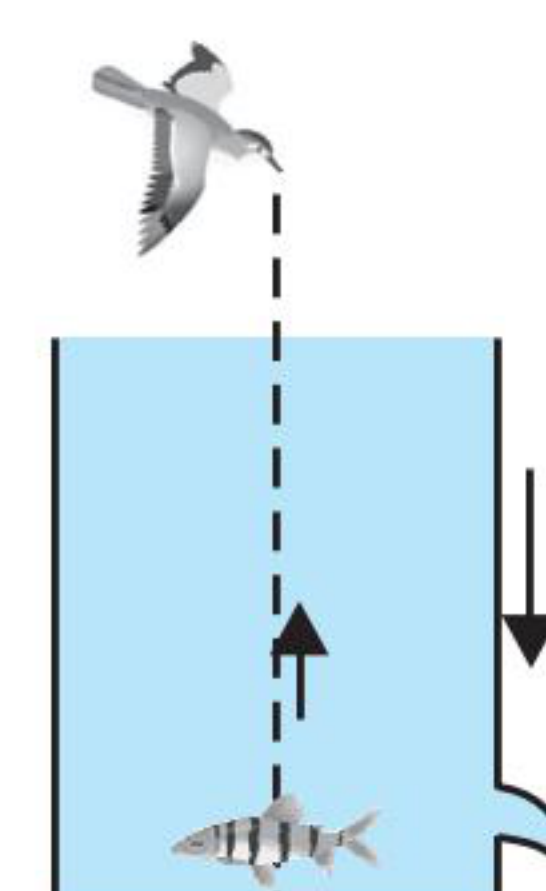
2. For a prism of refracting angle A and refractive index 2. Assume rays are incident at all angles of incidence $0^\circ \leq i \leq 90^\circ$. Ignore partial reflection.



Column I	Column II
i. $A = 15^\circ$	a. All rays are reflected back from the second surface
ii. $A = 45^\circ$	b. All rays are refracted into air from the second face
iii. $A = 70^\circ$	c. Some rays are reflected back from second surface
iv. $A = 50^\circ$	d. Some rays are refracted into air from the second surface

3. A bird in air is diving vertically over a tank with speed 5 cm s^{-1} . Base of tank is silvered. A fish in the tank is rising upward along the same line with speed 2 cm s^{-1} . Water level is falling at rate of 2 cm s^{-1} .

[Take: $\mu_{\text{water}} = 4/3$]

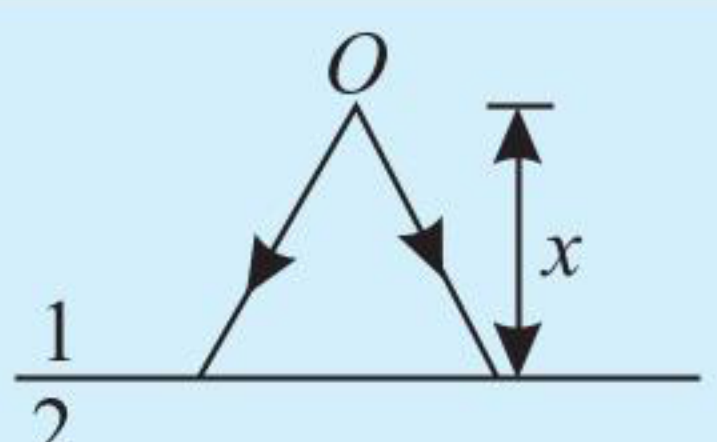
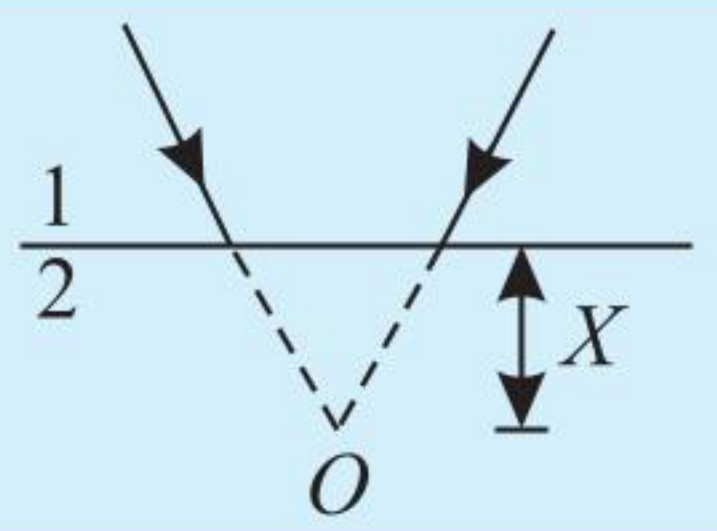
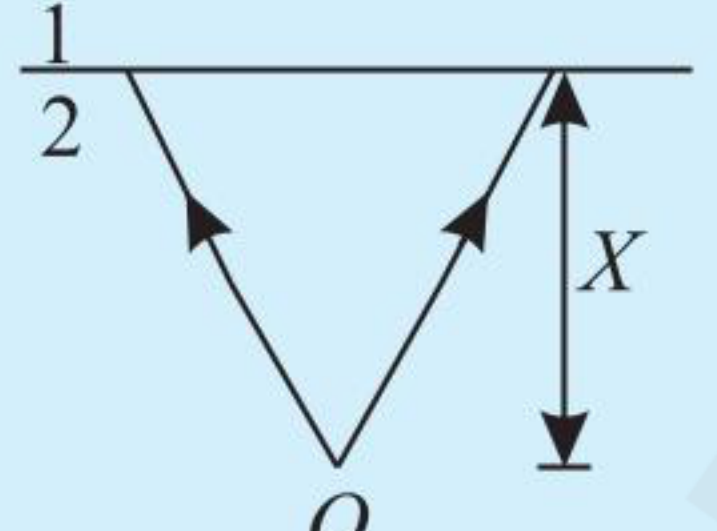
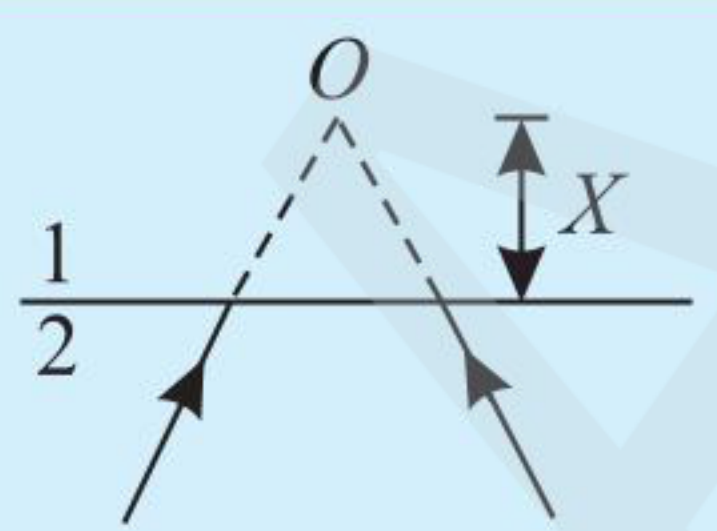


Column I	Column II (in cm s^{-1})
i. Speed of the fish as seen by the bird directly	a. 8
ii. Speed of the image of fish formed after reflection in the mirror as seen by the bird	b. 6
iii. Speed of bird relative to the fish looking upwards	c. 3
iv. Speed of image of bird relative to the fish looking downwards in the mirror	d. 4

4. In column I, types of the mirrors are given and in column II, types of the image for virtual objects are listed, match the following two columns.

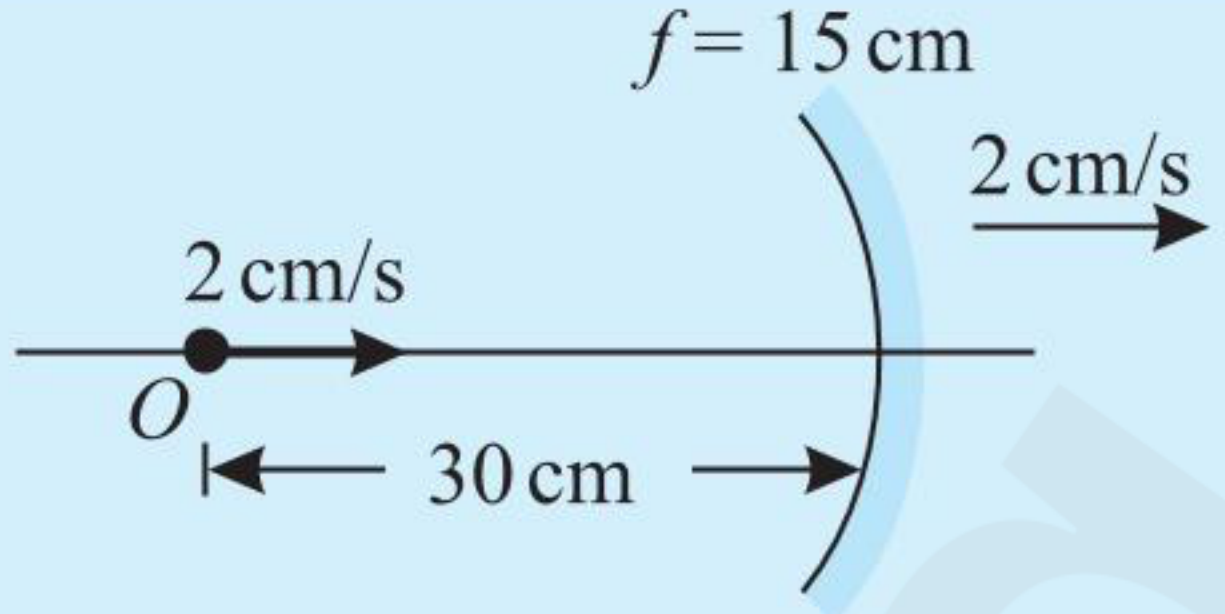
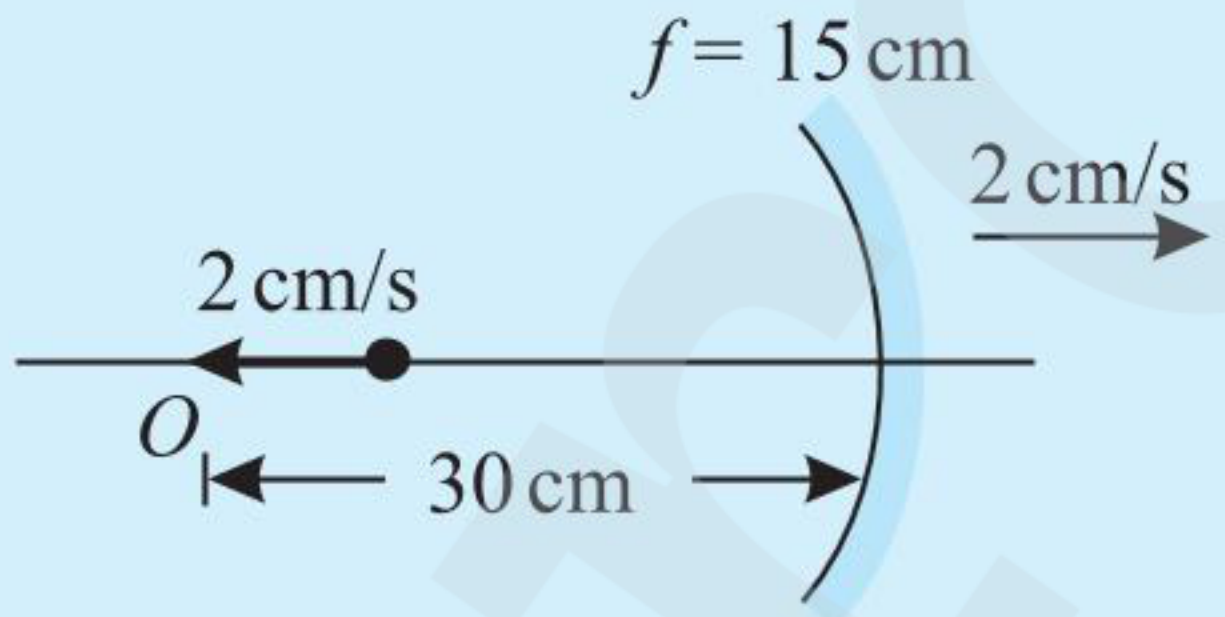
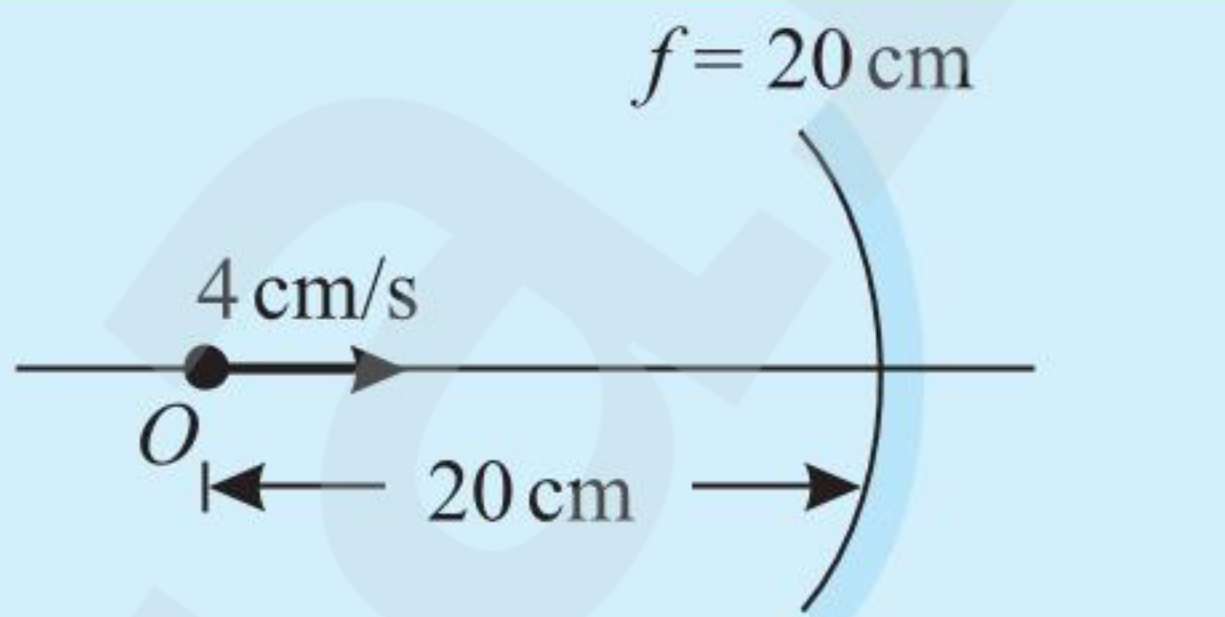
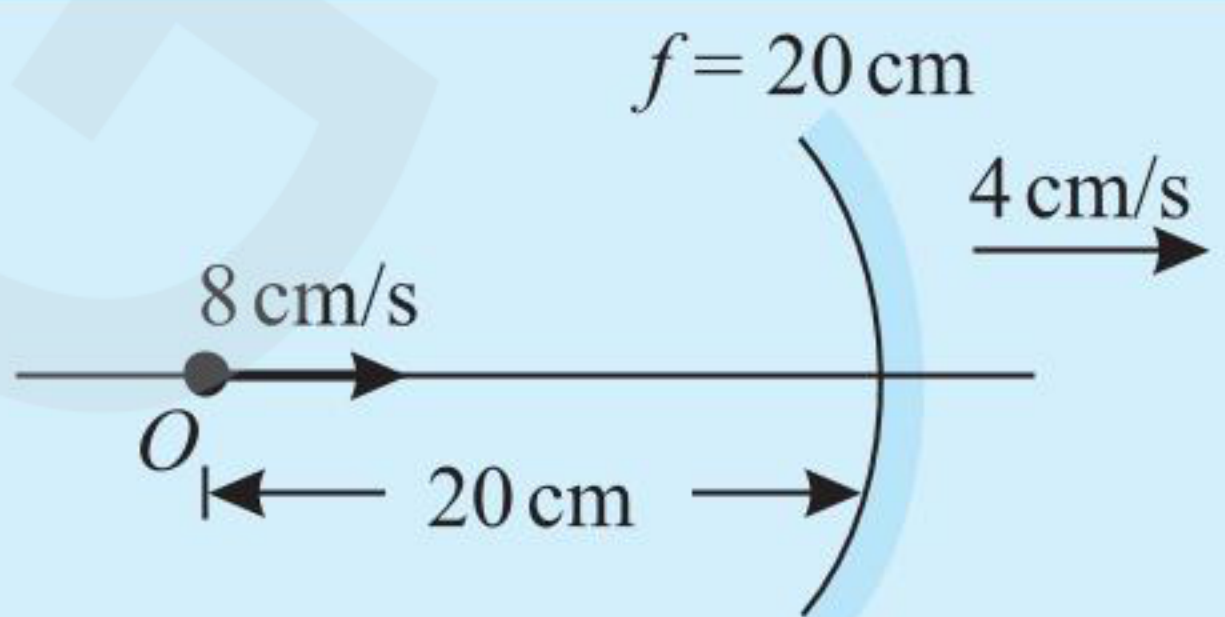
Column I	Column II
i. Concave mirror	a. only real image
ii. Convex mirror	b. only virtual image
iii. Plane mirror	c. may be real or virtual image

- (1) i. $\rightarrow$ a.; ii. $\rightarrow$ c.; iii. $\rightarrow$ a.
(2) i. $\rightarrow$ b.; ii. $\rightarrow$ c.; iii. $\rightarrow$ a.
(3) i. $\rightarrow$ c.; ii. $\rightarrow$ b.; iii. $\rightarrow$ a.
(4) i. $\rightarrow$ a.; ii. $\rightarrow$ c.; iii. $\rightarrow$ b.
5. Match the following two columns corresponding to single refraction from plane surface. In cases shown in Column I, $\mu_1 > \mu_2$.

Column I	Column II
i. 	a. Image distance greater than x from plane surface
ii. 	b. Image distance less than x from plane surface
iii. 	c. Real image
iv. 	d. Virtual image

- (1) i. $\rightarrow$ b., d.; ii. $\rightarrow$ b., c.; iii. $\rightarrow$ a., d.; iv. $\rightarrow$ a., c.
(2) i. $\rightarrow$ b., c.; ii. $\rightarrow$ b., c.; iii. $\rightarrow$ a., d.; iv. $\rightarrow$ a.
(3) i. $\rightarrow$ b.; ii. $\rightarrow$ b., c.; iii. $\rightarrow$ a., d.; iv. $\rightarrow$ a., c.
(4) i. $\rightarrow$ a., d.; ii. $\rightarrow$ b., c.; iii. $\rightarrow$ a., c.; iv. $\rightarrow$ a., c.
6. Column I shows an optical system, in which velocity of object and mirror is shown. Column II has value of magnitude of velocity of image formed by mirror w.r.t. ground. Match the

proper entries from column II to column I. Object is moving on principal axis of mirror.

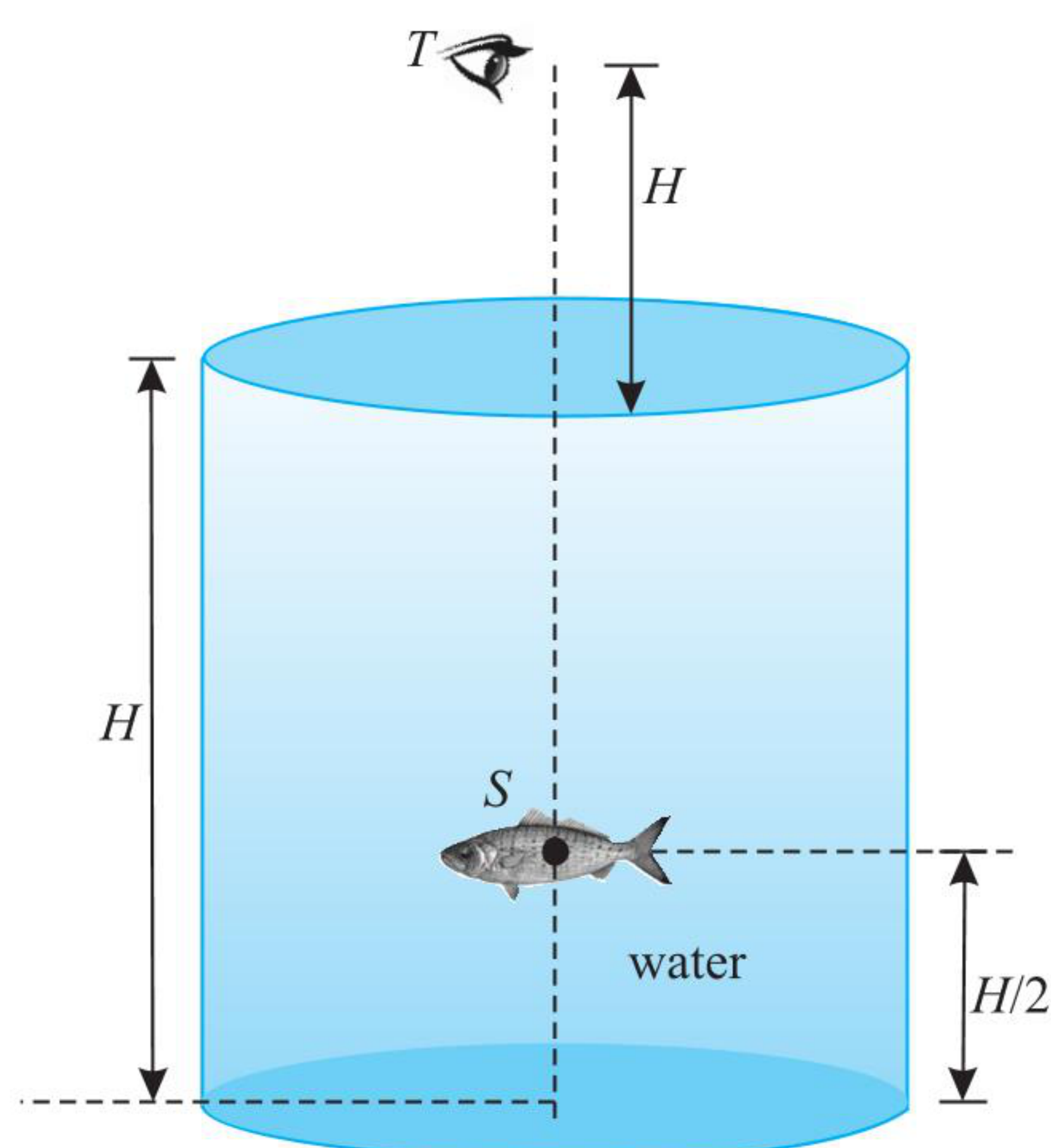
Column I	Column II
i. 	a. 2 m/s
ii. 	b. 1 m/s
iii. 	c. 6 m/s
iv. 	d. 3 m/s

- (1) i. $\rightarrow$ d.; ii. $\rightarrow$ b.; iii. $\rightarrow$ a.; iv. $\rightarrow$ c.
(2) i. $\rightarrow$ a.; ii. $\rightarrow$ c.; iii. $\rightarrow$ d.; iv. $\rightarrow$ d.
(3) i. $\rightarrow$ b.; ii. $\rightarrow$ a.; iii. $\rightarrow$ d.; iv. $\rightarrow$ c.
(4) i. $\rightarrow$ d.; ii. $\rightarrow$ d.; iii. $\rightarrow$ a.; iv. $\rightarrow$ c.
7. In column I different object positions w.r.t. a concave or convex mirror are given. In column II corresponding magnifications are given. Match the situations given in column I with the situations given in column II.

Column I	Column II
i. An object is placed at a distance equal to focal length from pole before convex mirror	a. ∞
ii. An object is placed at focus before a concave mirror	b. 0.5
iii. An object is placed at the centre of curvature before a concave mirror	c. $\frac{1}{3}$
iv. An object is placed at a distance equal to radius of curvature before a convex mirror	d. -1

- (1) i. $\rightarrow$ a.; ii. $\rightarrow$ b.; iii. $\rightarrow$ d.; iv. $\rightarrow$ c.
(2) i. $\rightarrow$ b.; ii. $\rightarrow$ a.; iii. $\rightarrow$ d.; iv. $\rightarrow$ c.
(3) i. $\rightarrow$ c.; ii. $\rightarrow$ a.; iii. $\rightarrow$ d.; iv. $\rightarrow$ b.
(4) i. $\rightarrow$ d.; ii. $\rightarrow$ b.; iii. $\rightarrow$ a.; iv. $\rightarrow$ c.
8. The bottom of the container is reflecting plane mirror, S is a small fish and T is a human eye as shown in figure. Refractive index of water is μ . Fish can see two images of human eye, first due to refraction from top surface only and other due to refraction from top surface and then reflection

at bottom. Distance of these images from fish are d_1 and d_2 respectively. Similarly, human eye can also see the two images of fish, first due to refraction only and other due to, reflection and then refraction. Distance of these images from human eye are d_3 and d_4 respectively. Match the quantities of column I with their values in column II.

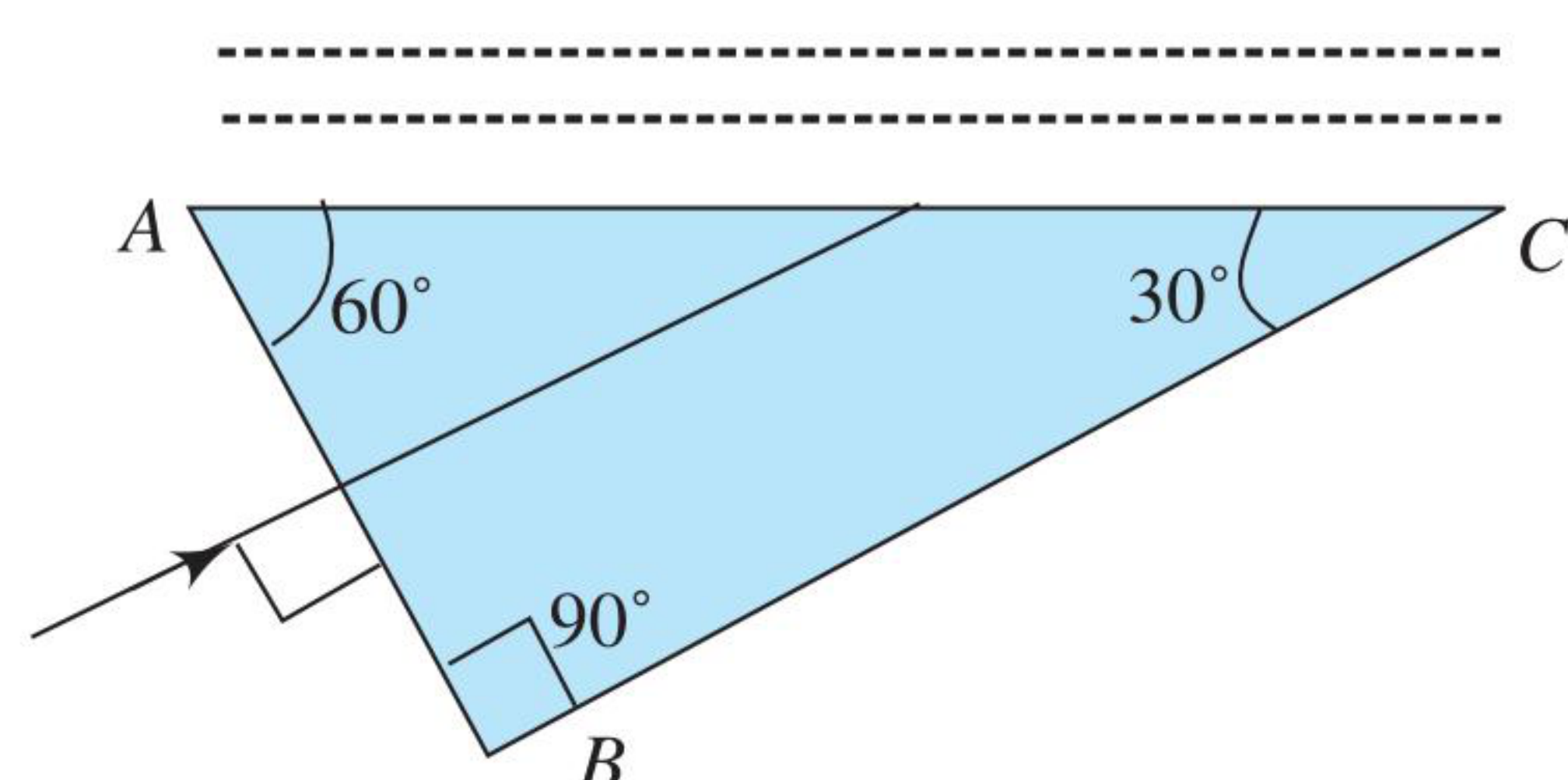


Column I	Column II
i. d_1	a. $H \left[1 + \frac{1}{2\mu} \right]$
ii. d_2	b. $H \left[\mu + \frac{1}{2} \right]$
iii. d_3	c. $H \left[1 + \frac{3}{2\mu} \right]$
iv. d_4	d. $H \left[\mu + \frac{3}{2} \right]$

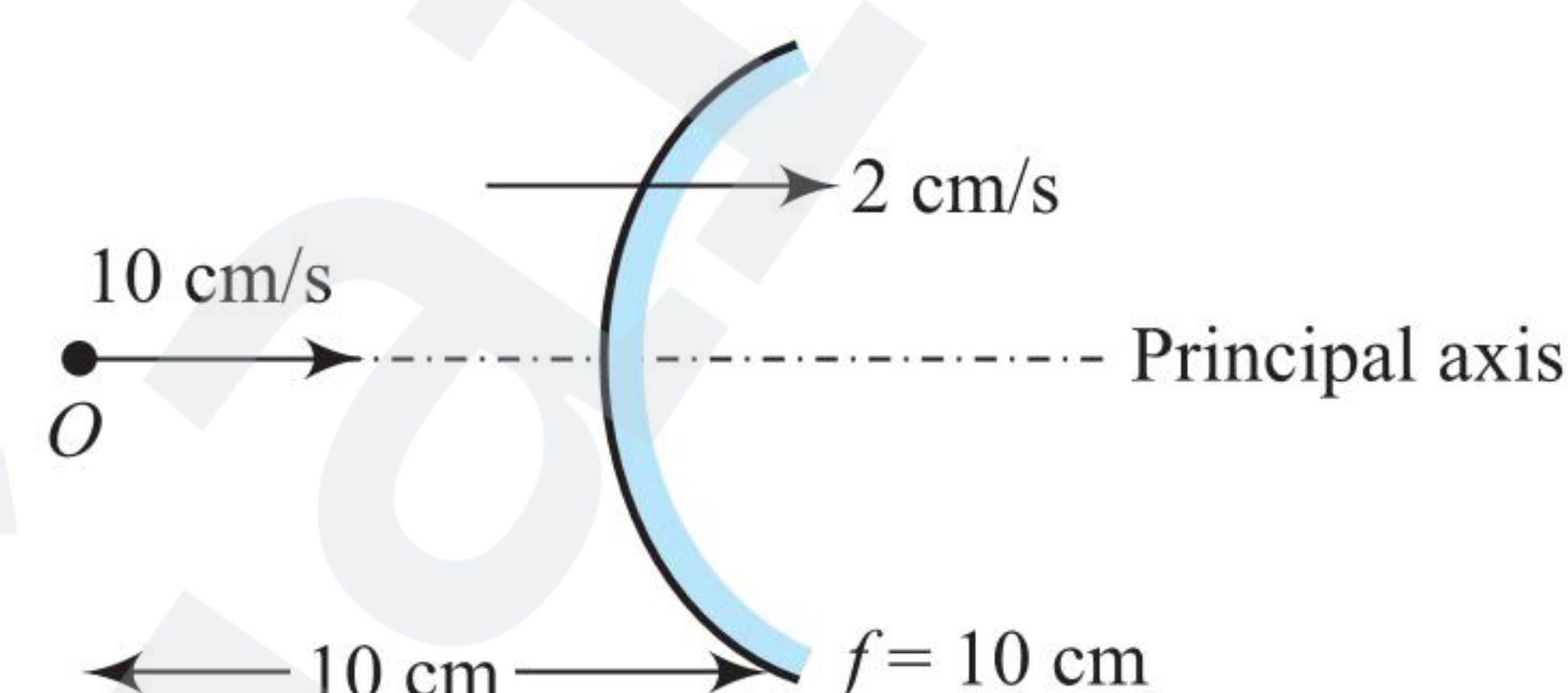
- (1) i. $\rightarrow$ a.; ii. $\rightarrow$ d.; iii. $\rightarrow$ b.; iv. $\rightarrow$ c.
 (2) i. $\rightarrow$ d.; ii. $\rightarrow$ b.; iii. $\rightarrow$ a.; iv. $\rightarrow$ c.
 (3) i. $\rightarrow$ c.; ii. $\rightarrow$ a.; iii. $\rightarrow$ d.; iv. $\rightarrow$ b.
 (4) i. $\rightarrow$ b.; ii. $\rightarrow$ d.; iii. $\rightarrow$ a.; iv. $\rightarrow$ c.

Numerical Value Type

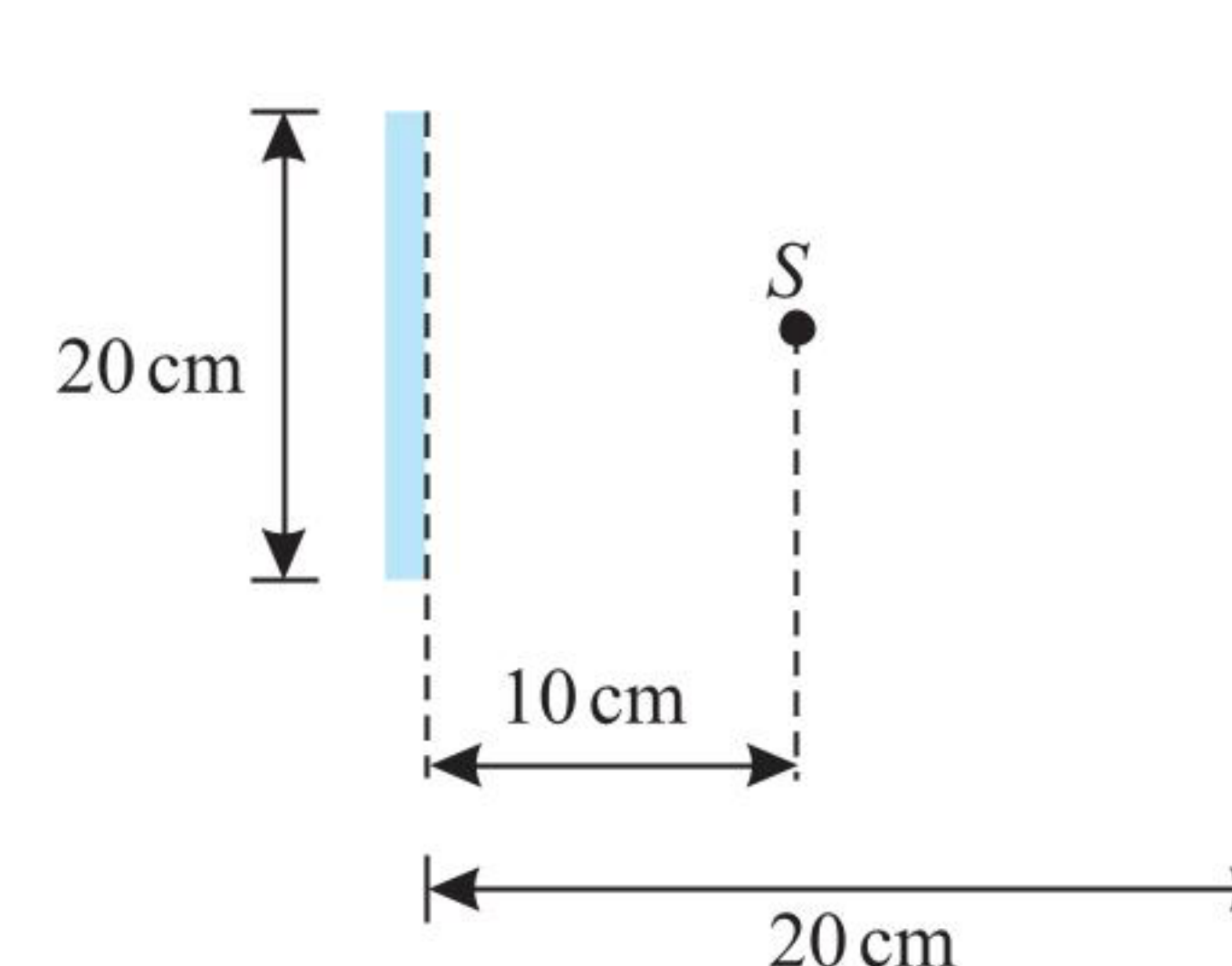
1. As shown in figure, light is incident normally on one face of the prism. A liquid of refractive index μ is placed on the horizontal face AC . The refractive index of prism is $3/2$. If total internal reflection takes place on face AC , μ should be less than $\frac{I\sqrt{3}}{4}$, where I is an integer. Find the value of I .



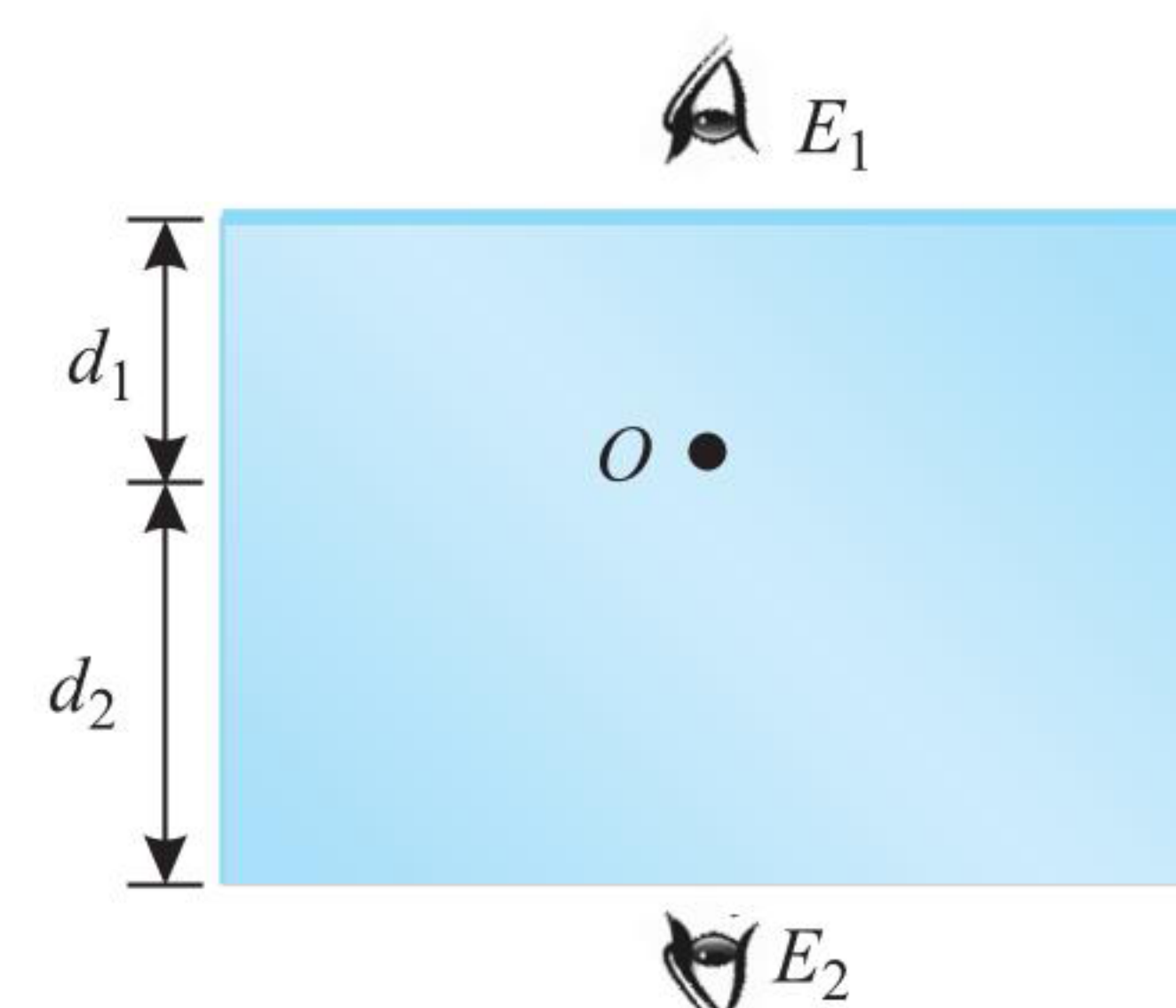
2. A ray of light is incident at an angle of 60° on one face of a prism which has refracting angle of 30° . The ray emerging out of the prism makes an angle of 30° with the incident ray. If the refractive index of the material of the prism is $\mu = \sqrt{a}$, find the value of a .
3. A container of uniform cross-section has a height of 14 m. Up to what height (in metre) water of refractive index $4/3$ should be filled inside the container so that container seems to be half filled for normal viewing.
4. What is the velocity (in cm s^{-1}) of image in situation shown in figure (O = object, f = focal length)? Object moves with velocity 10 cm/s and mirror moves with velocity 2 cm s^{-1} as shown.



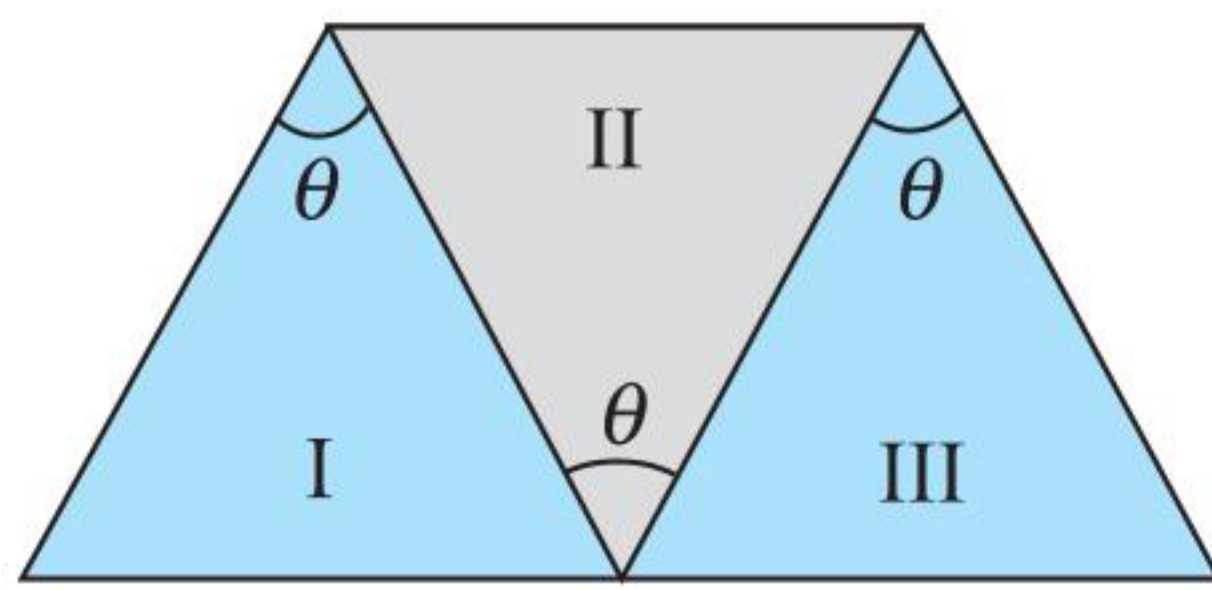
5. A point source of light S is placed at a distance 10 cm in front of the centre of a mirror of width 20 cm suspended vertically on a wall. An insect walks with a speed 10 cm/s in front of the mirror along a line parallel to the mirror at a distance 20 cm from it as shown in figure. Find the maximum time (in s) during which the insect can see the image of the source S in the mirror.



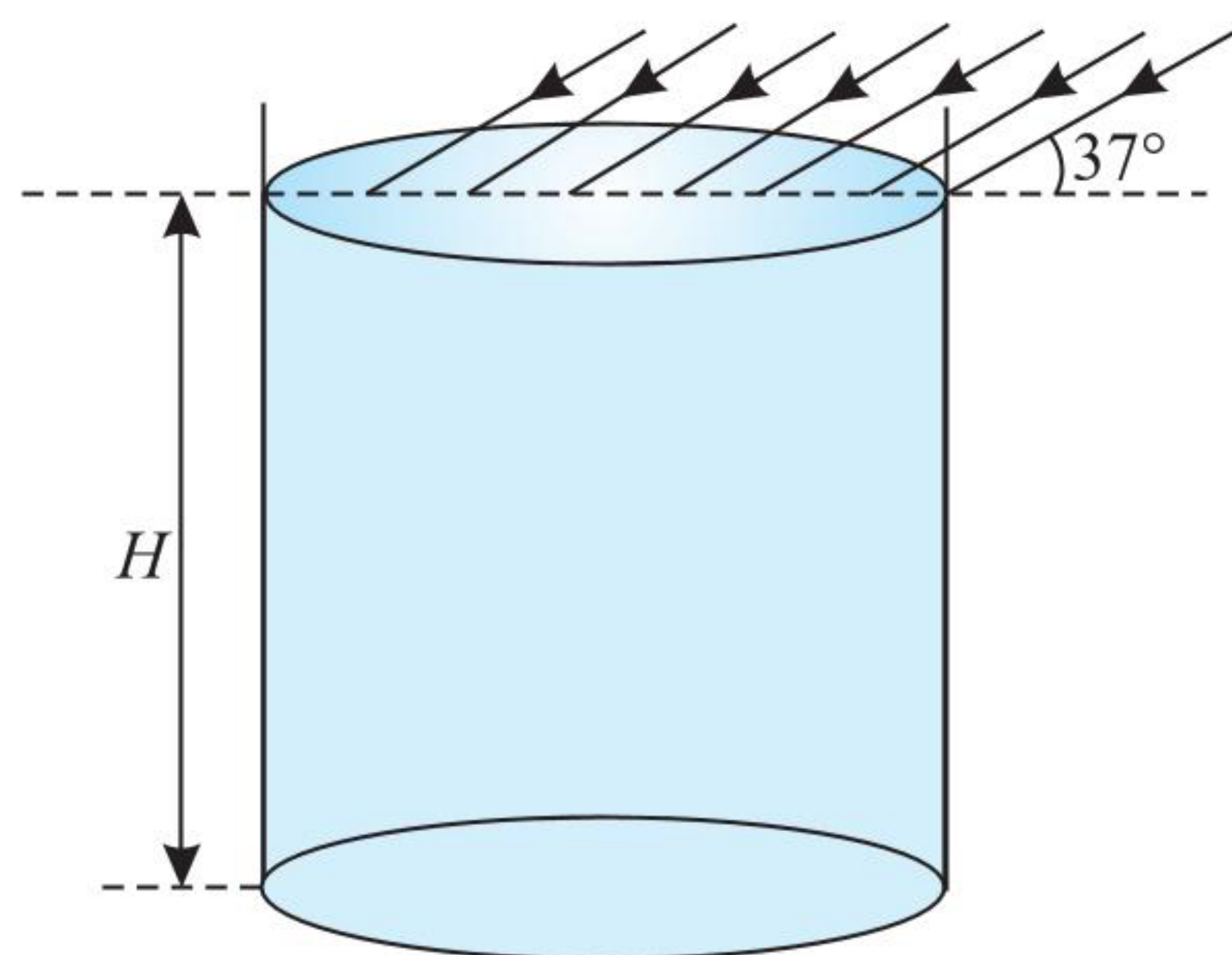
6. Refractive index of the glass slab is 1.5. There is a point object O inside the slab as shown. To eye E_1 object appears at a distance of 6 cm (from the top surface) and to eye E_2 it appears at a distance of 8 cm (from the bottom surface). Find thickness of the glass slab (in cm).



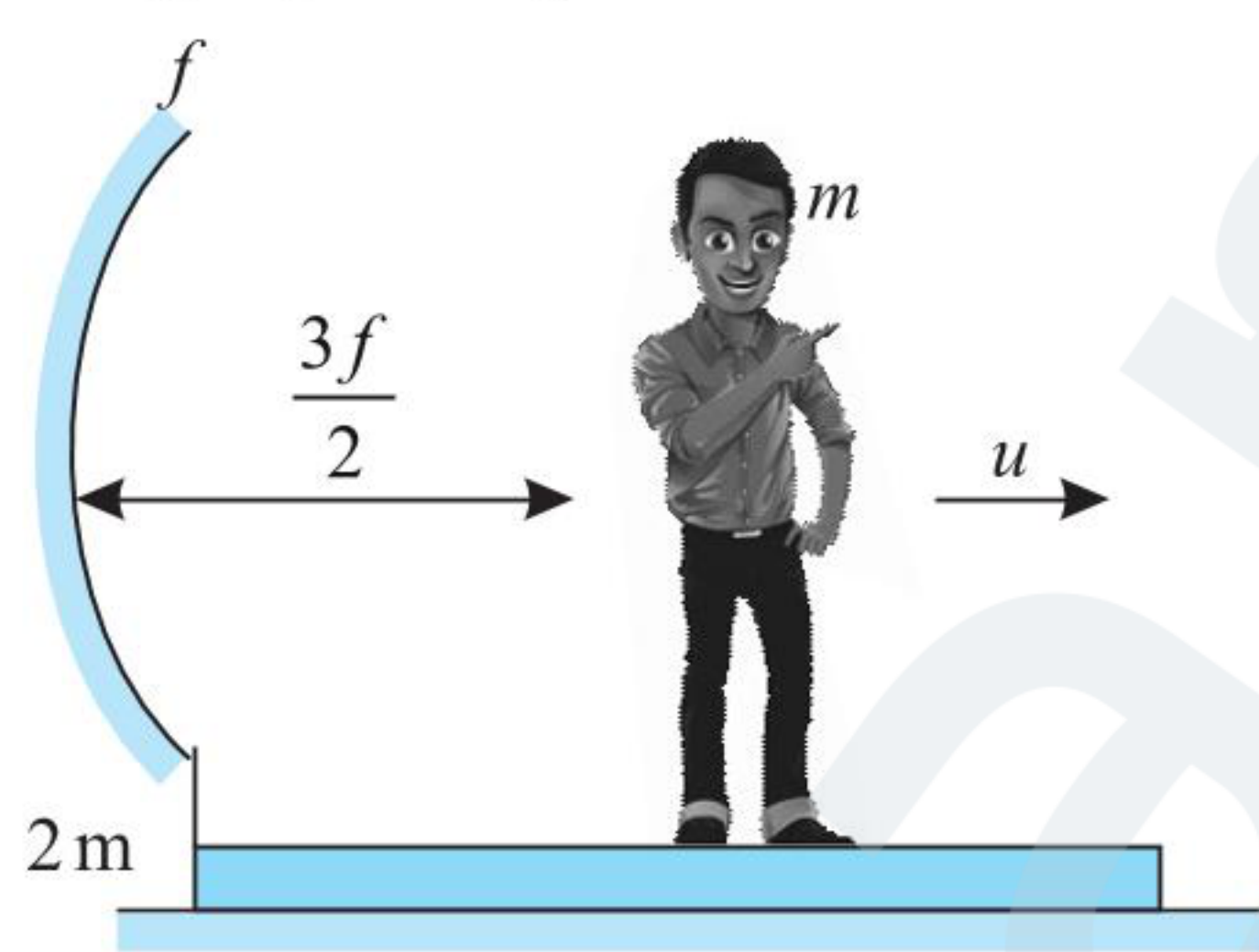
7. A direct-vision prism is made out of three prisms, each with a refracting angle of $\theta = 60^\circ$, attached to each other as shown in figure. Light of a certain wavelength is incident on the first prism. The angle of incidence is 30° and the ray leaves the third prism parallel to the direction of incidence. The refractive index of the glass of the first and third prisms is 1.5. Find the refractive index of the material of the middle prism. ($\sqrt{6} = 2.45$)



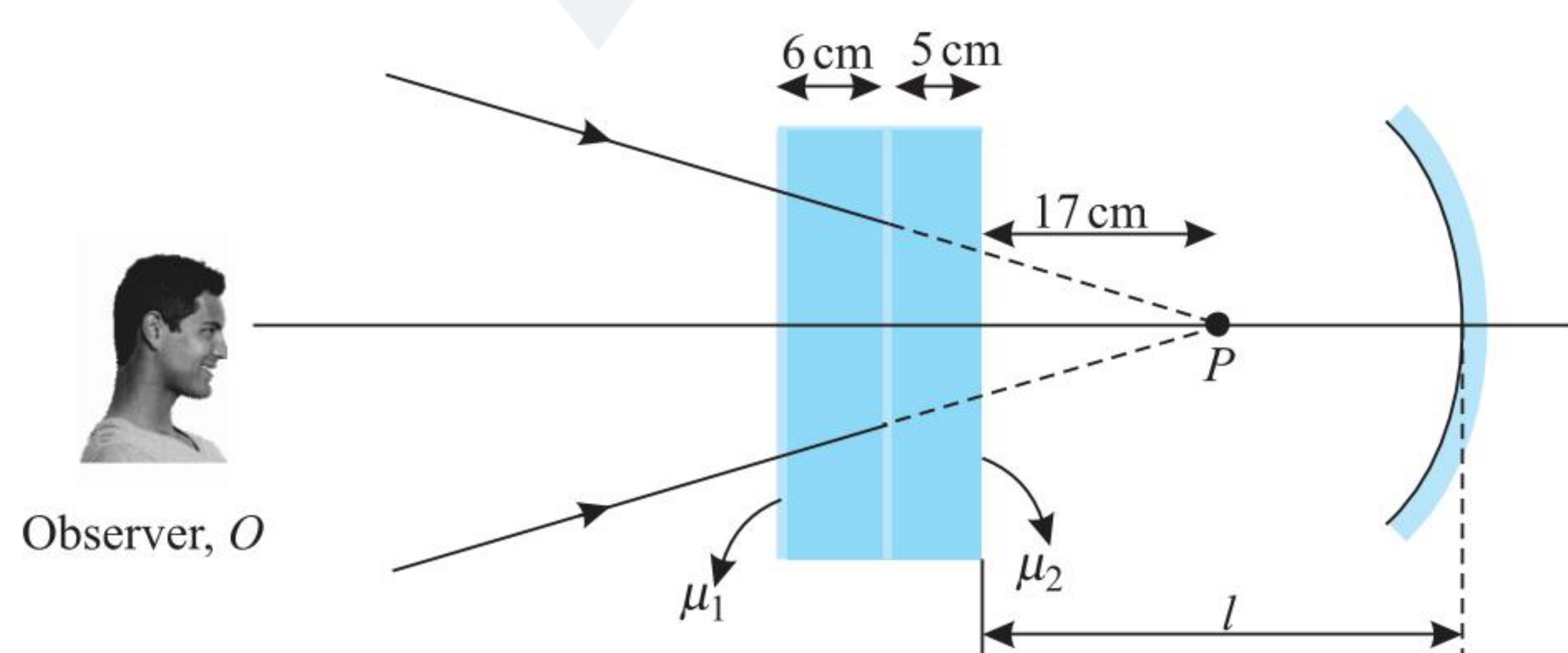
8. A point object is placed 33 cm from a convex mirror which curvature radius 40 cm. A glass plate of thickness 6 cm and index 2.0 is placed between the object and the mirror, closer to the mirror. Find the distance of final image from the object (in cm).
9. An opaque cylindrical tank with an open top has a diameter of 3.00 m and is completely filled with water. When the setting sun reaches an angle of 37° above the horizon, sunlight ceases to illuminate any part of the bottom of the tank. How deep is the tank (in m)?



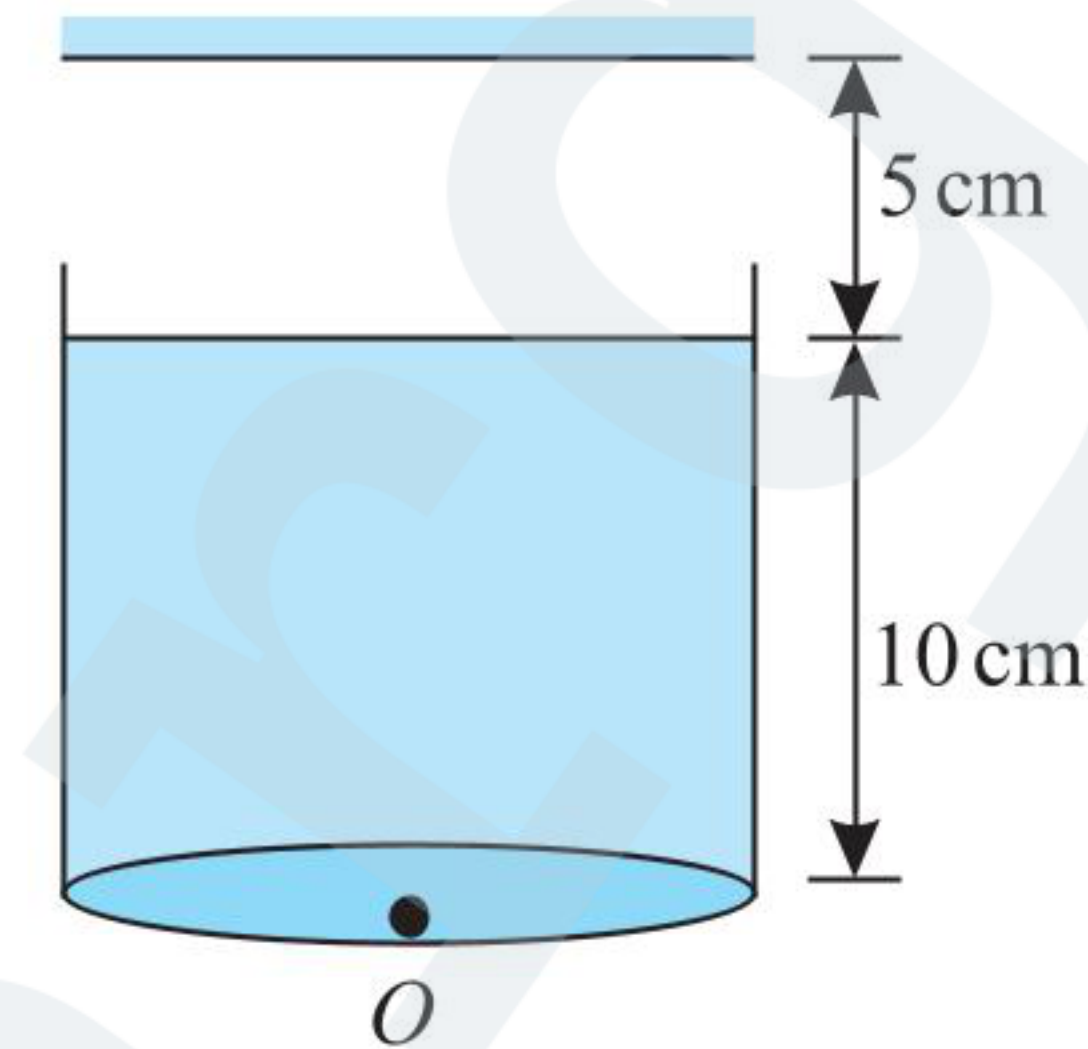
10. A prism of refractive index $\sqrt{2}$ has a refracting angle of 30° . One of the refracting surfaces of the prism is polished. For the beam of monochromatic light to retrace its path, find the angle of incidence (in degree) on the refracting surface.
11. A man of mass m starts moving w.r.t. a platform of mass 2 m with a velocity $\mu = 9/13$ m/s as shown in the figure. The platform is fitted with a concave mirror of focal length f . The velocity of image (m m/s) at the initial moment is



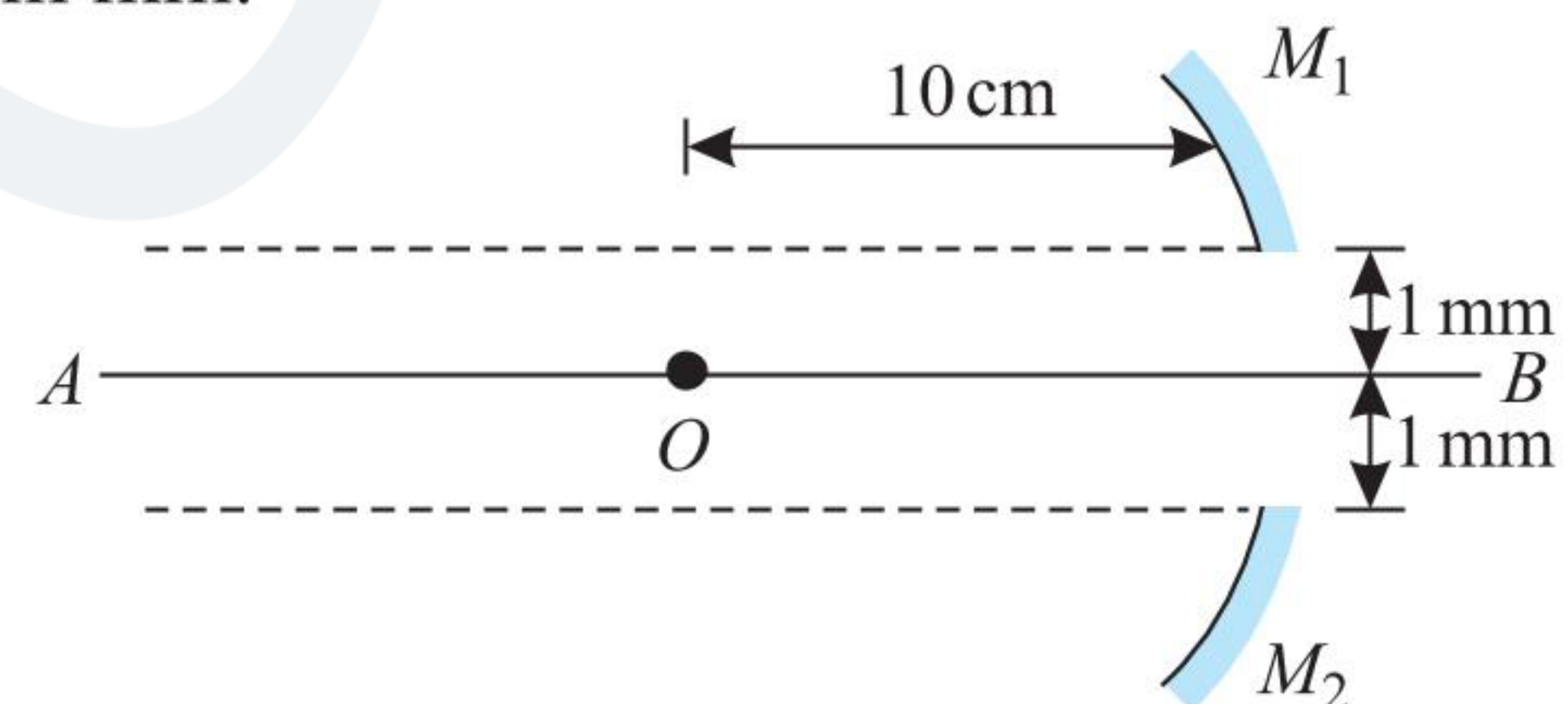
12. A converging beam is incident on a system of two glass slabs with refractive indices $\mu_1 = (3/2)$, $\mu_2 = (5/4)$ and a concave mirror placed at a distance ' l ' from the closest face of the slab 2. The observer O receives the reflected rays from the system and perceives that the image is formed at P itself (see figure) which is also the point of convergence of the incident beam (extrapolated). If the concave mirror's focal length is 20 cm. Find the value of l in cm.



13. Consider the situation shown in figure. Water ($\mu_w = \frac{4}{3}$) is filled in a beaker up to a height of 10 cm. A plane mirror is fixed at a height of 5 cm from the surface of water. Find the distance of image from the mirror after reflection from it of an object O at the bottom of the beaker (in cm)

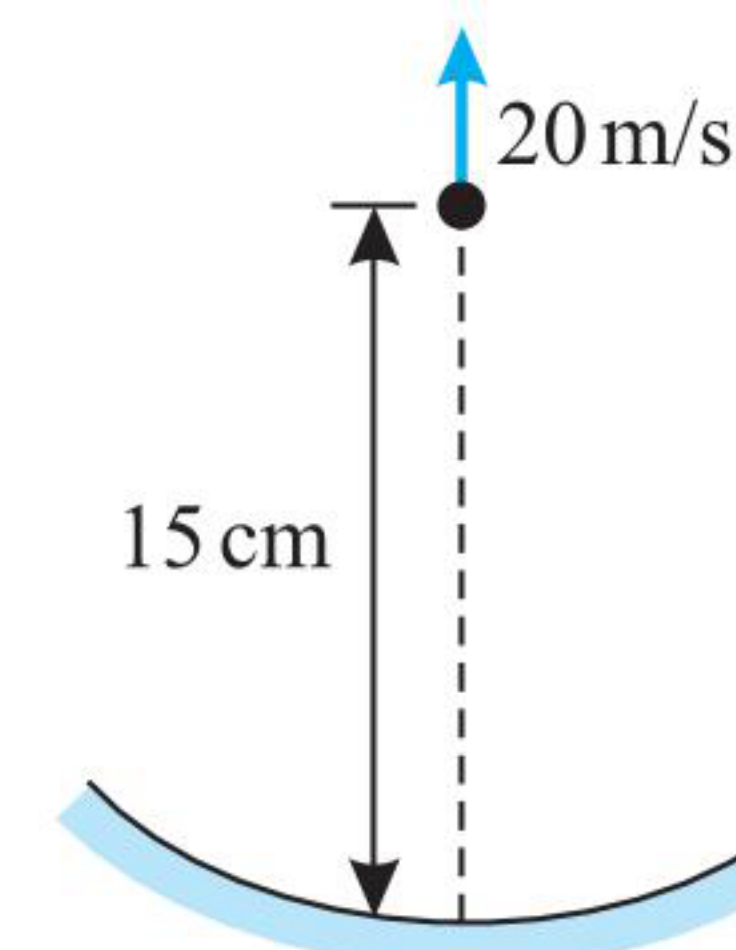


14. A concave mirror of focal length 20 cm is cut into two parts from the middle and the two parts are moved perpendicularly by a distance 1 mm from the previous principal axis AB . Find the distance between the images formed by the two parts in mm.

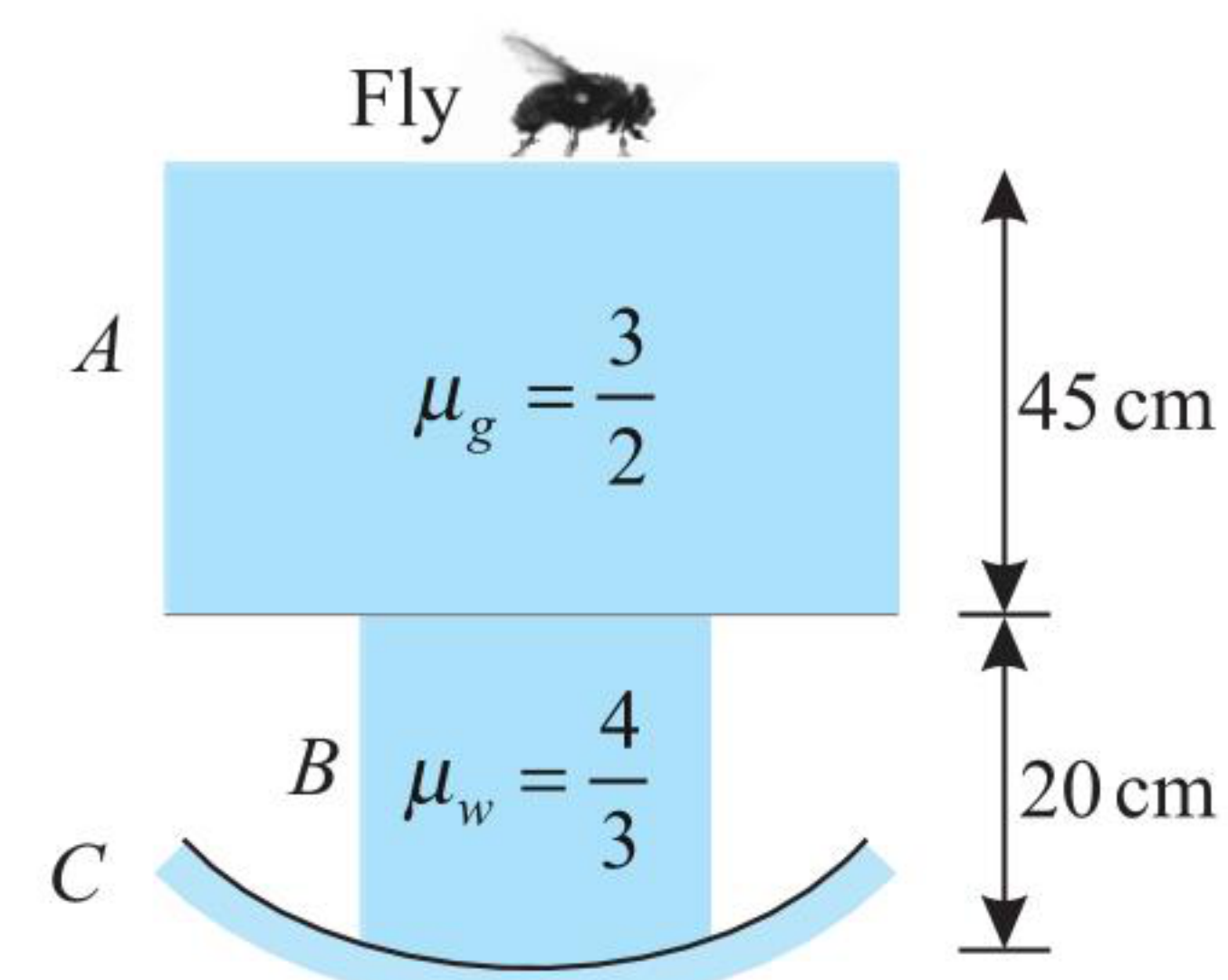


15. A balloon is rising up along the axis of a concave mirror of radius of curvature 20 m. A ball is dropped from the balloon at a height 15 m from the mirror when the balloon has velocity 20 m/s. Find the speed of the image of the ball (in m/s) formed by concave mirror after 4 second?

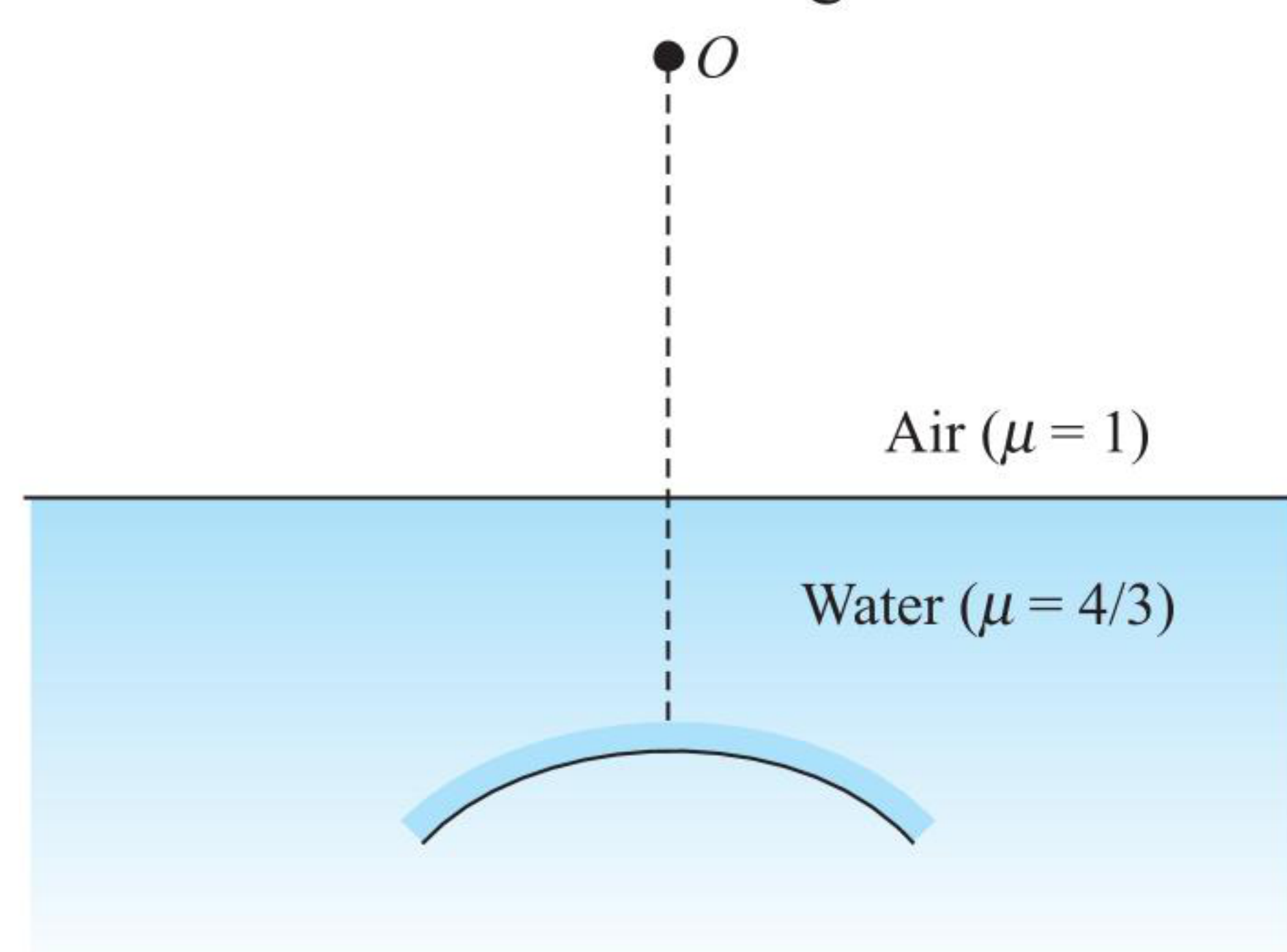
[Take: $g = 10 \text{ m/s}^2$]



16. A fly F is sitting on a glass slab A , 45 cm thick and of refractive index $3/2$. The slab covers the top of a container B containing water (Refractive index $4/3$) up to a height of 20 cm. The bottom of container is closed by a concave mirror C of radius of curvature 40 cm. Find the distance of the final image from the top surface of slab A in cm, formed by all refractions and reflection assuming paraxial rays.



17. A slab of glass of thickness 6 cm and index 1.5 is placed somewhere in between a concave mirror and a point object, perpendicular to the mirror's optical axis. The radius of curvature of the mirror is 40 cm. If the reflected final image coincides with the object, then find the distance of the object from the mirror (in cm).
18. A point object lies 30 cm above water on the axis of a convex mirror of focal length 40 cm lying 20 cm below water surface. Consider two images:

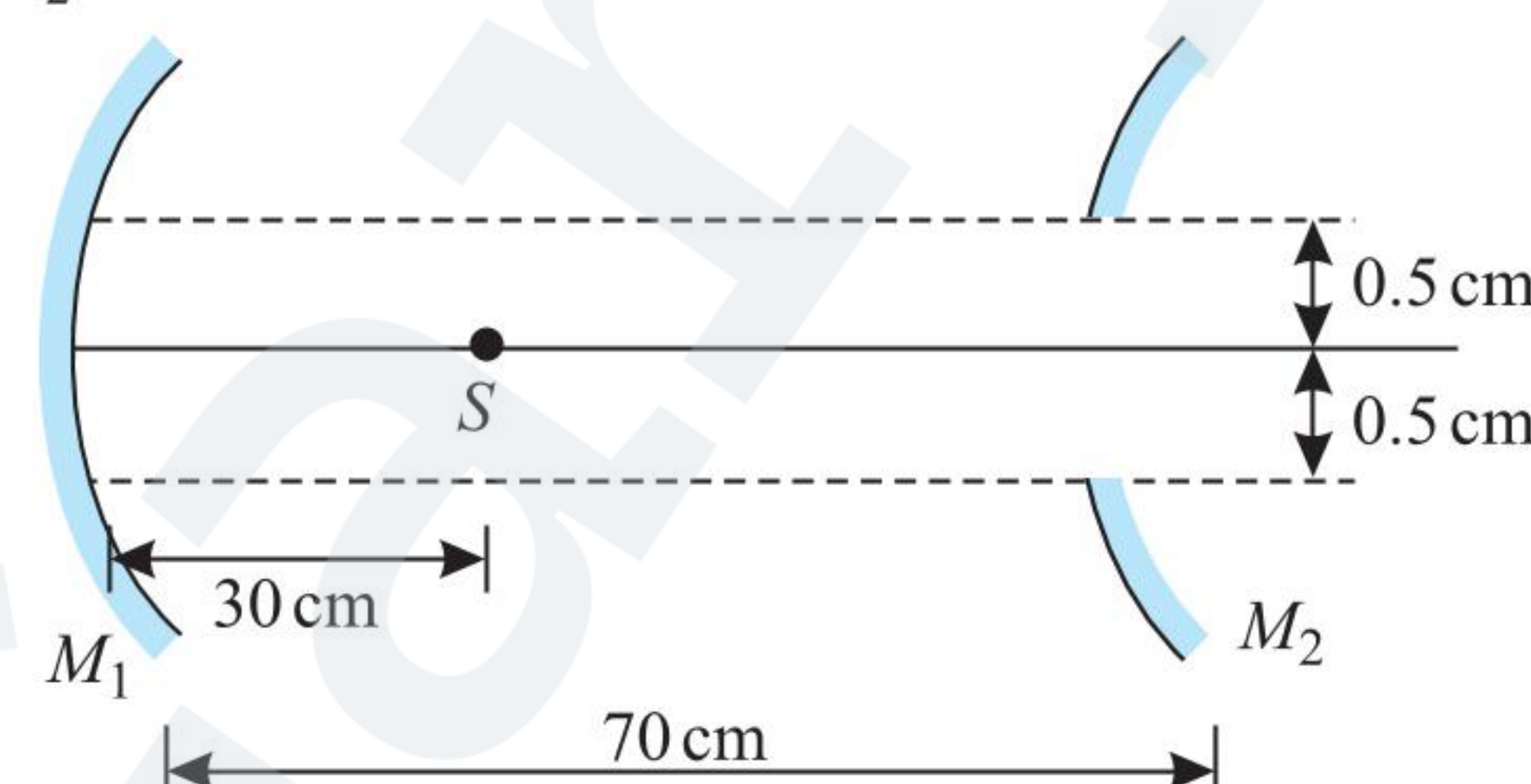


(i) Image formed by partial reflection from water surface

- (ii) Image formed by refraction from water surface followed by reflection from mirror and again refraction out of the water surface.

Find the distance between these two images in cm.

19. Two spherical mirrors M_1 and M_2 are arranged as shown in the figure. The mirror M_2 is cut into two halves and its halves are drawn symmetrically at a distance 1 cm apart in a direction perpendicular to optical axis. A point source S is placed at 30 cm from mirror M_1 , whose focal length is 20 cm. The focal length of mirror M_2 is 10 cm. If we consider the image formation due to successive reflection through M_1 and M_2 . Find the separation between image (in mm).



Archives

JEE ADVANCED

Single Correct Answer Type

1. Two beams of red and violet colors are made to pass separately through a prism (angle of the prism is 60°). In the position of minimum deviation, the angle of refraction will be
 (1) 30° for both the colors
 (2) greater for the violet color
 (3) greater for the red color
 (4) equal but not 30° for both the colors (IIT-JEE 2008)

2. A ball is dropped from a height of 20 m above the surface of water in a lake. The refractive index of water is $4/3$. A fish inside the lake, in the line of fall of the ball, is looking at the ball. At an instant when the ball is 12.8 m above the water surface, the fish sees the speed of ball as
 (1) 9 m s^{-1} (2) 12 m s^{-1}
 (3) 16 m s^{-1} (4) 21.33 m s^{-1}

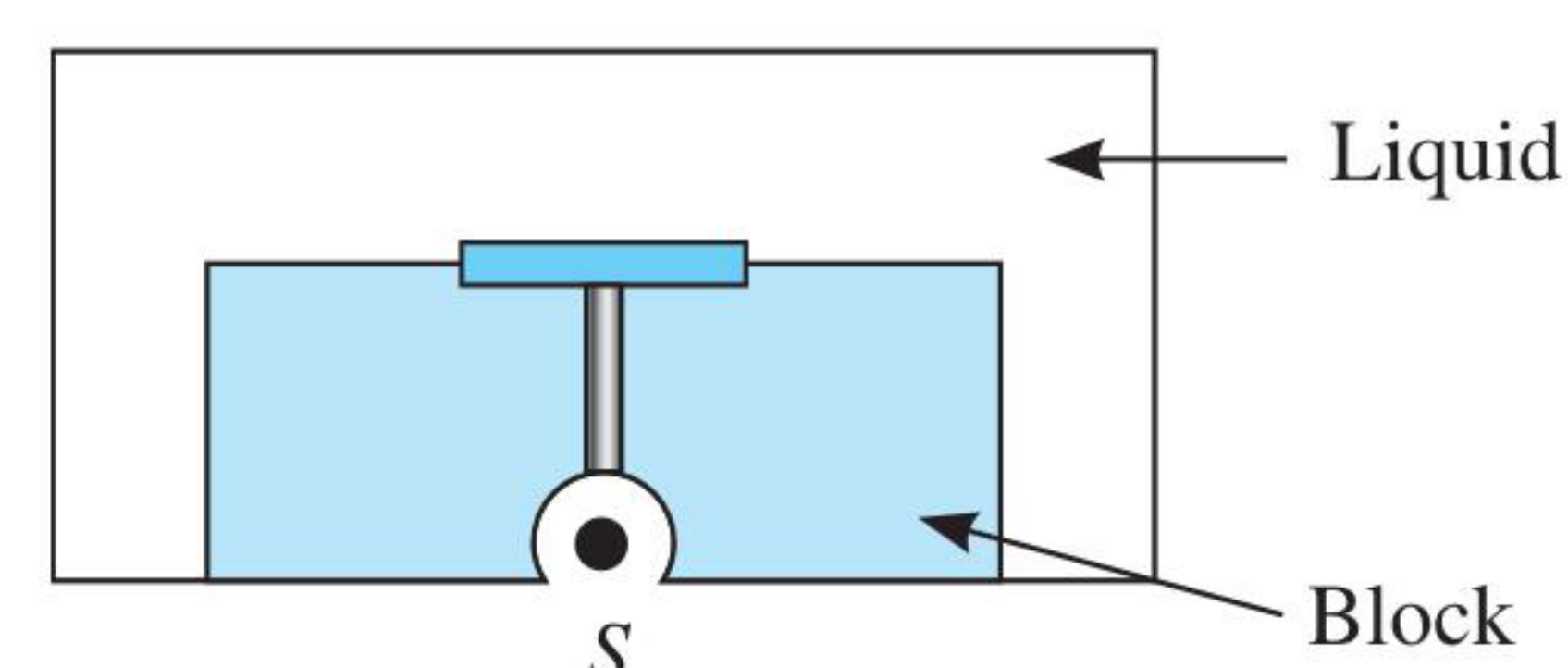
(IIT-JEE 2009)

3. A ray of light travelling in the direction $\frac{1}{2}(\hat{i} + \sqrt{3}\hat{j})$ is incident on a plane mirror. After reflection, it travels along the direction $\frac{1}{2}(\hat{i} - \sqrt{3}\hat{j})$. The angle of incidence is
 (1) 30° (2) 45°
 (3) 60° (4) 75°

(JEE Advanced 2013)

4. A point source S is placed at the bottom of a transparent block of height 10 mm and refractive index 2.72. It is immersed in a lower refractive index liquid as shown in

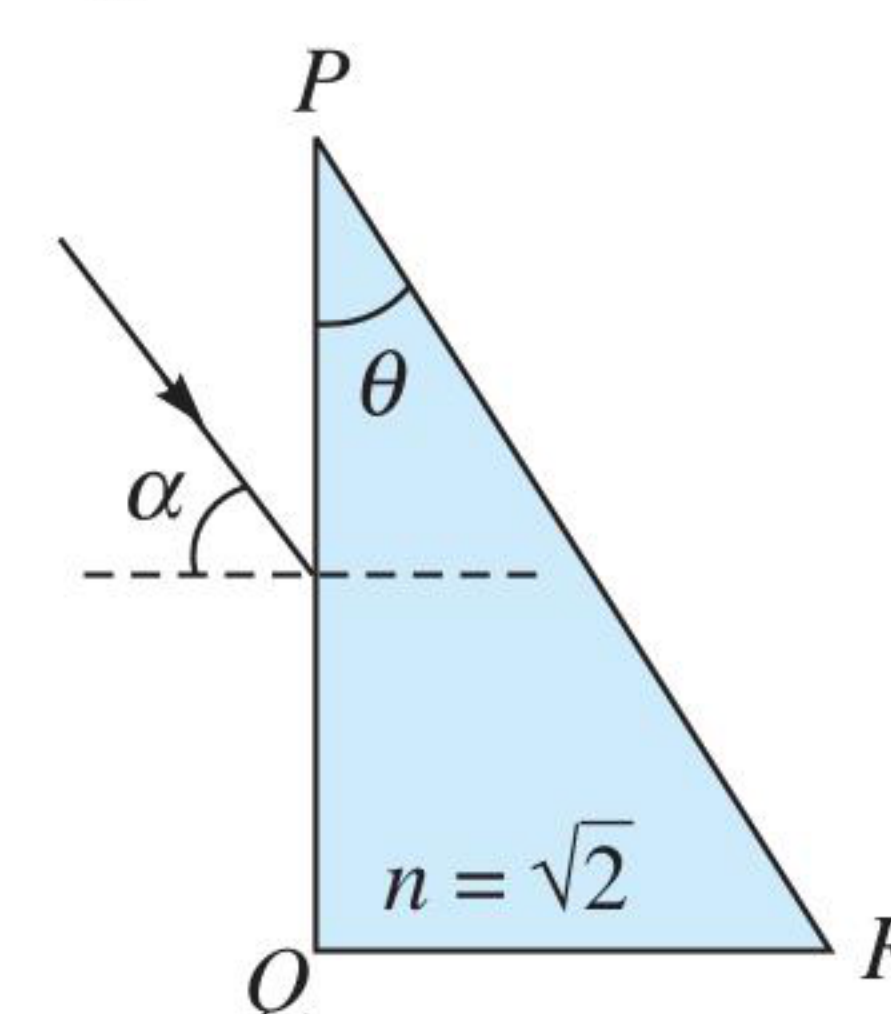
the figure. It is found that the light emerging from the block to the liquid forms a circular bright spot of diameter 11.54 mm on the top of the block. The refractive index of the liquid is



- (1) 1.21 (2) 1.30
 (3) 1.36 (4) 1.42

(JEE Advanced 2014)

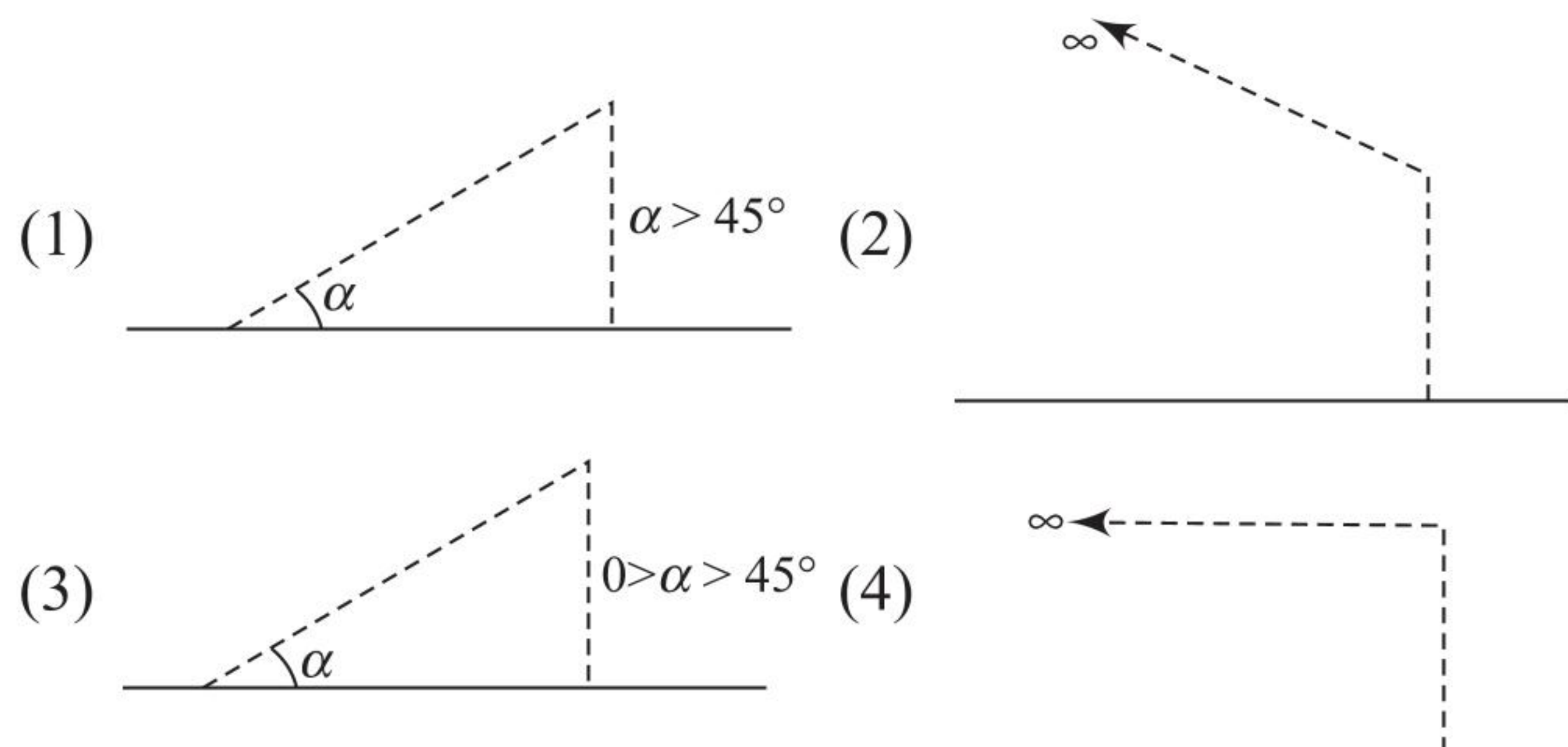
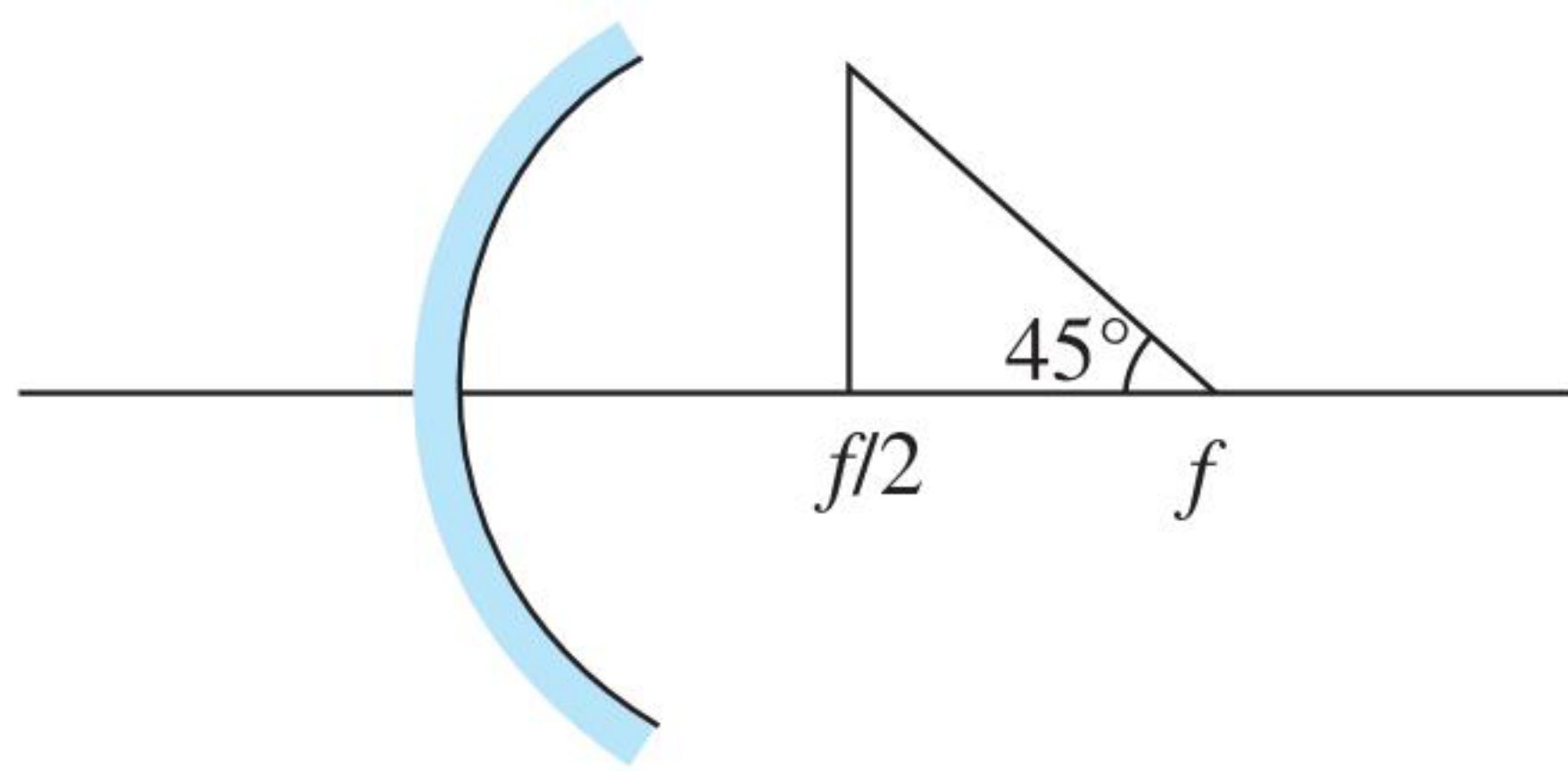
5. A parallel beam of light is incident from air at an angle α on the side PQ of a right angled triangular prism of refractive index $n = \sqrt{2}$. Light undergoes total internal reflection in the prism at the face PR when has a minimum value of 45° . The angle θ of the prism is



- (1) 15° (2) 22.5°
 (3) 30° (4) 45°

(JEE Advanced 2016)

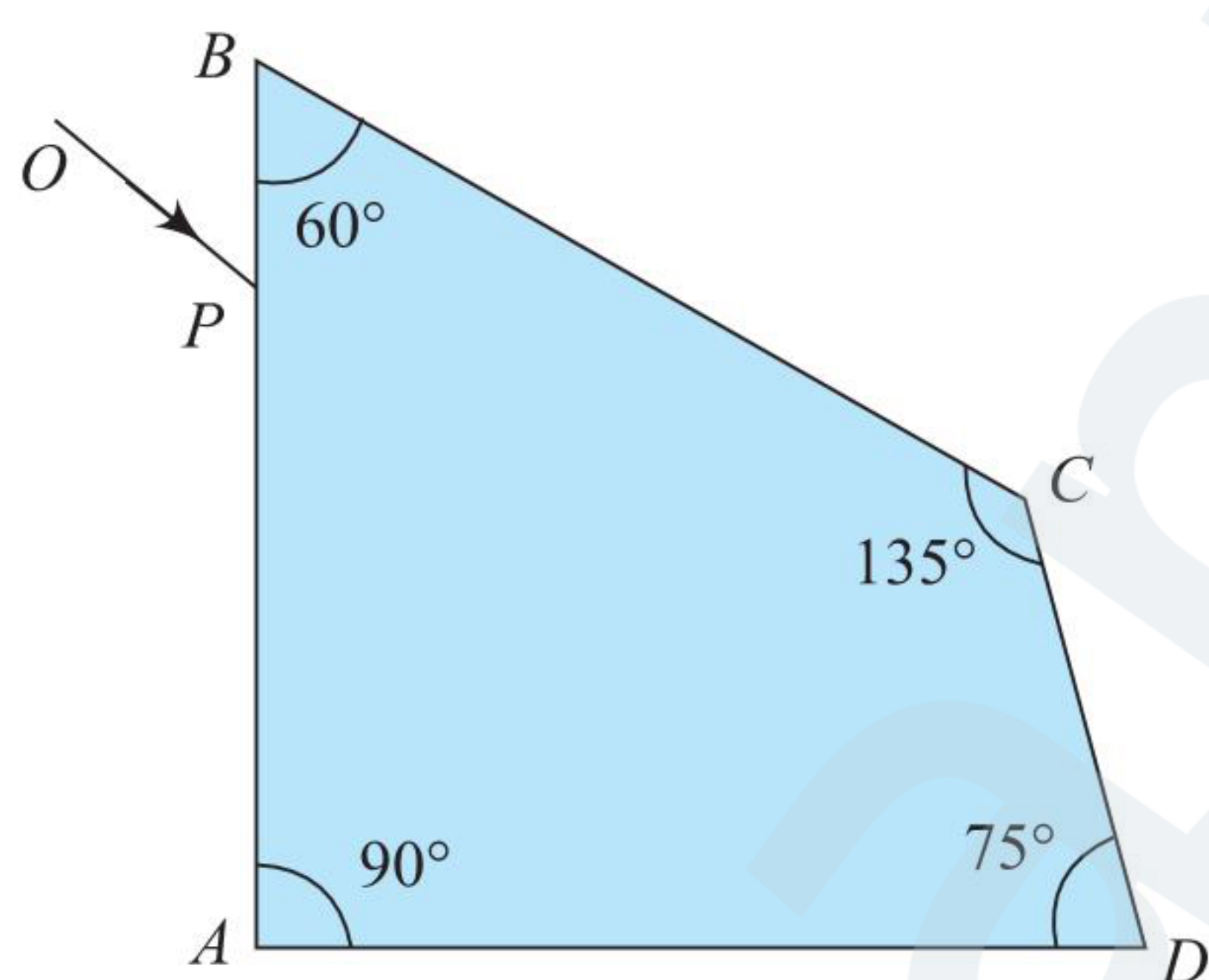
6. A wire is bent in the shape of a right angled triangle and is placed in front of a concave mirror of focal length f , as shown in the figure. Which of the figures shown in the four options qualitatively represent(s) the shape of the image of the bent wire? (These figures are not to scale.)



(JEE Advanced 2018)

Multiple Correct Answers Type

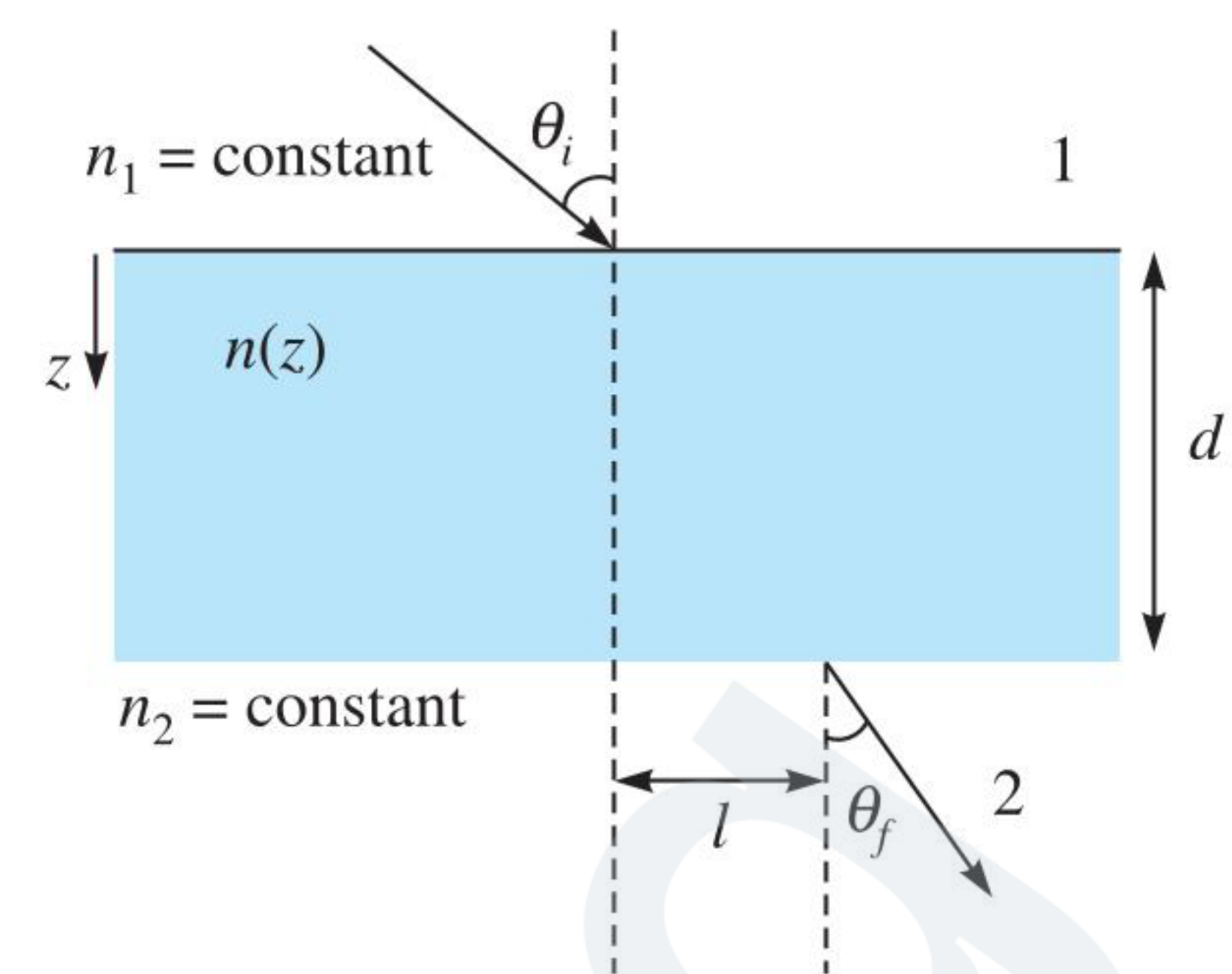
1. A ray OP of monochromatic light is incident on the face AB of prism $ABCD$ near vertex B at an incident angle of 60° (see figure). If the refractive index of the material of the prism is $\sqrt{3}$, which of the following is (are) correct?



- (1) The ray gets totally internally reflected at face CD
- (2) The ray comes out through face AD
- (3) The angle between the incident ray and the emergent ray is 90°
- (4) The angle between the incident ray and the emergent ray is 120°

(IIT-JEE 2010)

2. A transparent slab of thickness d has a refractive index $n(z)$ that increases with z . Here z is the vertical distance inside the slab, measured from the top. The slab is placed between two media with uniform refractive indices n_1 and n_2 ($n_2 > n_1$), as shown in the figure. A ray of light is incident with angle θ_i from medium 1 and emerges in medium 2 with refraction angle θ_f with a lateral displacement l . Which of the following statement(s) is (are) true?



- (1) l is independent of n_2
- (2) l is dependent on $n(z)$
- (3) $n_1 \sin \theta_i = (n_2 - n_1) \sin \theta_f$
- (4) $n_1 \sin \theta_i = n_2 \sin \theta_f$

(JEE Advanced 2016)

3. For an isosceles prism of angle A and refractive index μ , it is found that the angle of minimum deviation $\delta_m = A$. Which of the following options is/are correct?

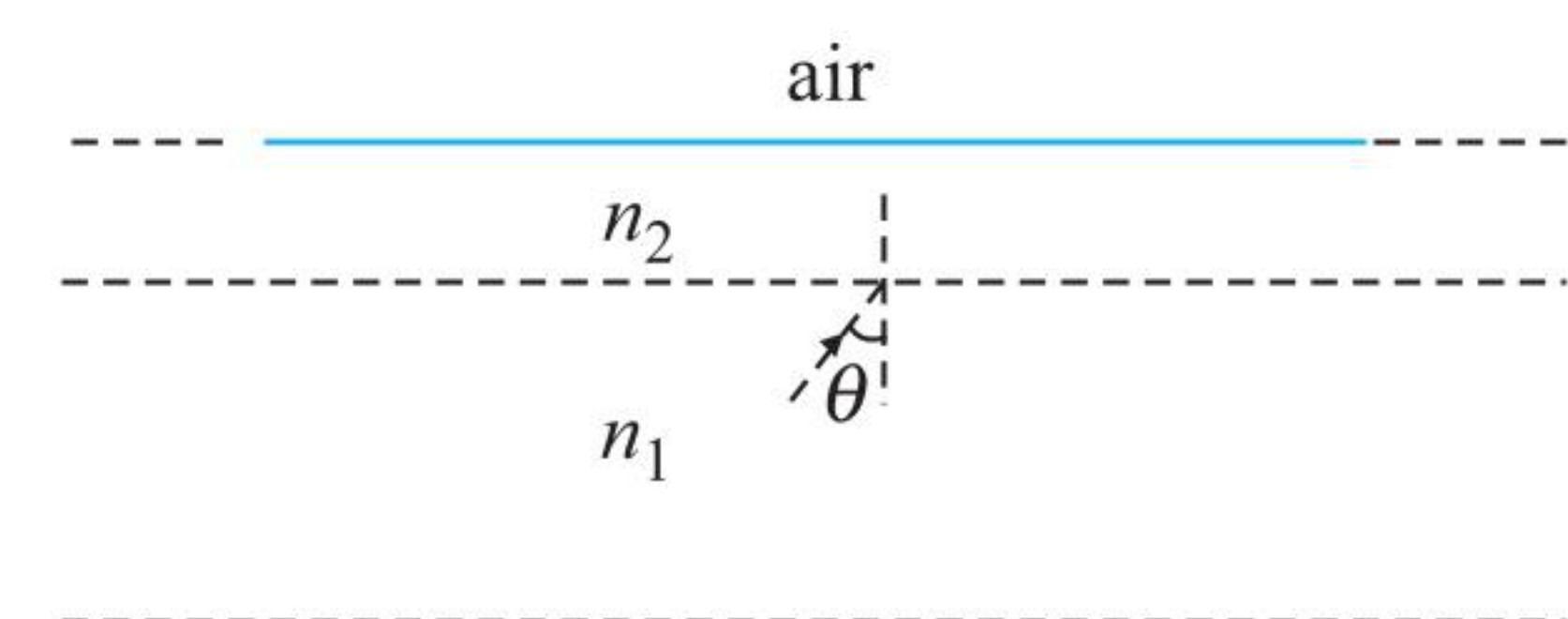
- (1) At minimum deviation, the incident angle i_1 and the refracting angle r_1 at the first refracting surface are related by $r_1 = (i_1 / 2)$
- (2) For this prism, the refractive index μ and the angle of prism A are related as $A = \frac{1}{2} \cos^{-1} \left(\frac{\mu}{2} \right)$

- (3) For the angle of incidence $i_1 = A$, the ray inside the prism is parallel to the base of the prism

- (4) For this prism, the emergent ray at the second surface will be tangential to the surface when the angle of incidence at the first surface is $i_1 = \sin^{-1} \left[\sin A \sqrt{4 \cos^2 \frac{A}{2} - 1} - \cos A \right]$

(JEE Advanced 2017)

4. A wide slab consisting of two media of refractive indices n_1 and n_2 is placed in air as shown in the figure. A ray of light is incident from medium n_1 to n_2 at an angle θ , where $\sin \theta$ is slightly larger than $1/n_1$. Take refractive index of air as 1. Which of the following statement(s) is (are) correct?



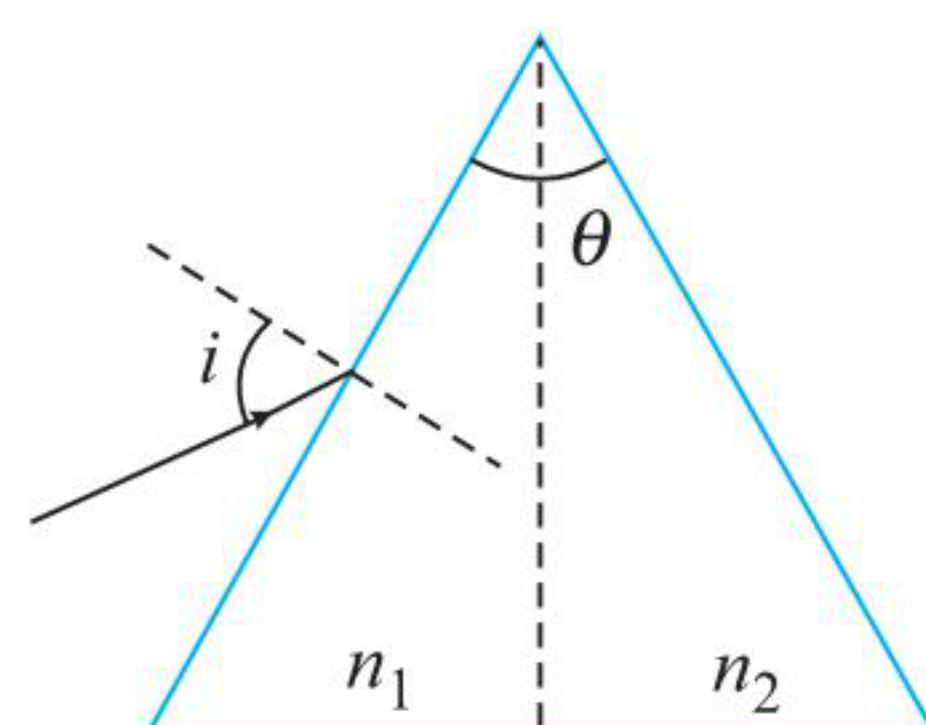
- (1) The light ray enters air if $n_2 = n_1$
- (2) The light ray is finally reflected back into the medium of refractive index n_1 if $n_2 < n_1$
- (3) The light ray is finally reflected back into the medium of refractive index n_1 if $n_2 > n_1$
- (4) The light ray is reflected back into the medium of refractive index n_1 if $n_2 = 1$

(JEE Advanced 2021)

5. For a prism of prism angle $\theta = 60^\circ$, the refractive indices of the left half and the right half are, respectively, n_1 and n_2 ($n_2 \geq n_1$) as shown in the figure. The angle of incidence i is chosen such that the incident light rays will have

minimum deviation if $n_1 = n_2 = n = 1.5$. For the case of unequal refractive indices, $n_1 = n$ and $n_2 = n + \Delta n$ (where $\Delta n \ll n$), the angle of emergence $e = i + \Delta e$.

Which of the following statement(s) is (are) correct?



- (1) The value of Δe (in radians) is greater than that of Δn
- (2) Δe is proportional to Δn
- (3) Δe lies between 2.0 and 3.0 milliradians, if $\Delta n = 2.8 \times 10^{-3}$
- (4) Δe lies between 1.0 and 1.6 milliradians, if $\Delta n = 2.8 \times 10^{-3}$

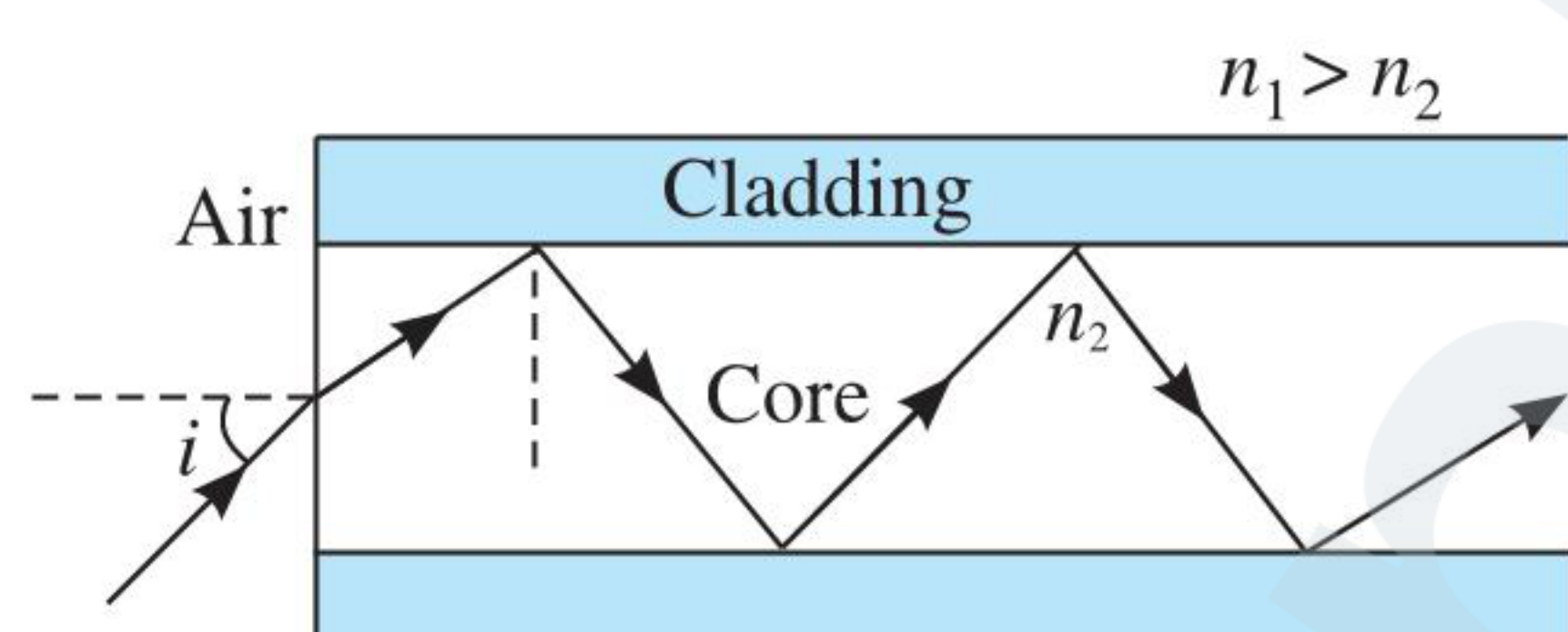
(JEE Advanced 2021)

Linked Comprehension Type

For Problems 1–2

Light guidance in an optical fiber can be understood by considering a structure comprising of thin solid glass cylinder of refractive index n_1 surrounded by a medium of lower refractive index n_2 . The light guidance in the structure takes place due to successive total internal reflections at the interface of the media n_1 and n_2 as shown in the figure. All rays with the angle of incidence i less than a particular value i_m are confined in the medium of refractive index n_1 . The numerical aperture (NA) of the structure is defined as $\sin i_m$.

(JEE Advanced 2015)



1. For two structures namely S_1 with $n_1 = \sqrt{45}/4$ and $n_2 = 3/2$, and S_2 with $n_1 = 8/5$ and $n_2 = 7/5$ and taking the refractive index of water to be $4/3$ and that of air to be 1, the correct option(s) is (are)

- (1) NA of S_1 immersed in water is the same as that of S_2 immersed in a liquid of refractive index $\frac{16}{3\sqrt{15}}$

- (2) NA of S_1 immersed in liquid of refractive index $\frac{16}{\sqrt{15}}$ is that as that of S_2 immersed in water

- (3) NA of S_1 placed in air is the same as that of S_2 immersed in liquid of refractive index $\frac{4}{\sqrt{15}}$

- (4) NA of S_1 placed in air is the same as that of S_2 placed in water

2. If two structures of same cross-sectional area, but different numerical apertures NA_1 and NA_2 ($NA_2 < NA_1$) are joined longitudinally, the numerical aperture of the combined structure is

$$(1) \frac{NA_1 NA_2}{NA_1 + NA_2}$$

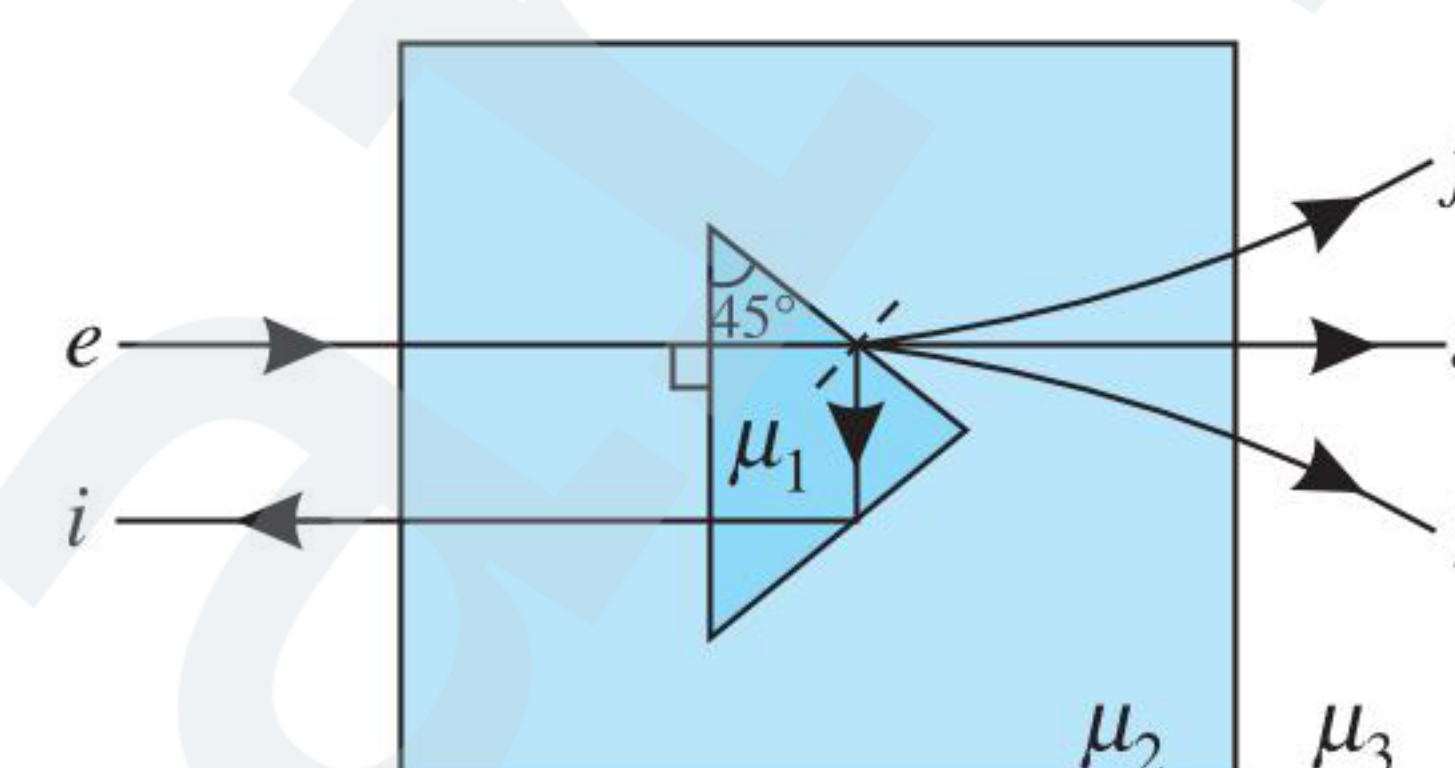
$$(2) NA_1 + NA_2$$

$$(3) NA_1$$

$$(4) NA_2$$

Matrix Match Type

1. A right angled prism of refractive index μ_1 is placed in a rectangular block of refractive index μ_2 , which is surrounded by a medium of refractive index μ_3 , as shown in the figure. A ray of light e enters the rectangular block at normal incidence. Depending upon the relationships between μ_1 , μ_2 and μ_3 , it takes one of the four possible paths ef , eg , eh , or ei .



Match the paths in Column I with conditions of refractive indices in Column II and select the correct answer using the codes given below the lists:

Column I	Column II
i. $e \rightarrow f$	a. $\mu_1 > \sqrt{2} \mu_2$
ii. $e \rightarrow g$	b. $\mu_2 > \mu_1$ and $\mu_2 > \mu_3$
iii. $e \rightarrow h$	c. $\mu_1 = \mu_2$
iv. $e \rightarrow i$	d. $\mu_2 < \mu_1 < \sqrt{2} \mu_2$ and $\mu_2 > \mu_3$

Codes:

	i.	ii.	iii.	iv.
(1)	b.	c.	a.	d.
(2)	a.	b.	d.	c.
(3)	d.	a.	b.	c.
(4)	b.	c.	d.	a.

(JEE Advanced 2013)

Numerical Value Type

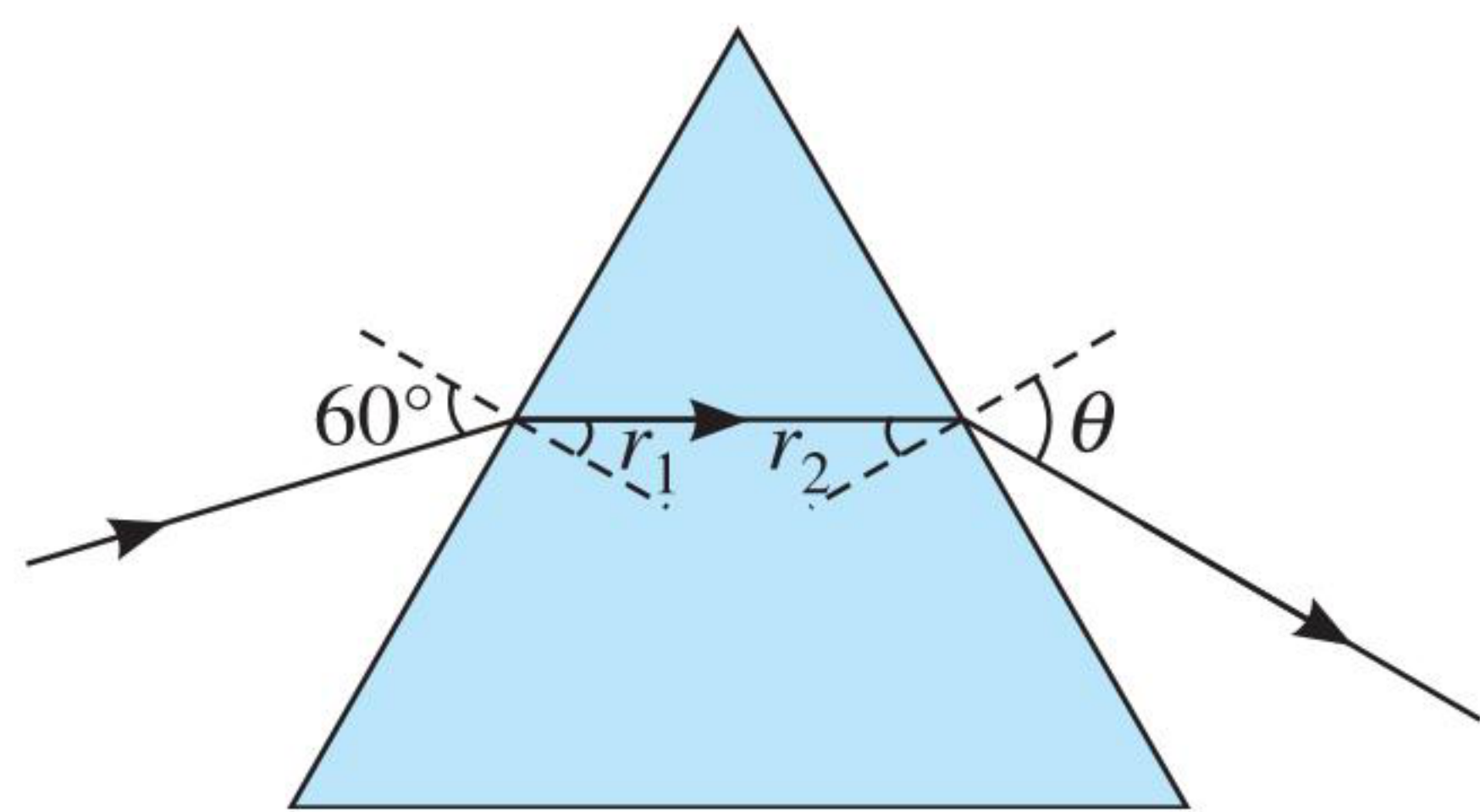
1. A large glass slab ($\mu = 5/3$) of thickness 8 cm is placed over a point source of light on a plane surface. It is seen that light emerges out of the top surface of the slab from a circular area of radius R cm. What is the value of R ?

(IIT-JEE 2010)

2. Image of an object approaching a convex mirror of radius of curvature 20 m along its optical axis is observed to move from $25/3$ m to $50/7$ m in 30 s. What is the speed of the object in km per hour?

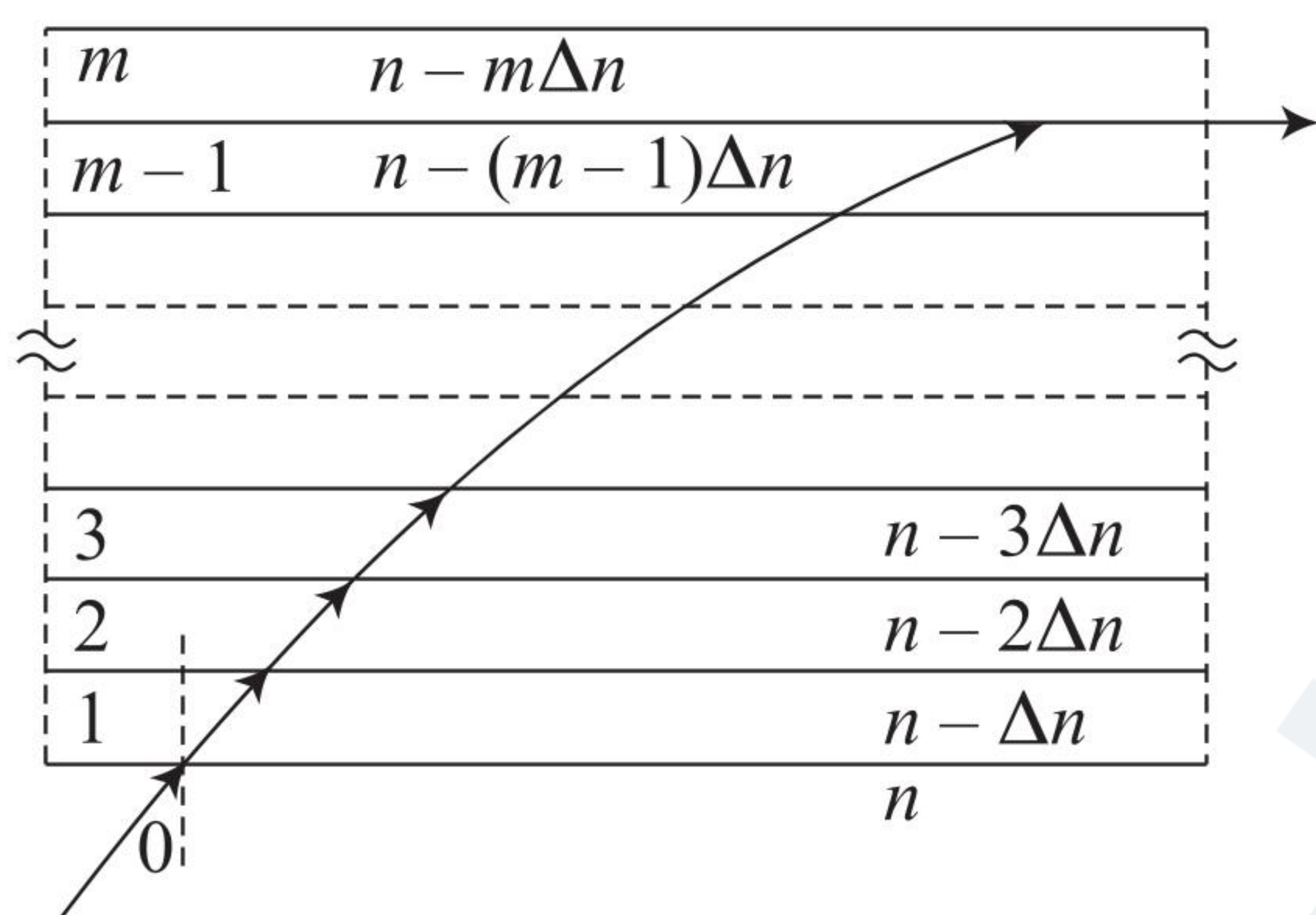
(IIT-JEE 2010)

3. A monochromatic beam of light is incident at 60° on one face of an equilateral prism of refractive index n and emerges from the opposite face making an angle $\theta(n)$ with the normal (see the figure). For $n = \sqrt{3}$ the value of θ is 60° and $\frac{d\theta}{dn} = m$. The value of m is



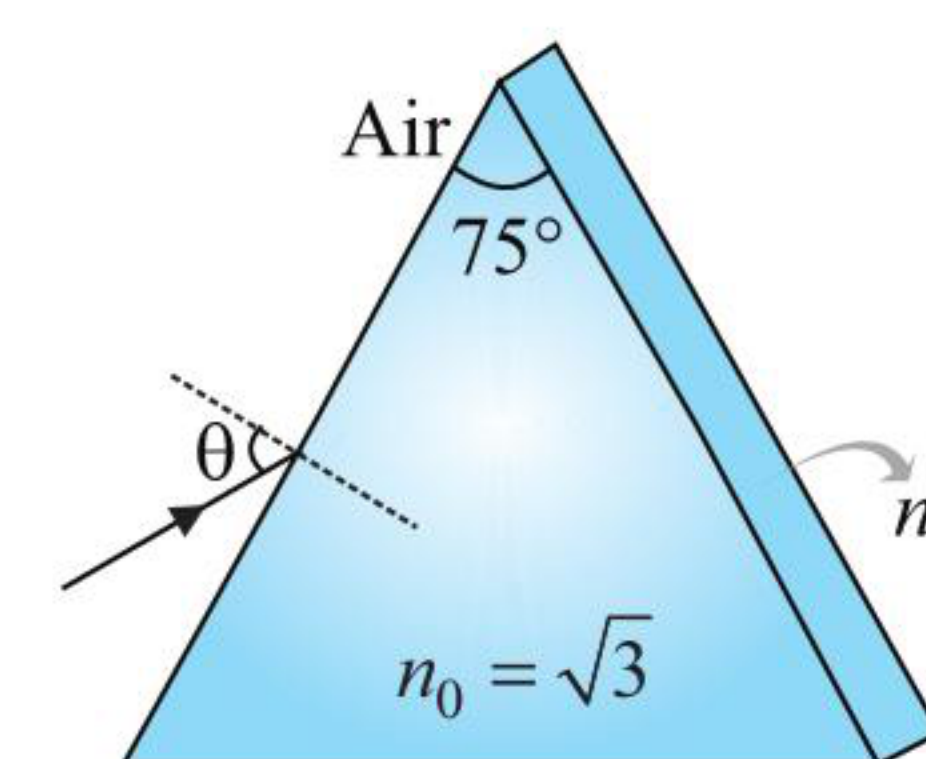
(JEE Advanced 2015)

4. A monochromatic light is travelling in a medium of refractive index $n = 1.6$. It enters a stack of glass layers from the bottom side at an angle $\theta = 30^\circ$. The interfaces of the glass layers are parallel to each other. The refractive indices of different glass layers are monotonically decreasing as $n_m = n - m\Delta n$, where n_m is the refractive index of the m^{th} slab and $\Delta n = 0.1$ (see the figure). The ray is refracted out parallel to the interface between the $(m-1)^{\text{th}}$ and m^{th} slabs from the right side of the stack. What is the value of m ?



(JEE Advanced 2017)

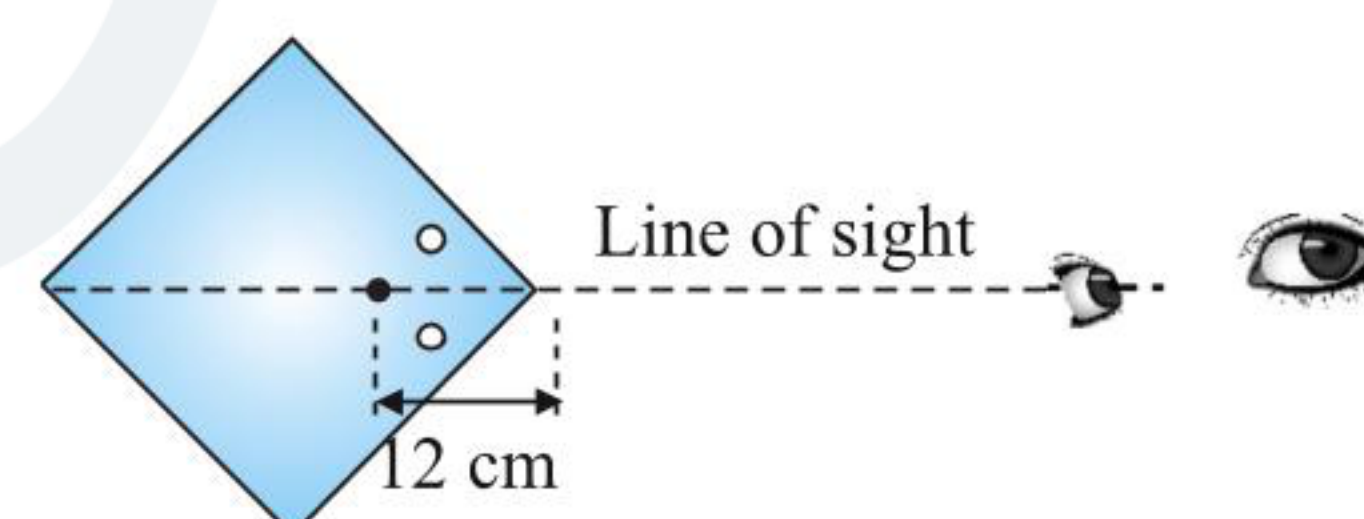
5. A monochromatic light is incident from air on a refracting surface of a prism of angle 75° and refractive index $n_0 = \sqrt{3}$. The other refracting surface of the prism is coated by a thin film of material of refractive



index n as shown in figure. The light suffers total internal reflection at the coated prism surface for an incidence angle of $\theta \leq 60^\circ$. The value of n^2 is _____.

(JEE Advanced 2019)

6. A large square container with thin transparent vertical walls and filled with water (refractive index $4/3$) is kept on a horizontal table. A student holds a thin straight wire vertically inside the water 12 cm from one of its corners, as shown schematically in the figure. Looking at the wire from this corner, another student sees two images of the wire, located symmetrically on each side of the line of sight as shown. The separation (in cm) between these images is _____.



(JEE Advanced 2020)

Answers Key

EXERCISES

Single Correct Answer Type

- | | | | | |
|---------|---------|---------|---------|---------|
| 1. (3) | 2. (4) | 3. (1) | 4. (1) | 5. (2) |
| 6. (3) | 7. (1) | 8. (3) | 9. (4) | 10. (3) |
| 11. (2) | 12. (4) | 13. (2) | 14. (4) | 15. (2) |
| 16. (1) | 17. (2) | 18. (3) | 19. (2) | 20. (4) |
| 21. (1) | 22. (1) | 23. (1) | 24. (4) | 25. (4) |
| 26. (4) | 27. (1) | 28. (2) | 29. (1) | 30. (2) |
| 31. (1) | 32. (4) | 33. (1) | 34. (2) | 35. (3) |
| 36. (3) | 37. (2) | 38. (3) | 39. (2) | 40. (3) |
| 41. (1) | 42. (1) | 43. (1) | 44. (2) | 45. (3) |
| 46. (3) | 47. (2) | 48. (2) | 49. (1) | 50. (1) |
| 51. (2) | 52. (1) | 53. (4) | 54. (4) | 55. (4) |
| 56. (4) | 57. (4) | 58. (4) | 59. (2) | 60. (4) |
| 61. (3) | 62. (3) | 63. (4) | 64. (4) | 65. (1) |
| 66. (4) | 67. (1) | 68. (1) | 69. (1) | 70. (2) |
| 71. (4) | 72. (2) | 73. (1) | 74. (2) | 75. (2) |
| 76. (1) | 77. (2) | 78. (1) | 79. (3) | 80. (2) |

Multiple Correct Answers Type

- | | | |
|---------------------|-----------------|---------------------|
| 1. (3),(4) | 2. (1),(3) | 3. (1),(2) |
| 4. (1),(3),(4) | 5. (1),(2),(3) | 6. (1),(3) |
| 7. (1),(2),(3),(4) | 8. (2),(3) | 9. (1),(2),(3),(4) |
| 10. (2),(3),(4) | 11. (1),(4) | 12. (2),(4) |
| 13. (1),(2),(3),(4) | 14. (2),(3) | 15. (2),(4) |
| 16. (1),(3) | 17. (2),(4) | 18. (2),(4) |
| 19. (2),(3) | 20. (1),(3) | 21. (1),(3) |
| 22. (2),(3) | 23. (1),(3) | 24. (2),(3) |
| 25. (1),(2) | 26. (1),(2),(3) | 27. (1),(2),(3),(4) |
| 28. (1),(2),(3),(4) | 29. (2),(3) | 30. (2),(4) |
| 31. (1),(3) | 32. (1),(3) | 33. (1),(3) |
| 34. (3),(4) | | |

Linked Comprehension Type

- | | | | | |
|---------|---------|---------|---------|---------|
| 1. (2) | 2. (1) | 3. (4) | 4. (3) | 5. (3) |
| 6. (1) | 7. (4) | 8. (1) | 9. (4) | 10. (2) |
| 11. (4) | 12. (3) | 13. (4) | 14. (3) | 15. (4) |
| 16. (2) | | | | |

Matrix Match Type

1. i. $\rightarrow$ a.; ii. $\rightarrow$ c.; iii. $\rightarrow$ b.; iv. $\rightarrow$ d.
 2. i. $\rightarrow$ b.; ii. $\rightarrow$ c., d.; iii. $\rightarrow$ a.; iv. $\rightarrow$ c., d.
 3. i. $\rightarrow$ b.; ii. $\rightarrow$ c.; iii. $\rightarrow$ a.; iv. $\rightarrow$ d.
 4. (1) 5. (1) 6. (2) 7. (2) 8. (4)

Numerical Value Type

- | | | | | |
|-------------|-----------|------------|---------|----------|
| 1. (3) | 2. (3) | 3. (8) | 4. (0) | 5. (6) |
| 6. (21) | 7. (1.95) | 8. (42) | 9. (4) | 10. (45) |
| 11. (3) | 12. (60) | 13. (12.5) | 14. (2) | 15. (80) |
| 16. (22.50) | 17. (42) | 18. (3) | 19. (5) | |

ARCHIVES

JEE Advanced

Single Correct Answer Type

- | | | | | |
|--------|--------|--------|--------|--------|
| 1. (1) | 2. (3) | 3. (1) | 4. (3) | 5. (1) |
| 6. (4) | | | | |

Multiple Correct Answers Type

- | | | |
|----------------|----------------|----------------|
| 1. (1),(2),(3) | 2. (1),(2),(4) | 3. (1),(3),(4) |
| 4. (2),(3),(4) | 5. (2),(3) | |

Linked Comprehension Type

- | | |
|------------|--------|
| 1. (1),(3) | 2. (4) |
|------------|--------|

Matrix Match Type

- | |
|--------|
| 1. (4) |
|--------|

Numerical Value Type

- | | | | | |
|--------|--------|--------|--------|-----------|
| 1. (6) | 2. (3) | 3. (2) | 4. (8) | 5. (1.50) |
| 6. (2) | | | | |

Solutions

Chapter 1

Concept Application Exercises

Exercise 1.1

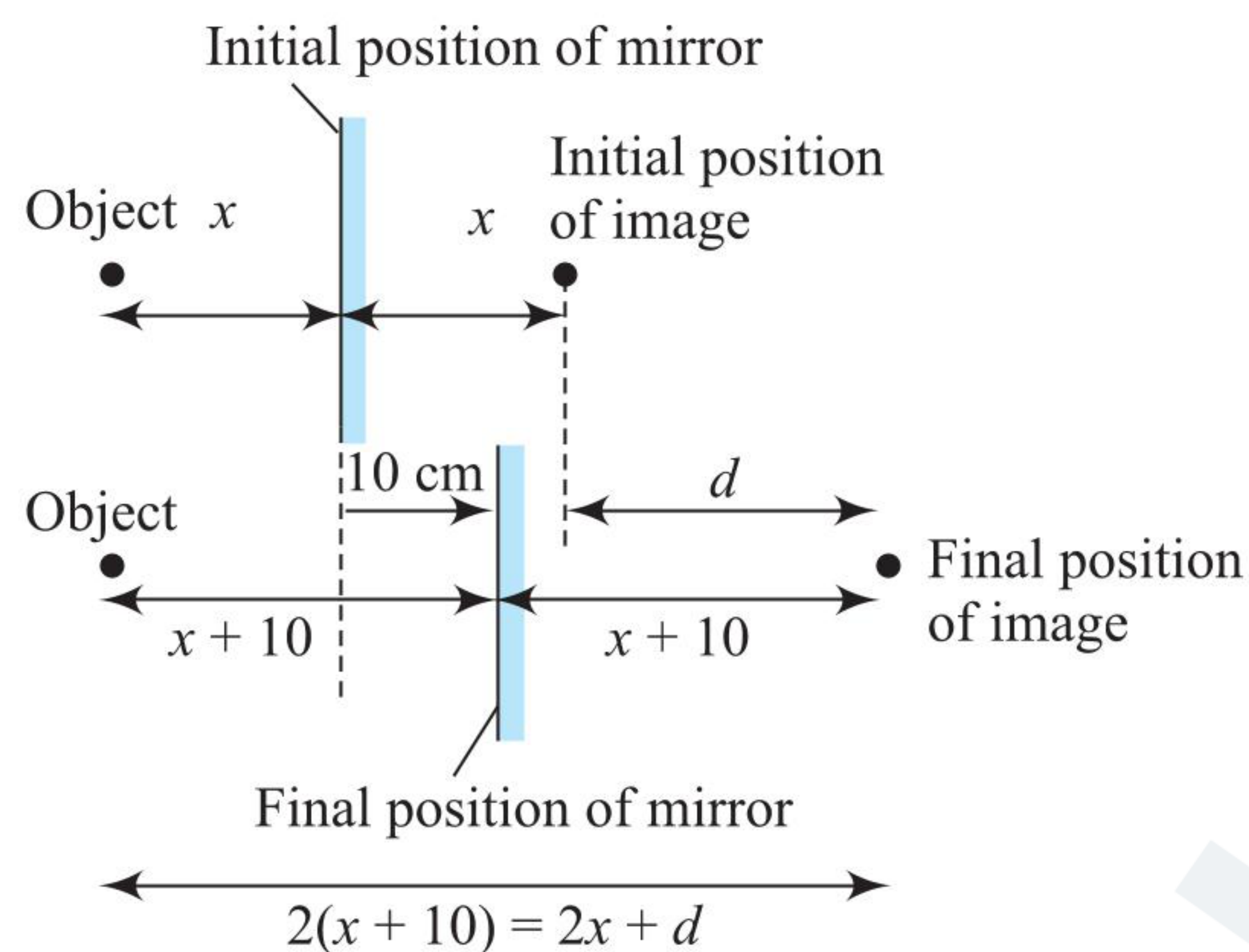
1. We know that

$$x_{im} = -x_{om} \text{ or } x_i - x_m = x_m - x_o$$

$$\text{or } \Delta x_i - \Delta x_m = \Delta x_m - \Delta x_o$$

$$\text{Here } \Delta x_o = 0; \Delta x_m = 10 \text{ cm}$$

$$\text{Therefore, } \Delta x_i = 2\Delta x_m - \Delta x_o = 20 \text{ cm}$$



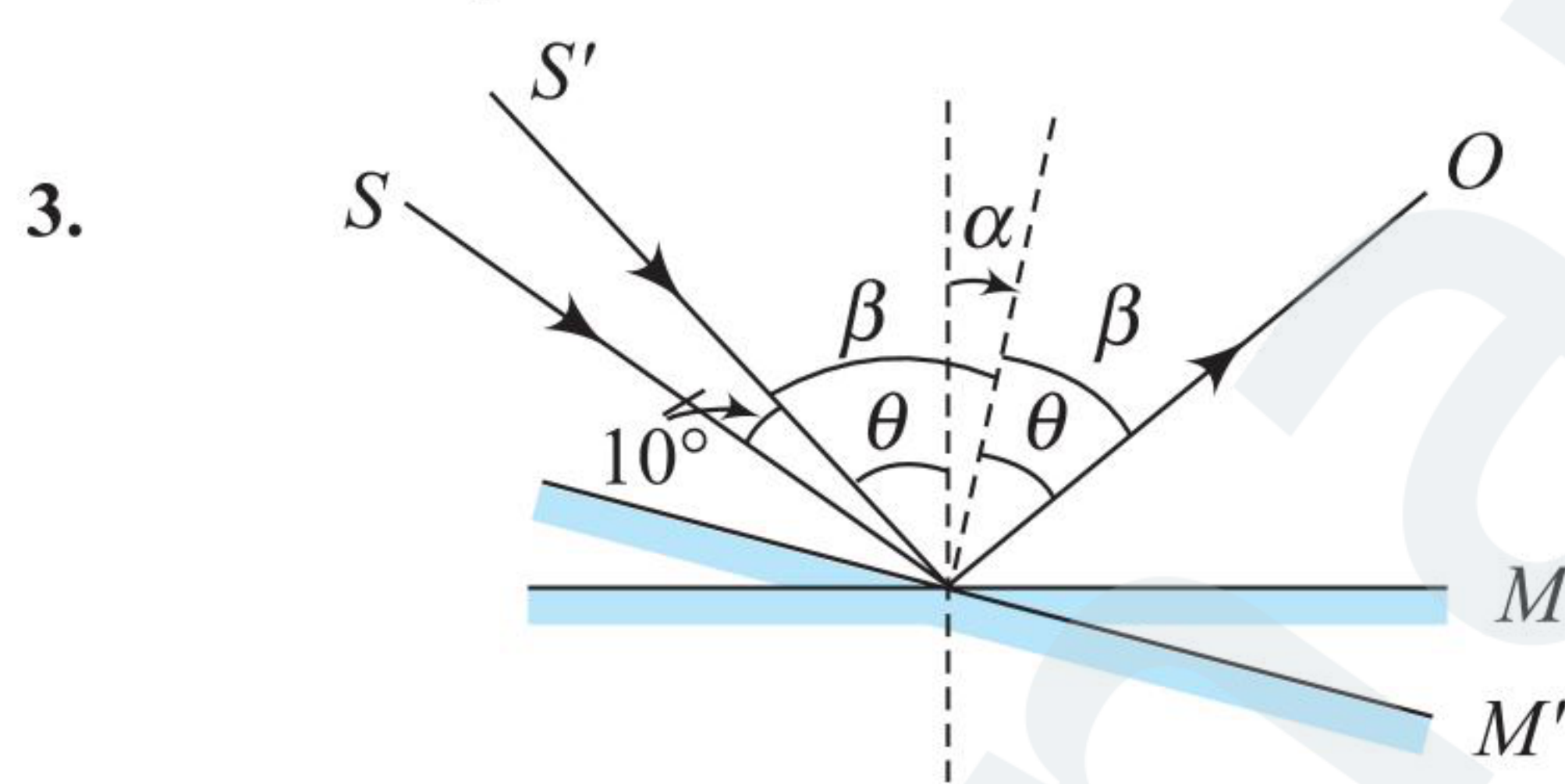
$$\therefore d = 20 \text{ cm}$$

2. The first image at M_1 is formed at a distance of 4 cm behind it.

The first image at M_2 is 6 cm behind it.

The second image at M_2 is at a distance of $4 + 10 = 14$ cm behind M_2 .

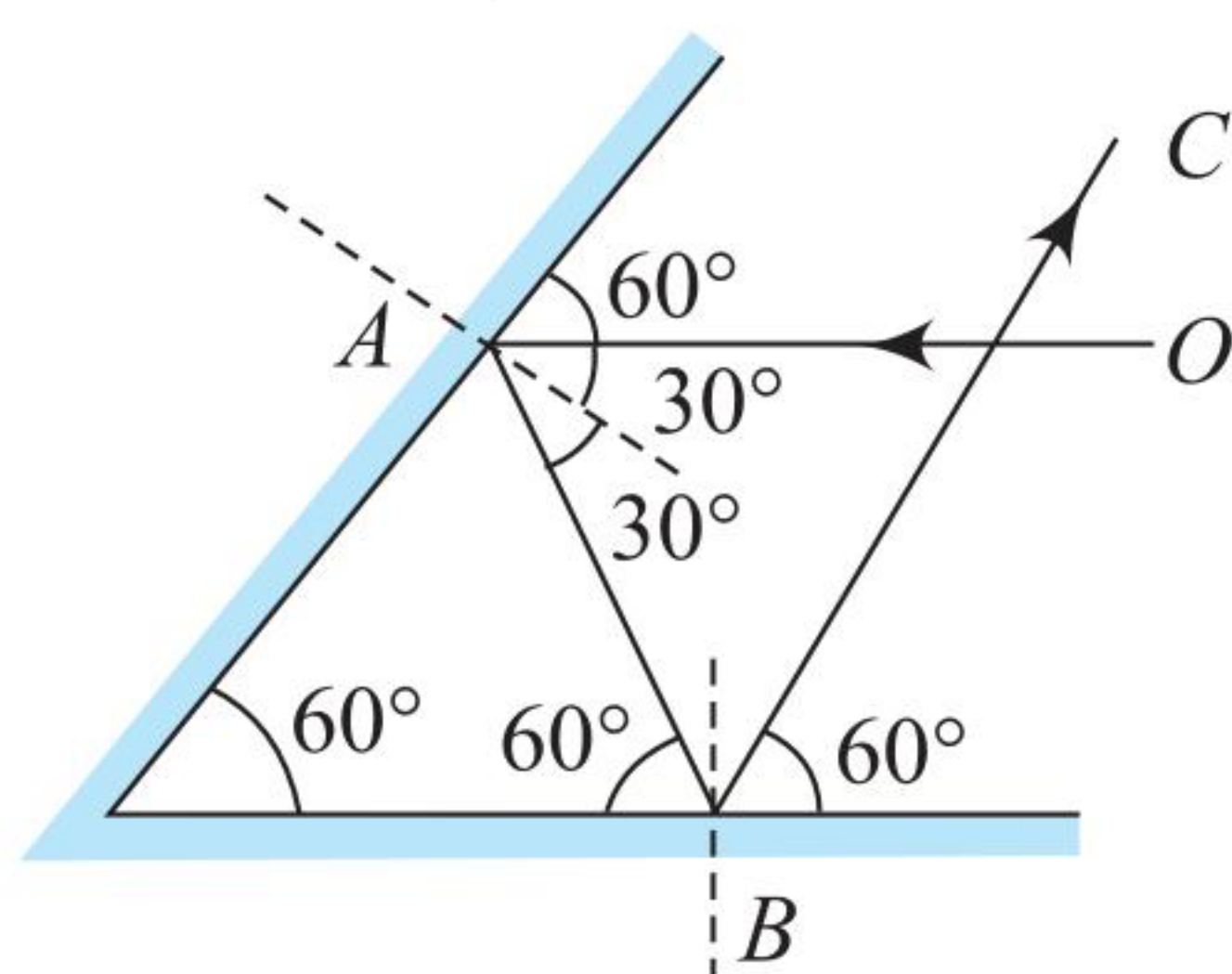
Therefore, the distance between the first image at M_1 and second image at M_2 is $4 + 14 + 10 = 28$ cm.



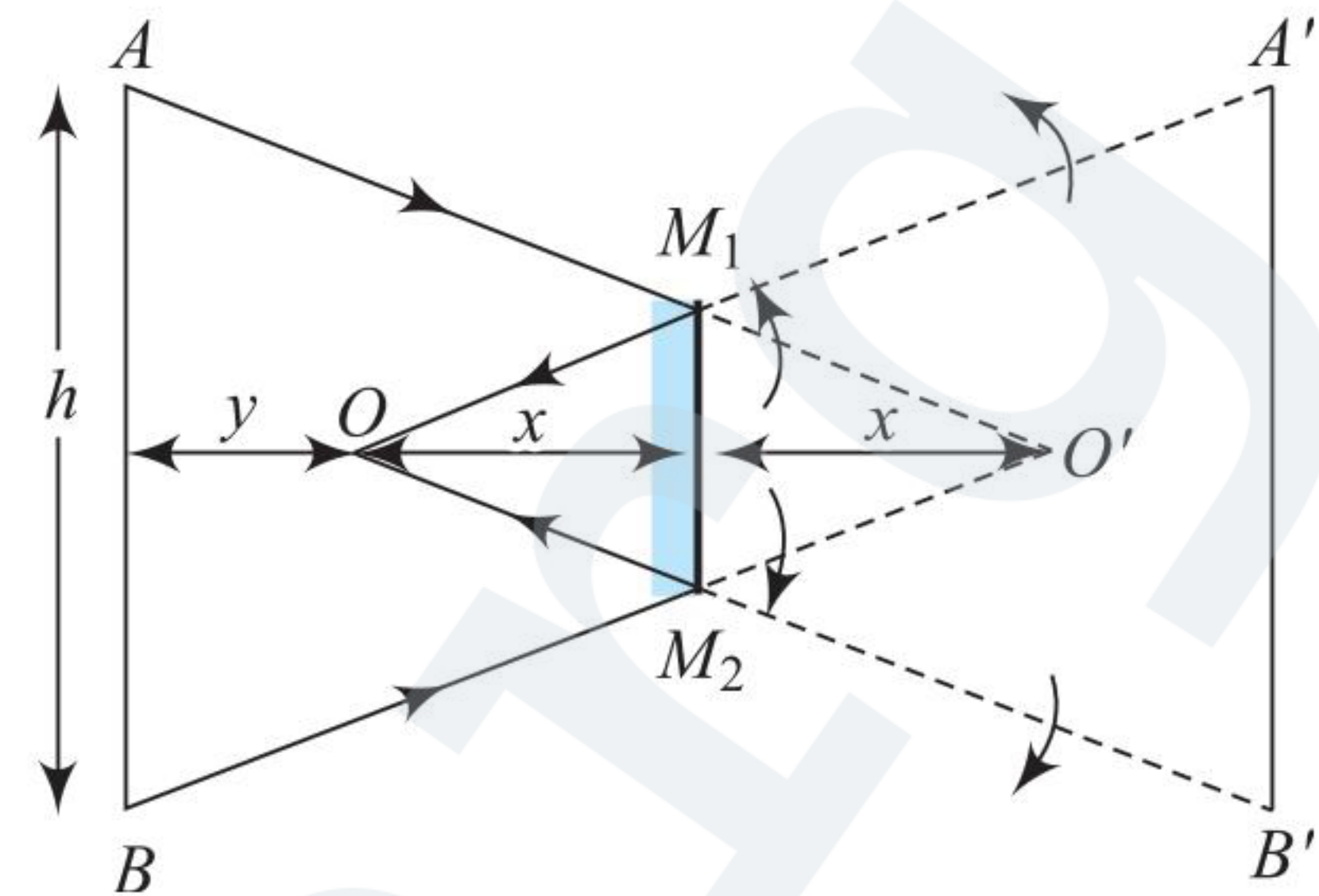
$$\theta + \alpha - 10^\circ = \beta \text{ and } \theta = \beta + \alpha$$

$$\Rightarrow \alpha = 5^\circ \text{ clockwise}$$

4. The angle that the ray makes after each reflection is clearly labeled in the following figure. It is clear that final ray makes an angle of 60° with the horizontal, or in other words it emerges parallel to mirror 1.



5.



From triangles $Q'M_1M_2$ and $O'AB$

$$\frac{M_1M_2}{h} = \frac{x}{2x + y}$$

$$\text{Size of mirror, } M_1M_2 = \frac{hx}{y + 2x}$$

6. The image of point A in the mirror is at A' (6, 0).

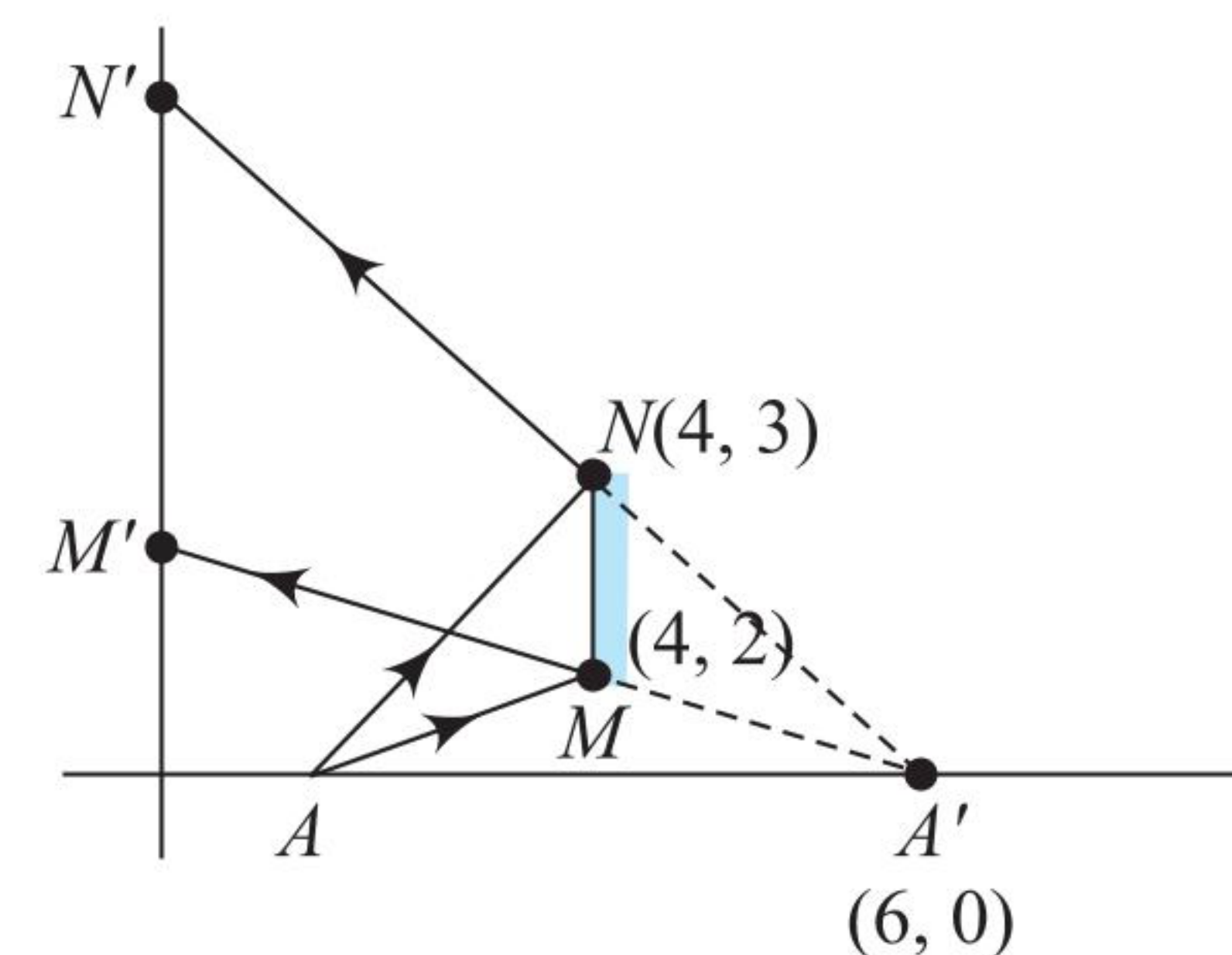
Join $A'M$ and extend to cut Y -axis at M' (ray originating from A which strikes the mirror at M gets reflected as the ray MM' which appears to come from A'). Join $A'N$ and extend to cut Y -axis at N' (ray originating from A which strikes the mirror at N gets reflected as the ray NN' which appears to come from A').

From Geometry:

$$M' \equiv (0, 6)$$

$$\text{and } N' \equiv (0, 9)$$

$M'N'$ is the region on Y -axis in which reflected rays are present.



7. Take direction toward right as positive.

$$v_i - v_m = v_m - v_o$$

$$v_i - (-1) = (-1) - 5$$

$$\therefore v_i = -7 \text{ m s}^{-1}$$

$\therefore$ The velocity of image is 7 m s^{-1} and direction is toward left.

8. Velocity of point P , $v_p = 10(\hat{i} + \hat{j})$

$$\text{Velocity of mirror } v_m = -5\hat{i}$$

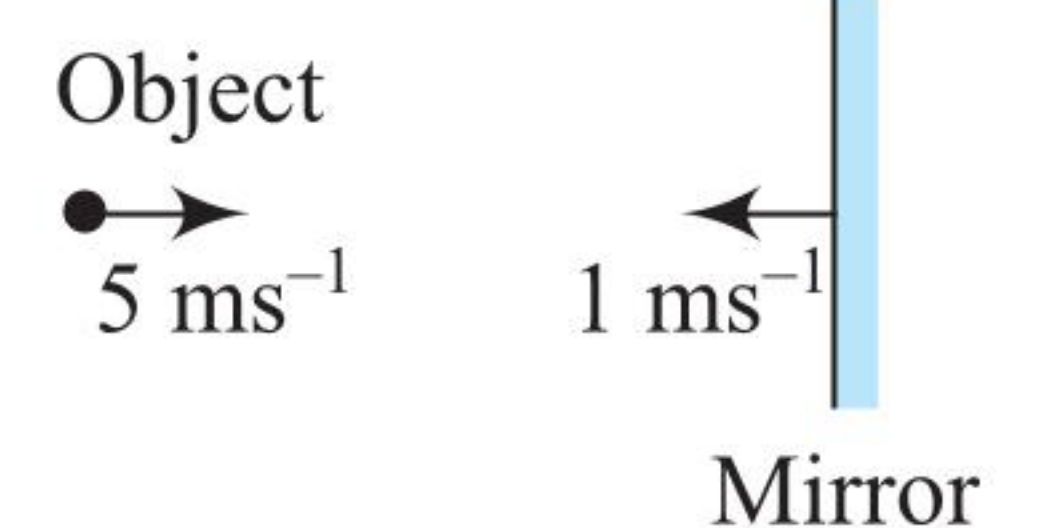
Velocity of particle relative to mirror is

$$\vec{v}_{pm} = -15\hat{i} + 10\hat{j}$$

Velocity of image relative to observer

$$\begin{aligned} \vec{v}_{PO} &= \vec{v}_{pm} - \vec{v}_{Om} = (-15\hat{i} + 10\hat{j}) - (\vec{v}_O - \vec{v}_m) \\ &= (-30\hat{i} + 10\hat{j}) \text{ ms}^{-1} \end{aligned}$$

$$\text{Angle with } X\text{-axis, } \alpha = \tan^{-1}(-1/3)$$



Exercise 1.2

1. (a) True. According to the magnification formula,

$$m = \frac{f_0}{f_0 + x}$$

where x is the distance of a real object. For all positive values of x , m is always positive.

- (b) False. For a real object, the image formed by a convex mirror is always erect and diminished.

$$m = \frac{f_0}{f_0 + x}$$

For a virtual object lying between pole and focus, the image formed is real, enlarged, and erect.

$$m = \frac{f_0}{f_0 + x}$$

- (c) True. For a concave mirror, we know that

$$m = \frac{f_0}{f_0 + x}$$

where x is the object distance.

If $x < f_0$, then $m > 1$, which implies a virtual enlarged image.

- (d) False. In case of mirrors (convex, concave, and plane), the object and its image always move in opposite directions.

$$\text{That is, } \frac{V}{U} = -\left(\frac{f}{u - f}\right)^2$$

where V = velocity of image and U = velocity of object.

- (e) False. When the object is beyond the centre of curvature, its image moves slower.

- (f) True. For a convex mirror, $m = f_0/f_0 + x$
i.e., if $x = f_0$, then $m = 1/2$

- (g) True. A concave mirror forms an enlarged image in two situations:

(i) Object is placed between C and F .

(ii) Object is placed between F and P .

- (h) False. Convex mirror is used as a rear view mirror.

- (i) True. Complete image will be formed, but of less brightness because now the incident rays will be reflected from reduced area to form the image.

- (j) True. $m = -v/u$. For a plane mirror, $v = -u$
 $\Rightarrow m = +1$. Hence, image is erect and of same size.

- (k) True. We see our left hand as right hand and right as left hand in a plane mirror.

- (l) True. Virtual object is that point where incident rays seem to be converging.

$$2. \quad u = -15 \text{ cm}, f = \frac{+20}{2} = 10 \text{ cm}$$

$$\frac{1}{-15} + \frac{1}{v} = \frac{1}{10}$$

$$\frac{1}{v} = \frac{1}{10} + \frac{1}{15} \Rightarrow v = 6 \text{ cm}$$

$$m = -\frac{v}{u} = \frac{h_2}{h_1}$$

$$\frac{-6}{-15} = \frac{h_2}{5} \Rightarrow h_2 = 2 \text{ cm}$$

So, radius of circular path of the image is 2 cm.

3. Consider the situation shown in the figure below which also includes the coordinate system.

Here, $u = -10 \text{ cm}$

$$f = -20 \text{ cm}$$

Substituting in the equation,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}, \text{ we get}$$

$$v = +20 \text{ cm}$$

Thus, the image is formed 20 cm behind the mirror, magnification +2, erect and virtual.

4. Consider the situation shown in the following figure which also indicates the coordinate system.

Here, $u = -15 \text{ cm}$ and $m = -2$.

Note the negative sign for m . The fact that the image is captured on the screen implies the image is real. Therefore, the magnification is to be taken negative, though the problem simply states that the magnification is 2.

Substituting in equation

$$m = -\frac{v}{u}, \text{ we get } v = -30 \text{ cm}$$

Substituting in equation

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}, \text{ we get } f = -10 \text{ cm}$$

Conclusion: The focal length of the mirror is 10 cm. The negative sign implies it is concave.

5. Consider the situation shown in the following figure where the optical element is represented by a box as we do not know its nature.

Here $u = -15 \text{ cm}$,

$$h_i = +5 \text{ cm, and}$$

$$h_o = 15 \text{ cm}$$

Substituting in equation

$$m = \frac{h_i}{h_o} = +\frac{1}{3}$$

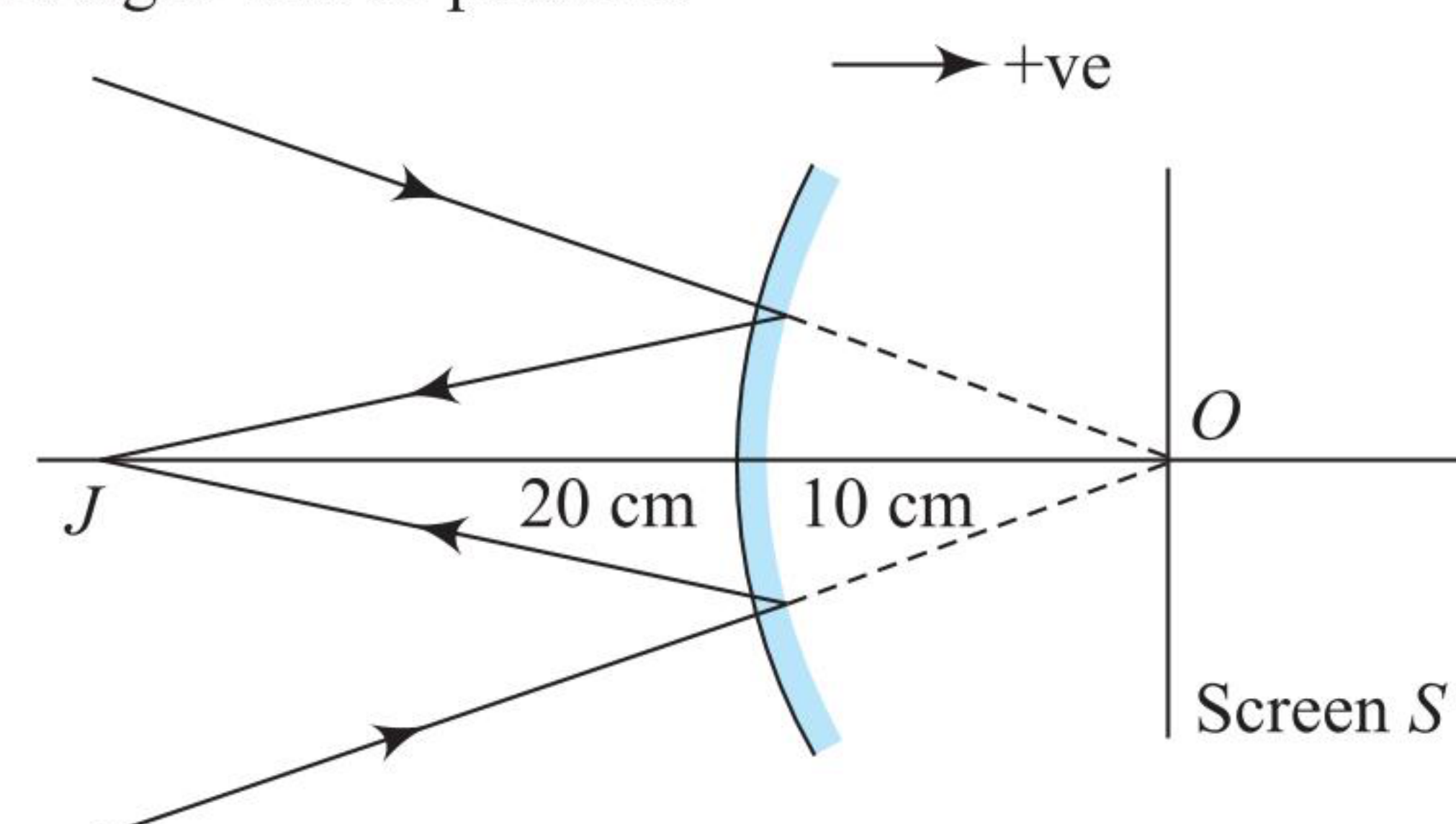
$$\text{and } m = \frac{-v}{u}, \text{ we get } v = +5 \text{ cm}$$

Substituting in equation

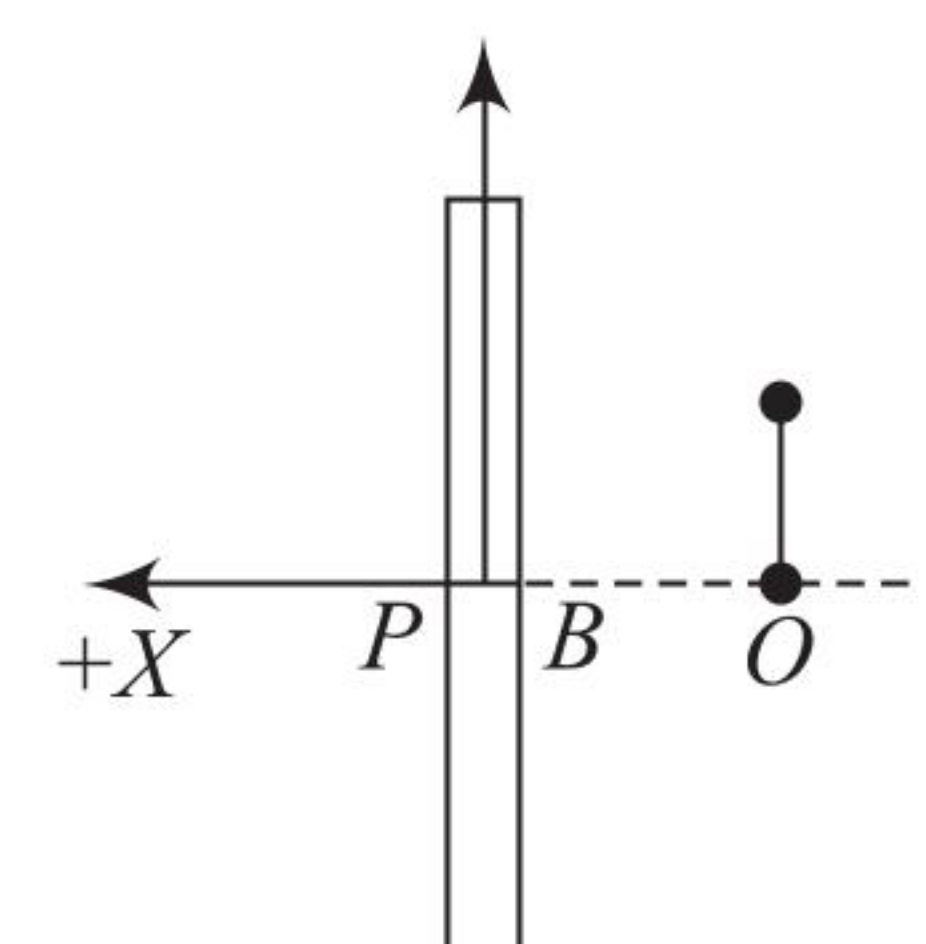
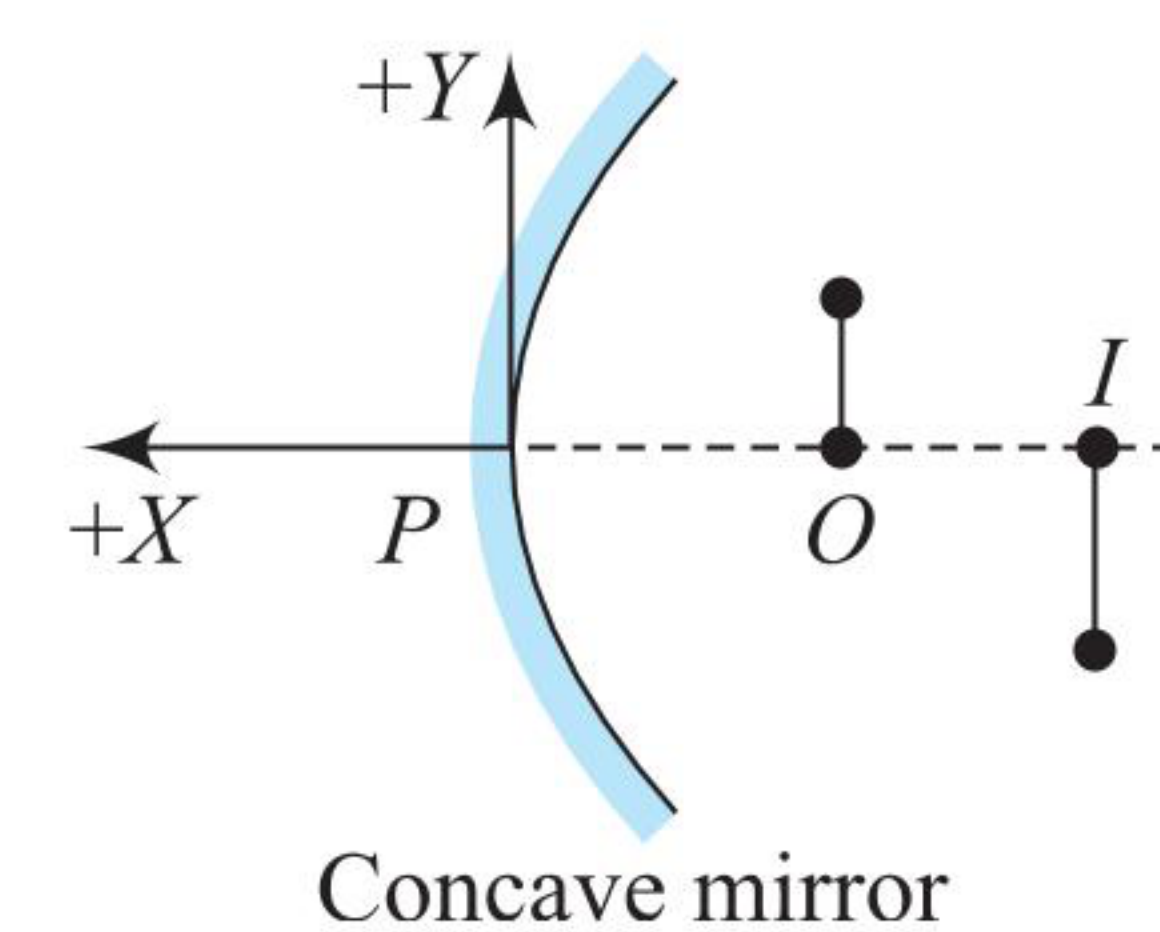
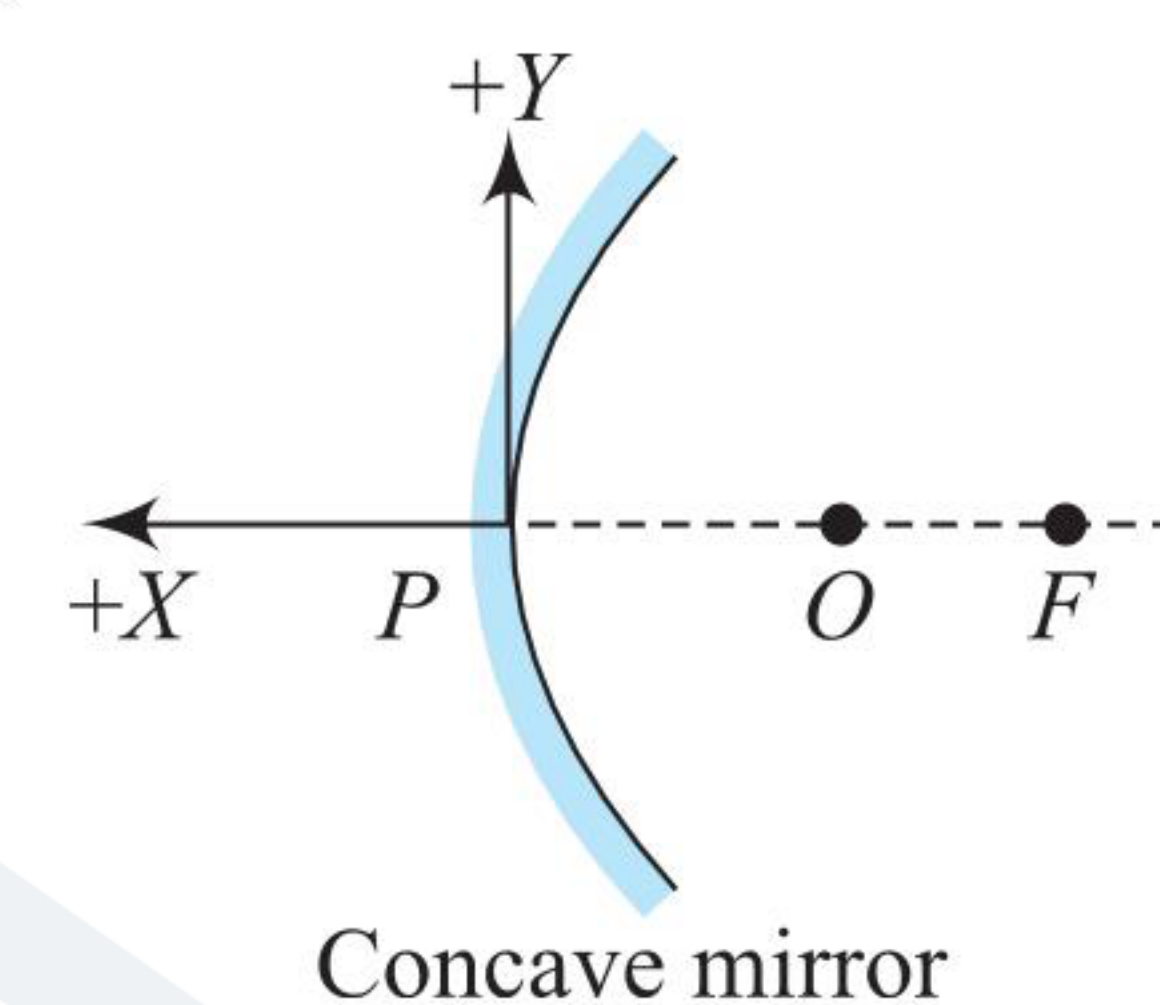
$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}, \text{ we get } f = +7.5$$

Conclusion: The focal length of the mirror is 7.5 cm. The positive sign implies that the mirror is convex.

6. The mirror can be concave or convex. Let f be the focal length. Let the incident rays be directed toward right as shown. All quantities toward right will be positive.



The initial point of convergence (O) now acts as a virtual object for the mirror.



$$u = -\left(2R - \frac{3R}{4}\right) = -\frac{5R}{4}$$

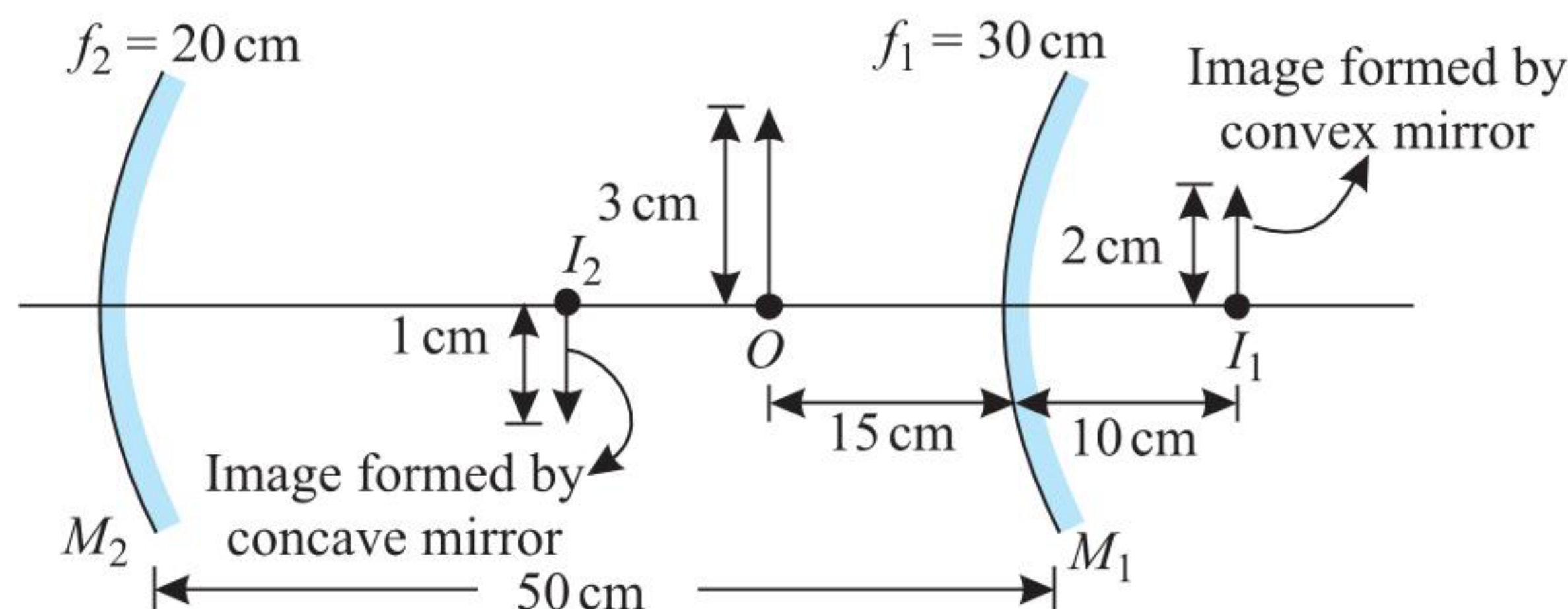
$$\therefore \frac{1}{v} = \frac{-2}{R} + \frac{4}{5R}$$

$$v = -\frac{5R}{6} \text{ (Image at } I_2\text{)}$$

$\therefore$ Distance of image from the nearest wall is $\frac{5R}{6}$.

4. For reflection at the convex mirror, $f = +30$ cm, $u = -15$ cm

$$\text{Using } \frac{1}{v} + \frac{1}{u} = \frac{1}{f}; \frac{1}{v} + \frac{1}{(-15)} = \frac{1}{(+30)} \Rightarrow v = +10 \text{ cm}$$



I_1 is the image formed by the convex mirror, which acts as the object for the concave mirror. For reflection at the concave mirror, for sign convention we are taking initial ray travelling direction as positive

$$f = +20 \text{ cm, } u = +(50 + 10) \text{ cm} = +60 \text{ cm}$$

$$\text{Using } \frac{1}{v} + \frac{1}{u} = \frac{1}{f}; \frac{1}{v} + \frac{1}{(+60)} = \frac{1}{(+20)} \Rightarrow v = +30 \text{ cm}$$

$$\text{Magnification produced by convex mirror, } m_1 = -\frac{(+10)}{(-15)} = \frac{2}{3}$$

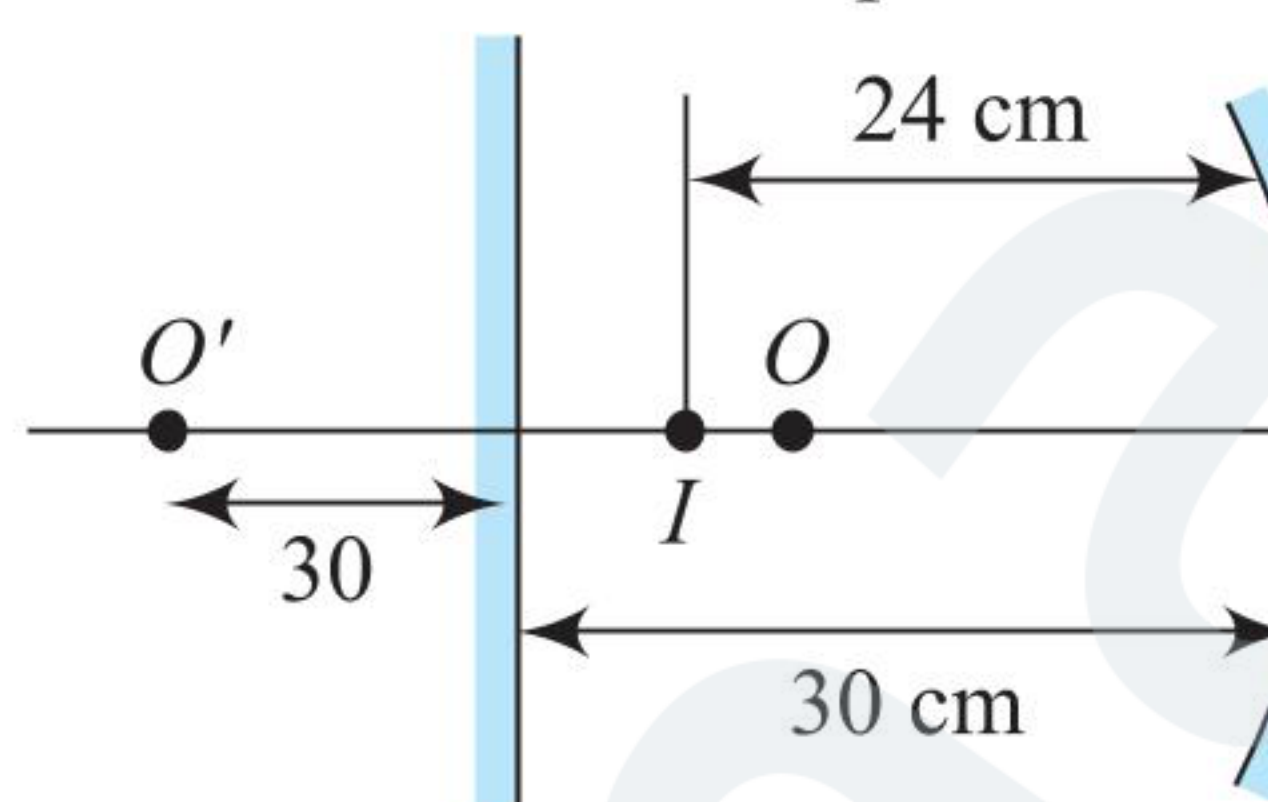
$$\text{Magnification produced by concave mirror, } m_2 = -\frac{(+30)}{(+60)} = -\frac{1}{2}$$

$$\text{Total magnification } m = m_1 m_2 = \frac{2}{3} \times \left(-\frac{1}{2}\right) = -\frac{1}{3}$$

$$\text{Size of image} = m \times \text{size of the object} = m_1 m_2 = -\frac{1}{3} \times 3 = -1 \text{ cm}$$

The final image is inverted, real and of height 1 cm.

5. (a) If reflection is first taken on the plane mirror:



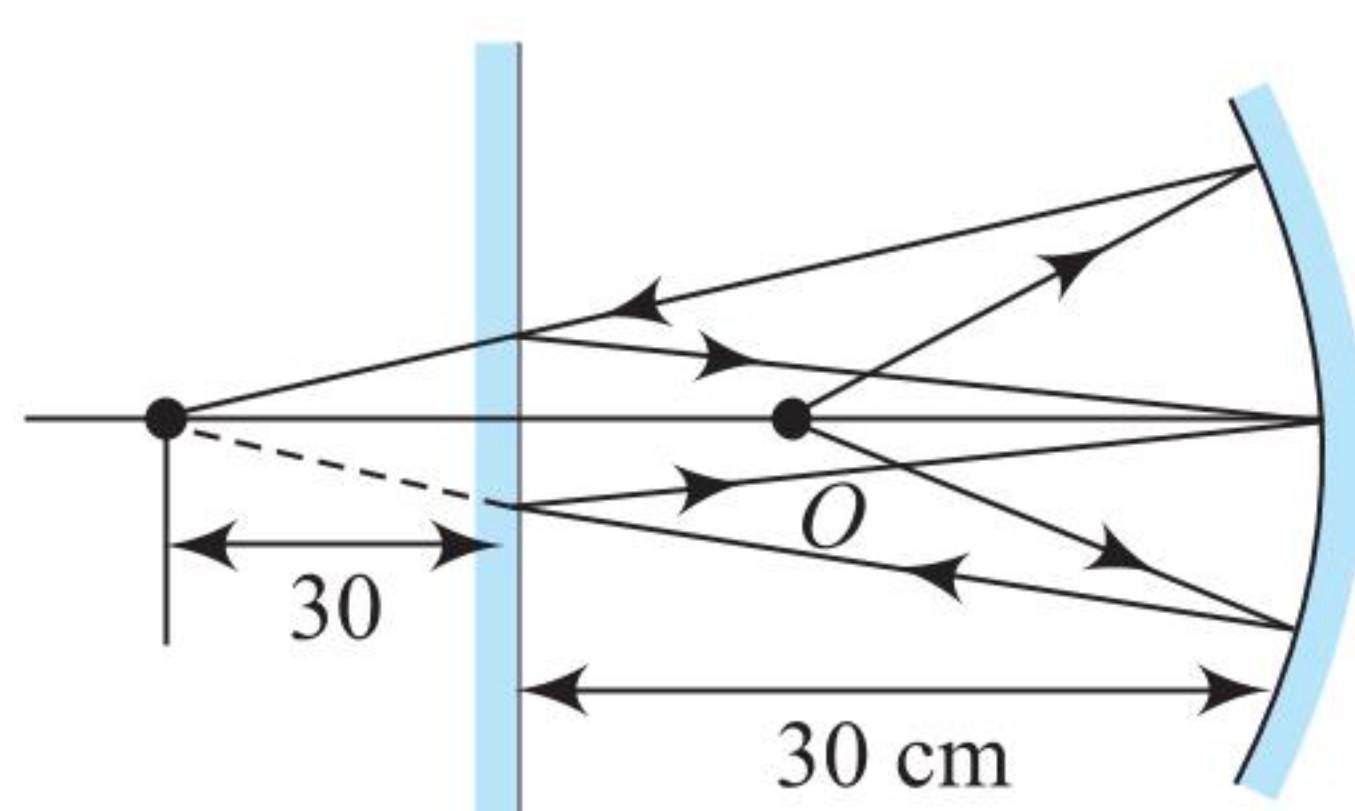
For second reflection at the concave mirror:

$$u = -40 \text{ cm, } f = -15 \text{ cm}$$

$$\frac{1}{v} + \frac{1}{-40} = -\frac{1}{15} \Rightarrow v = -24 \text{ cm}$$

- (b) If reflection is first taken on the concave mirror: $u = -20$ cm

$$\frac{1}{v} + \frac{1}{-20} = -\frac{1}{15} \Rightarrow v = -60 \text{ cm}$$



Final image will be formed at the pole of concave mirror.

6. Reflection from concave mirror

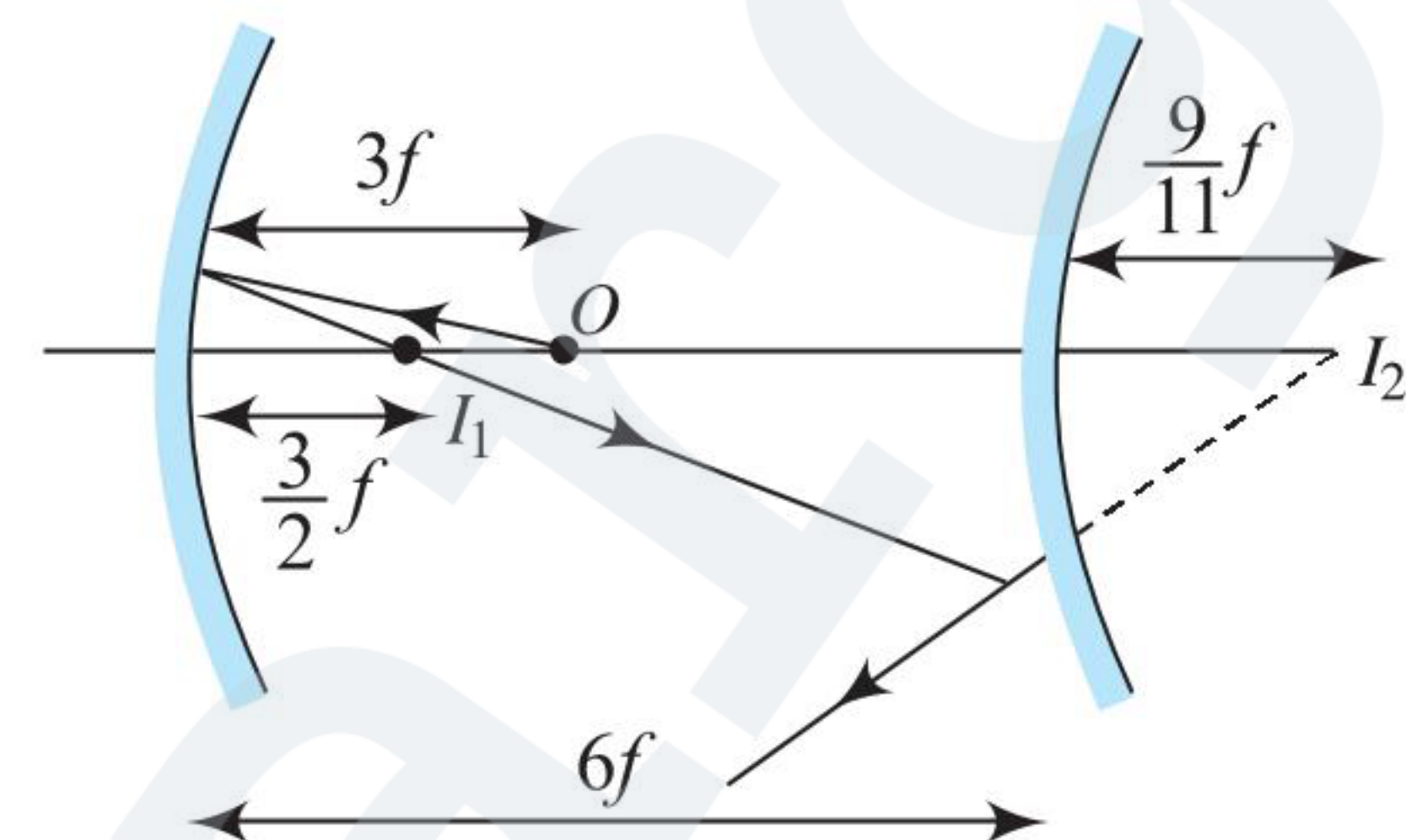
$$u = -3f, f = -f$$

$$\text{Using } \frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow v = -\frac{3}{2}f$$

Now reflection from convex mirror

$$u = -\left(6f - \frac{3}{2}f\right) = -\frac{9}{2}f, f = f$$

Again using mirror formula, we get $v = \frac{9f}{11}$



7. First reflection:

Focus of mirror = -10 cm

$$u = -15 \text{ cm}$$

$$\text{Applying mirror formula: } \frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$v = -30 \text{ cm}$$

For second reflection on plane mirror:

$$u = -10 \text{ cm} \Rightarrow v = 10 \text{ cm}$$

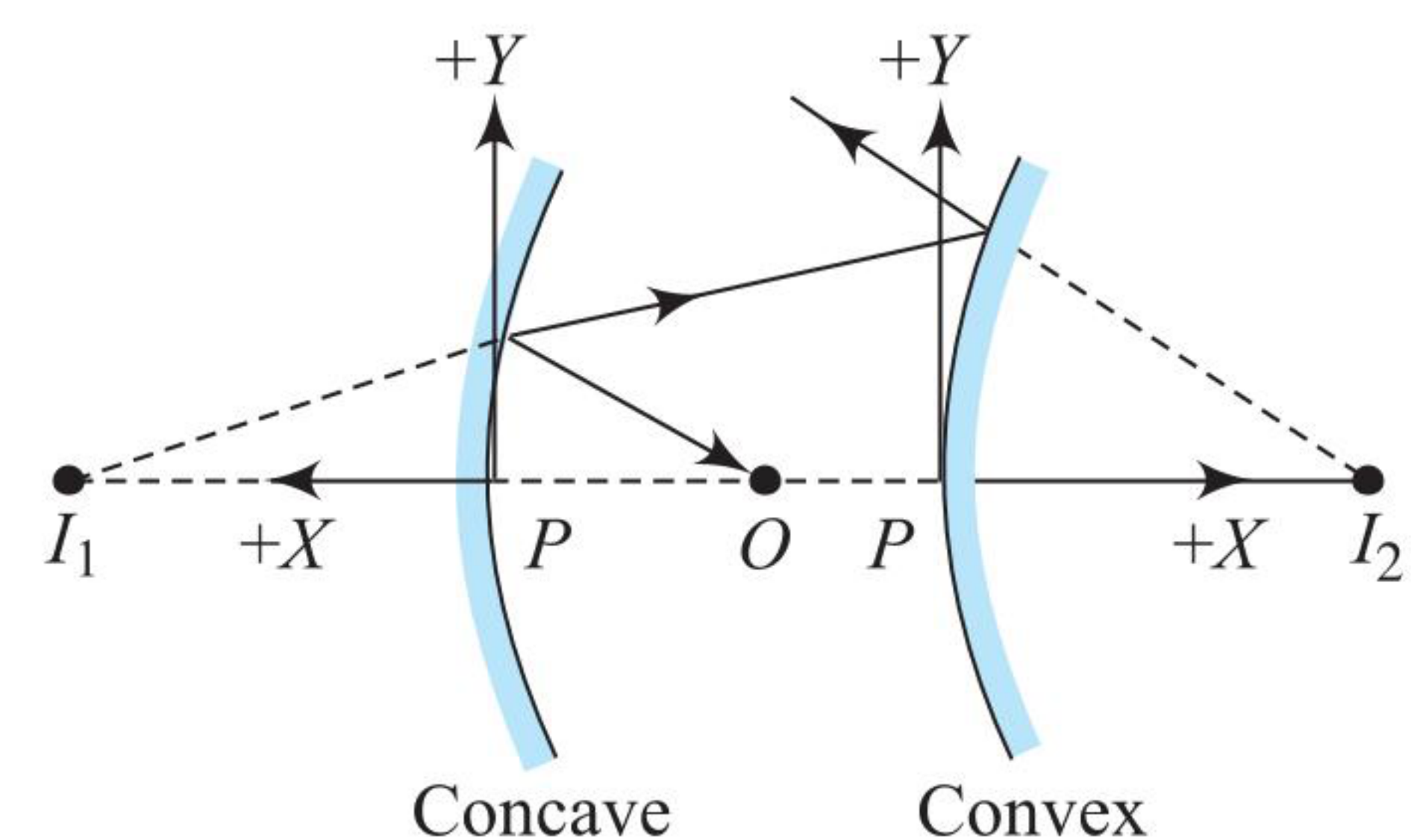
For third reflection on curved mirror again:

$$u = -50 \text{ cm and } f = -10 \text{ cm}$$

$$\text{Applying mirror formula: } \frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow v = -12.5 \text{ cm}$$

8. Let us consider the case of rays first striking the concave mirror and then the convex mirror.



Concave mirror: Here, $u = -5$ cm and $f = -10$ cm

Substituting in equation

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}, \text{ we get } v = 10 \text{ cm}$$

Thus, the image is formed 10 cm behind the mirror.

This image now serves as the object for the convex mirror. However, keep in mind that for convex mirror, all measurements will now be from the pole of this mirror. Furthermore, the sign convention for the convex mirror is shown in the figure.

Convex mirror: Here, $u = -30$ cm and $f = +20$ cm

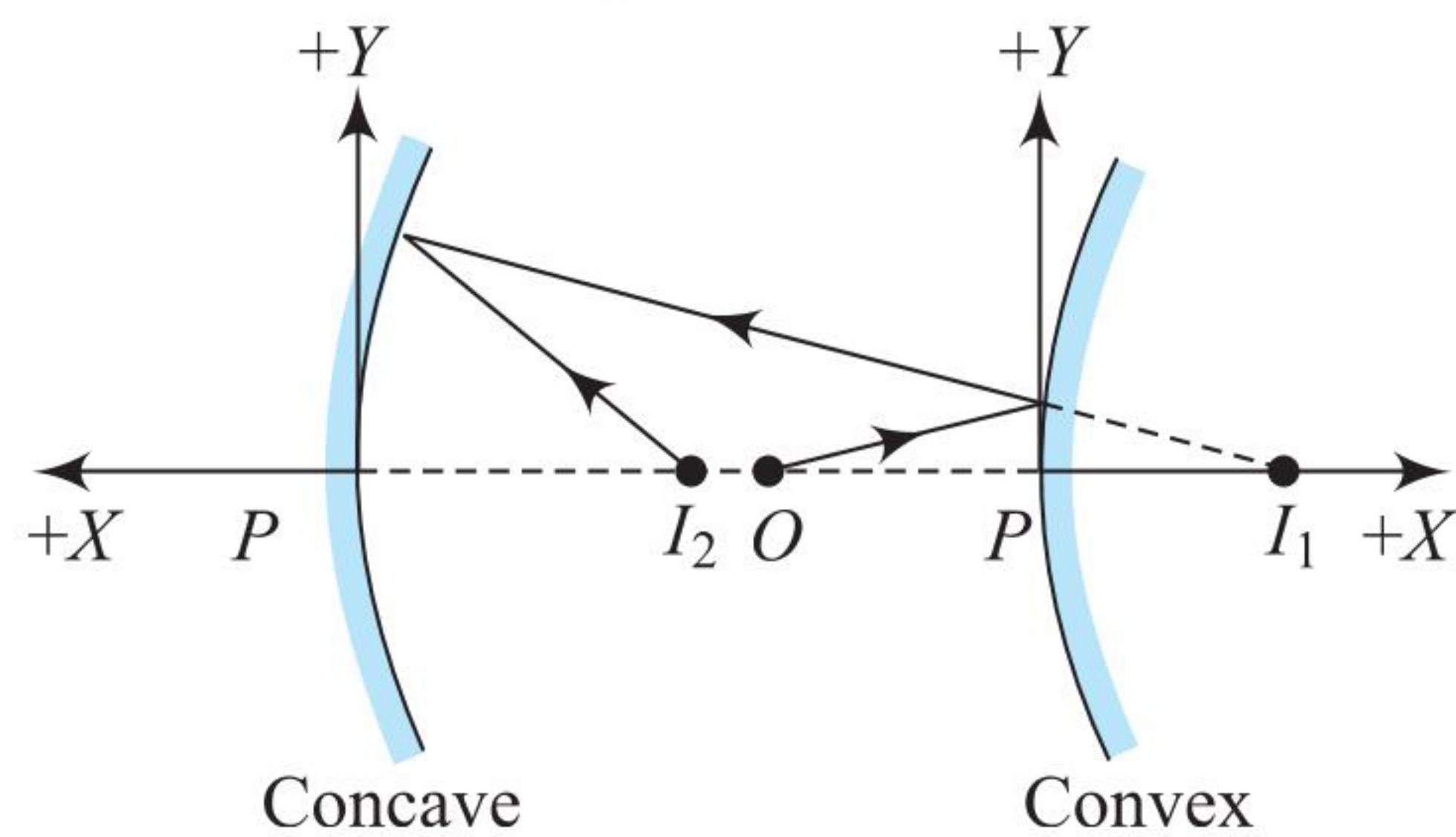
Substituting in equation

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}, \text{ we get } v = 12 \text{ cm}$$

Thus, the image is formed 12 cm behind the convex mirror. This image will now be the object for the concave mirror and the process can be repeated.

Refer to the following figure. Here, **for the convex mirror:**

$$u = -15 \text{ cm} \quad \text{and} \quad f = +20 \text{ cm}$$



Substituting in the equation $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$,
we get $v = +(60/7) \text{ cm}$

Thus, the image is formed $(60/7) \text{ cm}$ behind the convex mirror. The image now serves as the object for the concave mirror.

For the concave mirror:

$$u = -(20 + 60/7) = -200/7 \quad \text{and} \quad f = -10 \text{ cm}$$

Substituting in the equation

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}, \quad \text{we get} \quad v = -200/13$$

The image is formed $(200/13) \text{ cm}$ in front of the concave mirror.

9. Now, using $\vec{V}_{i,m} = -m^2 \vec{V}_{o,m}$

$$\text{or} \quad (\vec{V}_i - \vec{V}_m) = -m^2 (\vec{V}_o - \vec{V}_m)$$

Here, $u = -15 \text{ m}$ and $f = -10 \text{ m}$

Using mirror formula: $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$$\frac{1}{v} + \frac{1}{(-15)} = \frac{1}{(-10)} \Rightarrow v = -30 \text{ m}$$

$$\text{Magnification } m = -\frac{v}{u} = -\frac{(-30)}{(-15)} = -2$$

$$\Rightarrow [\vec{V}_i - (-1\hat{i})] = -(-2)^2 [(1\hat{i}) - (-1\hat{i})]$$

$$\Rightarrow \vec{V}_i = -9\hat{i} \text{ m/s}$$

So, the image will move with velocity 9 ms^{-1} towards left.

10. We have mirror formula $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$... (i)

Substituting the values, $u = -30 \text{ cm}$ and $f = -20 \text{ cm}$

$$\text{In Eq. (i), we have, } \frac{1}{v} + \frac{1}{-30} = \frac{1}{-20}$$

Solving this equation, we get $v = -60 \text{ cm}$

$$\text{Further, } m = -\frac{v}{u} = -\frac{(-60)}{(-30)} = -2$$

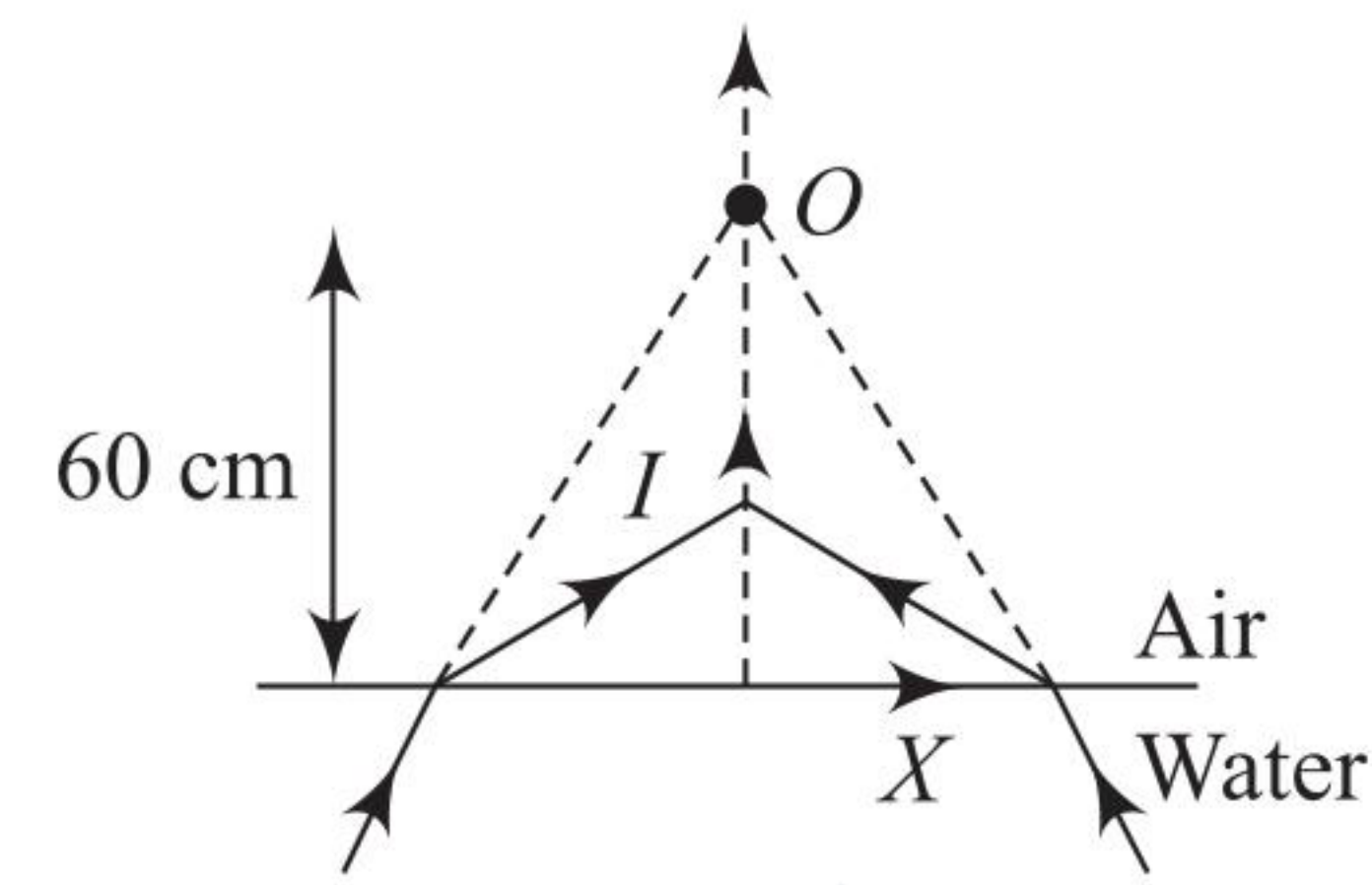
The velocity of image in the direction perpendicular to principal axis,

$$(V_{i,m})_y = m(V_{o,m})_y = (-2) \times 5\hat{j} = -10\hat{j} \text{ cm/s}$$

So, the image will move with velocity 10 cms^{-1} in downward direction.

Exercise 1.4

1. A ray of light now travels through water, across the interface through air and converges to form an image at I . Here, medium 1 is water and medium 2 is air.



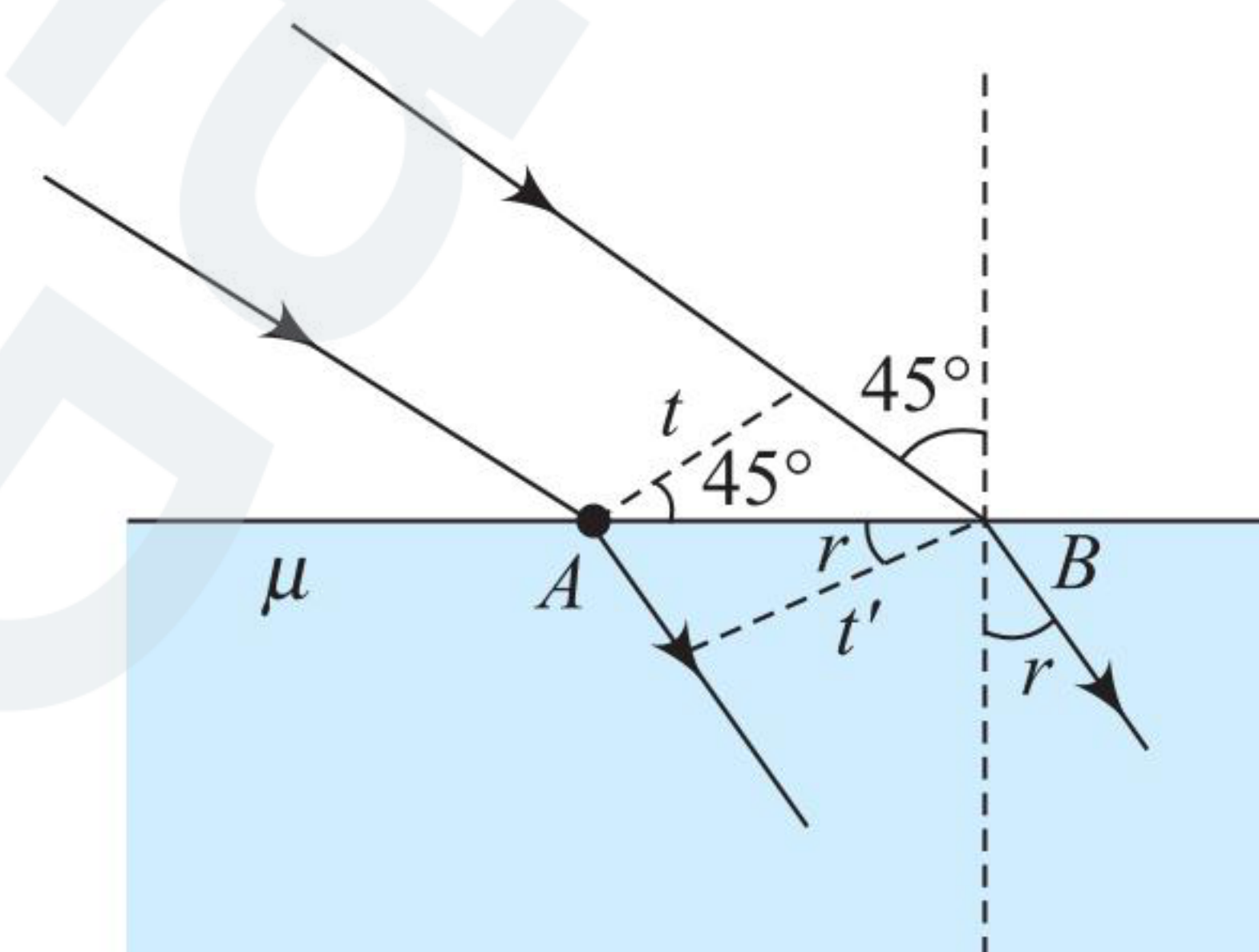
$$d_1 = +60 \text{ cm}, \mu_1 = \frac{4}{3}, \mu_2 = 1$$

$$\text{As } \frac{\mu_1}{d_1} = \frac{\mu_2}{d_2} \quad \text{or} \quad d_2 = \frac{\mu_2}{\mu_1} d_1, \text{ we get}$$

$$d_2 = +45 \text{ cm}$$

Thus, the image is formed 45 cm above the interface.

$$2. AB = \frac{t}{\cos 45^\circ} = \frac{t'}{\cos r} \Rightarrow t' = t \frac{\cos r}{\cos 45^\circ}$$

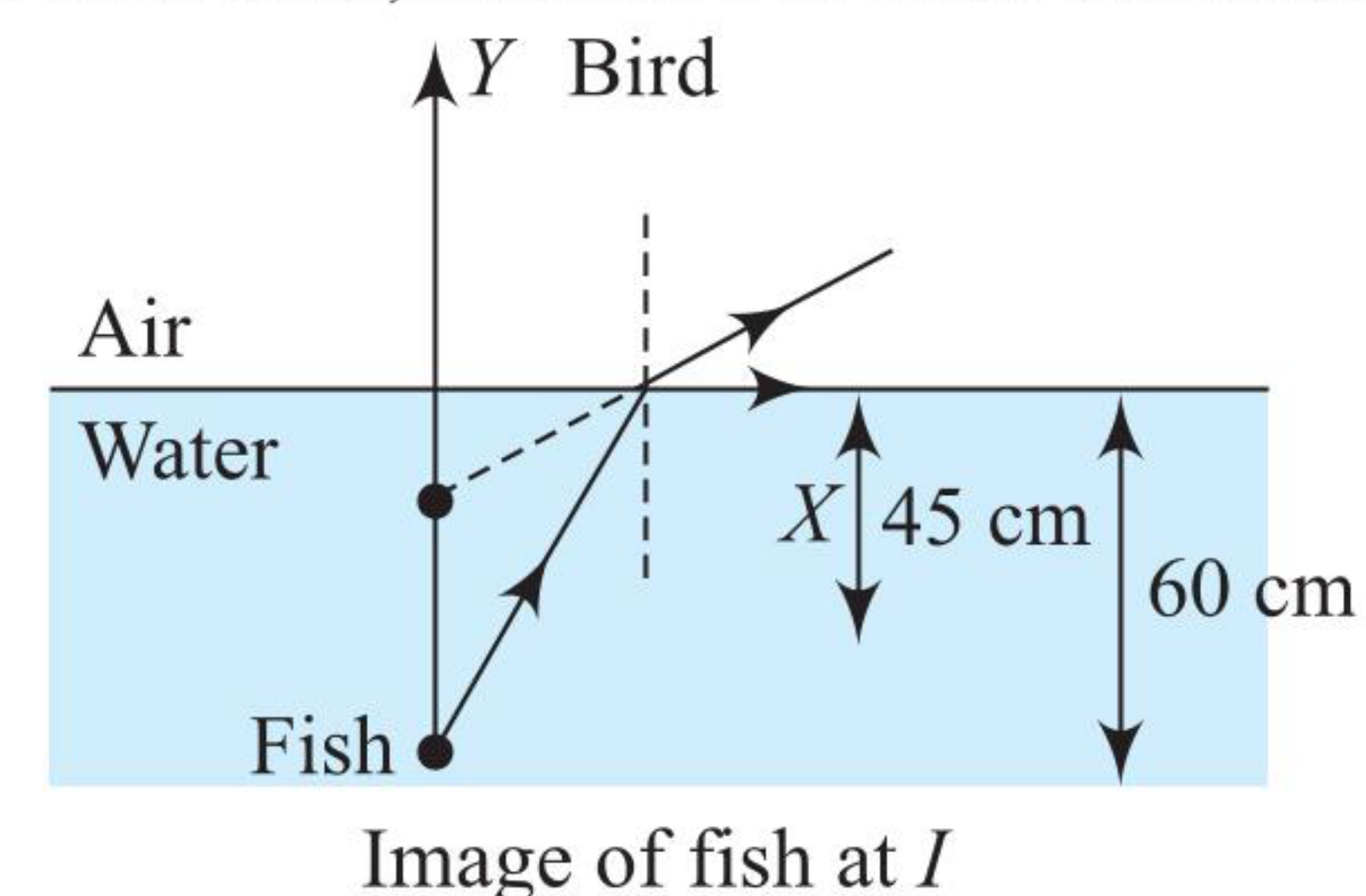


$$\mu = \frac{\sin 45^\circ}{\sin r} \Rightarrow \sin r = \frac{1}{\sqrt{2} \mu}$$

$$\text{Solving we get, } t' = \frac{t \sqrt{2\mu^2 - 1}}{\mu}$$

3. (a) **When the bird sees the fish:**

The fish and the bird are shown in the following figure along with the corresponding coordinate system. A ray of light now travels from the fish, through water, across the interface, through air and reaches the bird. Here, medium 1 is water and medium 2 is air.



$$d_1 = -60 \text{ cm}, \mu_1 = \frac{4}{3}, \mu_2 = 1$$

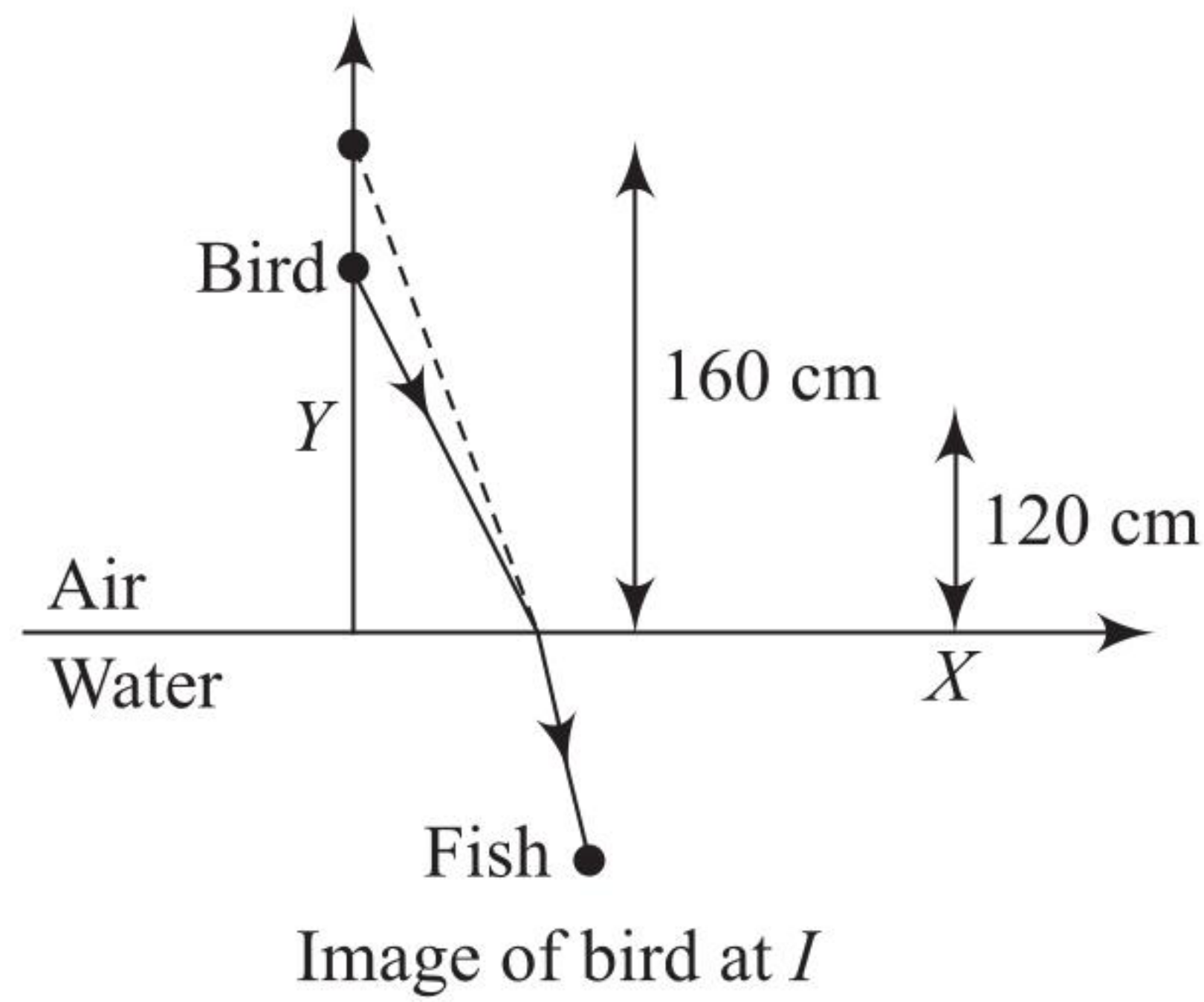
$$\text{As } \frac{\mu_1}{d_1} = \frac{\mu_2}{d_2} \quad \text{or} \quad d_2 = \frac{\mu_2}{\mu_1} d_1, \text{ we get}$$

$$d_2 = -45 \text{ cm}$$

Thus, the fish appears to be 45 cm below the water-air interface. Image of fish is formed at I .

(b) **When the fish sees the bird:**

The fish and the bird are shown in the following figure along with the corresponding coordinate system. A ray of light now travels from the bird, through air, across the interface, through water and reaches the fish. Here, medium 1 is air and medium 2 is water.



$$d_1 = +120 \text{ cm}, \mu_1 = 1, \mu_2 = \frac{4}{3}$$

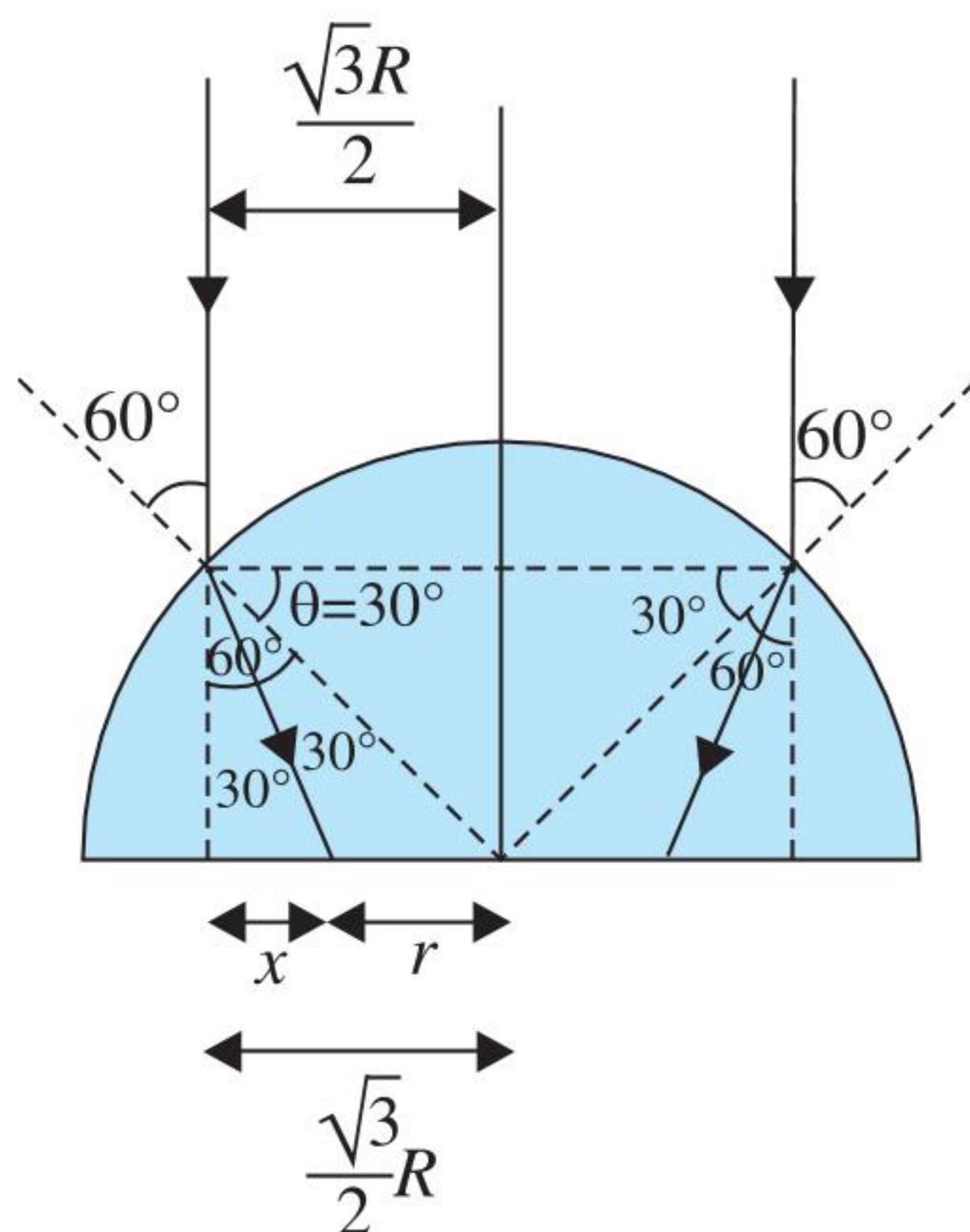
$$\text{As } \frac{\mu_1}{d_1} = \frac{\mu_2}{d_2} \text{ or } d_2 = \frac{\mu_2}{\mu_1} d_1, \text{ we get } d_2 = 160 \text{ cm}$$

Thus, the bird appears to be 160 cm above the water–air interface.
Image of bird is formed at I .

$$4. \mu_1 \sin i = \mu_2 \sin r$$

$$1 \times \sin 60^\circ = \sqrt{3} \times \sin r; r = 30^\circ$$

$$\text{In } \triangle MNP, \tan 30^\circ = x/(R/2)$$

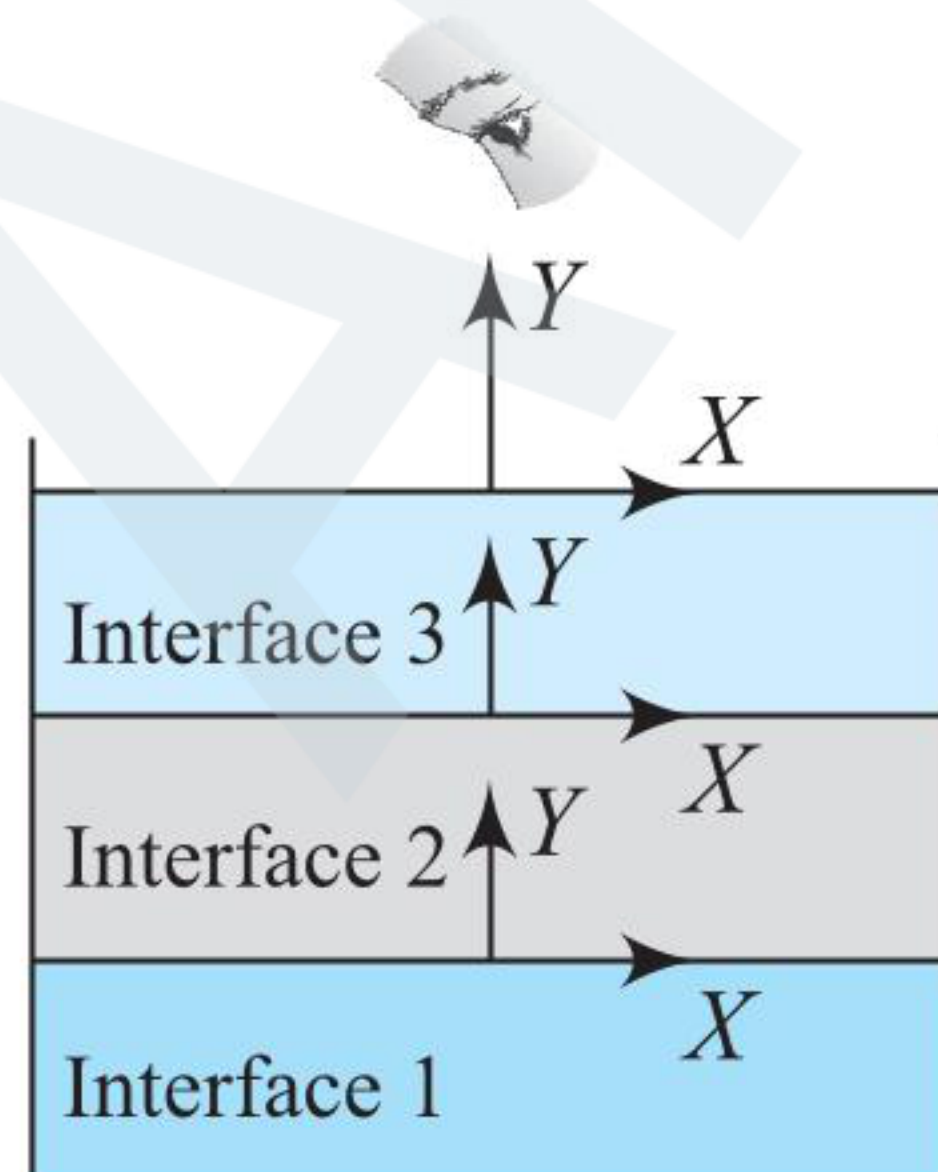


$$x = \frac{R}{2\sqrt{3}}$$

$$r = \frac{\sqrt{3}}{2} R - x = \frac{\sqrt{3}}{2} R - \frac{R}{2\sqrt{3}} = \left(\frac{3-1}{2\sqrt{3}} \right) R = \frac{R}{\sqrt{3}}$$

5. Method 1: Method of interfaces:

A ray of light from the object undergoes refraction at three interfaces: (1) Water–oil, (2) Oil–glycerine, and (3) Glycerine–air. The coordinate system for each of the interfaces is shown in the figure below.



(1) Water–oil interface:

$$d_1 = -8 \text{ cm}, \mu_1 = \frac{4}{3}, \mu_2 = 1.5$$

$$\frac{\mu_1}{d_1} = \frac{\mu_2}{d_2} \text{ or } d_2 = \frac{\mu_2}{\mu_1} d_1, \text{ we get } d_2 = -9 \text{ cm}$$

(2) Oil–glycerine interface:

$$d_1 = -(9 + 9) = -18 \text{ cm}, \mu_1 = 1.5, \mu_2 = 2$$

$$\text{As } \frac{\mu_1}{d_1} = \frac{\mu_2}{d_2} \text{ or } d_2 = \frac{\mu_2}{\mu_1} d_1$$

$$d_2 = -24 \text{ cm}$$

(3) Glycerine–air interface:

$$d_1 = -(4 + 24) = -28 \text{ cm}, \mu_1 = 2, \mu_2 = 1$$

$$\text{As } \frac{\mu_1}{d_1} = \frac{\mu_2}{d_2} \text{ or } d_2 = \frac{\mu_2}{\mu_1} d_1, \text{ we get } d_2 = -14 \text{ cm}$$

Thus, the final image is 14 cm below the glycerine–air interface.

Method 2: Method of shifting

Net shifting

$$s = d_1 \left(1 - \frac{1}{\mu_1} \right) + d_2 \left(1 - \frac{1}{\mu_2} \right) + d_3 \left(1 - \frac{1}{\mu_3} \right)$$

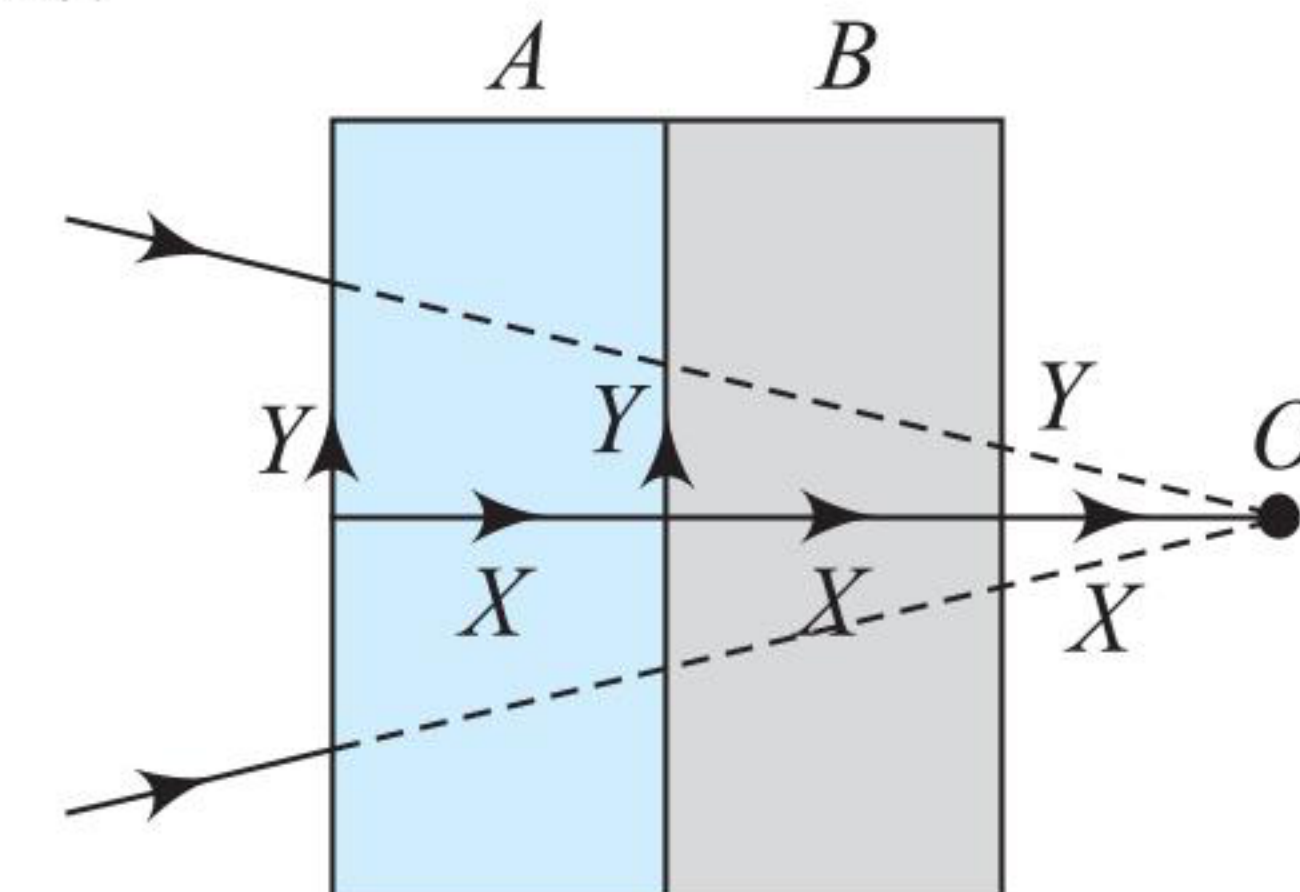
$$= 8 \left(1 - \frac{1}{4/3} \right) + 9 \left(1 - \frac{1}{3/2} \right) + 4 \left(1 - \frac{1}{2} \right) = 7 \text{ cm}$$

The shifting will be in the direction of ray travelling, i.e., upwards.

Hence, apparent depth $h' = (21 - 7) = 14 \text{ cm}$.

6. Method 1: Method of interfaces

A ray of light from the object undergoes refraction at three interfaces: (1) Air–medium A, (2) Medium A–medium B, (3) Medium B–air. The coordinate system for each of the interface is shown in the following figure.



(1) Air–medium A interface:

$$d_1 = +14 \text{ cm}, \mu_1 = 1, \mu_2 = 1.5$$

$$\text{As } \frac{\mu_1}{d_1} = \frac{\mu_2}{d_2} \text{ or } d_2 = \frac{\mu_2}{\mu_1} d_1, \text{ we get}$$

$$d_2 = +21 \text{ cm}$$

(2) Medium A–medium B interface:

$$d_1 = (21 - 6) = 15 \text{ cm}, \mu_1 = 1.5, \mu_2 = 2$$

$$\text{As } \frac{\mu_1}{d_1} = \frac{\mu_2}{d_2} \text{ or } d_2 = \frac{\mu_2}{\mu_1} d_1, \text{ we get}$$

$$d_2 = +20 \text{ cm}$$

(3) Medium B–air interface:

$$d_1 = +(20 - 4) = +16 \text{ cm}, \mu_1 = 2, \mu_2 = 1$$

$$\text{As } \frac{\mu_1}{d_1} = \frac{\mu_2}{d_2} \text{ or } d_2 = \frac{\mu_2}{\mu_1} d_1, \text{ we get}$$

$$d_2 = +8 \text{ cm}$$

Thus, the final image is 8 cm in front of the medium B–air interface.

Method 2: Method of shifting

$$\text{Net shifting } S = 6 \left(1 - \frac{1}{3/2} \right) + 4 \left(1 - \frac{1}{2} \right) = 4 \text{ cm}$$

The shifting will be in the direction of ray travelling, i.e., towards right.

Hence, rays will converge finally = 14 + 4 = 18 cm from left surface or 8 cm from right surface.

7. For the light ray to be reflected from the lower surface RS , the angle of incidence must be greater than the critical angle for the glass–air interface (at N).

$$i > \sin^{-1} \left(\frac{1}{2} \right) \text{ or } i > 30^\circ$$

At the water–glass interface, $\mu_1 = \frac{4}{3}$, $\mu_2 = 2$

$$\theta_1 = \theta = ?, \theta_2 = i = 30^\circ$$

Since $\mu_1 \sin \theta_1 = \mu_2 \sin \theta_2$, we get

$$\theta = \sin^{-1} \left[\frac{\mu_2 \sin \theta_2}{\mu_1} \right]$$

$$\Rightarrow \theta = \sin^{-1} \left(\frac{3}{4} \right)$$

$$8. \mu = \frac{\sin i}{\sin r} \Rightarrow \sin r = \frac{\sin i}{\mu}$$

Now, $r' > \text{critical angle}$

$$\sin r' > \frac{1}{\mu}$$

$$\sin [90^\circ - r] > \frac{1}{\mu}$$

$$\cos r > \frac{1}{\mu}$$

$$\sqrt{1 - \sin^2 r} > \frac{1}{\mu}$$

$$\sqrt{1 - \frac{\sin^2 i}{\mu^2}} > \frac{1}{\mu}$$

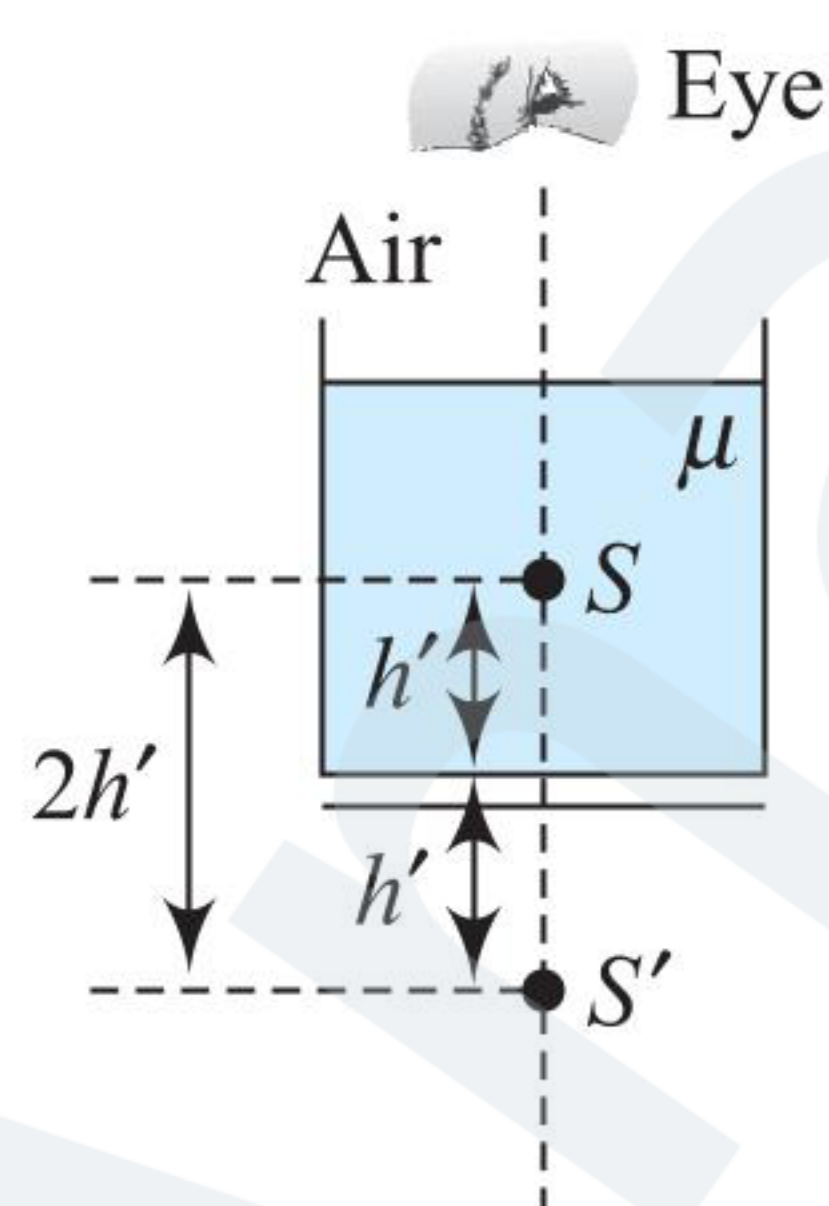
$$\mu^2 - \sin^2 i > 1$$

$$\mu > \sqrt{1 + \sin^2 i}$$

$$i_{\max} = 90^\circ, \text{ so } \mu > \sqrt{1 + \sin^2 90^\circ}$$

$$\mu > \sqrt{2}$$

9. (a) To observer in air, there are two objects placed in the medium of refractive index μ . One is the actual object S and the other object is S' which is the image of S in the plane mirror.



The observer is able to see two images, one for each object.

- (b) For observer in air, apparent depth $h' = \frac{h}{\mu}$

Hence distance between two images will be $2h' = \frac{2h}{\mu}$

10. The bottom of the beaker appears to be shifted up by a distance

$$\Delta t = \left(1 - \frac{1}{\mu} \right) d$$

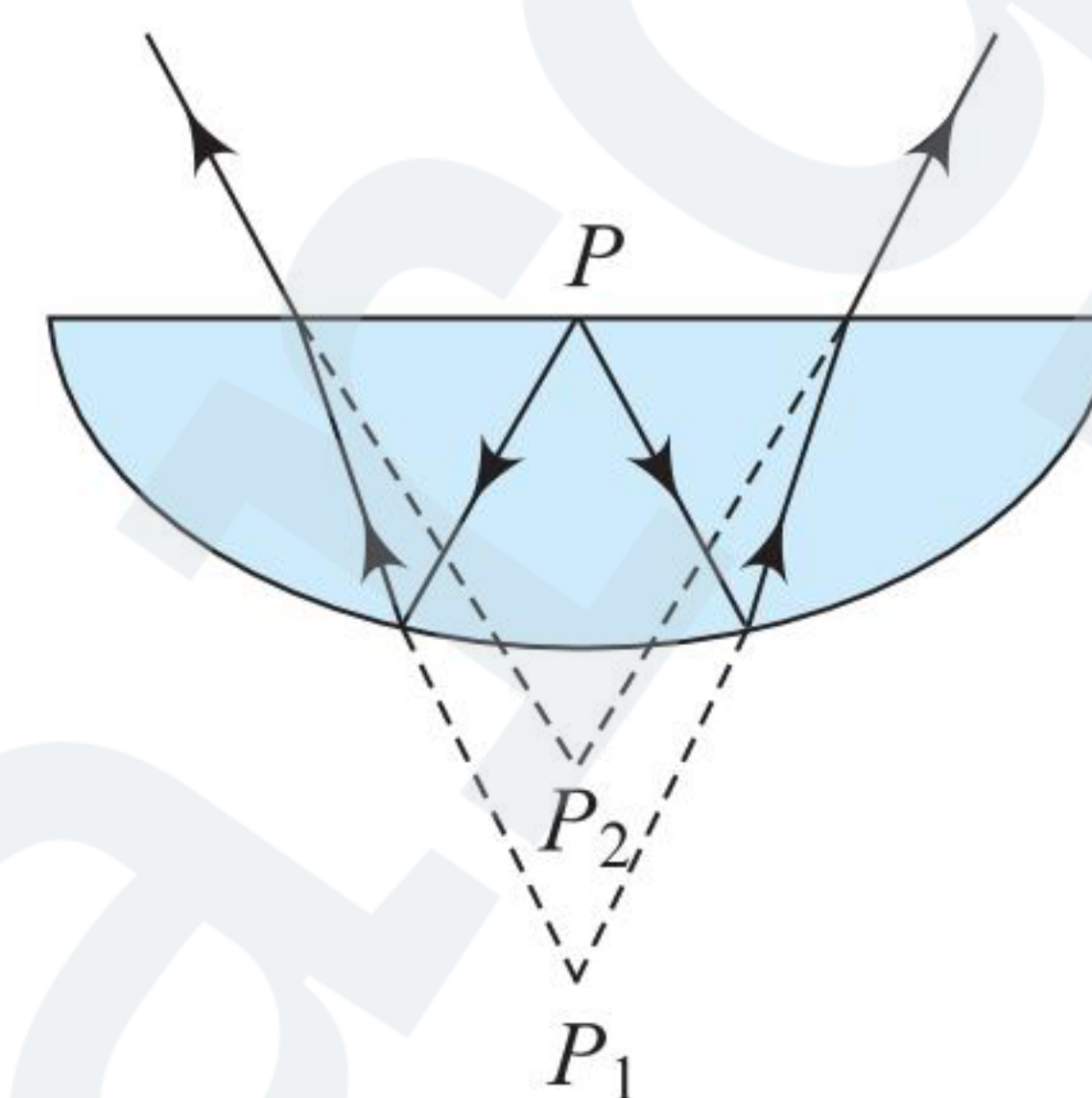
Thus, the apparent distance of the bottom from the mirror is $h - \Delta t = h - [1 - (1/\mu)]d = h - d + (d/\mu)$. The image is formed behind the mirror at a distance $h - d + (d/\mu)$.

11. The ray diagram is shown in the following figure. Let us first locate the image formed by the concave mirror. Let us take vertically upward as the negative axis. Then, $R = -40$ cm. The object distance is $u = -5$ cm. Using the mirror equation:

$$\frac{1}{u} + \frac{1}{v} = \frac{2}{R}$$

$$\frac{1}{v} = \frac{2}{R} - \frac{1}{u} = \frac{2}{-40 \text{ cm}} - \frac{1}{-5 \text{ cm}} = \frac{6}{40} \text{ cm}$$

$$\text{or } v = 6.67 \text{ cm}$$



The positive sign shows that the image P_1 is formed below the mirror and hence, it is virtual. These reflected rays are refracted at the water surface and go to the observer. The depth of the point P_1 from the surface is $6.67 \text{ cm} + 5.00 \text{ cm} = 11.67 \text{ cm}$. Due to refraction at the water surface, the image P_1 will be shifted above by a distance

$$(11.67 \text{ cm}) \times \left(1 - \frac{1}{1.33} \right) = 2.92 \text{ cm}$$

Thus, the final image is formed at a point $(11.67 - 2.92) \text{ cm} = 8.75 \text{ cm}$ below the water surface.

12. Let the object be placed at a height ' x ' above the surface of water. The apparent position of the object with respect to the mirror should be at the centre of curvature so that the image is formed at the same position.

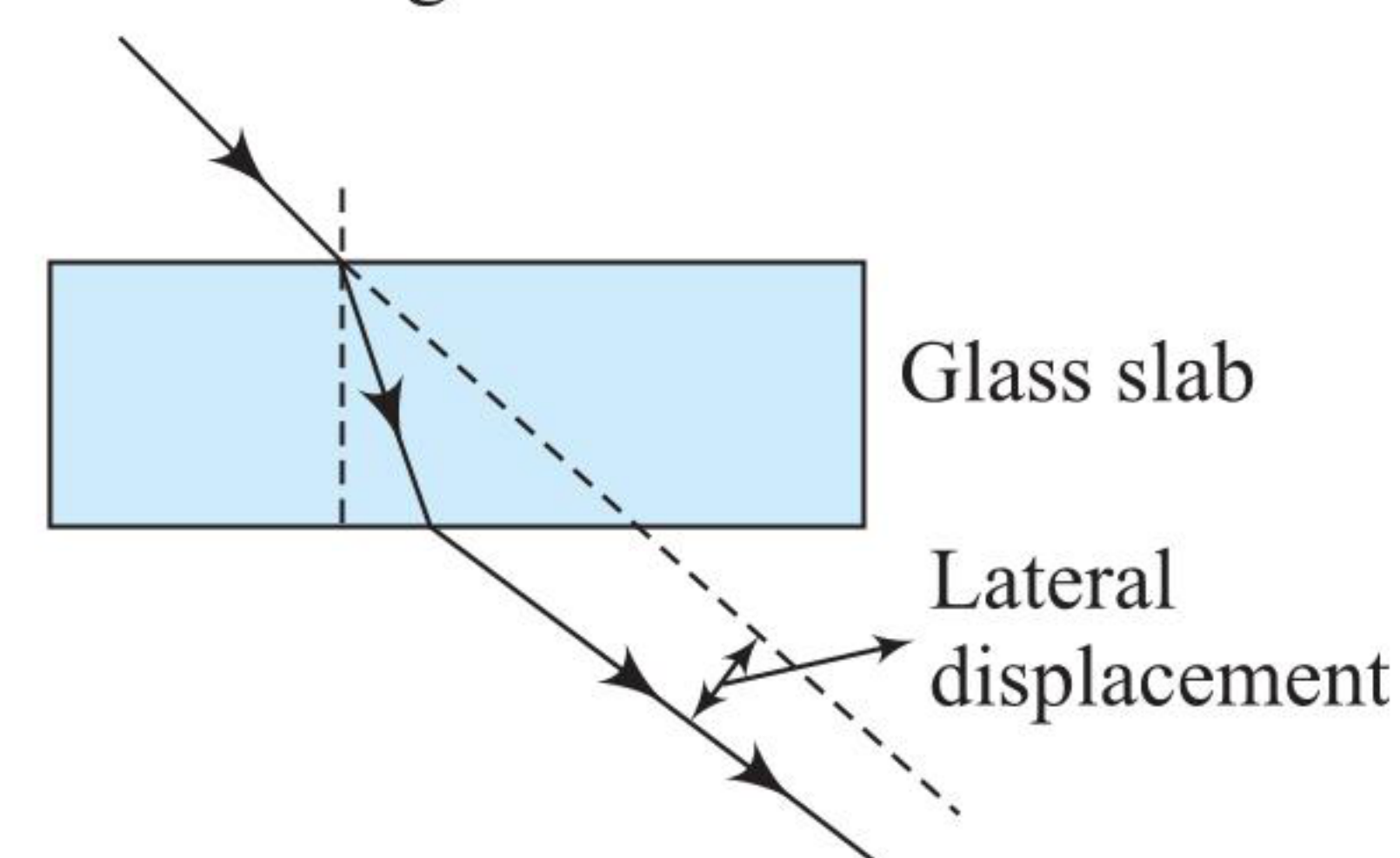
$$\text{Since } \frac{\text{Real depth}}{\text{Apparent depth}} = \frac{1}{\mu}$$

$$(\text{with respect to the mirror}), \text{ therefore } \frac{x}{R - h} = \frac{1}{\mu}$$

$$\text{or } x = \frac{R - h}{\mu}$$

Exercise 1.5

1. (a) True. The emergent ray is always parallel to the incident ray.
(b) True. See the figure below.



- (c) False. The shift produced by a glass slab is always in the direction of incident light, irrespective of the converging or diverging nature of beam.

- (d) False. As shifting $s = t \left(1 - \frac{1}{\mu_{\text{rel}}} \right)$

$$\mu_{\text{rel}} = \frac{\mu_{\text{slab}}}{\mu_{\text{medium}}}$$

If $\mu_{\text{medium}} > \mu_{\text{slab}}$, the value of μ_{rel} will be less than 1 and shifting will be negative, i.e., opposite to direction of ray travelling.

(e) True. As we know shifting $s = t \left(1 - \frac{1}{\mu_{\text{rel}}} \right)$

$$\mu_{\text{rel}} = \frac{\mu_{\text{slab}}}{\mu_{\text{medium}}} = \frac{\mu_2}{\mu_1}$$

If $\mu_2 > \mu_1$, then $s < t$

If $\mu_1 > \mu_2$, then shift (s) can be greater than t only if $\mu_1/\mu_2 > 2$.

2. (a) Distance of the watcher as seen from fish

$$8 \times \frac{4}{3} + 3 \times \frac{4/3}{8/5} + 10 = \frac{139}{6} \text{ cm}$$

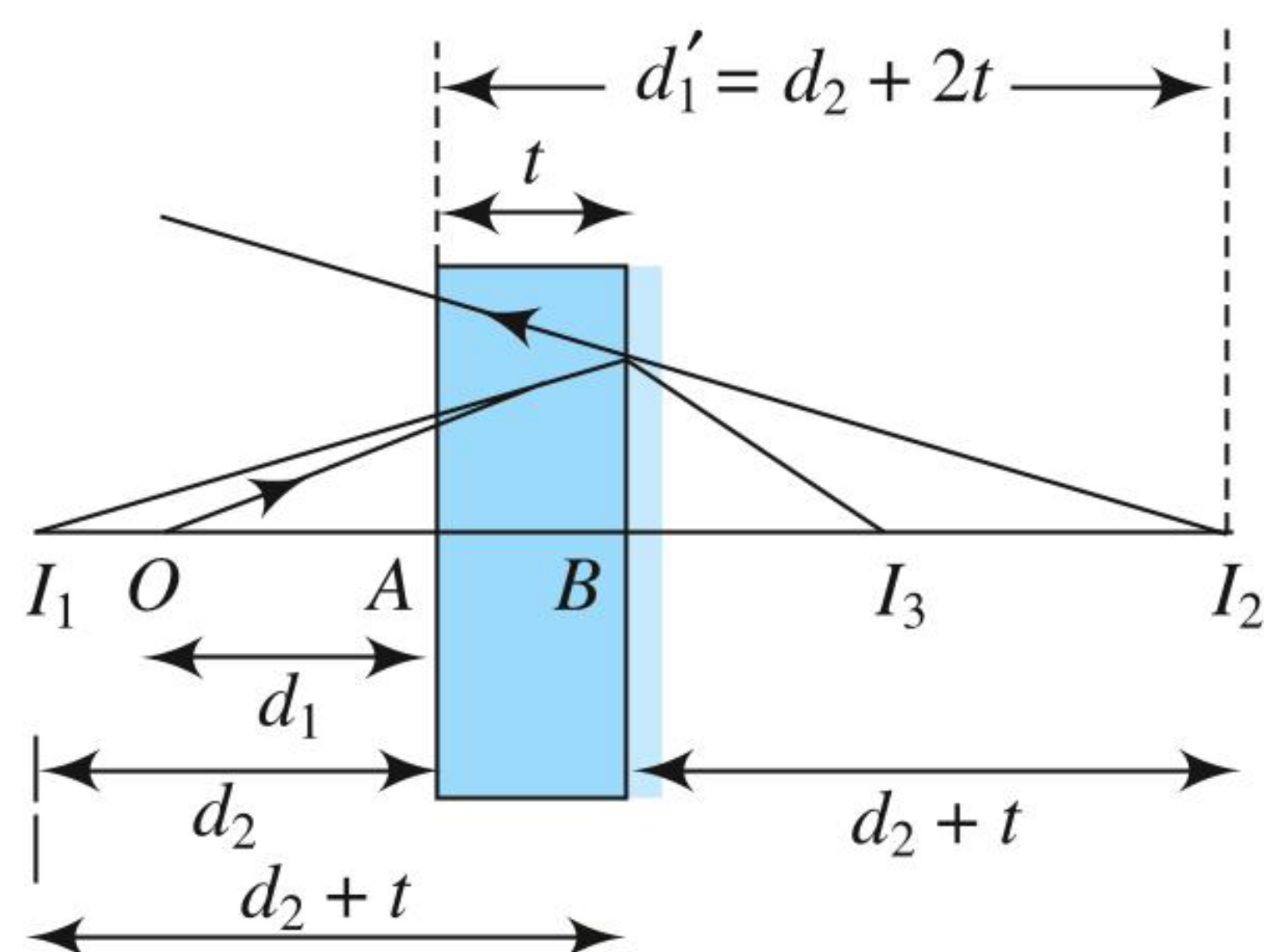
(b) Distance of the fish as seen from watcher is

$$\frac{10}{4/3} + \frac{3}{8/5} + 8 = \frac{139}{8} \text{ cm}$$

3. At first surface,

$$\frac{\mu_1}{d_1} = \frac{\mu_2}{d_2} \Rightarrow \frac{1}{(-8)} = \frac{\mu_2}{d_2} \Rightarrow d_2 = -8\mu$$

Image I_1 will serve as an object for the mirror and form an image I_2 behind it at a distance of $(8\mu + 6)$ cm.



I_2 will serve as an object for the first surface.

The rays will reflect from the mirror.

Again using $\frac{d_1'}{\mu} = \frac{d_2'}{1}$

$$d_1' = d_2 + 2t = -(8\mu + 12)$$

$$d_2' = -(10 + 6) = -16 \text{ cm}$$

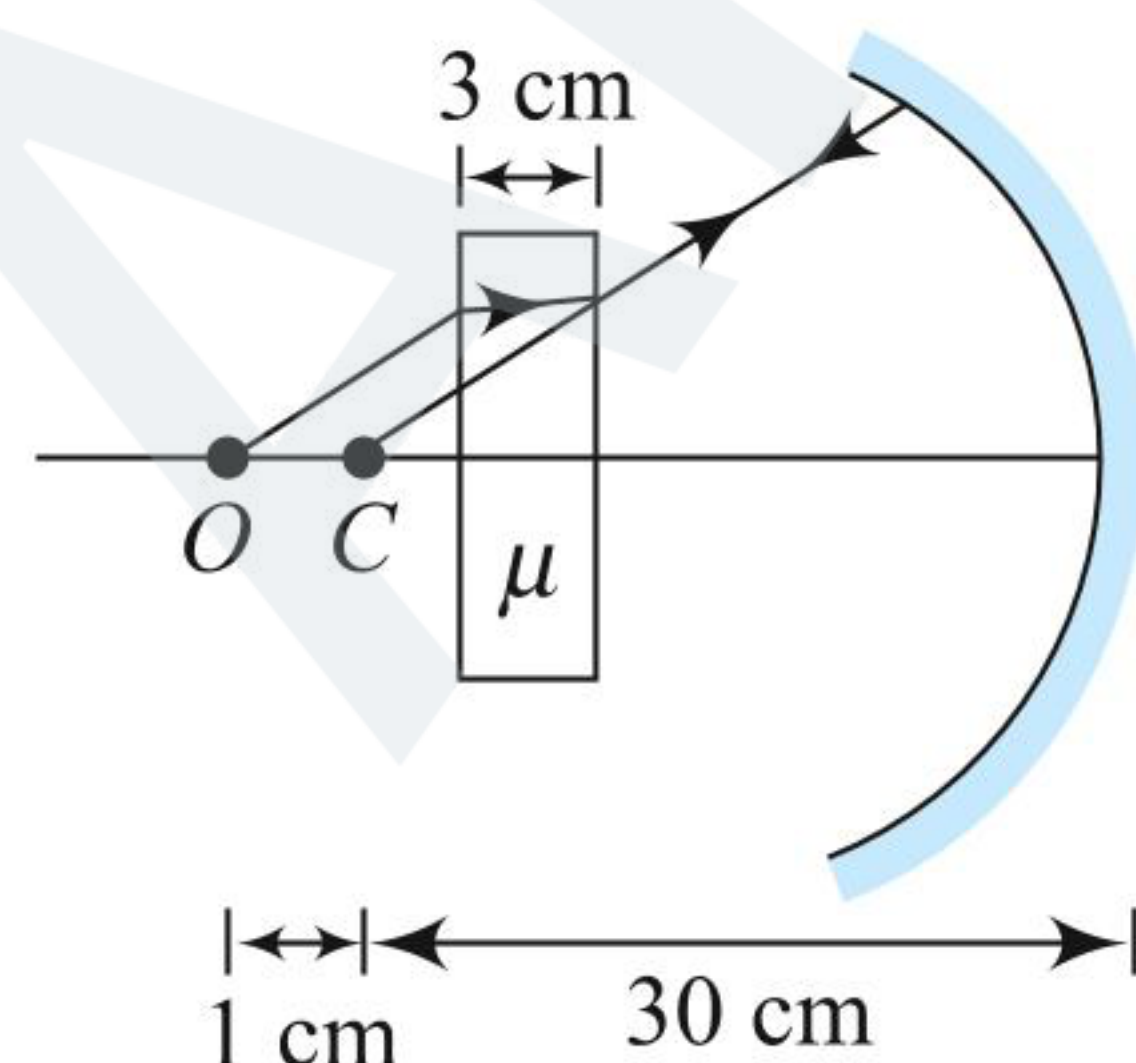
After substituting the values, $\mu = 1.5$.

4. Shift = $3 \left(1 - \frac{1}{3/2} \right)$

For the mirror, object is at a distance = $21 - 3 \left(1 - \frac{1}{3/2} \right) = 20$ cm

Therefore, object is at the centre of curvature of the mirror. Hence, the light rays will retrace and image will be formed on the object itself.

5. (a) Since the apparent shift occurs in the direction of incident light, therefore the mirror should be displaced away from the object.



(b) The magnitude of displacement is equal to the apparent shift,

$$\text{i.e., } s = t \left(1 - \frac{1}{\mu} \right) = 3 \left(1 - \frac{1}{3/2} \right) = 1 \text{ cm}$$

6. $\mu = \mu_0 \sqrt{x+1}$, at $x = 0 \rightarrow \mu = \mu_0$

$$\mu \sin \theta = \text{constant}$$

$$\mu_0 \sin 90^\circ = \mu_0 \sqrt{x+1} \sin \theta$$

$$\sin \theta = \frac{1}{\sqrt{x+1}}$$

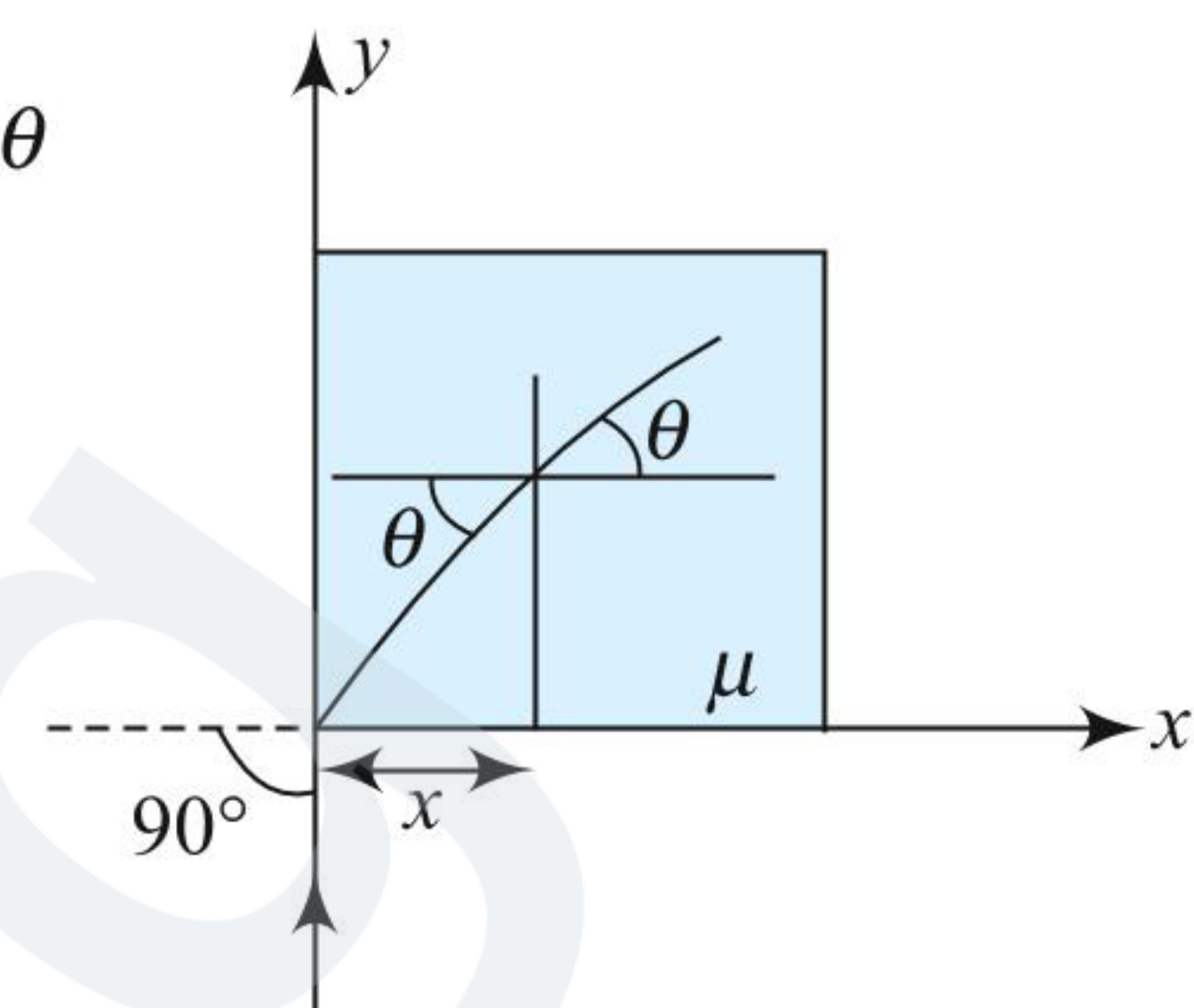
$$\Rightarrow \cos \theta = \sqrt{\frac{x}{x+1}}$$

$$\frac{dy}{dx} = \tan \theta$$

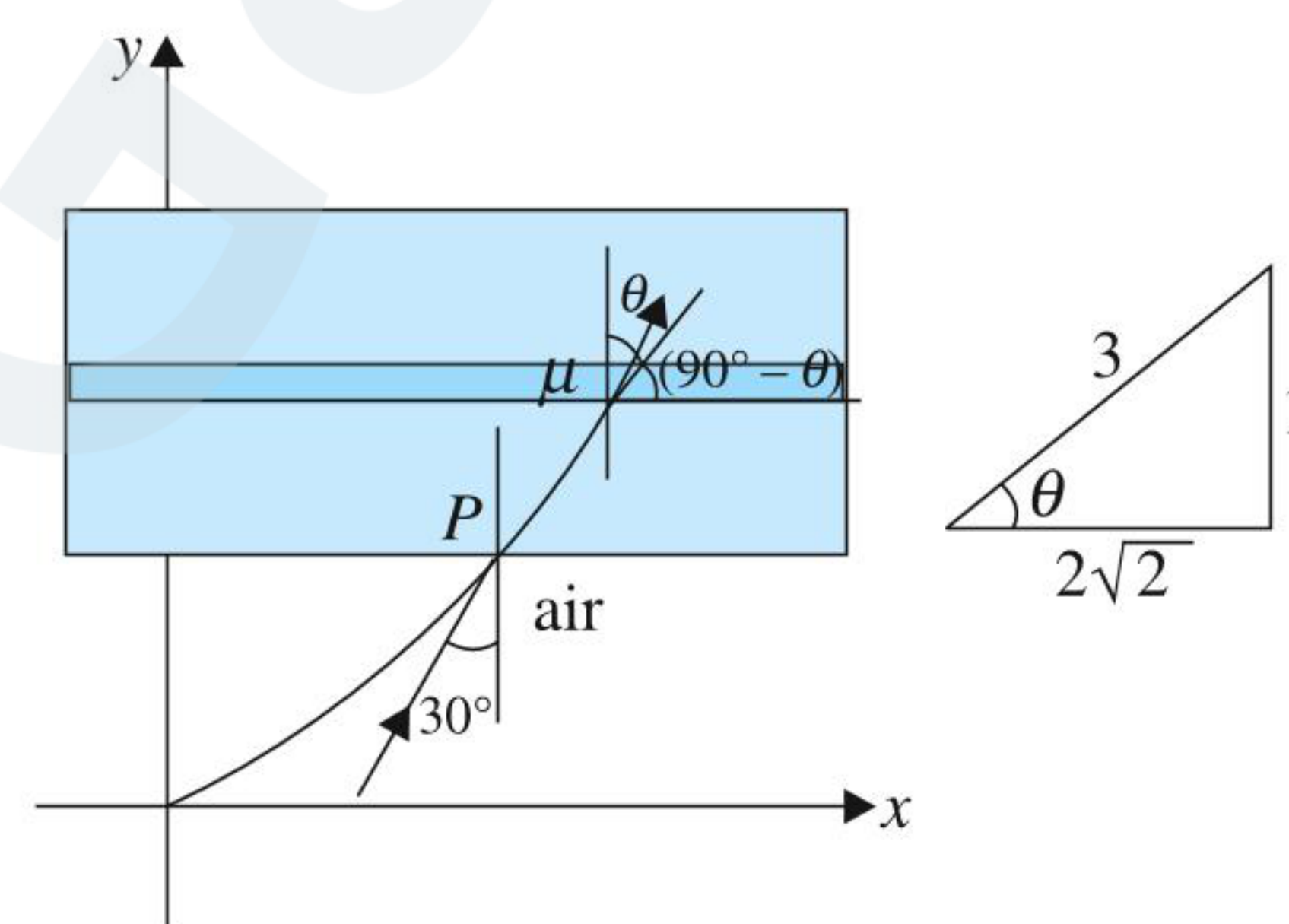
$$\Rightarrow \frac{dy}{dx} = \frac{\sin \theta}{\cos \theta} = \frac{1}{\sqrt{x}}$$

$$\therefore \int dy = \int \frac{1}{\sqrt{x}} dx$$

$$\text{or } y = 2\sqrt{x}$$



7. Let RI at $y = y$ is μ and corresponding angle of refraction is θ .



$$\mu \sin \theta = 1 \sin 30^\circ \quad \dots(i)$$

$$\text{and } \tan \left(\frac{\pi}{2} - \theta \right) = \frac{dy}{dx} \Rightarrow \cot \theta = 8x \Rightarrow \cot \theta = \frac{8y^{1/2}}{2}$$

$$\cot \theta = 4y^{1/2}$$

$$\text{At } y = \frac{1}{2} \text{ m} \Rightarrow \cot \theta = \frac{4}{\sqrt{2}} = 2\sqrt{2} \text{ or } \sin \theta = \frac{1}{3}$$

$$\text{From (i), } \mu \times \frac{1}{3} = \frac{1}{2} \Rightarrow \mu = \frac{3}{2}$$

$$8. \text{ We have } \frac{dy}{dx} = \cot(\theta_y) = \sqrt{\frac{\mu_y^2 - \sin^2 \theta}{\sin \theta}}$$

$$\text{Here, } \mu = \sqrt{1+y} \text{ and } \theta = 90^\circ. \text{ Therefore, } \frac{dy}{dt} = y^{1/2}$$

Integrating with the boundary condition that $y = 0$ at $x = 0$, we get $y = x^2/4$ to be the equation of the path of the ray through the slab.

The ray will obviously exit at the point $(2\sqrt{2} \text{ m}, 2 \text{ m})$.

9. The action of slab is to cause a shift in the direction of incident ray.

$$\text{Shift } s = t \left(1 - \frac{1}{\mu} \right) = 6 \left(1 - \frac{2}{3} \right) = 2 \text{ cm}$$

After refraction from the slab, image is formed at distance of 60 cm from the mirror. This acts as object for mirror.

$$\frac{1}{v} + \frac{1}{-60} = \frac{1}{-20}$$

$$\Rightarrow v = -30 \text{ cm}$$

After reflection from the mirror, image is formed at a distance of 30 cm to the left of the mirror. This act as an object for the slab and its image is shifted by 2 cm. Final image is at a distance of 32 cm from the mirror.

Exercise 1.6

1. Angle of dispersion: $\delta_v - \delta_r = (\mu_v - \mu_r)A = (1.632 - 1.613)5 = 0.095^\circ$

2. (a) $\mu_y = \frac{\mu_v + \mu_r}{2} = \frac{1.6 + 1.5}{2} = 1.55$

(b) Dispersive power: $\omega = \frac{\mu_v - \mu_r}{\mu_y - 1} = \frac{1.6 - 1.5}{1.55 - 1} = \frac{2}{11}$

3. At minimum deviation, $2r = A$ or $r = 30^\circ$

Also, $\mu = \frac{\sin(i)}{\sin(r)}$

Substituting the values, we get $1.5 = \frac{\sin(i)}{\sin(30^\circ)}$ or $i = 48.6^\circ$

The angle of deviation, $\delta = 2i - A = 37.2^\circ$

At maximum deviation: $A = 60^\circ$, $\mu = 1.5$ and $i_1 = 90^\circ$

Since $\mu = \frac{\sin(i_1)}{\sin(r_1)}$, we get $1.5 = \frac{\sin(90^\circ)}{\sin(r_1)}$

or $r_1 = 41.8^\circ$

But $r_1 + r_2 = A$, therefore, $r_2 = 18.2^\circ$

Once again, $\mu = \frac{\sin(i_2)}{\sin(r_2)}$

$\therefore 1.5 = \frac{\sin(i_2)}{\sin(18.2^\circ)}$ or $i_2 = 27.9^\circ$

Therefore, the angle of deviation, $\delta = i_1 + i_2 - A = 57.9^\circ$

4. Given, $\mu = 1.5$ and $A = \delta_{\min}$

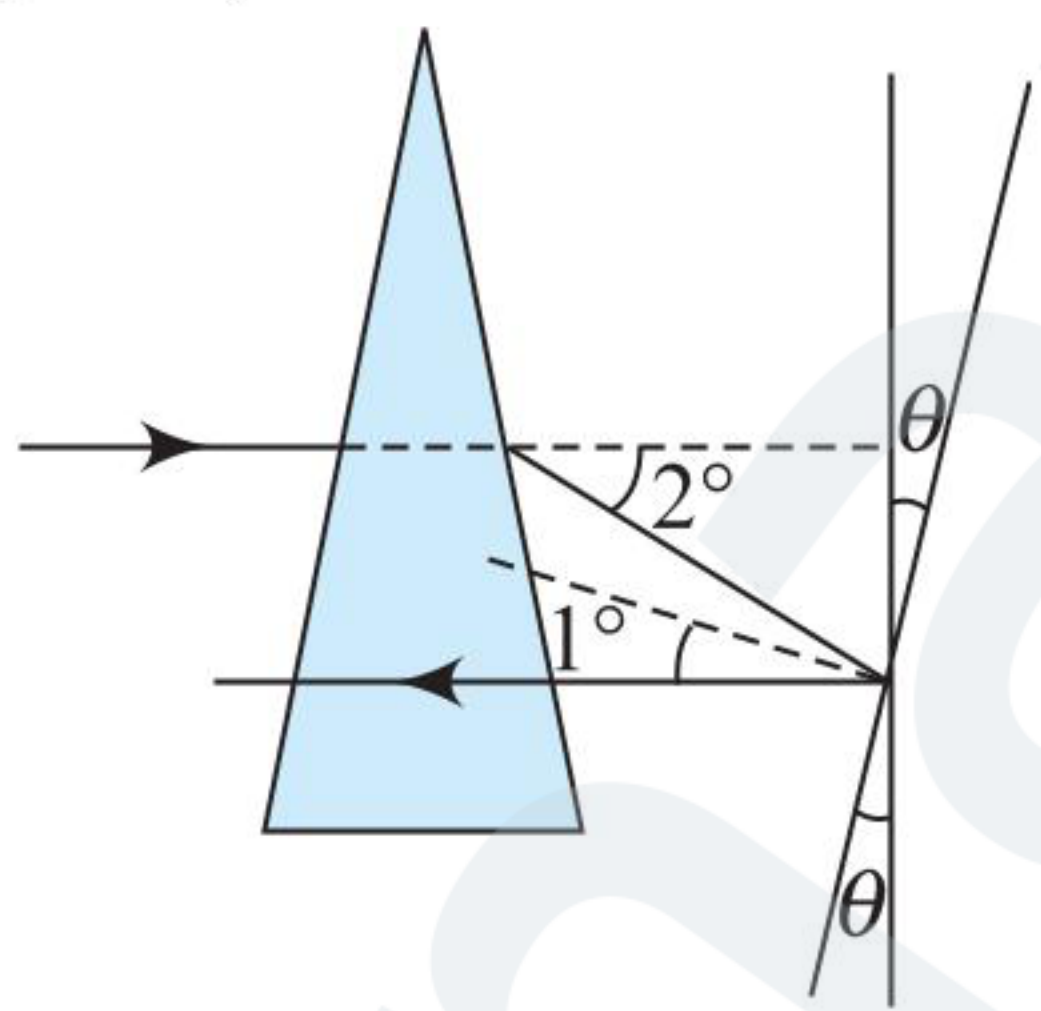
At minimum deviation, $r = A/2$ and $\delta_{\min} = 2i - A$

Substituting for A , we get $i = \delta_{\min}$

But $\mu = \frac{\sin\left(\frac{\delta_{\min} + A}{2}\right)}{\sin\left(\frac{A}{2}\right)} = \frac{\sin A}{\sin\left[\frac{A}{2}\right]} = 2 \cos\left[\frac{A}{2}\right]$

Therefore, $1.5 = 2 \cos\left[\frac{A}{2}\right]$ or $A = 2 \cos^{-1}\left(\frac{3}{4}\right)$

5. $\delta = A(\mu - 1) = 4\left(\frac{3}{2} - 1\right) = 2^\circ$



Hence, angle of rotation of mirror = 1° .

6. Critical angle between glass and liquid face is

$$\sin \theta_c = \frac{3/2}{\mu} = \frac{3}{2\mu} \quad \dots(i)$$

Angle of incidence at face AC is 60° , i.e., $i = 60^\circ$.

For total internal reflection to take place, $i = \theta_c$ or $\sin i > \sin \theta_c$

or $\sin 60^\circ > \frac{3}{2\mu}$ or $\frac{\sqrt{3}}{2} > \frac{3}{2\mu}$

or $\mu > \sqrt{3}$

7. In the case of minimum deviation,

$$i_1 = i_2, r_1 = r_2$$

$$\delta = 2i - A$$

$$r = A/2$$

According to problem, $i = 2r = A$

$$\delta_{\min} = 2A - A = A$$

$$n = \frac{\sin\left(\frac{A + \delta_{\min}}{2}\right)}{\sin\left(\frac{A}{2}\right)} = \frac{\sin A}{\sin\frac{A}{2}} = \frac{2 \sin\frac{A}{2} \cos\frac{A}{2}}{\sin\frac{A}{2}}$$

$$\cos\frac{A}{2} = \frac{n}{2} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$$

$$\therefore A = 90^\circ$$

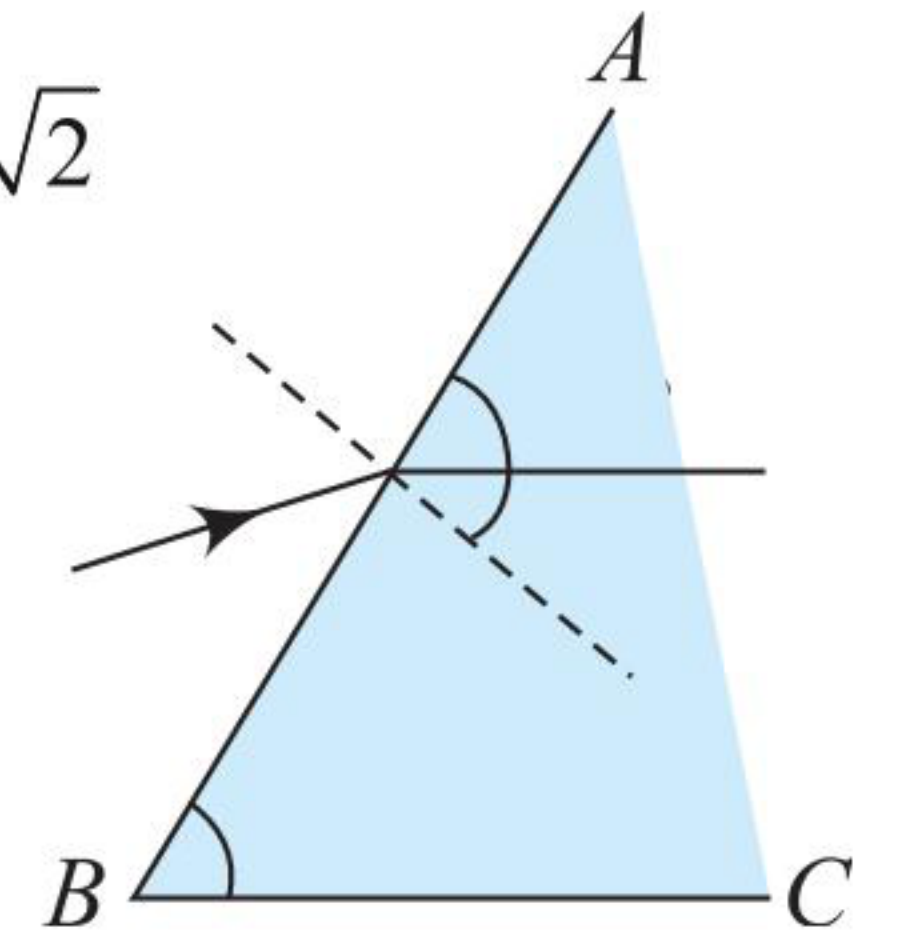
8. The given parameters are $\delta = 30^\circ$ and $A = 60^\circ$. Let us test whether the prism is in the position of minimum deviation.

$$n = \frac{\sin\left(\frac{30^\circ + 60^\circ}{2}\right)}{\sin\left(\frac{60^\circ}{2}\right)} = \frac{\sin 45^\circ}{\sin 30^\circ} = \frac{1}{\sqrt{2}} \times 2 = \sqrt{2}$$

As $n = \sqrt{2}$, the ray suffers minimum deviation through the prism. Thus,

$$r_1 = r_2 = r = \frac{A}{2} = 30^\circ$$

Inside the prism, the ray makes an angle of 60° with the face AB, so it is parallel to the base.



9. In the figure shown,

$$r_2 = C$$

$$r_1 = 60^\circ - r_2 = 60^\circ - C$$

$$r_3 = 60^\circ - r_2 = 60^\circ - C$$

$$r_1 = r_3 = r \text{ (say)}$$

$$i_1 = i_2 = i \text{ (say)}$$

Net deviation, $\delta = (i - r) + (180^\circ - 2r_2) + (i - r) = 108^\circ$

$$r_2 = r - i = 36^\circ$$

$$C + 60^\circ - C - i = 36^\circ$$

$$i = 24^\circ, \sin 24^\circ \approx 0.40$$

From Snell's law, we have

$$\sin 24^\circ = \mu \sin r$$

$$0.4 = \mu \sin(60^\circ - C)$$

$$0.4 = \mu \left[\frac{\sqrt{3}}{2} \cos C - \frac{1}{2\mu} \right]$$

$$0.8 = \sqrt{3}\mu \sqrt{1 - \frac{1}{\mu^2}} - 1$$

or $3(\mu^2 - 1) = (1.8)^2$

$\therefore \mu^2 - 1 = 1.08$ or $\mu = 1.447$

10. As the ray of light grazes the second surface, r_2 is the critical angle.

$$\sin r_2 = \frac{1}{\mu}$$

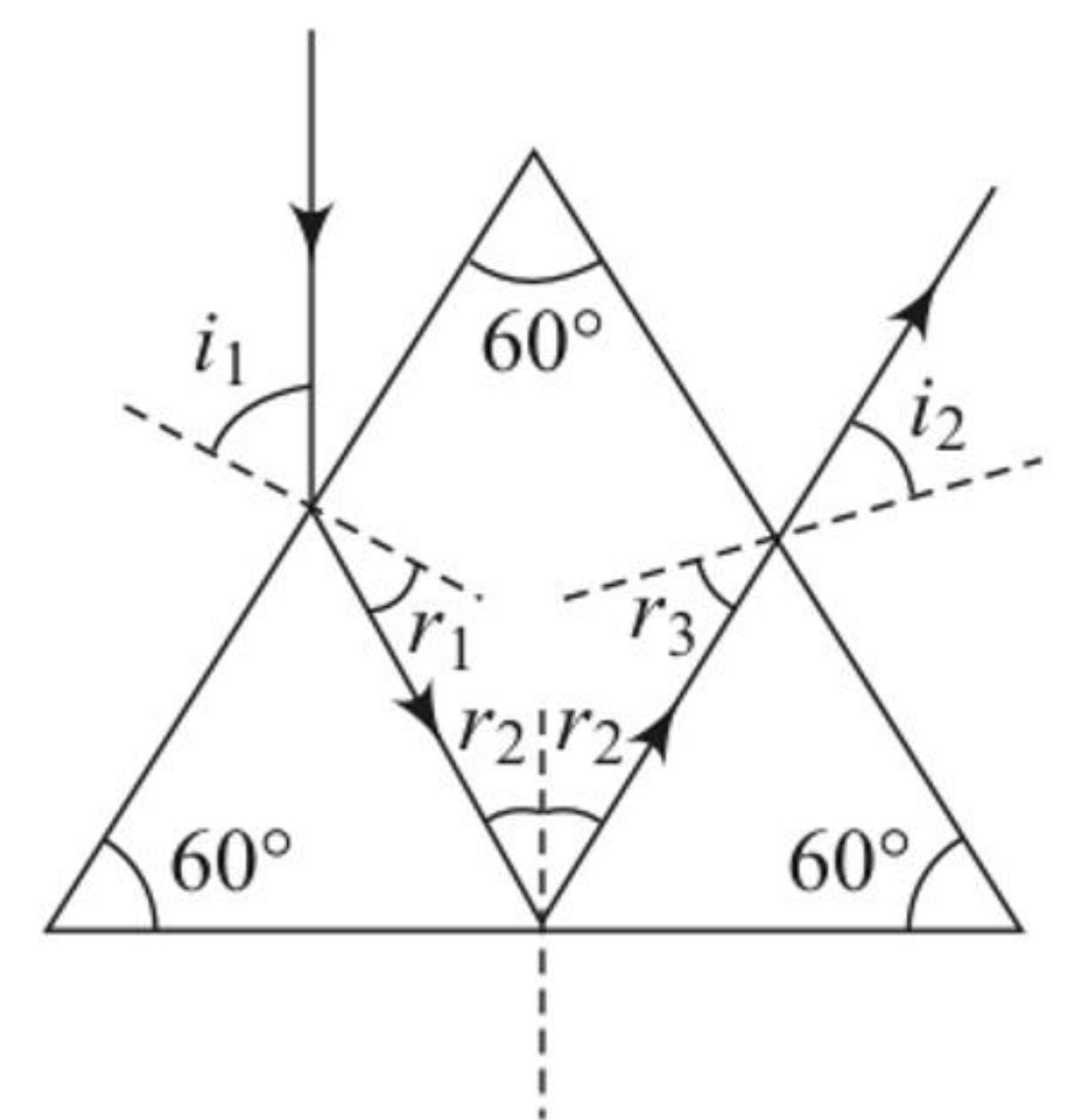
$$r_2 = (45^\circ - r_1)$$

$$\sin r_1 = \frac{\sin 45^\circ}{\mu} = \frac{1}{\sqrt{2}\mu}$$

$$\sin r_2 = \sin(45^\circ - r_1) = \frac{1}{\sqrt{2}} [\cos r_1 - \sin r_1]$$

$$\frac{1}{\mu} = \frac{1}{\sqrt{2}} \left[\sqrt{1 - \frac{1}{2\mu^2}} - \frac{1}{\sqrt{2}\mu} \right]$$

or $\frac{1}{\mu} = \frac{1}{\sqrt{2}} \left[\frac{\sqrt{2\mu^2 - 1}}{\sqrt{2}\mu} - \frac{1}{\sqrt{2}\mu} \right]$



$$2 = \sqrt{2\mu^2 - 1} - 1$$

$$\text{or } 2\mu^2 - 1 = 9$$

$$2\mu^2 = 10$$

$$\Rightarrow \mu^2 = 5 \quad \text{or} \quad \mu = \sqrt{5}$$

$$\text{At minimum deviation, } r_1 = r_2 \frac{45^\circ}{2} = 22.5^\circ$$

$$\mu = \frac{\sin i_1}{\sin r_1} \Rightarrow \sin i_1 = (\sqrt{5}) \sin (22.5^\circ)$$

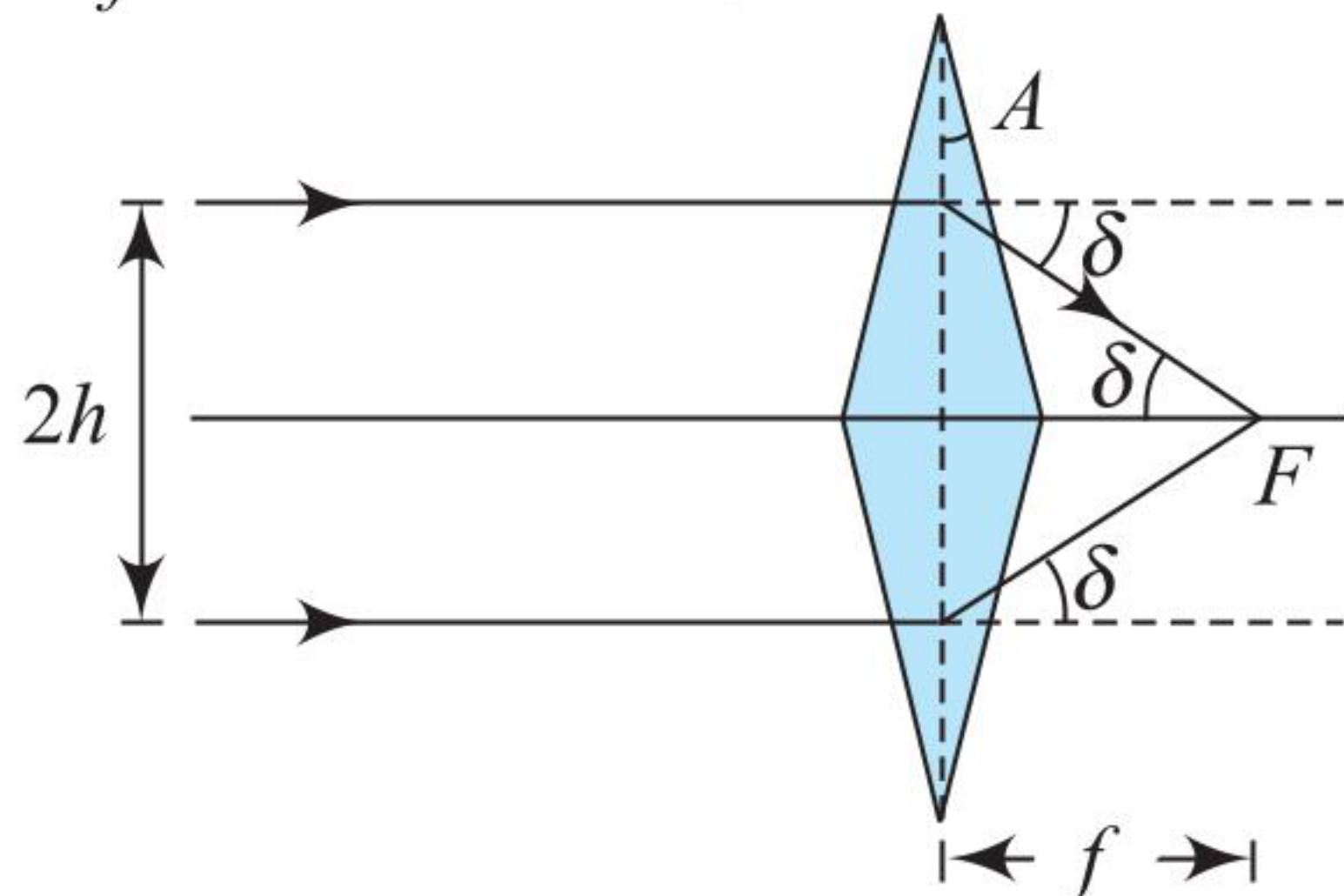
$$i_1 = 58.8^\circ$$

$$11. \text{ For minimum deviation: } \mu = \frac{\sin[(A + \delta_m)/2]}{\sin(A/2)}$$

$$\frac{3}{2} = \frac{\sin[(60^\circ + \delta_m)/2]}{\sin(60^\circ/2)} = \sin\left(\frac{60^\circ + \delta_m}{2}\right) = \frac{3}{4}$$

$$\Rightarrow i = \frac{A + \delta_m}{2} = \sin^{-1}\left(\frac{3}{4}\right) = 48.5^\circ$$

$$12. \text{ Since } \tan \delta = \frac{h}{f} \Rightarrow f = \frac{h}{\tan \delta}$$

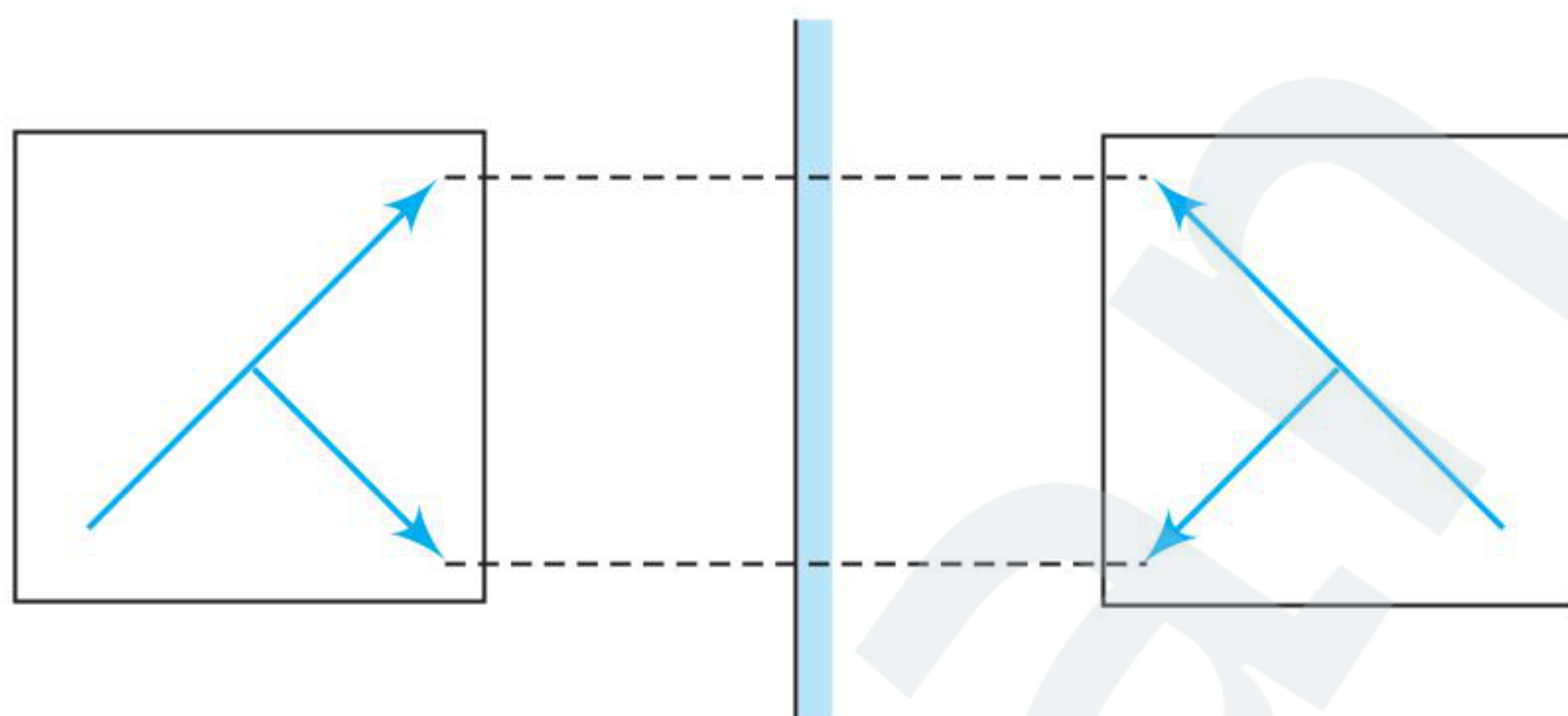


$$\text{Further, } \delta = (\mu - 1)A \Rightarrow f = \frac{h}{(\mu - 1)A}$$

Exercises

Single Correct Answer Type

1. (3)



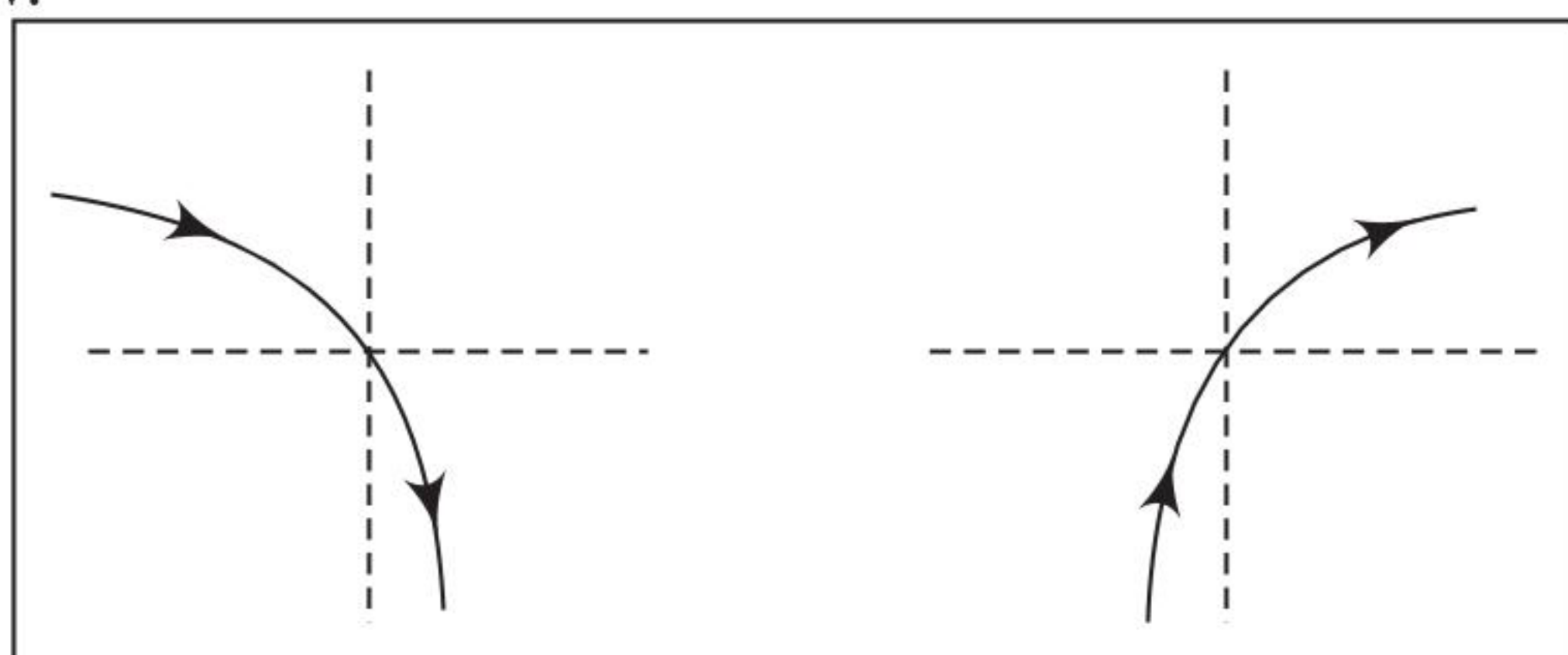
2. (4) As the ray moves toward the normal while entering medium 2 from 1, we have $n_2 > n_1$

For total internal reflection at interface of 2 and 3, $n_2 > n_3$.

Besides n_3 should also be less than n_1 or else ray would have emerged in medium 3, parallel to its path in medium 1.

Hence, $n_3 < n_1 < n_2$ is the correct order.

3. (1) Since the refractive index is changing, the light cannot travel in a straight line in the liquid as shown in options (3) and (4). Initially, it will bend towards normal and after reflecting from the bottom it will bend away from the normal as shown below.



$$4. (1) \text{ From Snell's law, } \mu_1 \sin i = \mu_2 \sin r \Rightarrow \frac{\mu_2}{\mu_1} = \frac{\sin i}{\sin r}$$

$$\text{From the graph, } \tan 60^\circ = \frac{\sin i}{\sin r} \Rightarrow \frac{\sin i}{\sin r} = \sqrt{3} = \frac{\mu_2}{\mu_1}$$

$$\mu = \frac{c}{v} \Rightarrow \frac{\mu_1}{\mu_2} = \frac{v_2}{v_1}$$

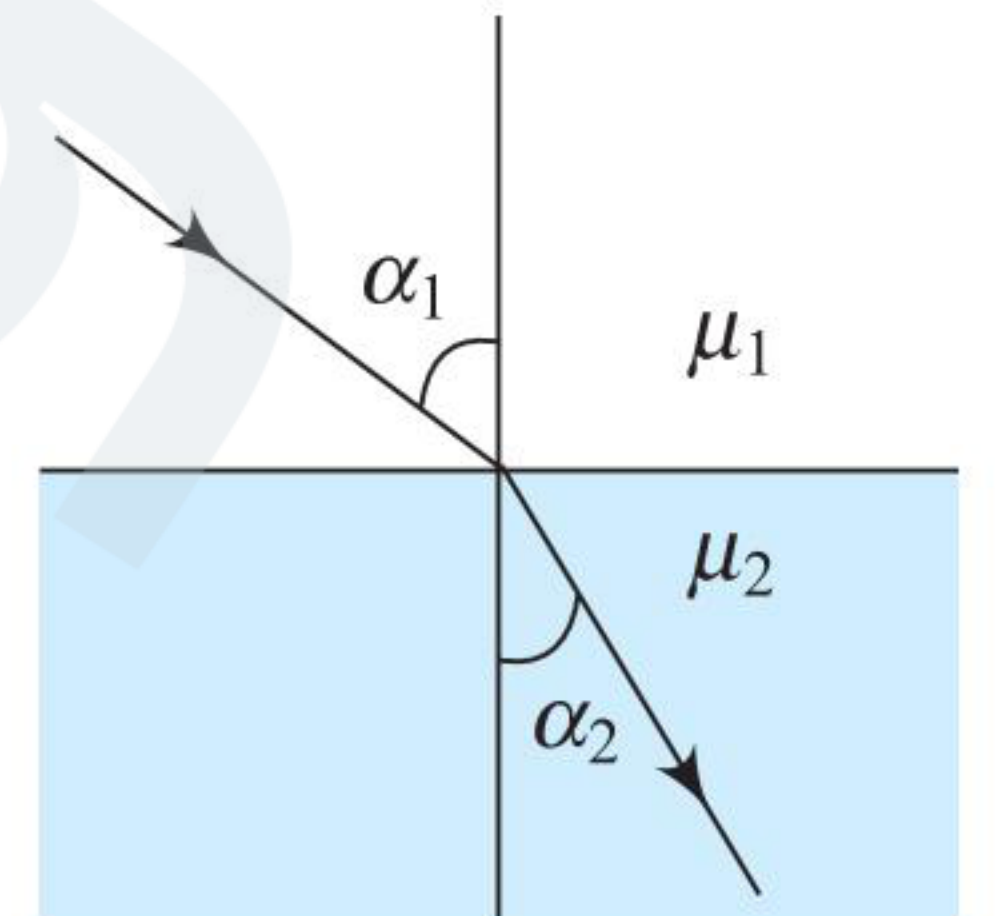
$$v_1 = \left(\frac{\mu_2}{\mu_1}\right) v_2 \Rightarrow v_1 = \sqrt{3} v_2$$

$$5. (2) \mu_1 \sin \alpha = \mu_2 \sin \alpha_2$$

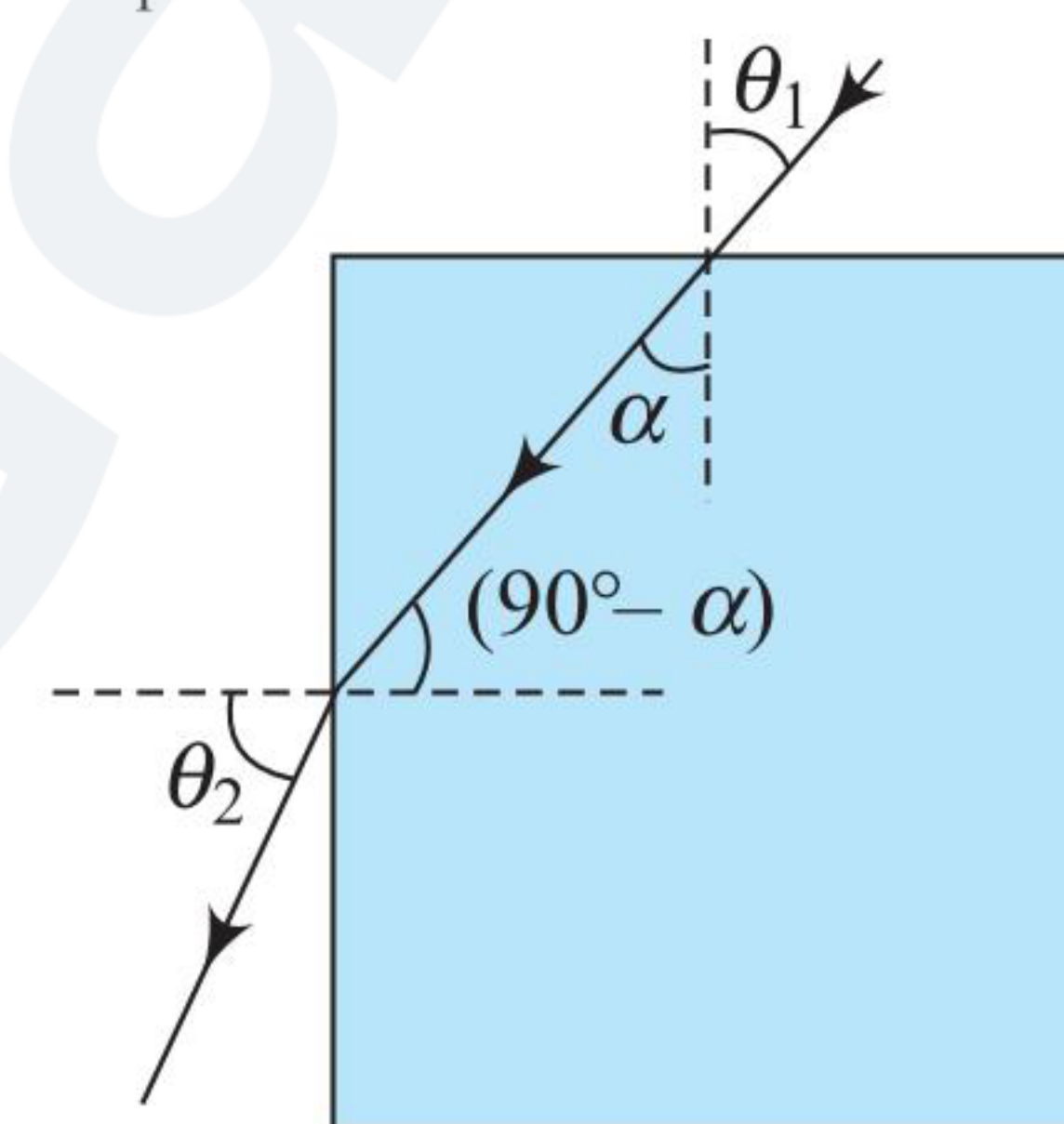
$$\frac{c}{v_1} \sin \alpha_1 = \frac{c}{v_2} \sin \alpha_2$$

$$\frac{\sin \alpha_1}{f \lambda_1} = \frac{\sin \alpha_2}{f \lambda_2}$$

$$\lambda_2 = \frac{\sin \alpha_2}{\sin \alpha_1} \lambda_1$$



6. (3)



As θ_1 increases, α also increases. So, $(90^\circ - \alpha)$ decreases and hence θ_2 decreases.

$$7. (1) \frac{\mu_1}{-u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}$$

For a plane surface, $R = \infty$

$$\therefore \frac{\mu_1}{-u} + \frac{\mu_2}{v} = 0$$

$$\text{or } \frac{\mu_2}{v} = \frac{\mu_1}{u} \quad \text{or } \frac{\mu}{v} = \frac{1}{u}$$

$$\text{or } v = \mu_2 u$$

Clearly, to the fish, the bird appears farther than its actual distance.

$$\text{Again, } \frac{dv}{dt} = \mu \frac{du}{dt}$$

or Apparent speed of bird = Refractive index $\times$ Actual speed of bird.

$$8. (3) \sqrt{3} = \frac{\sin\left(\frac{60^\circ + \delta_m}{2}\right)}{\sin\left(\frac{60^\circ}{2}\right)}$$

$$\frac{\sqrt{3}}{2} = \sin\left(\frac{60^\circ + \delta_m}{2}\right)$$

$$\sin 60^\circ = \sin\left(\frac{60^\circ + \delta_m}{2}\right)$$

$$\text{or } \frac{60^\circ + \delta_m}{2} = 60^\circ$$

$$\text{or } \delta_m = 60^\circ \Rightarrow i = \frac{A + \delta_m}{2} = \frac{60^\circ + 60^\circ}{2} = 60^\circ$$

9. (4) For A:

$$u = -3 \text{ m, } v_1 = ?, f = -1 \text{ m}$$

$$\frac{1}{v_1} = \frac{1}{f} - \frac{1}{u} = \frac{1}{-1} - \frac{1}{-3} = \frac{1}{3} - 1 = -\frac{2}{3}$$

or $v_1 = -\frac{3}{2}$

For B:

$$\frac{1}{v_2} = \frac{1}{-1} - \frac{1}{-5}$$

or $\frac{1}{v_2} = \frac{1}{5} - 1 = -\frac{4}{5}$ or $v_2 = -\frac{5}{4}$ m

Now, $v_1 - v_2 = -\frac{3}{2} - \left(-\frac{5}{4}\right) = -\frac{3}{2} + \frac{5}{4} = -\frac{1}{4}$ m = -0.25 m

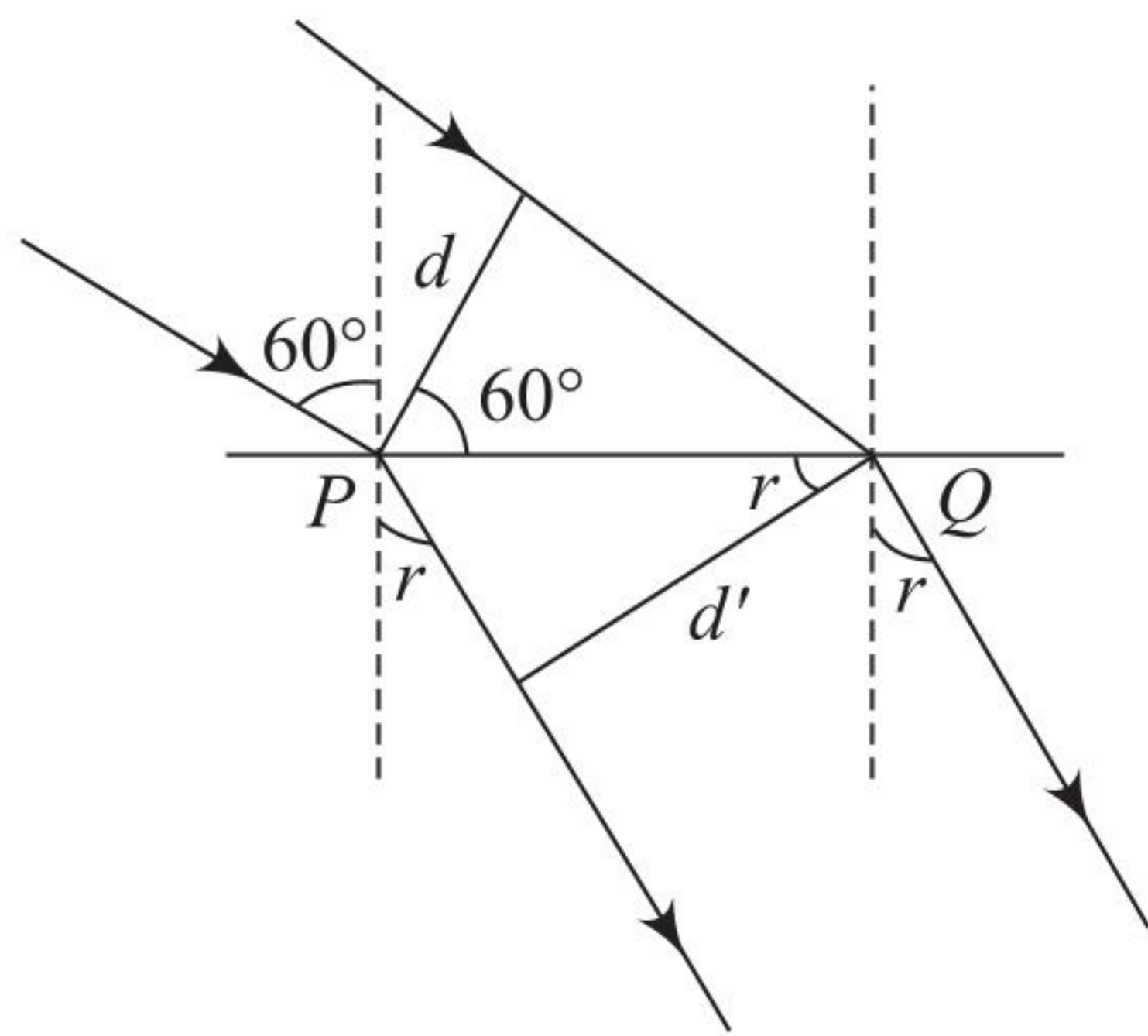
Again, $\frac{I_1}{O} = -\frac{v_1}{u}$

or $I_1 = -\frac{v_1}{u}O = -\left(\frac{-3}{2}\right)\left(\frac{-1}{3}\right) = -1$ m

Again, $\frac{I_2}{O} = -\frac{v_2}{u}$

or $I_2 = -\left(\frac{-5}{4}\right)\left(\frac{1}{-5}\right)2 = -0.5$ m

10. (3) Let d' be the diameter of refracted beam.



Then, $d = PQ \cos 60^\circ$

and $d' = PQ \cos r$

i.e. $\frac{d'}{d} = \frac{\cos r}{\cos 60^\circ} = 2 \cos r$

or $d' = 2d \cos r$

$$\sin r = \frac{\sin i}{\mu} = \frac{\sqrt{3}/2}{3/2} = \frac{1}{\sqrt{3}}$$

$$\therefore \cos r = \sqrt{1 - \sin^2 r} = \sqrt{\frac{2}{3}}$$

$$\therefore d' = (2)(2) \sqrt{\frac{2}{3}} = 4 \sqrt{\frac{2}{3}} \text{ cm} \approx 3.26 \text{ cm}$$

11. (2) $v_m = \frac{1}{2}c$

$$\mu = \frac{c}{v_m} = \frac{c}{\frac{1}{2}c} = 2 \text{ or } \frac{1}{\sin i_c} = 2$$

or $\sin i_c = \frac{1}{2}$ or $i_c = 30^\circ$

12. (4) For convex mirror, $u + v = 12 \times 2 = 24$ cm (because for plane mirror, distance of object = distance of image). Also, here for the convex mirror $u = 20$ cm, therefore $v = 4$ cm. Hence, using

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

We find $f = 5$ cm

13. (2) Distance of first image (I_1) formed after refraction from the plane surface of water is $\frac{10}{4/3} = 7.5$ cm from water surface
 $\left(d_{\text{app}} = \frac{d_{\text{actual}}}{\mu}\right)$.

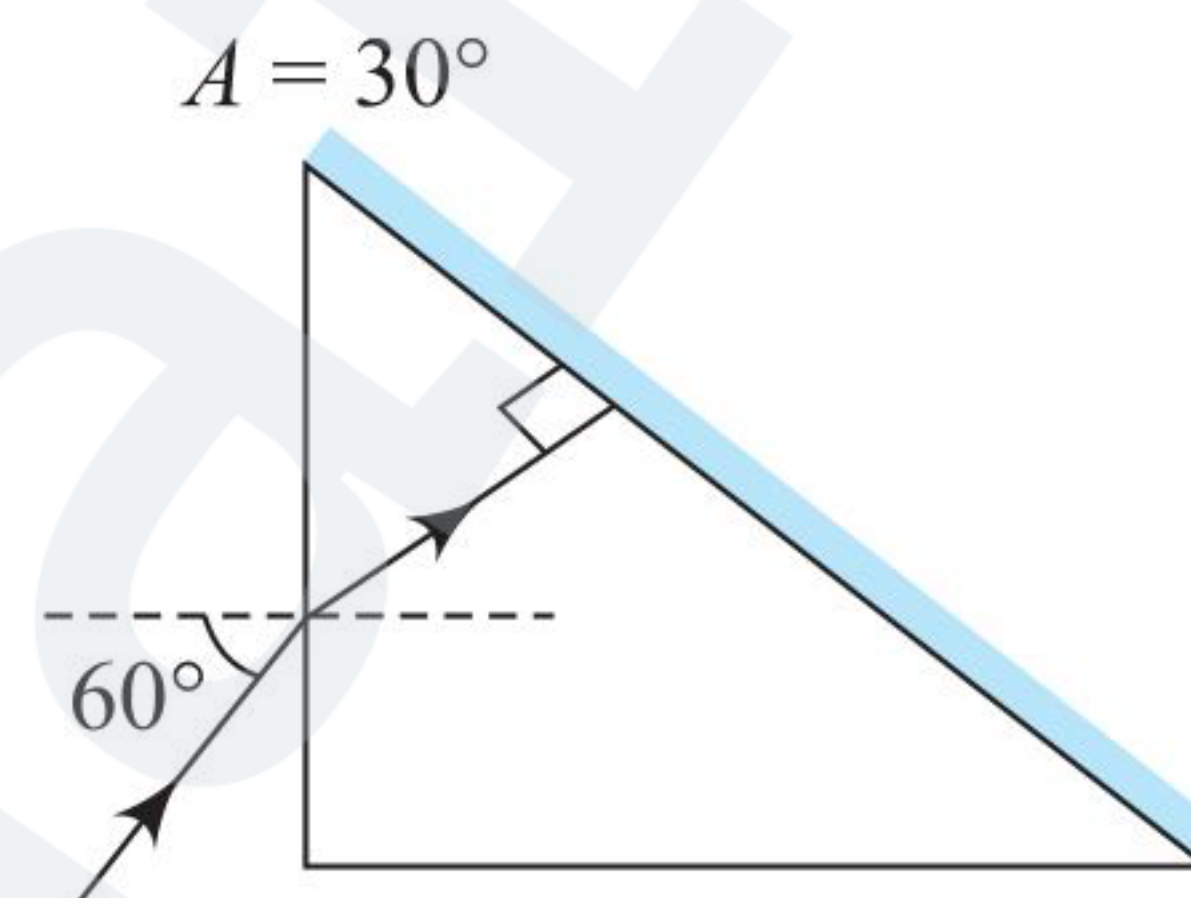
Now, distance of this image is $5 + 7.5 = 12.5$ cm from the plane mirror. Therefore, distance of second image (I_2) will also be equal to 12.5 cm from the mirror.

14. (4) The two slabs will shift the image a distance

$$d = 2\left(1 - \frac{1}{\mu}\right)t = 2\left(1 - \frac{1}{1.5}\right)(1.5) = 1.0 \text{ cm}$$

Therefore, final image will be 1 cm above point P .

15. (2) $r_2 = 0^\circ$



$$r_1 = A = 30^\circ \text{ and } i_1 = 60^\circ$$

$$\mu = \frac{\sin i_1}{\sin r_1} = \frac{\sin 60^\circ}{\sin 30^\circ} = \sqrt{3}$$

16. (1) $A = \delta_m = 60^\circ$

$$\text{At minimum deviation } i = \left(\frac{A + \delta_m}{2}\right) = 60^\circ$$

17. (2) $m = \frac{f}{f - u}$

$$3 = \frac{-24}{-24 - u}$$

or $-24 - u = -8$ or $u + 24 = 8$

or $u = (8 - 24) \text{ cm} = -16 \text{ cm}$

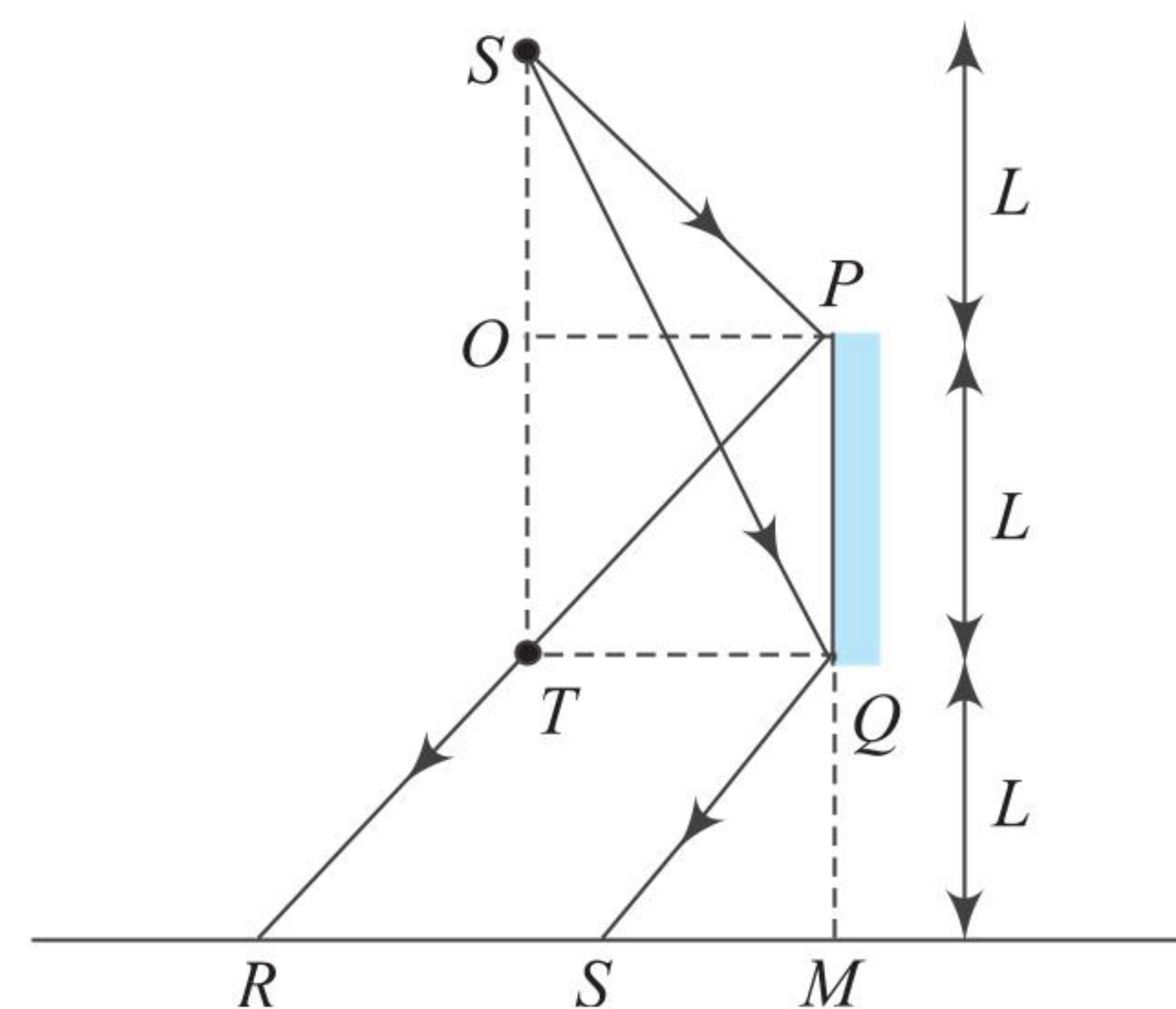
If $m = -3$, then $-3 = \frac{-24}{-24 - u}$

$$u + 24 = -8$$

or $u = -32 \text{ cm}$

Note that the magnification is greater than 1, so mirror cannot be convex.

18. (3) Ray diagram is as shown in figure:



$$\text{Here, } \frac{SO}{OP} = \frac{PM}{RM} \Rightarrow RM = 2L$$

$$\text{Also, } \frac{ST}{TQ} = \frac{QM}{SM} \Rightarrow SM = \frac{L}{2}$$

$$RS = RM - SM = \frac{3L}{2}$$

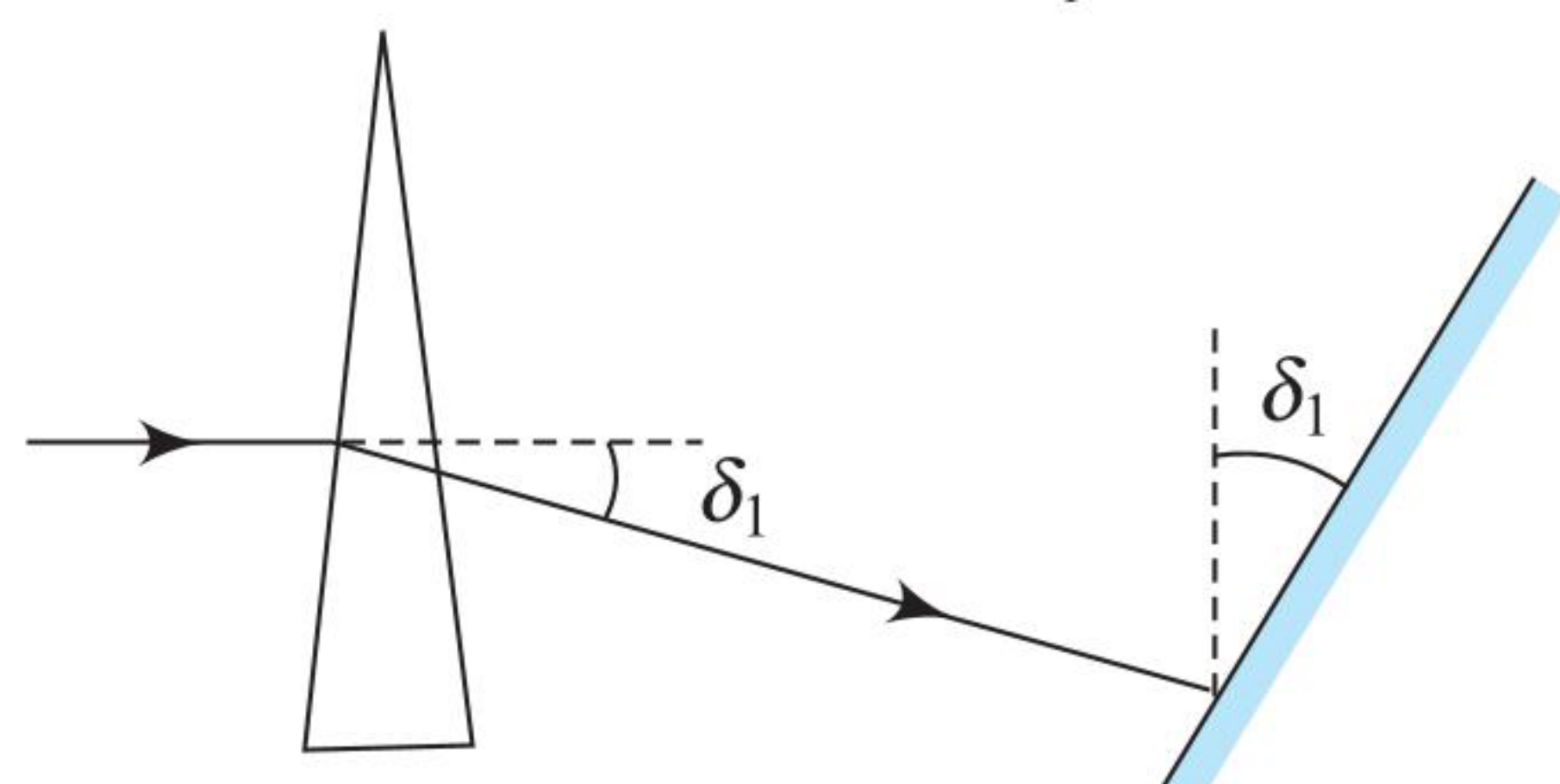
19. (2) The fish can observe the sky only if refraction takes place. If TIR takes place, then image of sky cannot be observed.

i.e., $i < i_c$ and $i_c = 45^\circ$

So, angle subtended $= \frac{\pi}{2} = 90^\circ$

20. (4) For light to retrace the path, the light ray falling on the mirror should be along the normal.

Deviation produced by prism, $\delta_1 = (1.5 - 1)4 = 2^\circ$ CW



So, mirror has to be rotated by 2° in CW direction.

21. (1) As object moves from infinity to the pole of convex mirror, image moves from focus to pole. So, $v_i < v_o$, always.

22. (1) In a prism: $r + r' = A \Rightarrow r = A - r'$

$$\therefore r = 60^\circ - (10 + t^\circ) = 50 - t^\circ$$

23. (1) Given $i = 60^\circ$ and $A = \delta = e$

$$\delta = i + e - A \Rightarrow \delta = i (\because e = A)$$

$$\mu = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin\frac{A}{2}}$$

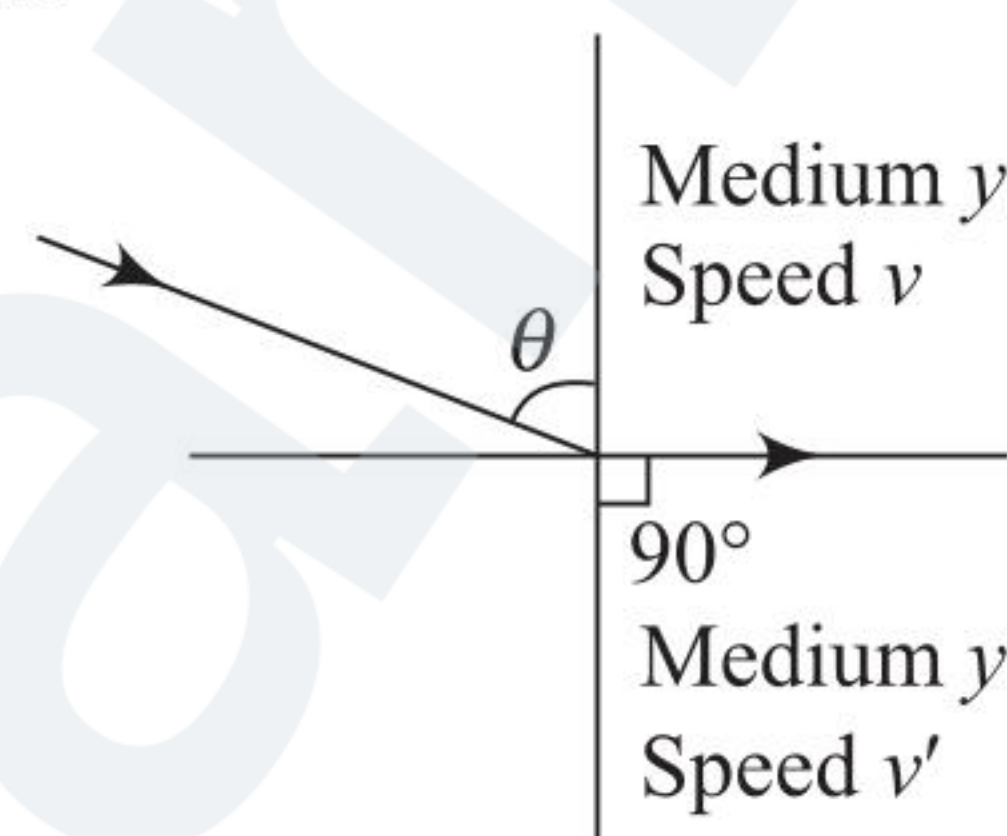
Here, angle of deviation is minimum ($\because i = e$)

$$\mu = \frac{\sin\left(\frac{60^\circ + 60^\circ}{2}\right)}{\sin(60^\circ/2)} = 1.73$$

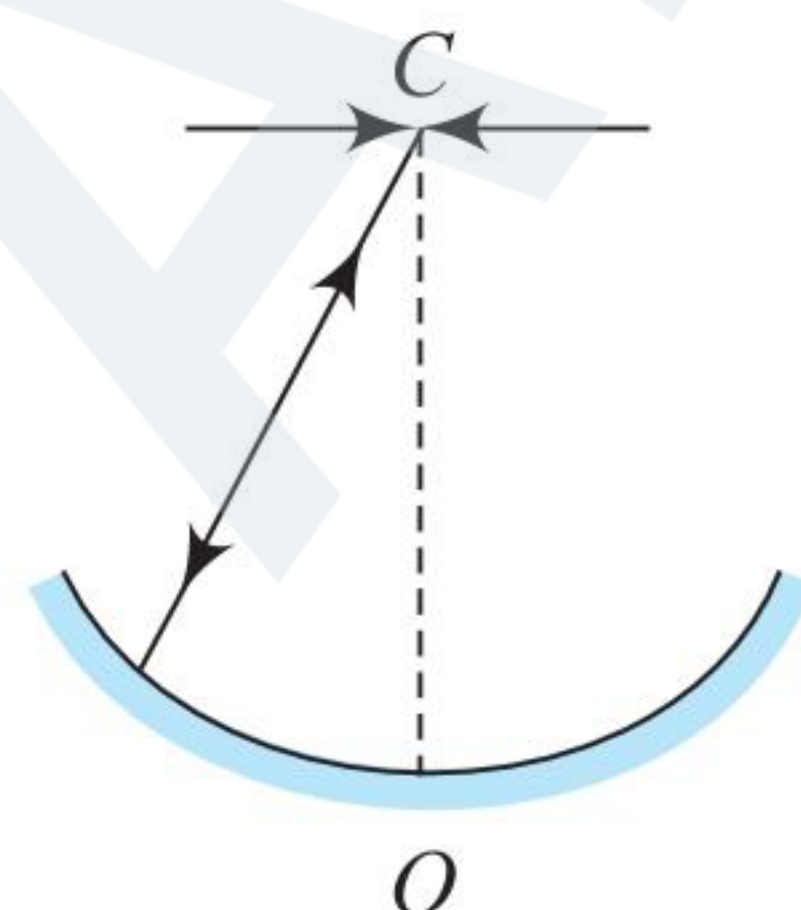
24. (4) Clearly, x is denser medium.

$$\text{Now, } \frac{\sin\theta}{\sin 90^\circ} = \frac{1}{\mu_X} = \mu_Y = \frac{v}{v'}$$

$$\text{or } v' = \frac{v}{\sin\theta}$$



25. (4) When there is no water in the mirror, the rays of light are incident normally on the mirror and retrace their path. So, we get an image coincident with the object as shown in the figure.



When the mirror is filled with water, then the equivalent focal length F is given by

$$\frac{1}{F} = \frac{1}{f_{\text{water lens}}} + \frac{1}{f_{\text{concave mirror}}} + \frac{1}{f_{\text{water lens}}}$$

$$\text{or } \frac{1}{F} = 2 \times \frac{1}{f_{\text{water lens}}} + \frac{1}{f_{\text{concave mirror}}}$$

$$\text{or } \frac{1}{F} = 2(\mu - 1)\left(\frac{1}{R}\right) + \frac{1}{R}$$

$$\text{or } \frac{1}{F} = \frac{2(\mu - 1)}{R} + \frac{2}{R}$$

$$\text{or } \frac{1}{F} = \frac{2\mu}{R} \text{ or } F = \frac{R}{2\mu}$$

Clearly, focal length of the new optical system is less than the original. So, the object is effectively at a distance greater than twice the focal length. So, the real image will be formed between F and $2F$.

26. (4) A thick glass mirror produces a number of images.

There is an apparent shift of actual silvered surface toward the unsilvered face.

Effective distance of the reflecting surface from unsilvered face

$$= \frac{d}{\mu} = \frac{3}{3/2} \text{ cm} = 2 \text{ cm.}$$

Distance of the point object from effective reflecting surface $= 9 \text{ cm} + 2 \text{ cm} = 11 \text{ cm}$.

Distance of image from the point object $= 11 \text{ cm} + 11 \text{ cm} = 22 \text{ cm}$

Distance of image from unsilvered face $= (22 - 9) \text{ cm} = 13 \text{ cm}$

27. (1) Clearly, the distance of I from P is 15 cm.

Now, $u = -25 \text{ cm}$, $v = 15 \text{ cm}$, $f = ?$

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \text{ or } \frac{1}{f} = \frac{1}{-25} + \frac{1}{15}$$

$$\text{or } \frac{1}{f} = \frac{-3 + 5}{75} \text{ or } f = \frac{75}{2} \text{ cm} = 37.5 \text{ cm}$$

28. (2) $\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \Rightarrow -\frac{du}{u^2} - \frac{dv}{v^2} = 0 \text{ or } -\frac{dv}{v^2} = \frac{du}{u^2}$

$$\text{or } dv = -\frac{v^2}{u^2} du = -\frac{10 \times 10}{30 \times 30} \times 9 \text{ m s}^{-1} = -1 \text{ m s}^{-1}$$

29. (1) For a concave mirror, $u = -\frac{15}{2} \text{ cm}$, $v = ?$

$$f = -\frac{10}{2} \text{ cm} = -5 \text{ cm}$$

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{-5} - \frac{1}{-15/2} = -\frac{1}{5} + \frac{2}{15} = \frac{-1}{15}$$

or $v = -15 \text{ cm}$

Clearly, the position of the final image is on the pole of the convex mirror.

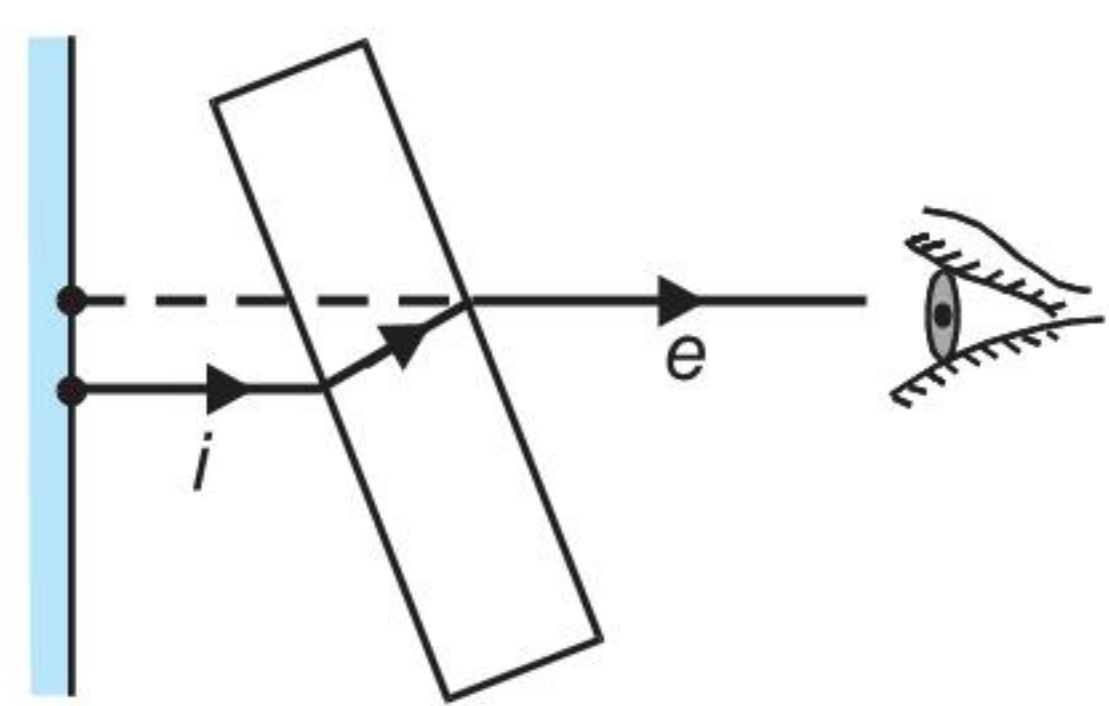
30. (2) For upright portion,

$$m = \frac{f}{f - u} = \frac{-10}{\frac{-10}{2} - (-20)} = \frac{-5}{-5 + 20} = \frac{-5}{15} = -\frac{1}{3}$$

For horizontal portion, magnification is $\left(-\frac{1}{3}\right)^2$ i.e., $\frac{1}{9}$

Required ratio is $\frac{-1/3}{1/9} = -3:1$

31. (1) Emergent ray e will be parallel to incident ray i . So both dots will appear to be shifted upward by same distance. Hence, distance between them remains same.



32. (4) Let $u = -x$

$$\frac{1}{-x} + \frac{1}{-x-10} = -\frac{1}{12}$$

or $\frac{1}{x} + \frac{1}{x+10} = \frac{1}{12}$

or $\frac{x+10+x}{x(x+10)} = \frac{1}{12}$

or $\frac{2x+10}{x^2+10x} = \frac{1}{12}$

or $x^2+10x = 24x+120$

or $x^2-14x-120=0$

$x^2-20x+6x-120=0$

or $x(x-20)+6(x-20)=0$

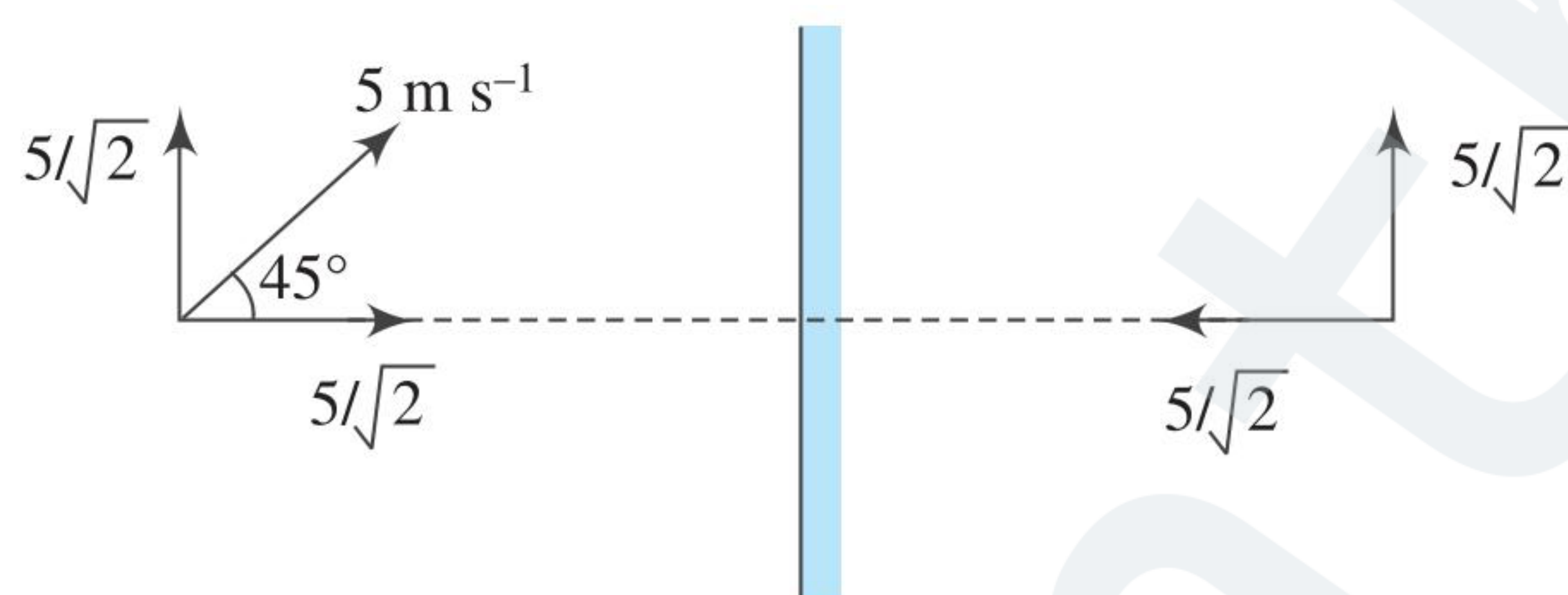
$(x+6)(x-20)=0$

Here $u = -20$ cm

$v = -(20+10)$ cm = -30 cm

Then magnification $m = -\frac{u}{v} = -\left(\frac{-30}{-20}\right) = -1.5$

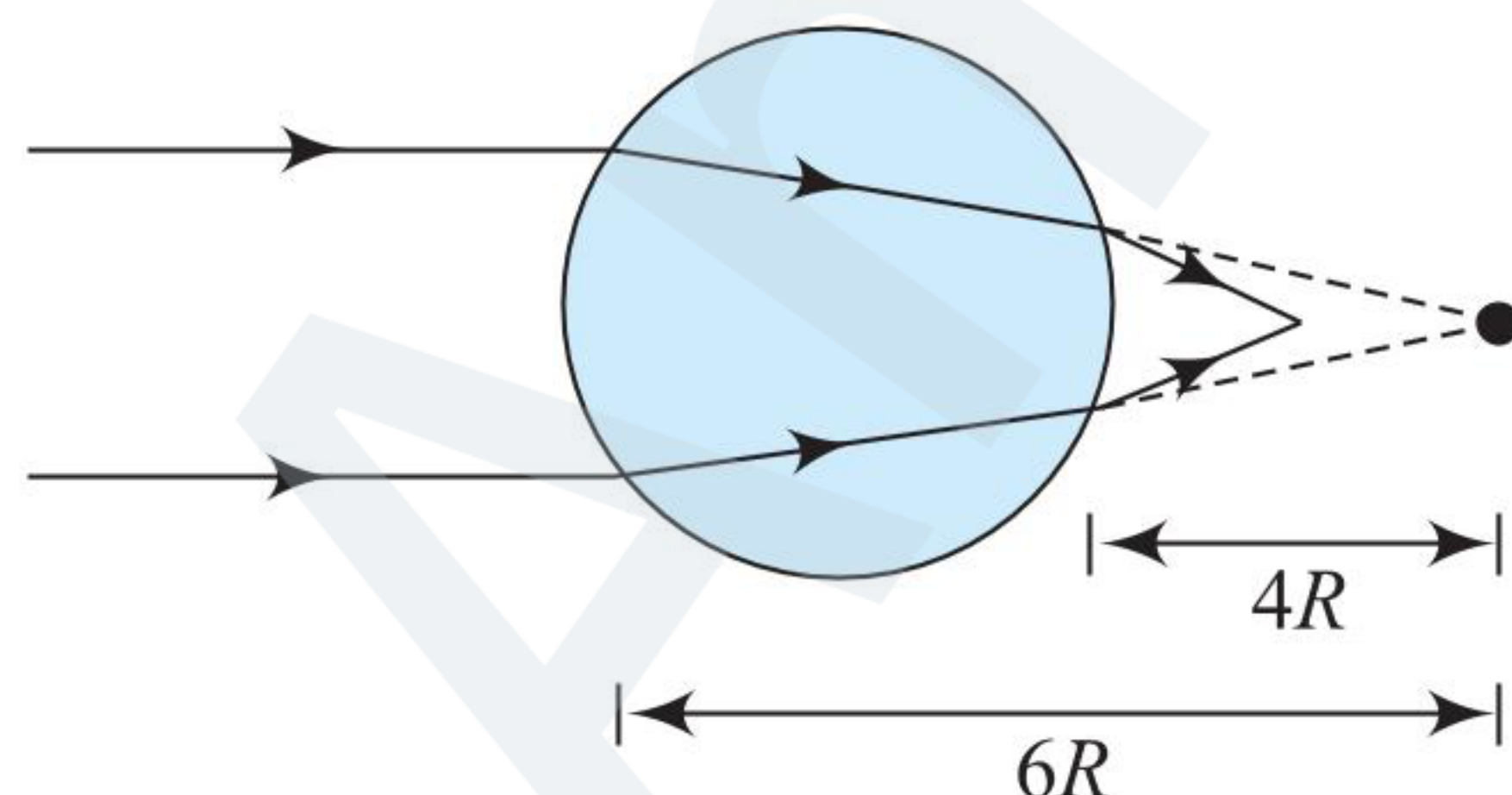
33. (1) Speed of image w.r.t. mirror = $\sqrt{\left(\frac{5}{\sqrt{2}}\right)^2 + \left(\frac{5}{\sqrt{2}}\right)^2} = 5 \text{ m s}^{-1}$



34. (2) $\frac{\mu_1}{-u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}$

$\frac{1.25}{-(-\infty)} + \frac{1.5}{v} = \frac{1.5-1.25}{R}$ or $\frac{1.5}{v} = \frac{0.25}{R}$

or $v = \frac{1.5R}{0.25} = 6R$



Again, $\frac{\mu_2}{-u} + \frac{\mu_1}{v} = \frac{\mu_1 - \mu_2}{R}$

$\frac{1.5}{-4R} + \frac{12.5}{v} = \frac{1.25-1.5}{-R}$

or $\frac{1.25}{v} = \frac{1}{4R} + \frac{1.5}{4R} = \frac{2.5}{4R}$

$v = \frac{1.25 \times 4R}{2.5} = \frac{5R}{2.5}$

or $v = 2R$

Distance from the centre = $3R$

35. (3) $\mu = \frac{\sin r'}{\sin i}$

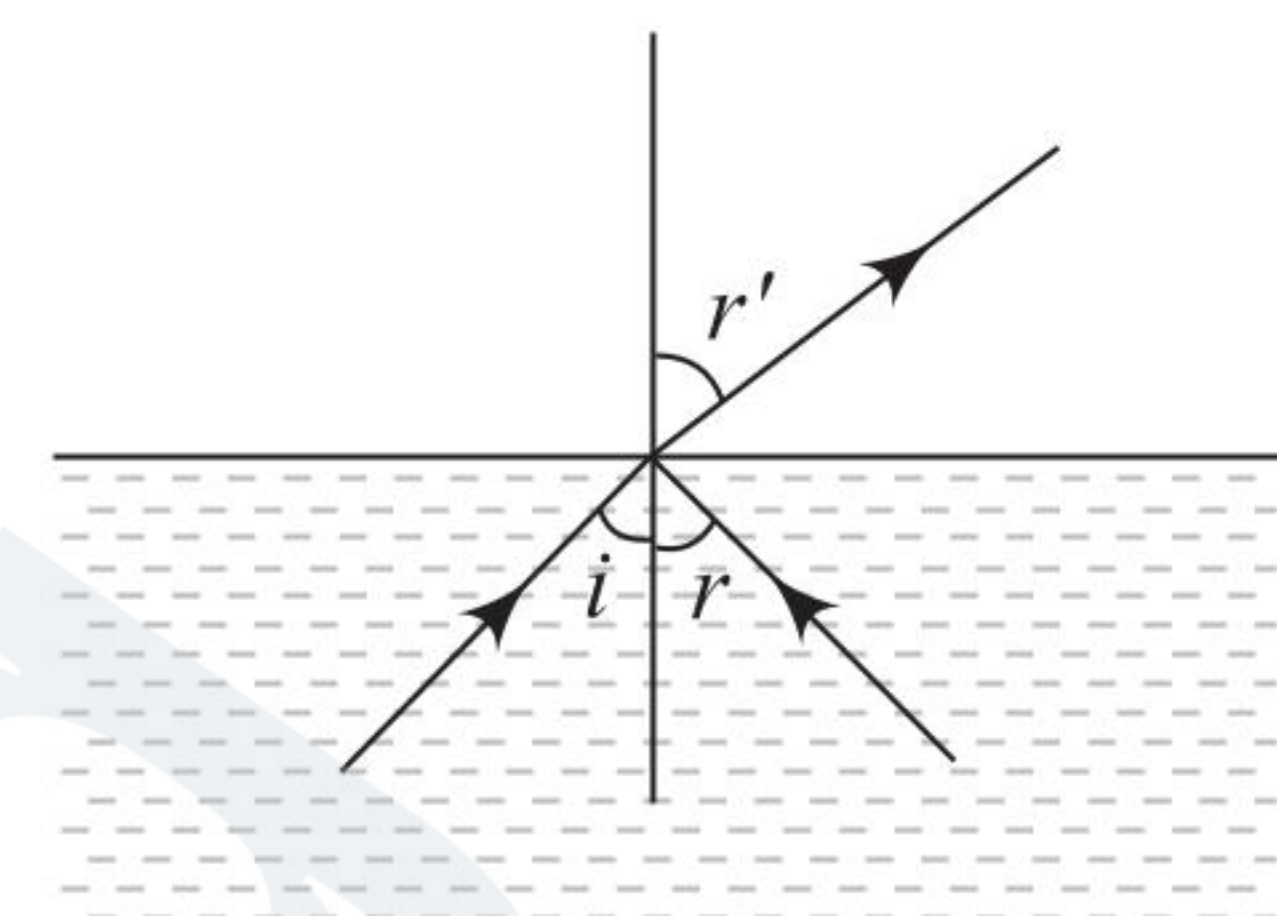
But $r' + r = 90^\circ$

or $r' = 90^\circ - r$

$r = 90^\circ - r'$

$\therefore \mu = \frac{\cos i}{\sin i}$

or $\frac{1}{\sin i_c} = \frac{1}{\tan i}$ or $i_c = \sin^{-1}(\tan i)$



36. (3) $\mu_g \sin \theta_c = \mu_l \sin 90^\circ$

or $\mu_g \sin \theta_c = 1$

When water is poured,

$\mu_w \sin r = \mu_g \sin \theta_c$

or $\mu_w \sin r = 1$

Again, $\mu_a \sin \theta = \mu_w \sin r$

or $\mu_a \sin \theta = 1$

or $\sin \theta = 1$ or $\theta = 90^\circ$

37. (2) $\frac{\sin 60^\circ}{\sin r_1} = \sqrt{3}$

or $\sin r_1 = \frac{\sin 60^\circ}{\sqrt{3}}$

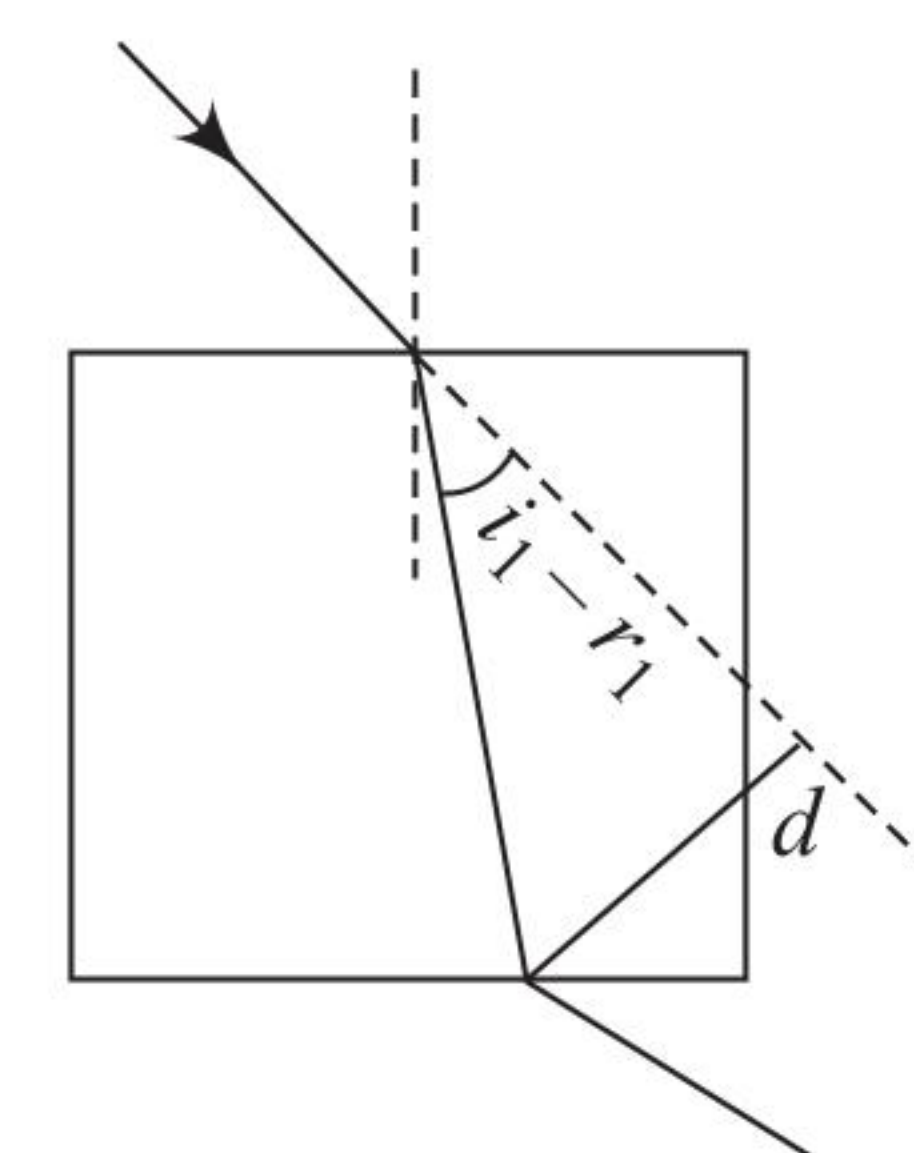
or $\sin r_1 = \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{3}} = \frac{1}{2}$

or $r_1 = 30^\circ$

Now, $\sin(i_1 - r_1) = \frac{d}{5}$

or $d = 5 \sin(i_1 - r_1)$

or $d = 5 \sin(60^\circ - 30^\circ) = 5 \sin 30^\circ = \frac{5}{2} \text{ cm}$



38. (3) $\sin i_c = \frac{1}{\sqrt{2}} \Rightarrow i_c = 45^\circ$

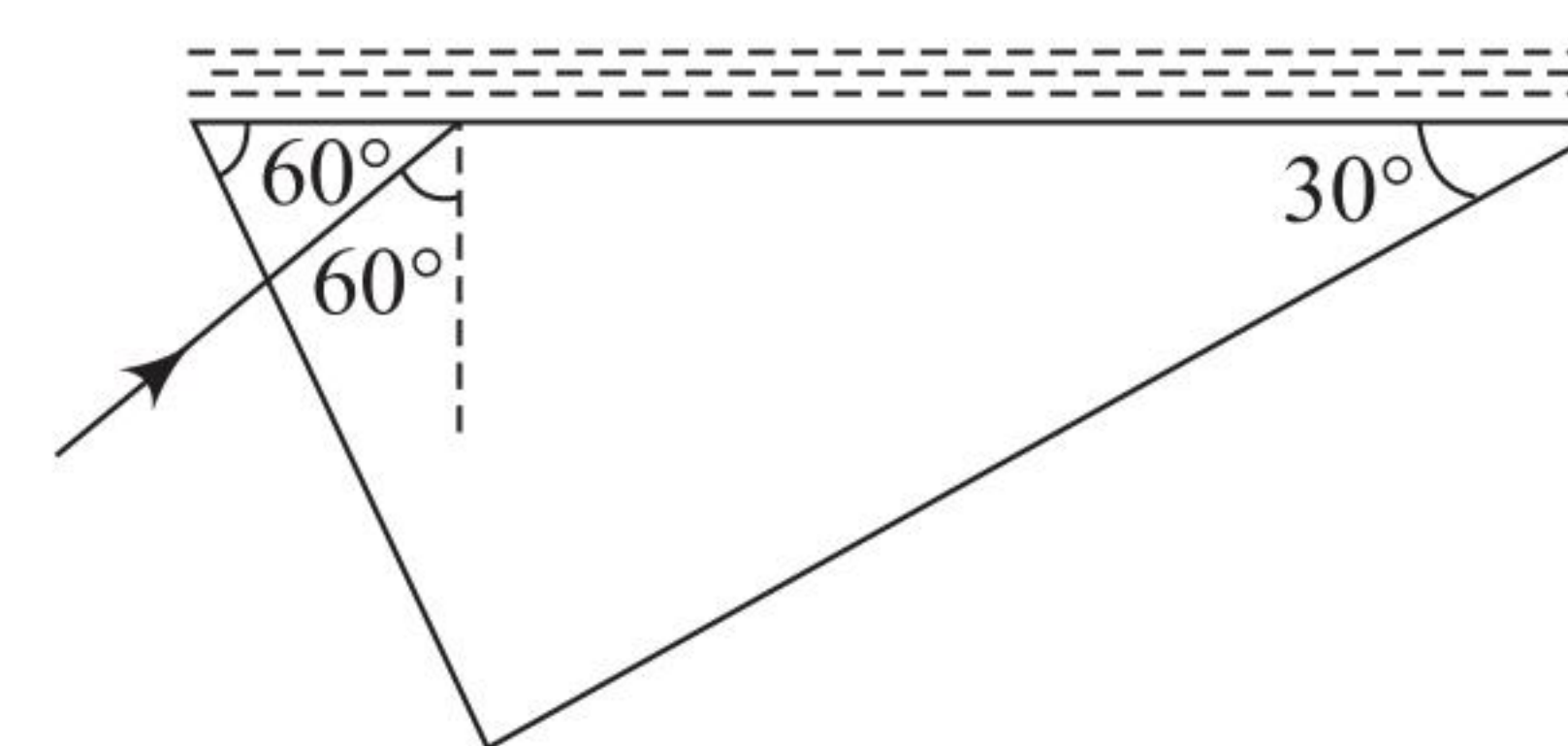
Now, $75^\circ = r + 45^\circ$ or $r = 30^\circ$

Now, $\frac{\sin i}{\sin 30^\circ} = \sqrt{2}$

or $\sin i = \sqrt{2} \times \frac{1}{2} = \frac{1}{\sqrt{2}}$

$\therefore i = 45^\circ$

39. (2)



Clearly, $i_c \leq 60^\circ$

So, maximum possible value of i_c is 60° .

Now, $\mu_g = \frac{1}{\sin i_c}$

$\frac{\mu_g}{\mu_l} = \frac{1}{\sin i_c}$

$$\begin{aligned} \text{or } \mu_1 &= \mu_g \sin i_c = 1.5 \sin 60^\circ = 1.5 \times \frac{\sqrt{3}}{2} \\ &= 1.5 \times 0.866 = 1.299 = 1.3 \end{aligned}$$

$$40. (3) A = r_1 + r_2$$

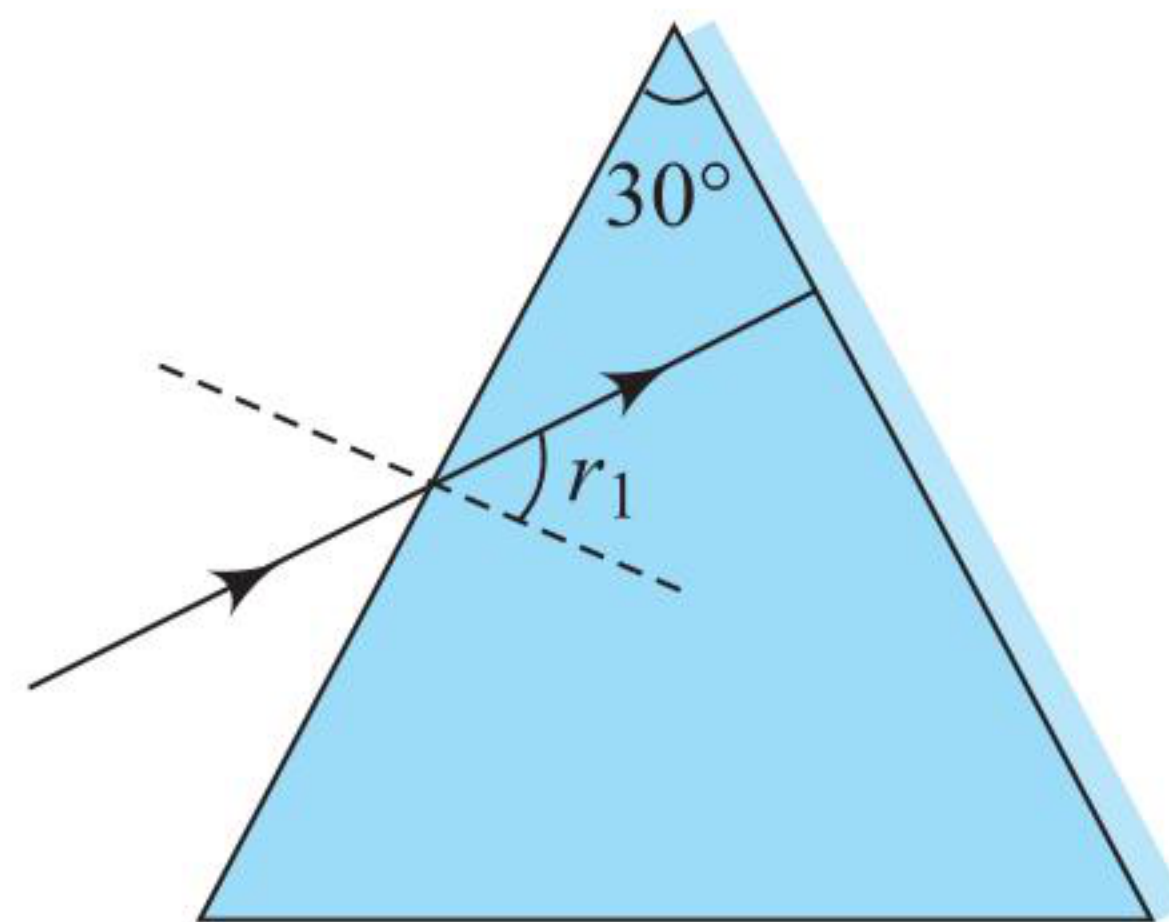
$$30^\circ = r_1 + 0$$

$$\text{or } r_1 = 30^\circ$$

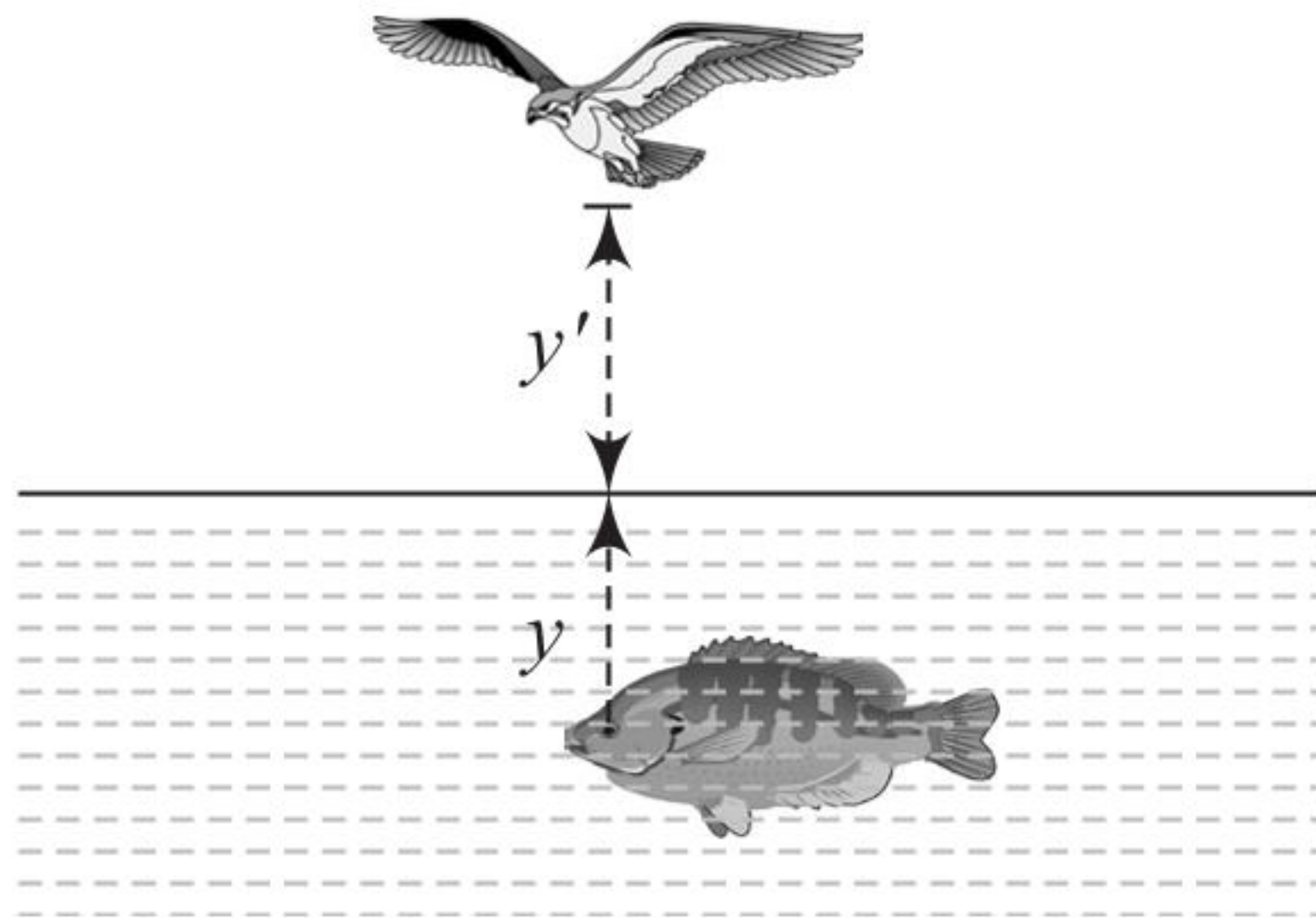
$$\text{Now, } \frac{\sin i}{\sin 30^\circ} = \sqrt{2}$$

$$\text{or } \sin i = \sqrt{2} \times \frac{1}{2}$$

$$\text{or } i = 45^\circ$$



$$41. (1) \text{ Effective height of the bird as seen by the fish, } Y = y + \mu y'$$



$$\frac{dY}{dt} = \frac{dy}{dt} + \mu \frac{dy'}{dt}$$

$$9 = 3 + \frac{4}{3} \frac{dy'}{dt}$$

$$\text{or } \frac{dy'}{dt} = \frac{6 \times 3}{4} = \frac{18}{4} = \frac{9}{2} = 4.5 \text{ m s}^{-1}$$

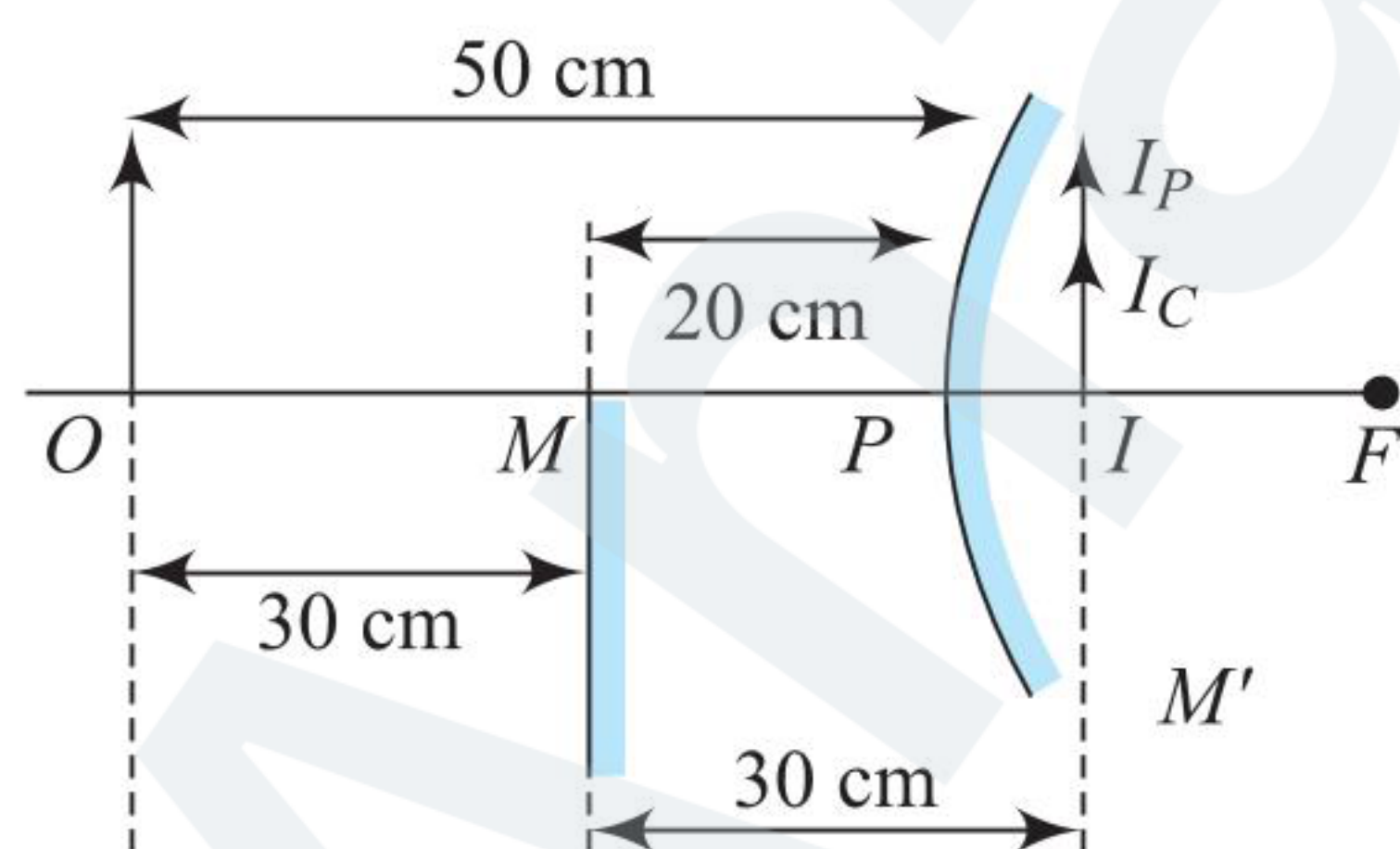
Therefore, actual velocity of bird = 4.5 m s^{-1}

$$42. (1) \sin i_c = \frac{1}{1.5} = \frac{2}{3}$$

$$i_c = \sin^{-1}\left(\frac{2}{3}\right) = \sin^{-1}(0.6667) = 41.8^\circ$$

The angle of incidence of face AC is 45° which is clearly greater than 41.8° .

43. (1) As shown in figure, the plane mirror will form erect and virtual image of same size at a distance of 30 cm behind it. So, the distance of image formed by plane mirror from convex mirror will be $PI = MI - MP$



But as $MI = MO$

$$\therefore PI = MO - MP = 30 - 20 = 10 \text{ cm}$$

Now, as this image coincides with the image formed by convex mirror, therefore for convex mirror,

$$u = -50 \text{ cm; } v = +10 \text{ cm}$$

$$\text{So, } \frac{1}{+10} + \frac{1}{-50} = \frac{1}{f} \quad \text{i.e., } f = \frac{50}{4} = 12.5 \text{ cm}$$

$$\text{So, } R = 2f = 2 \times 12.5 = 25 \text{ cm}$$

44. (2) In first case, if object distance is x , image distance = $x/4$. While it is 2nd case, object distance becomes $(x - 5 \text{ cm})$ and

image distance $(x - 5 \text{ cm})/2$. Using mirror formula, we get

$$\text{In first case, } \frac{1}{f} = -\frac{5}{x}$$

$$\text{In second case, } \frac{1}{f} = -\frac{3}{(x - 5)}$$

Solving we get, $x = 12.5 \text{ cm}$ and $f = -2.5 \text{ cm}$ and hence $|f| = 2.5 \text{ cm}$

45. (3) For concave mirror, if x and y are object distance and image distance, respectively, we have

$$-\frac{1}{x} - \frac{1}{y} = -\frac{1}{|f|}$$

$$\Rightarrow \frac{1}{x} + \frac{1}{y} = \frac{1}{|f|}$$

$$\Rightarrow -\frac{1}{x^2} \frac{dx}{dt} - \frac{1}{y^2} \frac{dy}{dt} = 0$$

$$\Rightarrow \left| \frac{V_x}{V_y} \right| = \frac{x^2}{y^2}$$

$$\text{For } \left| \frac{V_x}{V_y} \right| = \frac{1}{4}, \frac{x}{y} = \pm 2$$

$$\text{For } \frac{x}{y} = 2, \text{ we get } x = \frac{3|f|}{2} \quad [\text{for point A}]$$

$$\text{and for } \frac{x}{y} = -2, \text{ we get } x = \frac{|f|}{2} \quad [\text{for point B}]$$

As the middle point happens to be focus of the mirror, we get

$$|f| = L$$

$$46. (3) \frac{x}{1} = \frac{x_{\text{rel}}}{\mu} \Rightarrow x_{\text{rel}} = \mu x$$

$$\frac{d^2 x_{\text{rel}}}{dt^2} = \mu \frac{d^2 x}{dt^2} \Rightarrow a_{\text{rel}} = \mu g$$

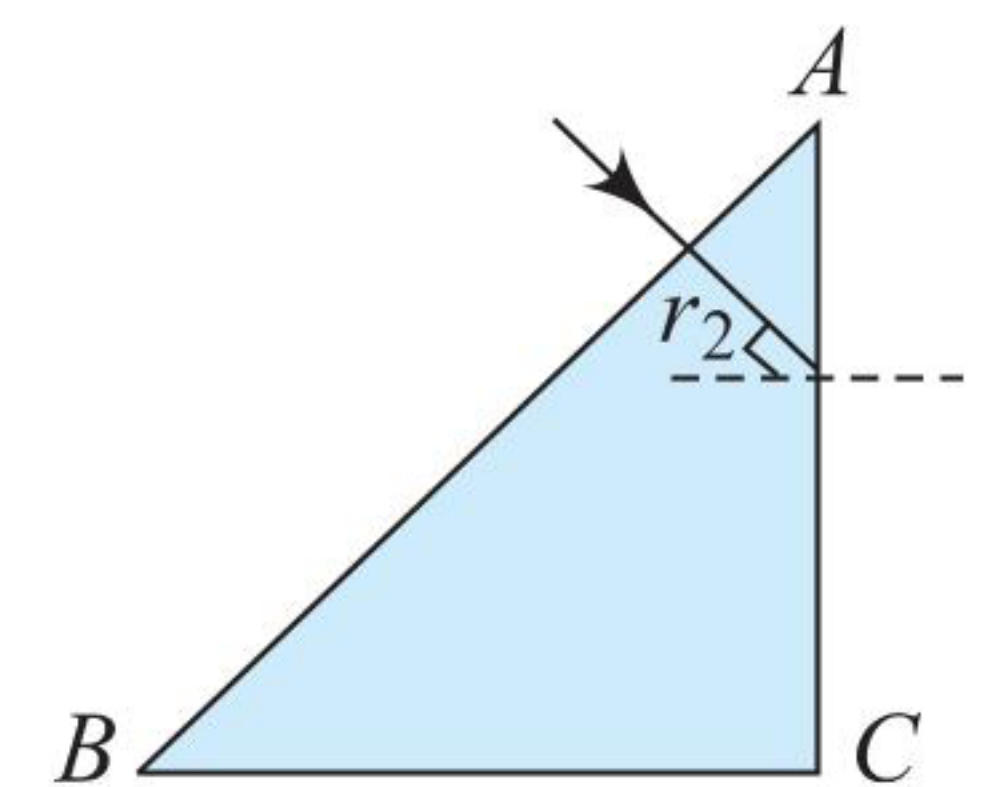
$$47. (2) A = 90^\circ - \theta$$

$$\Rightarrow r_2 = A = 90^\circ - \theta > \theta_c$$

$$\cos \theta > \sin \theta_c = \frac{6/5}{2/3} = \frac{4}{5}$$

(θ_c is critical angle)

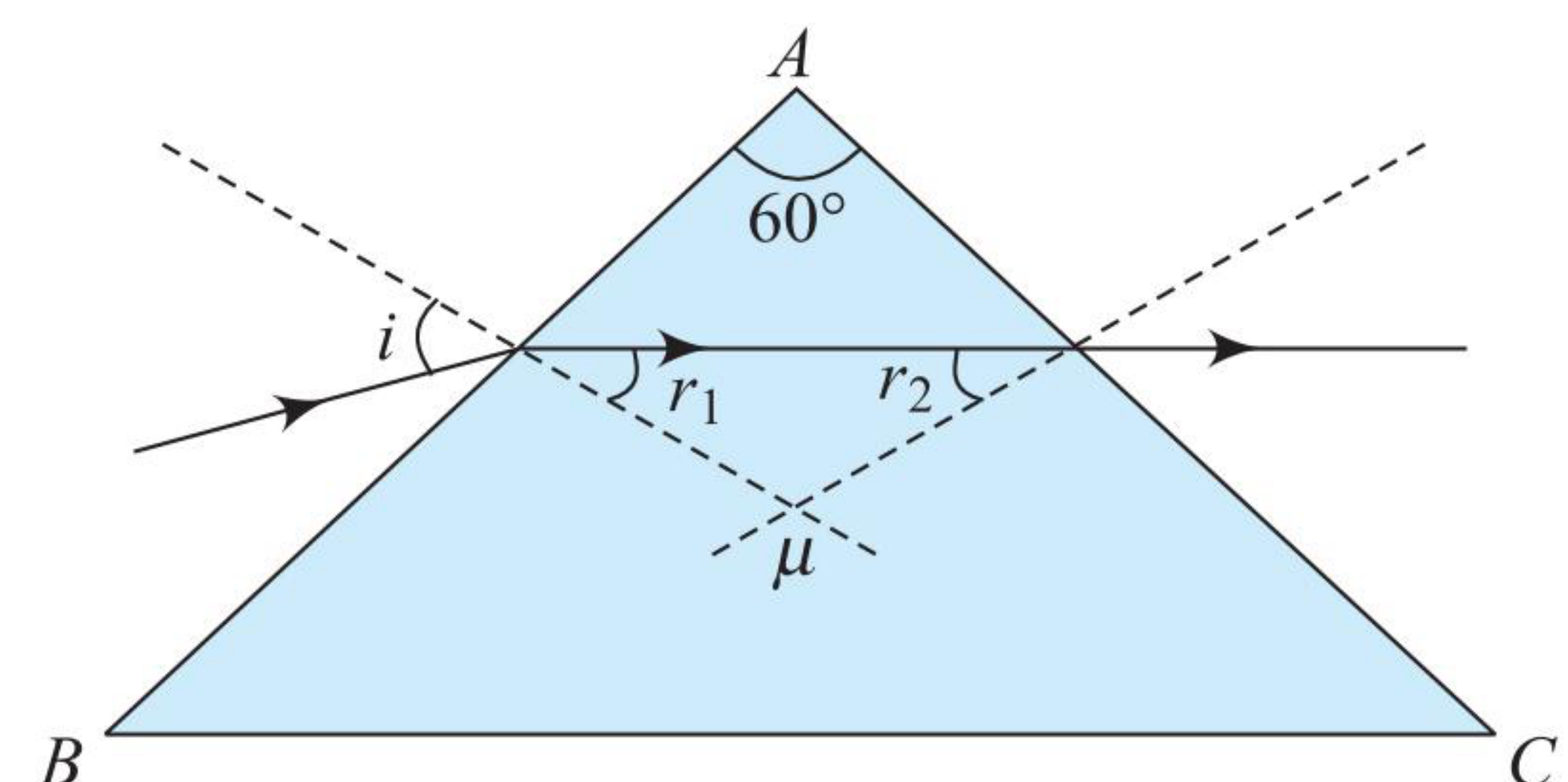
$$\theta < \cos^{-1} \frac{4}{5} = 37^\circ$$



48. (2) Image velocity (w.r.t. mirror) = $-m \times$ object velocity (w.r.t. mirror)

$$\text{Here } m = 1$$

$$49. (1) r_2 < \theta_c; A - r_1 < \theta_c$$



$$r_1 > A - \theta_c$$

$$\sin r_1 > \sin (A - \theta_c)$$

$$\frac{\sin i}{\mu} > \sin(A - \theta_c)$$

$$\sin i > \mu (\sin A \cos \theta_c - \cos A \sin \theta_c)$$

$$> \sqrt{\frac{7}{3}} \left(\frac{\sqrt{3}}{2} \sqrt{1 - \frac{3}{7}} - \sqrt{\frac{3}{7}} \cdot \frac{1}{2} \right) = 1 - \frac{1}{2} = \frac{1}{2}$$

$$\sin i > \frac{1}{2} \text{ or } i > 30^\circ$$

50. (1) Given $i = 60^\circ$, $A = \delta = e$

$$\delta = i + e - A \Rightarrow \delta = 1$$

$$(\because e = A) \text{ and } \delta = i = e$$

$$\mu = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin\frac{A}{2}}$$

Here, angle of deviation is minimum ($\because i = e$)

$$\mu = \frac{\sin\left(\frac{60^\circ + 60^\circ}{2}\right)}{\sin(60^\circ/2)} = \sqrt{3}$$

51. (2) Shift = $(l - m)\left(1 - \frac{1}{n_1}\right) + m\left(1 - \frac{1}{n_2}\right) = 0$

52. (1) By mirror formula: $\frac{1}{v} + \frac{1}{-10} = \frac{1}{10}$

$$\Rightarrow v = +5 \text{ cm}$$

$$\therefore m = +\frac{1}{2}$$

The image revolves in a circle of radius $1/2$ cm. Image of a radius is erect $\Rightarrow$ particle will revolve in the same direction as the particle. The image will complete one revolution in the same time 2 s.

$$\text{Velocity of image } v \text{ is } \omega r = \frac{2\pi}{2} \times \frac{1}{2} = \frac{\pi}{2} \text{ cm s}^{-1} = 1.57 \text{ cm s}^{-1}$$

53. (4) Mirror can be shifted to new position $C'D'$. Distances are shown in figure.

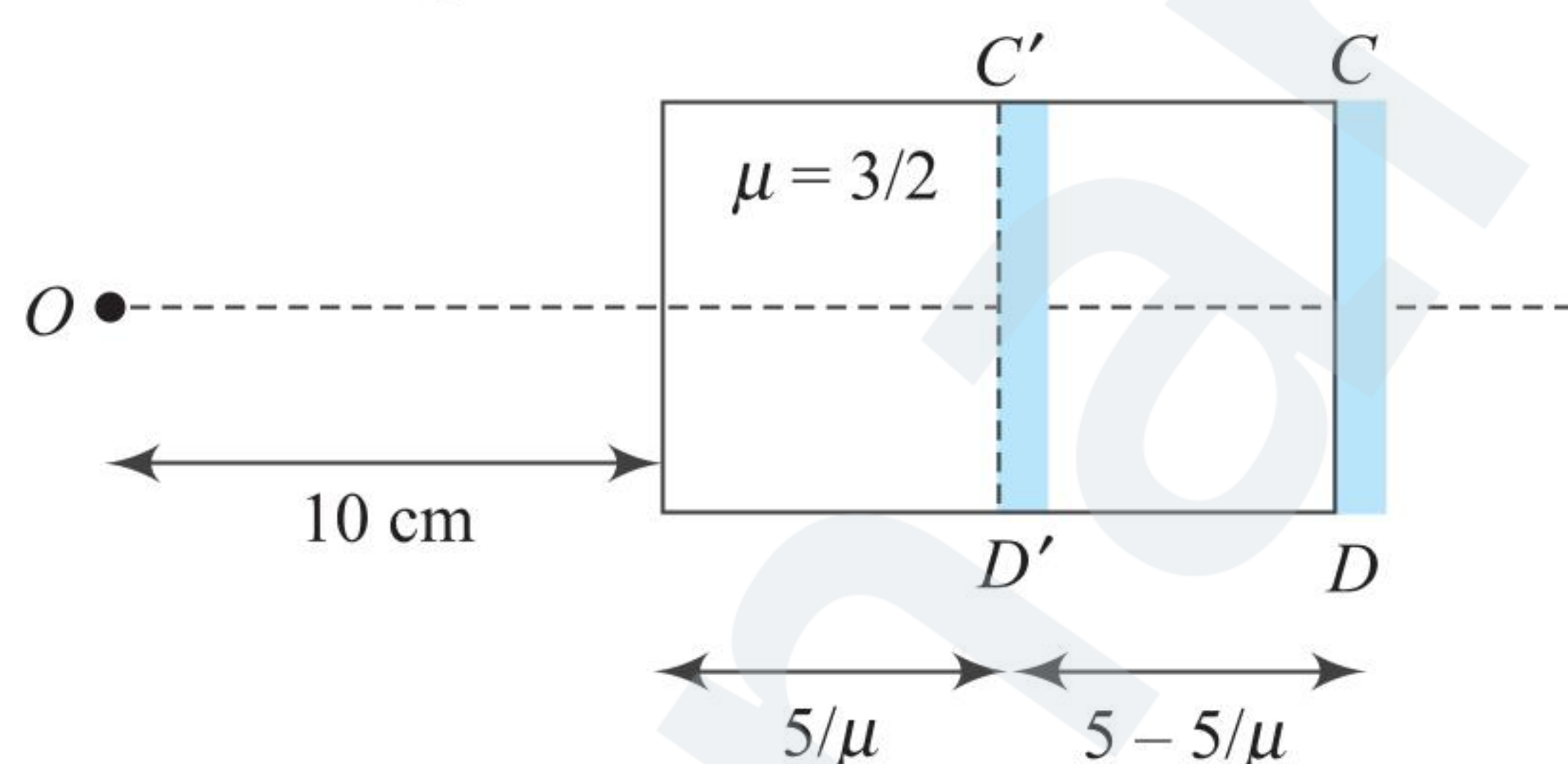


Image will be at equal distance from the mirror $C'D'$ as the object is.

$$\text{Image distance from } C'D' \text{ is } 10 + \frac{5}{3/2} = 10 + \frac{10}{3} = \frac{40}{3} \text{ cm}$$

$$\text{Separation between object and image is } \frac{80}{3} \text{ cm.}$$

54. (4) Deviation produced by prism is $\delta_1 = (\mu - 1)A = 2^\circ \text{ CW}$

Angle of incidence for mirror is δ_1 , so deviation produced by the mirror is $\delta_2 = \pi - 2\delta_1 = 176^\circ \text{ CW}$

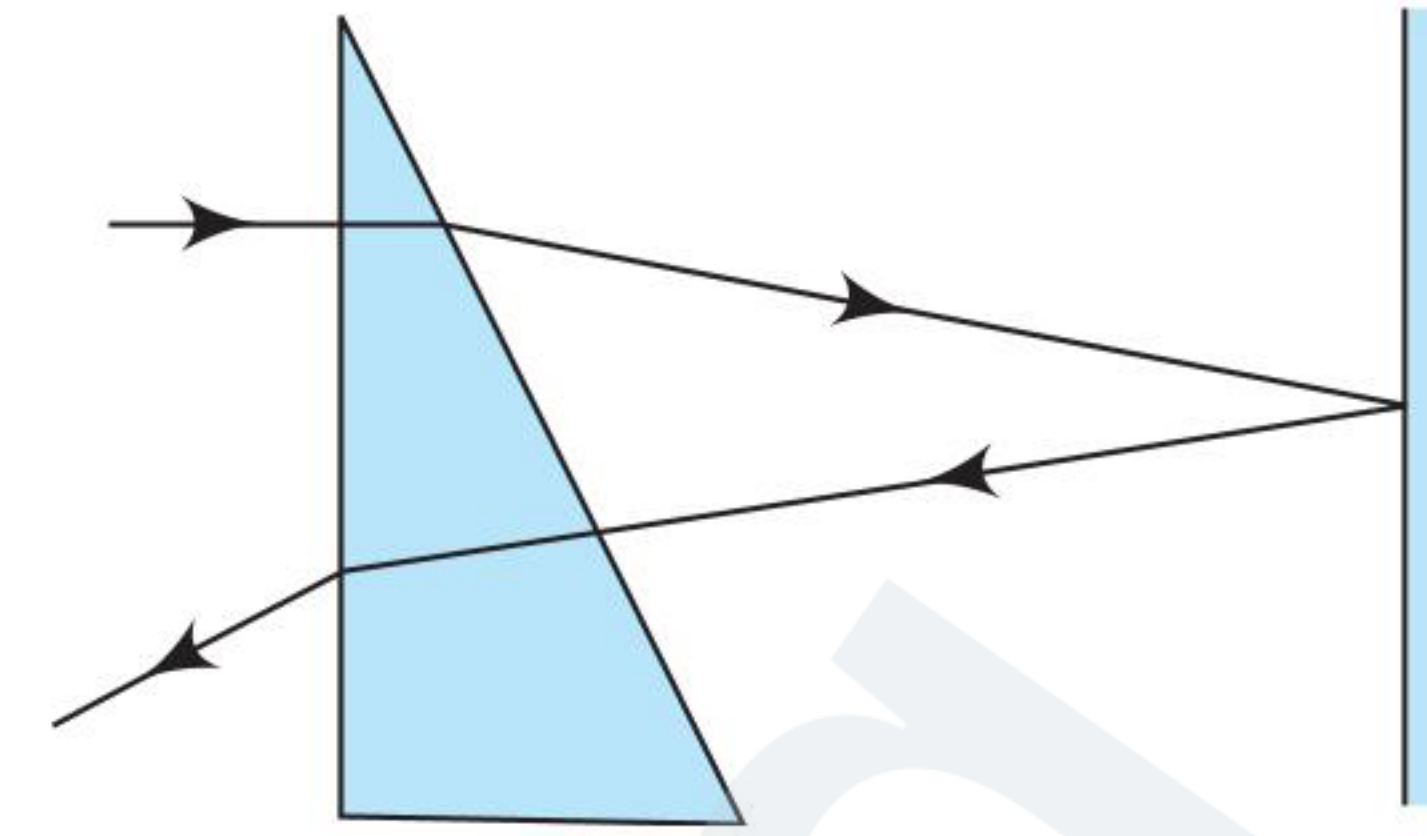
Deviation produced by the prism for 2nd refraction is

$$\delta_3 = 2^\circ \text{ ACW}$$

Net deviation = 174° CW

55. (4) Deviation produced by the mirror is

$$\delta_1 = \pi - 2 \times 60 = 60^\circ \text{ CW}$$



Deviation produced by the prism is

$$\delta_2 = (1 - 2) \times 6 = 6^\circ \text{ ACW}$$

So, net deviation, $\delta = \delta_1 + \delta_2 = (60^\circ - 6^\circ) \text{ CW} = 54^\circ \text{ CW}$

56. (4) From $\sin\left(\frac{A + \delta_m}{2}\right) = \mu \sin\frac{A}{2}$

$$\sin\left(\frac{A + \delta_m}{2}\right) = \frac{\cos\frac{A}{2}}{\sin\frac{A}{2}} \times \sin\frac{A}{2}$$

$$\frac{A + \delta_m}{2} = \frac{\pi}{2} - \frac{A}{2}$$

$$\delta_m = \pi - 2A = 180^\circ - 2A$$

57. (4) $u = 21 \text{ cm}$, $f = \frac{R}{2} = \frac{10}{2} = 5 \text{ cm}$

On introducing the glass slab, the object as well as the image will be shifted from the mirror through a distance

$$d = t\left(1 - \frac{1}{\mu}\right) = 3\left(1 - \frac{1}{1.5}\right) = 1 \text{ cm, so that apparent distance}$$

of the object = 20 cm i.e., $u = 20 \text{ cm}$

$$\text{By the mirror formula, } \frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$v = -\frac{20}{3} \text{ cm} = -6.67 \text{ cm}$$

Distance of the final image from the mirror = $6.67 + 1 = 7.67 \text{ cm}$

58. (4) No change as there would be no deviation in path of rays compared to what it was in air.

59. (2) From similar triangles APB and CBD , point P will be just visible if $CD = 4R$

$$\mu \sin i = 1 \times \sin r$$

$$\mu \sin(\angle CDQ) = 1 \times \sin(\angle CDB)$$

$$\mu \times \frac{R}{R\sqrt{17}} = 1 \times \frac{2R}{R\sqrt{20}}$$

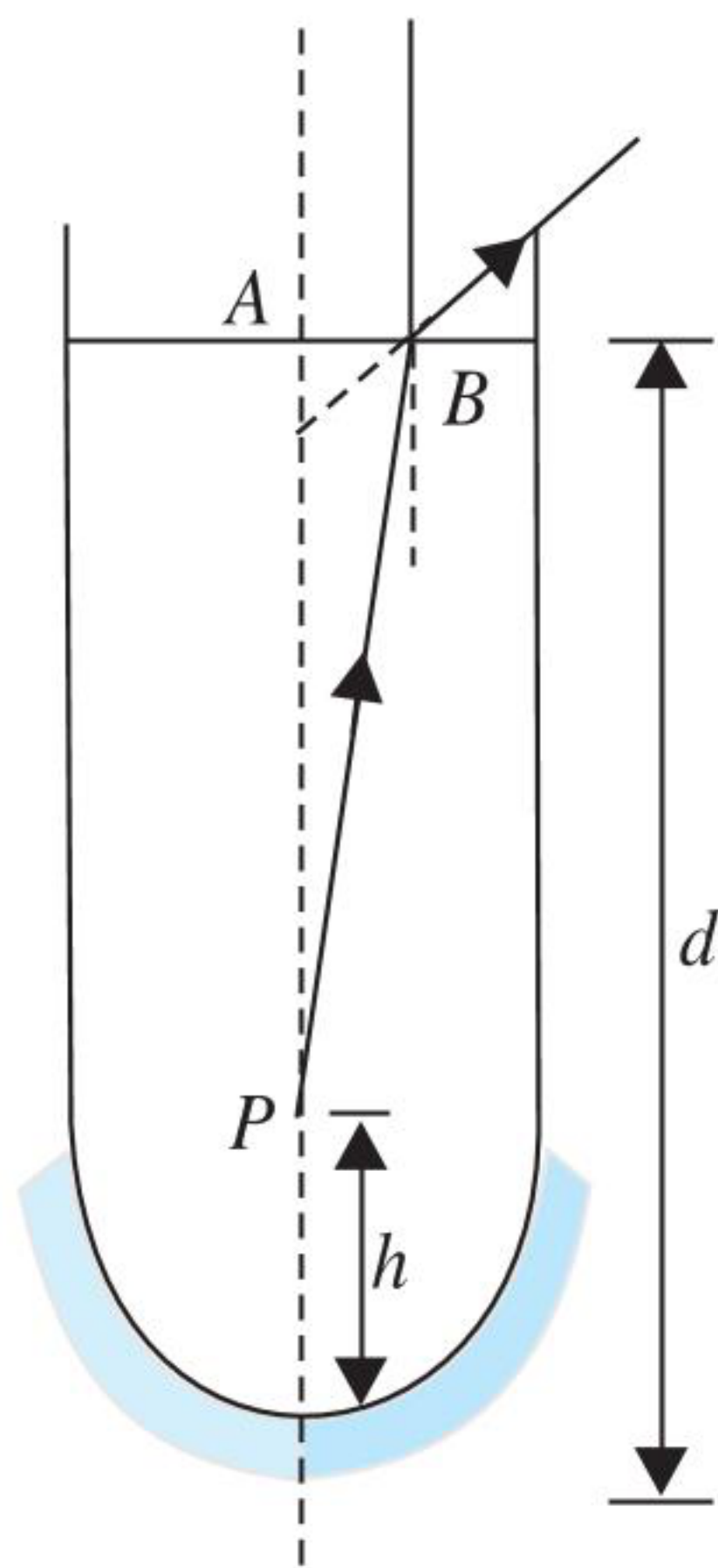
$$\Rightarrow \mu = \frac{2\sqrt{17}}{\sqrt{20}} = \frac{\sqrt{85}}{5}$$

60. (4) Any point to right of P on surface PA will not be visible when the vessel is filled with liquid.

61. (3) There will be two images on by direct observation other by reflection through mirror.

$$\text{Apparent depth} = \frac{\text{Real depth}}{\mu_{\text{rel}}} = \frac{\text{Real depth}}{\left(\frac{\mu_{\text{incident}}}{\mu_{\text{refractive}}}\right)} = \frac{(d - h)}{(\mu/1)}$$

$$d_{\text{app}} = \frac{(d - h)}{\mu}$$



Object P is placed at the centre of curvature of the mirror. Its image will be formed at P itself.

$$\text{So, for image also, } d_{\text{app}} = \frac{(d-h)}{\mu} - \frac{(d-h)}{\mu} = 0$$

62. (3) Given $r = 6$ cm, $r_1 = 4$ cm, $h_1 = 8$ cm

Let h = final height of water column

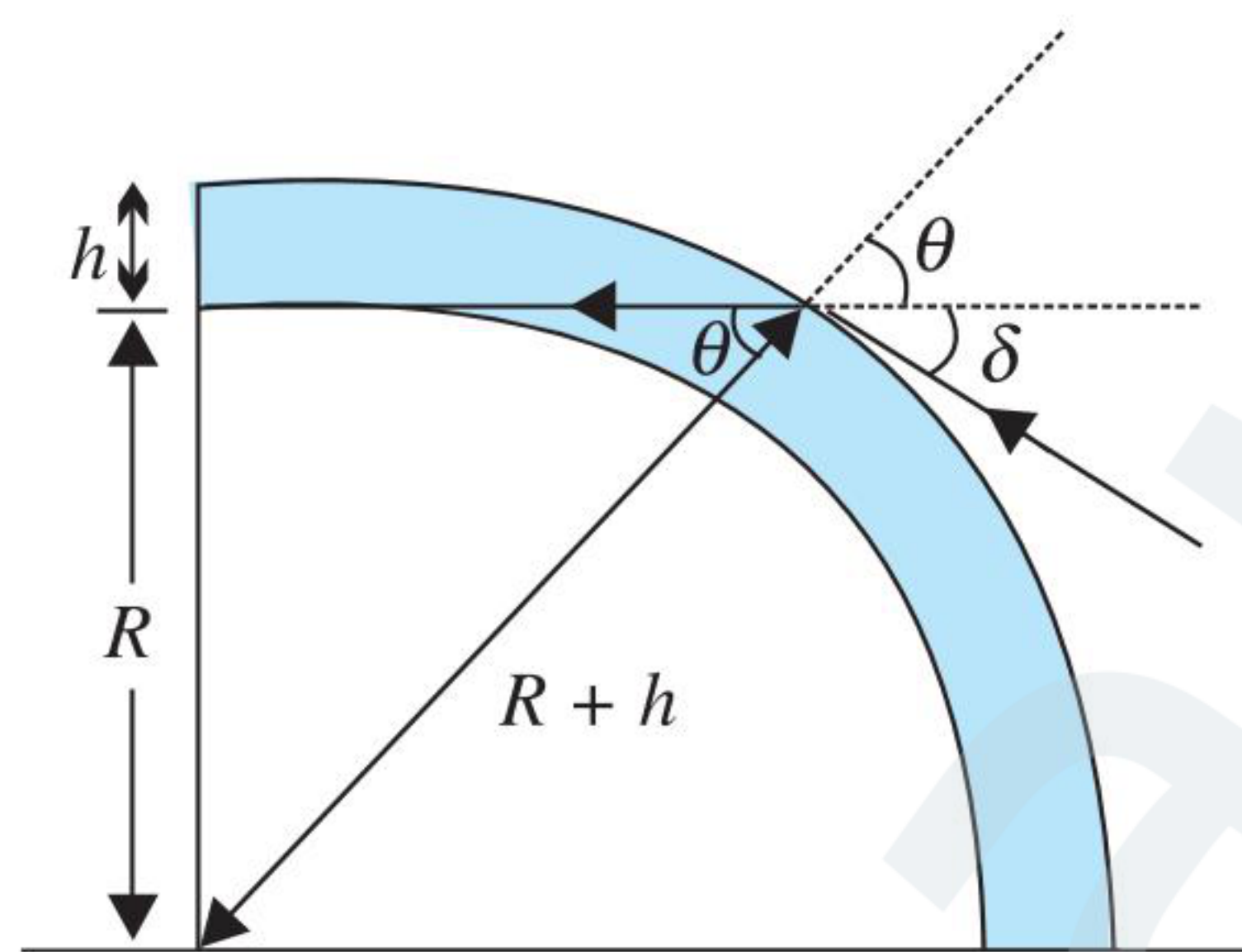
The volume of the cylindrical water column after the glass piece is put will be

$$\begin{aligned}\pi r^2 h &= 800\pi + \pi r_1^2 h_1 \\ r^2 h &= 800 + r_1^2 h_1 \\ (6)^2 h &= 800 + (4)^2 \cdot 8 \\ h &= \frac{800 + 128}{36} = \frac{928}{36} = 25.7 \text{ cm}\end{aligned}$$

There are two shifts due to glass block as well as water

Total shift = $(2.66 + 4.44) = 7.1$ cm above the bottom.

63. (4) By Snell's law



$$\frac{\sin(\theta + \delta)}{\sin \theta} = n$$

$$\sin \theta = \frac{R}{(R+h)}$$

$$\sin(\theta + \delta) = \frac{nR}{(R+h)}$$

$$\delta = \sin^{-1}\left(\frac{nR}{R+h}\right) - \sin^{-1}\left(\frac{R}{R+h}\right)$$

64. (4) Let v_1 be the speed of gun (or mirror) just after the firing of bullet. From conservation of linear momentum,

$$m_2 v_0 = m_1 v_1$$

$$v_1 = \frac{m_2 v_0}{m_1} \quad \dots(i)$$

Now rate at which distance between mirror and bullet is increasing,

$$\frac{du}{dt} = v_1 + v_0 \quad \dots(ii)$$

$$\frac{dv}{dt} = \left(\frac{v^2}{u^2}\right) \frac{du}{dt}$$

$$\text{Here, } \frac{v^2}{u^2} = m^2 = 1$$

(as at the time of firing, the bullet is at pole).

$$\frac{dv}{dt} = \frac{du}{dt} = v_1 + v_0 \quad \dots(iii)$$

Here, dv/dt is the rate at which distance between image (of bullet) and mirror is increasing. So, if v_2 is the absolute velocity of image (towards right), then

$$v_2 - v_1 = \frac{dv}{dt} = v_1 + v_0$$

$$\text{or } v_2 = 2v_1 + v_0 \quad \dots(iv)$$

Therefore, speed of separation of the bullet and the image will be

$$\begin{aligned}v_r &= v_2 + v_0 = 2v_1 + v_0 + v_0 \\ v_r &= 2(v_1 + v_0)\end{aligned}$$

Substituting value of v_1 from Eq. (i), we have

$$v_r = 2\left(1 + \frac{m_2}{m_1}\right)v_0$$

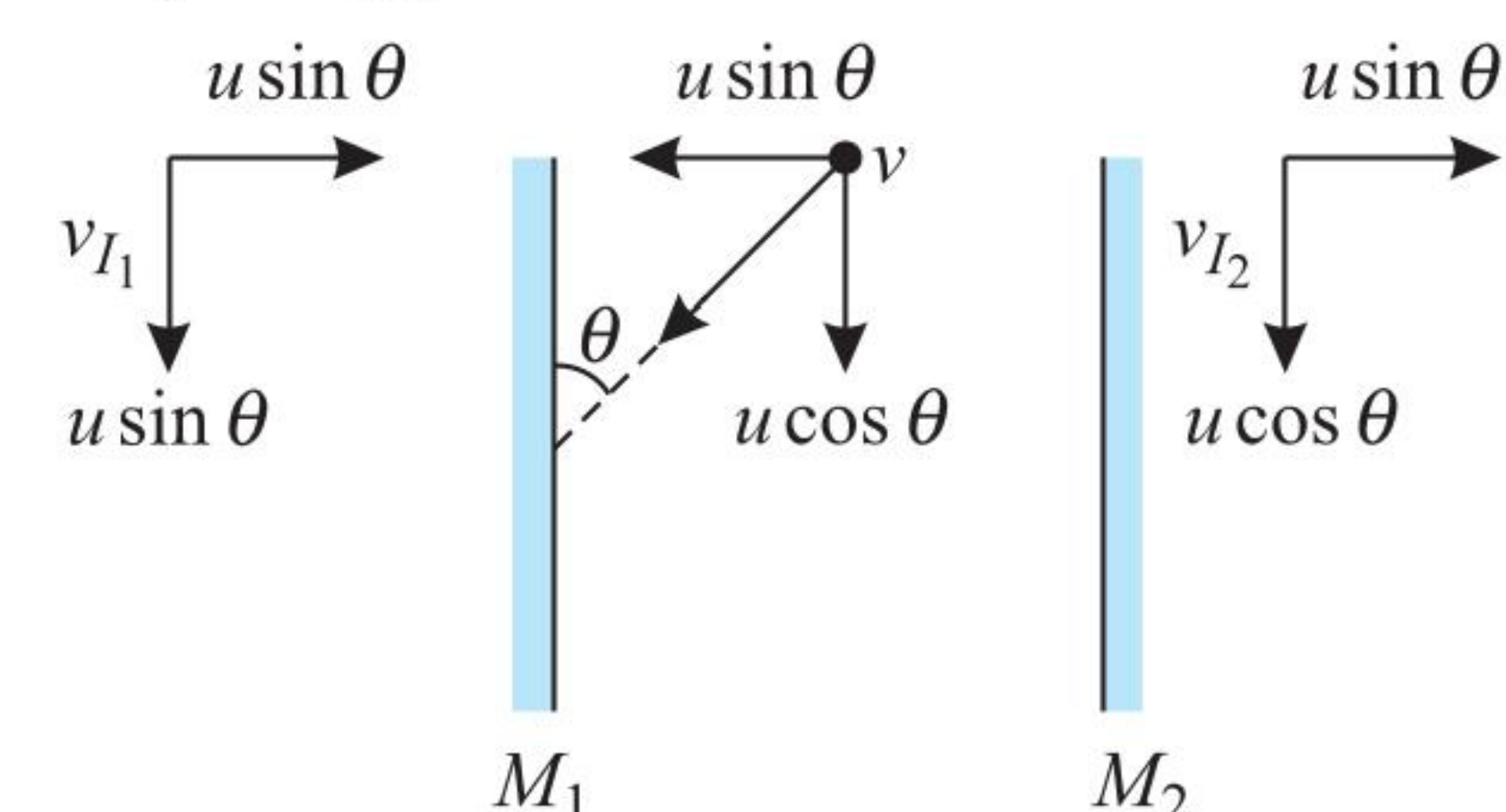
65. (1) For plane surface, $h_{\text{app}} = \mu h = \frac{3}{2} \times 10 = 15$ cm

This first image is at a distance $(15 + 3)$ cm from the plane mirror. So, mirror will make its second image at a distance 18 cm below the mirror or 21 cm below the plane surface.

$$\text{Now further applying, } d_{\text{app}} = \frac{d}{\mu} = \frac{21}{1.5} = 14 \text{ cm}$$

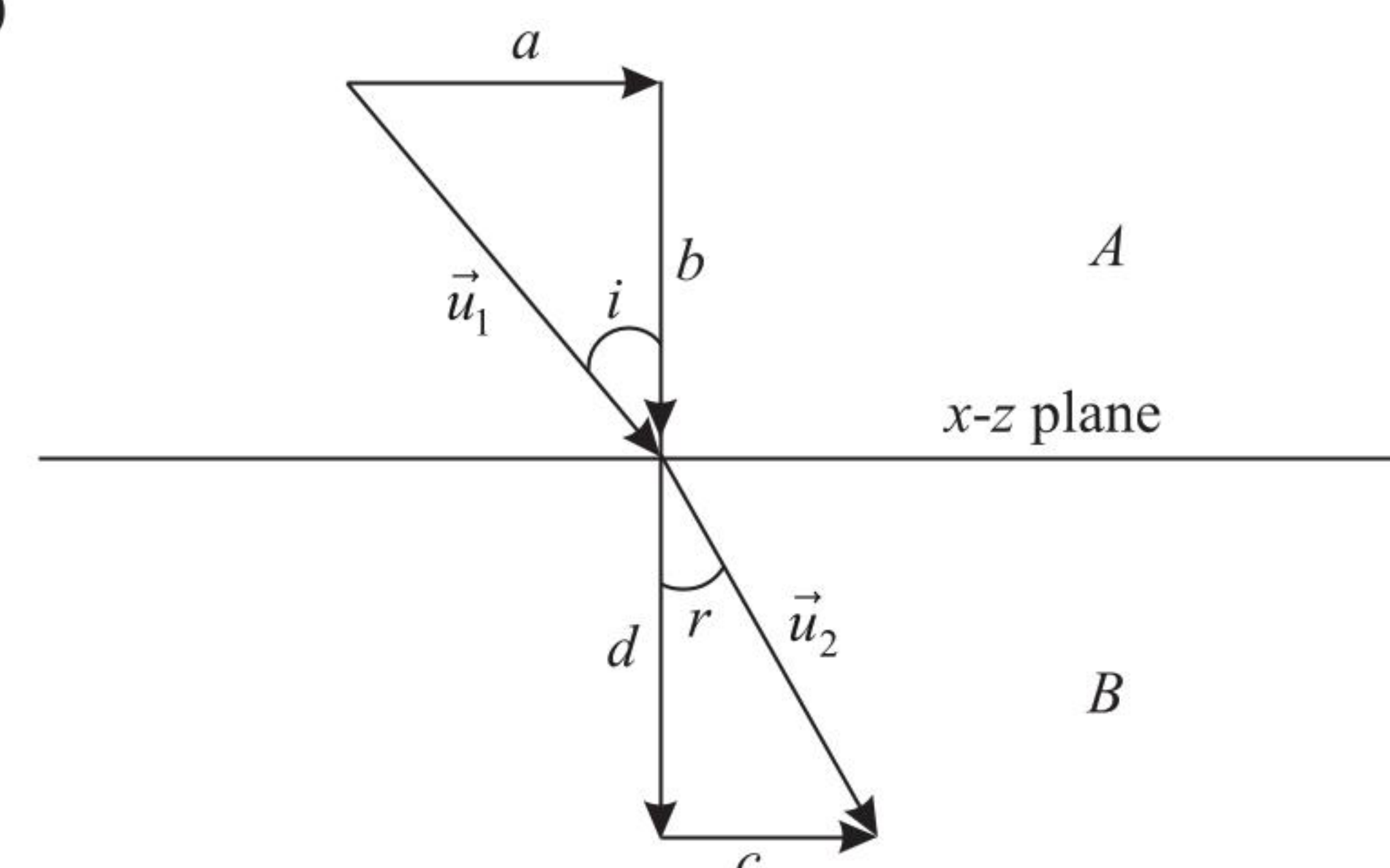
below the plane surface.

66. (4) The image velocity by both the mirror is equal hence the velocity of approach will be zero



$$\Rightarrow v_{I_1 I_2} = v_{I_2} - v_{I_1} = 0$$

67. (1)



From Snell's law: $1.5 \sin i = 2 \sin r$

$$1.5 \times \frac{a}{\sqrt{a^2 + b^2}} = 2 \times \frac{c}{\sqrt{c^2 + d^2}} \Rightarrow \frac{a}{c} = \frac{4}{3}$$

$$\left[\because \sqrt{a^2 + b^2} = \sqrt{c^2 + d^2} = 1(\text{unit vector}) \right]$$

68. (1) $\mu_{\text{red}} \sin i = 1 \sin r$

$$1.39 \times \frac{1}{1.414} = \sin r$$

$\sin r < 1$ possible (refraction is possible)

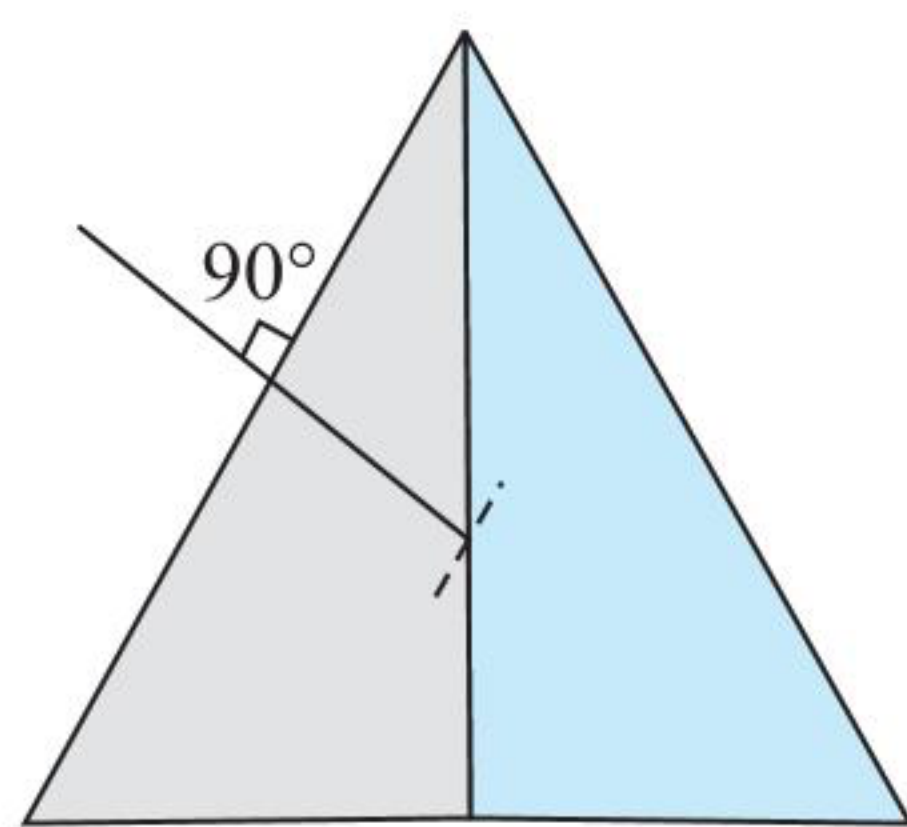
$$\mu_{\text{green}} \times \sin i = 1.44 \times \frac{1}{1.41} > 1 \Rightarrow \text{not possible}$$

$$\text{Same for blue } 1.47 \times \frac{1}{1.41} > 1$$

$\Rightarrow$ It also not possible.

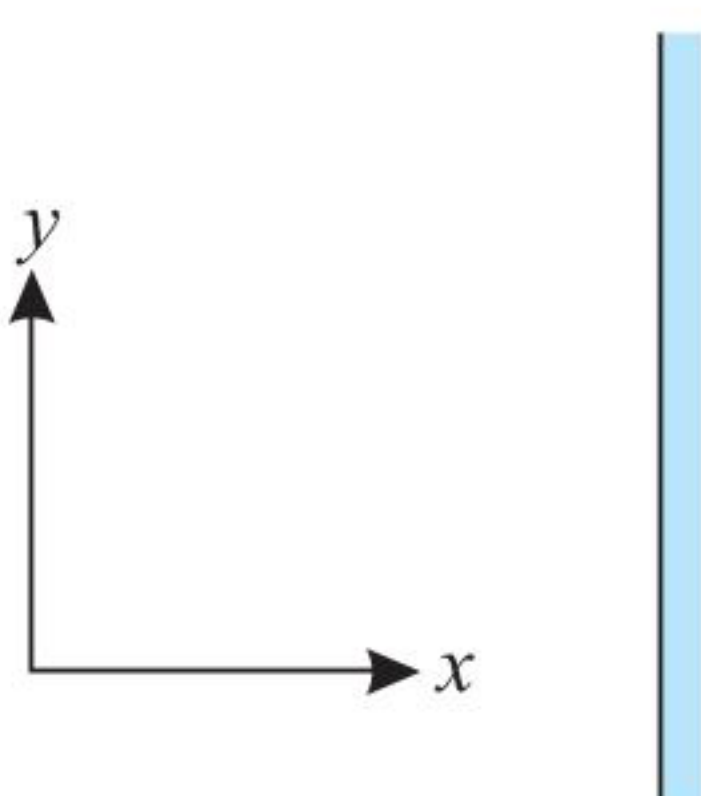
69. (1) $\mu \sin A = 1 \times \sin 90^\circ \Rightarrow \mu = \frac{1}{\sin A}$

$$\text{and } \mu \sin \frac{A}{2} = 1 \times \sin e = \frac{1}{\sin A} \times \sin \frac{A}{2} = \sin e$$



$$\sin e = \frac{\sin \frac{A}{2}}{2 \sin \frac{A}{2} \cos \frac{A}{2}} \Rightarrow e = \sin^{-1} \left[\frac{1}{2} \sec \frac{A}{2} \right]$$

70. (2)

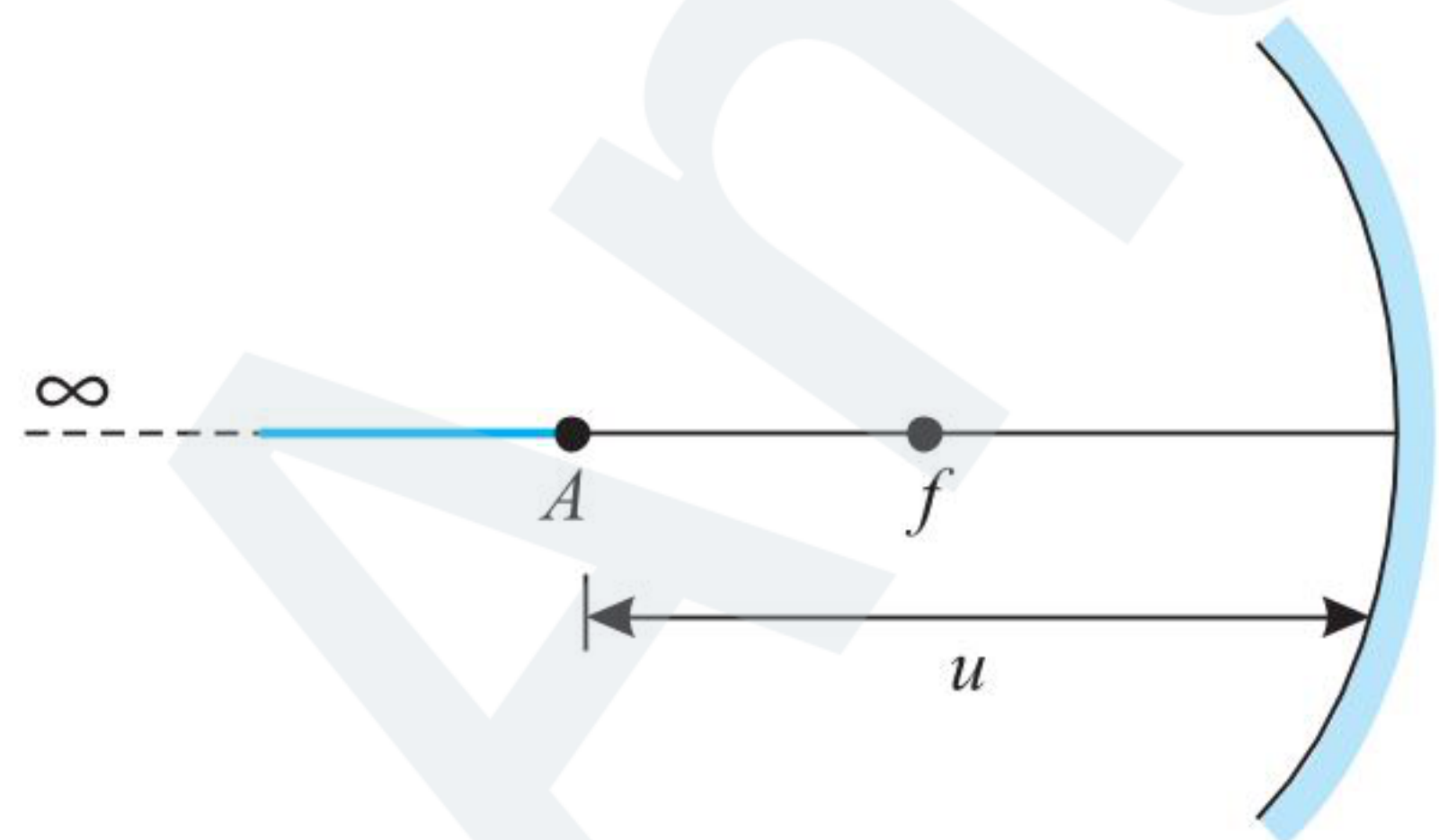


$$v_I = 2v_M - v_0 = 2 \times 0 - (5\hat{i} + (6+2t)\hat{j})$$

$$\text{At } t = 2 \text{ sec, } v_I = -5\hat{i} + 10\hat{j}; v_{I0} = v_I - v_0$$

$$\text{Hence } v_{I0} = -5\hat{i} + 10\hat{j} - (5\hat{i} + 10\hat{j}) = -10\hat{i}$$

71. (4)



$$u = -u, f = -f \text{ and } v_A = \frac{uf}{f-u}$$

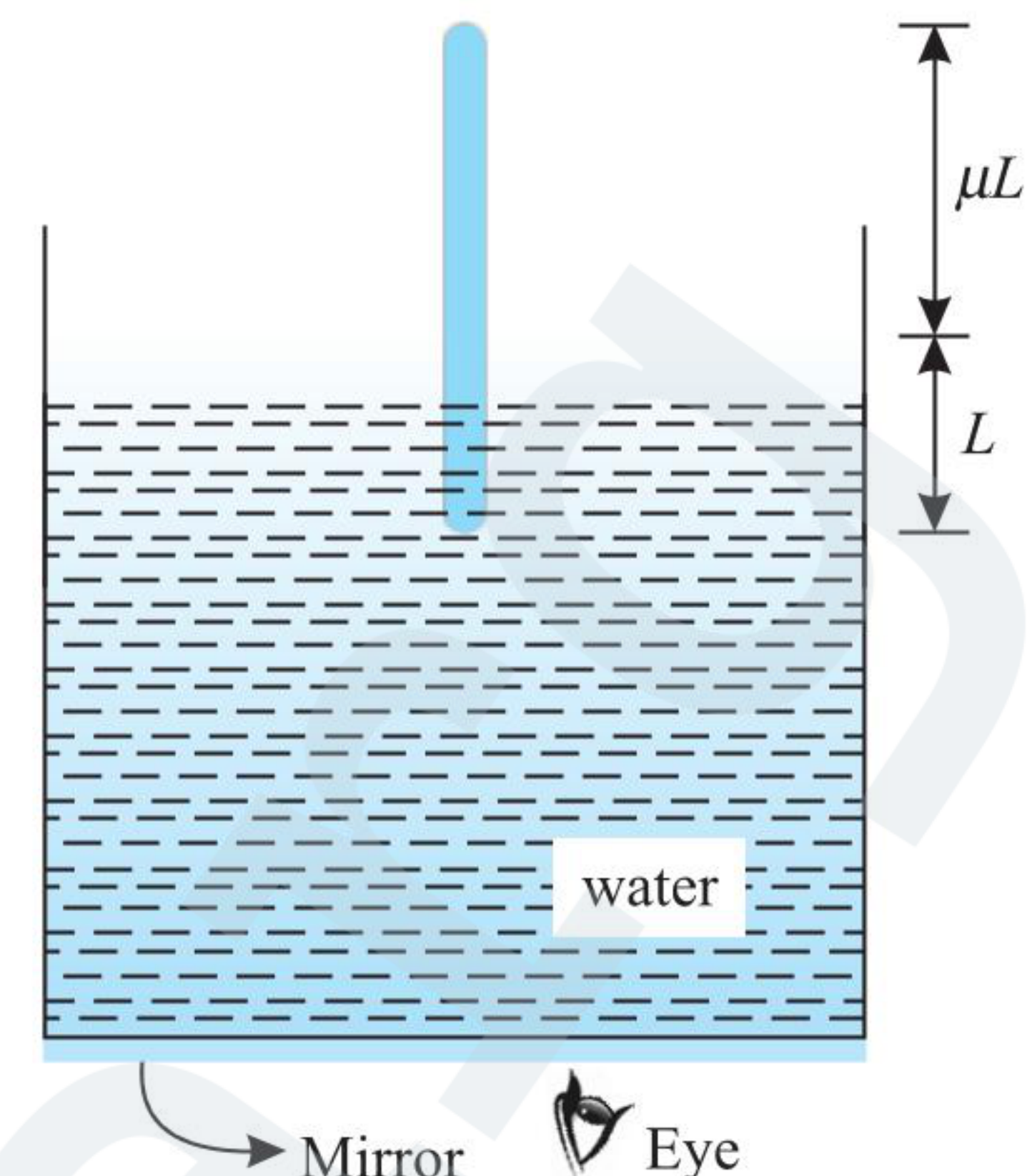
Image of last end of rod, $v_B = -f$

$$\text{Length} = v_B - v_A = -f - \frac{uf}{f-u} = \frac{-f^2 + uf - uf}{f-u}$$

$$\text{Length} = \frac{f^2}{u-f}. \text{ Hence, (4) is the correct option.}$$

72. (2) When observer in rarer medium and object is in denser medium then the length of immersed portion will be shorter (apparent)

Hence length of image of rod in water

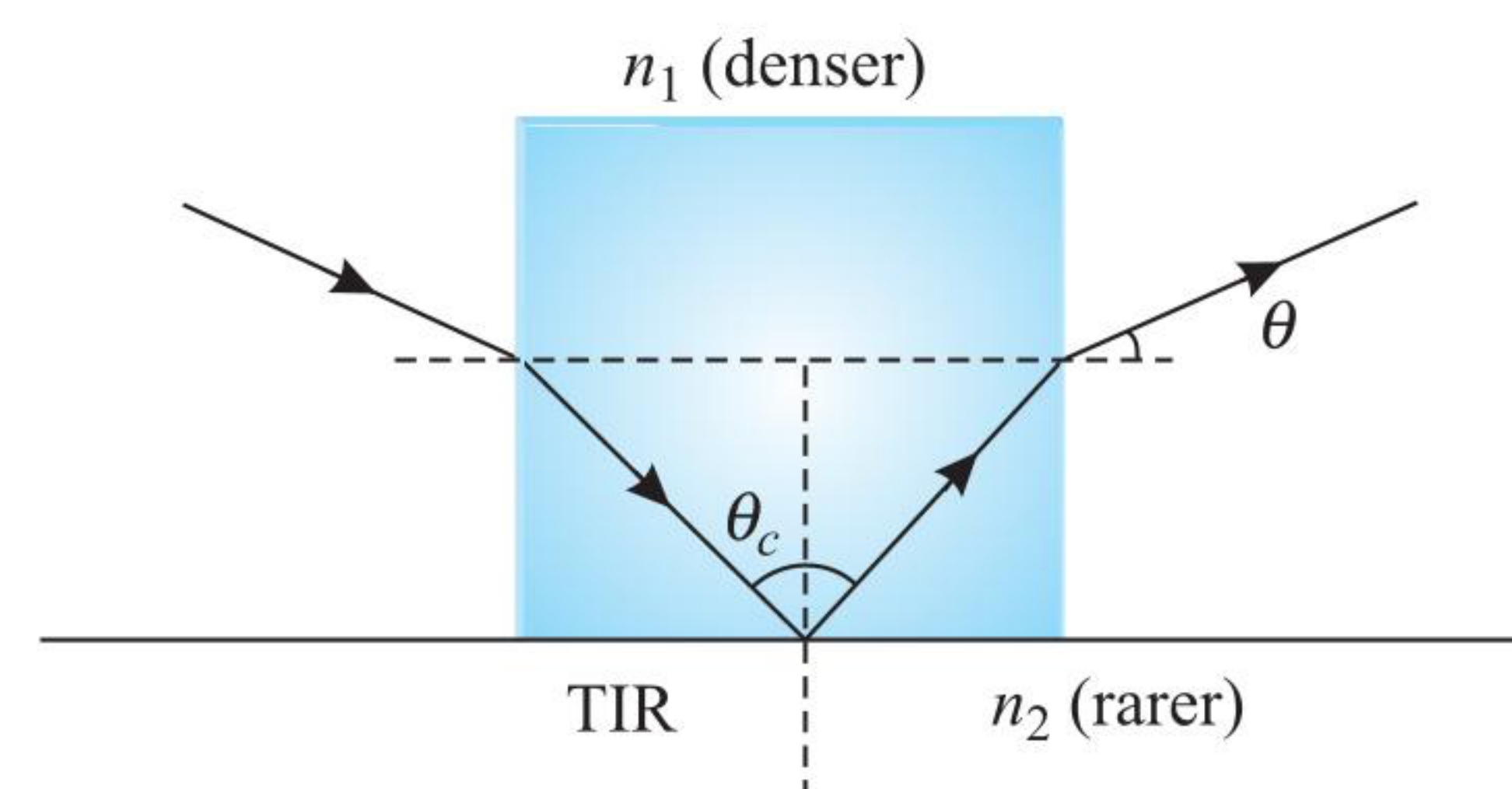


$$= 2L + \mu L - L = L + \mu L$$

Hence length of image of rod seen by observer in air is

$$= \frac{L + \mu L}{\mu} = \frac{L}{\mu} + L. \text{ Hence, (2) is the correct option.}$$

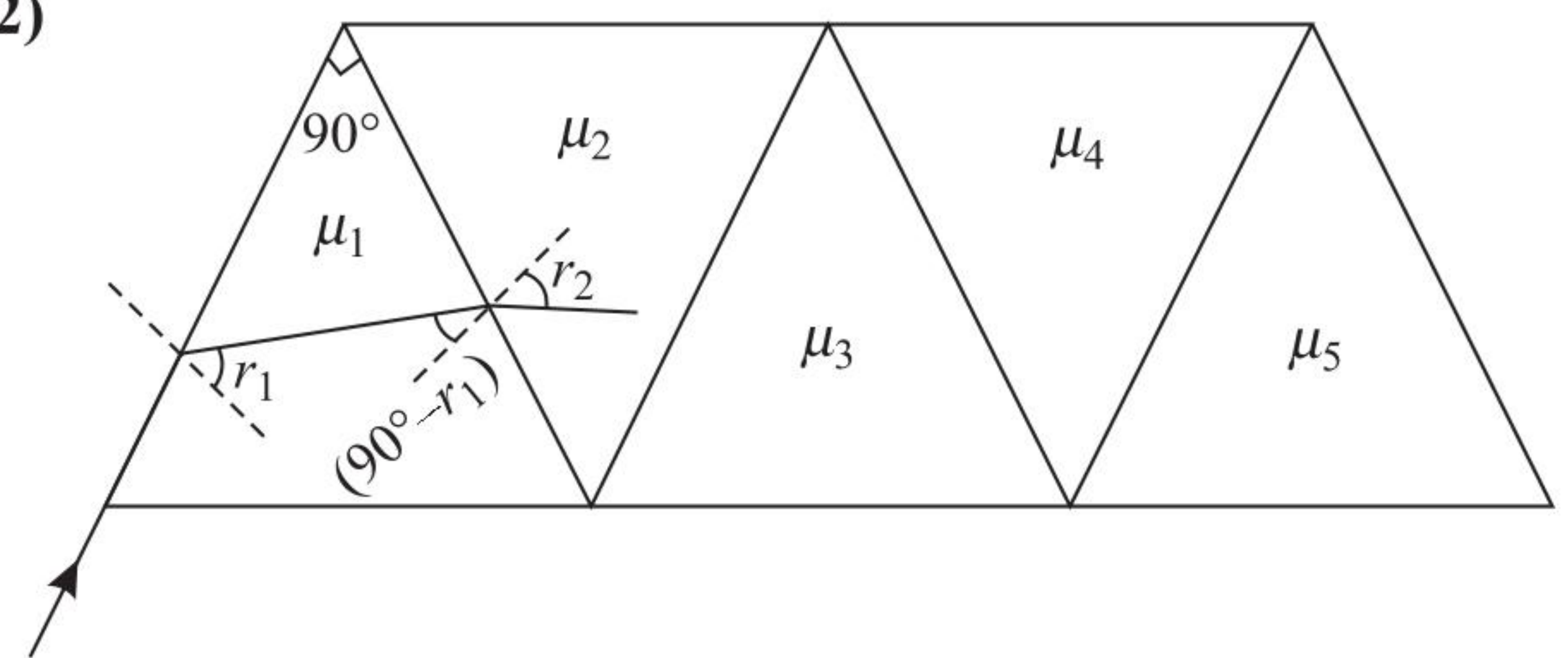
73. (1) $n_1 \sin \theta_C = n_2 \Rightarrow \sin \theta_C = \frac{n_2}{n_1}$



$$1 \times \sin \theta = n_1 \sin (90^\circ - \theta_c) = n_1 \cos \theta_c = n_1 \sqrt{1 - \frac{n_2^2}{n_1^2}}$$

$$\theta = \sin^{-1} \left(\sqrt{n_1^2 - n_2^2} \right); \sin \theta < \sqrt{n_1^2 - n_2^2}$$

74. (2)



$$1^{\text{st}} \text{ surface: } 1 \sin 90^\circ = \mu_1 \sin r_1$$

$$2^{\text{nd}} \text{ surface: } \mu_1 \cos r_1 = \mu_2 \sin r_2$$

$$3^{\text{rd}} \text{ surface: } \mu_2 \cos r_2 = \mu_3 \sin r_3$$

$$4^{\text{th}} \text{ surface: } \mu_4 \sin r_4 = \mu_3 \cos r_3$$

$$5^{\text{th}} \text{ surface: } \mu_4 \cos r_4 = \mu_5 \sin r_5$$

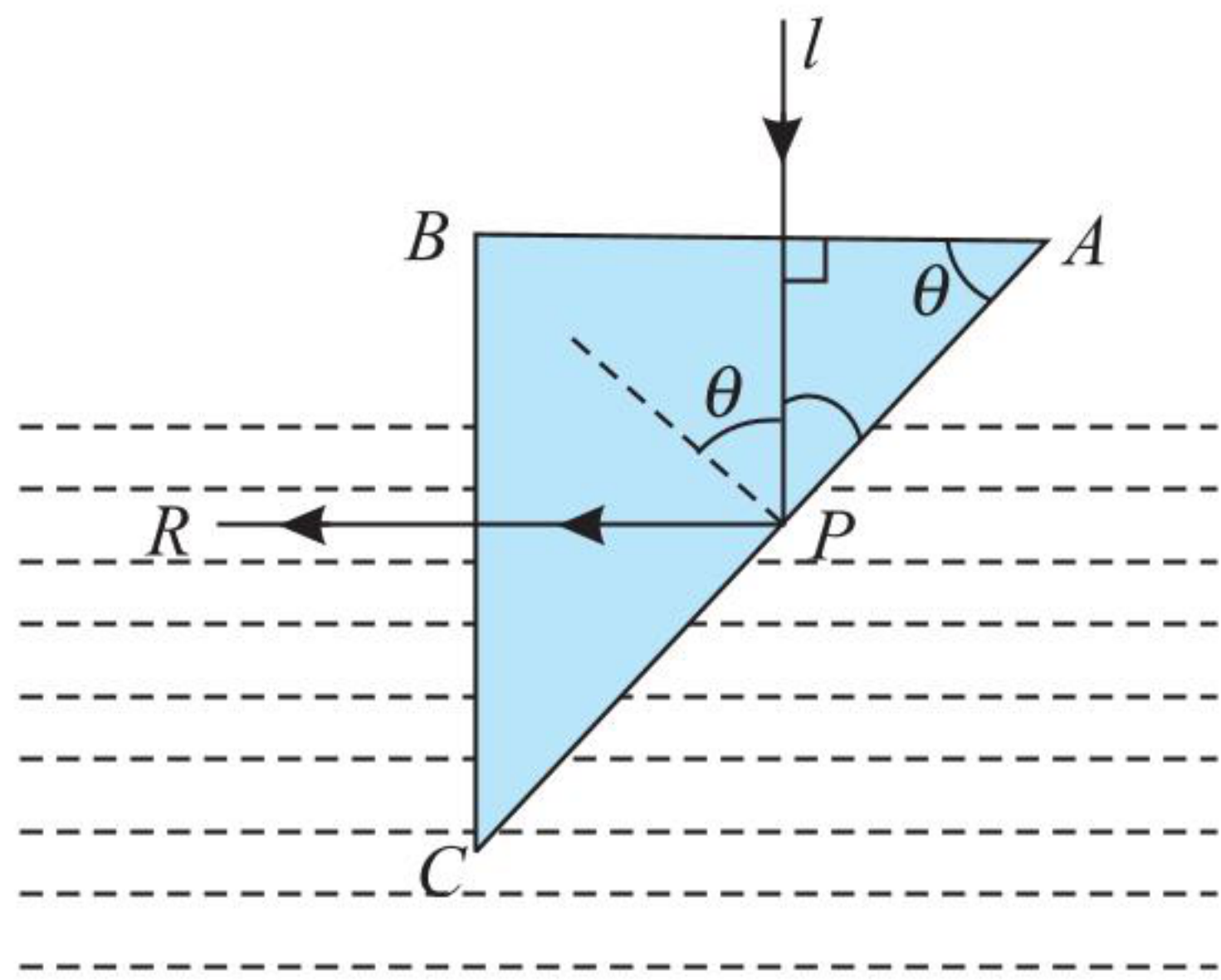
$$1 \cdot \sin 90^\circ = \mu_5 \cos r_5$$

$$\text{Square and add } 1 + \mu_2^2 + \mu_4^2 = \mu_1^2 + \mu_3^2 + \mu_5^2$$

75. (2) There will be infinite incident rays from point B, when we made a hole, only reflected of this hole is absent but other reflected rays will be present.

76. (1) The phenomenon of total internal reflection takes place during reflection at P .

$$\sin \theta = \frac{1}{{}_g\mu_w} \quad \dots(i)$$



where θ is the angle of incidence at P .

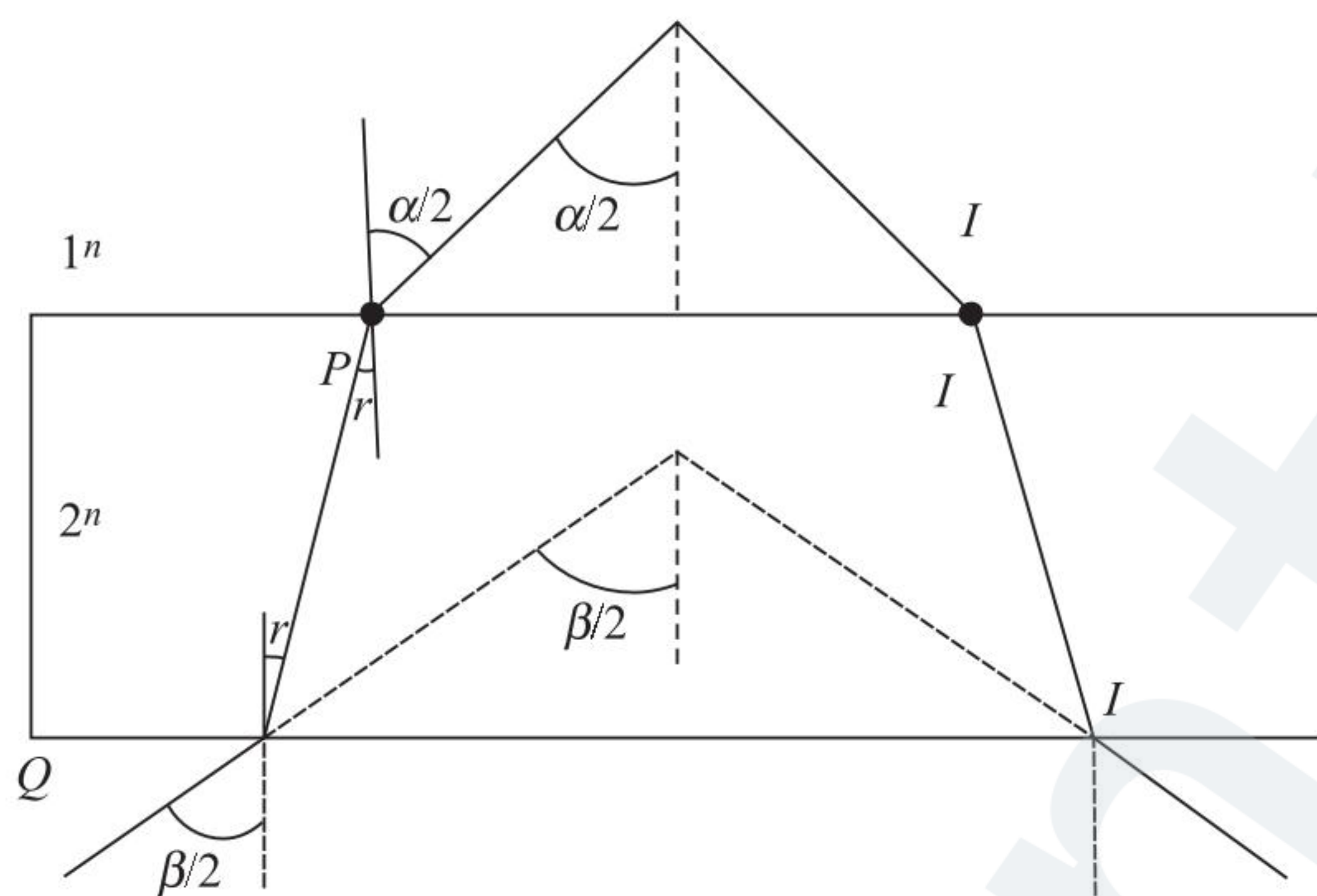
$$\text{Now, } {}_g\mu_w = \frac{{}_a\mu_h}{{}_a\mu_g} = \frac{1.5}{4/3} = 1.125$$

$$\text{Putting in (i), } \sin \theta = \frac{1}{1.125} = \frac{8}{9}$$

Therefore, $\sin \theta$ should be greater than $\frac{8}{9}$

77. (2) The path of rays becomes parallel to initial direction as they emerge.

$$\text{Applying Snell's law at } P, \frac{1}{2}n = \frac{\sin \alpha / 2}{\sin r} \quad \dots(i)$$



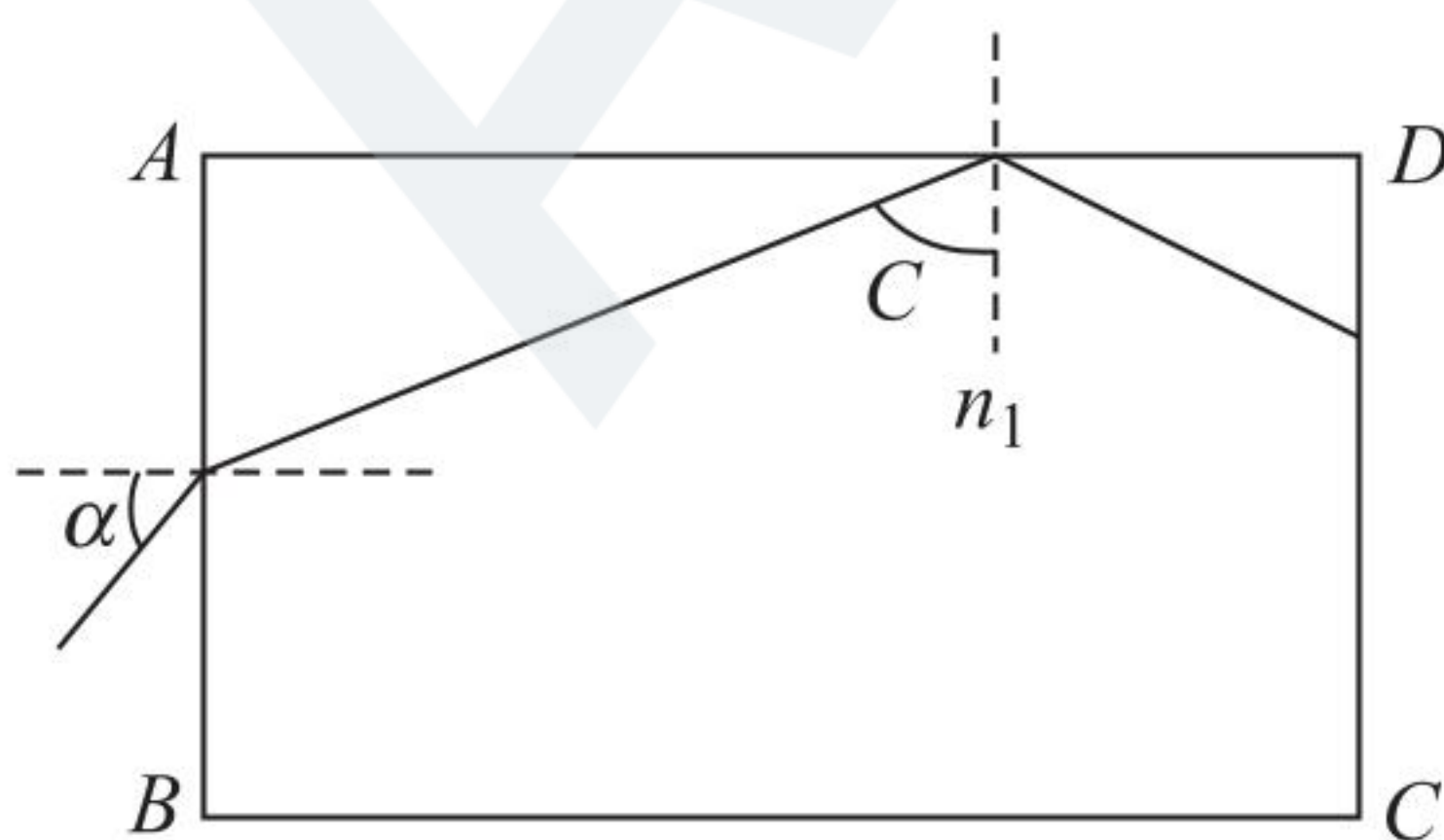
$$\text{Applying Snell's law at } Q, \frac{1}{2}n = \frac{\sin \beta / 2}{\sin r} \quad \dots(ii)$$

From Eqs. (i) and (ii), $a = b$

Therefore, (2) is the correct option.

78. (1) See figure. The ray will come out from CD if it suffers total internal reflection at surface AD , i.e., it strikes the surface AD at critical angle C (the limiting case). Applying Snell's law at P ,

$$n_1 \sin C = n_2 \text{ or } \sin C = \frac{n_2}{n_1}$$



Applying Snell's law at Q , $n_2 \sin \alpha = n_1 \cos C$;

$$\sin \alpha = \frac{n_1}{n_2} \cos \left\{ \sin^{-1} \left(\frac{n_2}{n_1} \right) \right\}$$

$$\text{or, } \alpha = \sin^{-1} \left[\frac{n_1}{n_2} \cos \left\{ \sin^{-1} \left(\frac{n_2}{n_1} \right) \right\} \right]$$

Hence, (1) is the correct option.

79. (3) The image I' for first refraction (i.e., when the ray comes out of liquid) is at a depth of $= \frac{33.25}{1.33} = 25 \text{ cm}$

$$\left[\because \text{Apparent depth} = \frac{\text{Real depth}}{\mu} \right]$$

Now, reflection will occur at concave mirror. For this I' behaves as an object

$$\therefore u = -(15 + 25) = -40 \text{ cm}, f = f, v = ?$$

Using mirror formula

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u} = \frac{1}{v} - \frac{1}{40} \Rightarrow v = \frac{40f}{40 + f} \quad \dots(i)$$

$$\text{But } v = \left[15 + \frac{25}{1.33} \right] \quad \dots(ii)$$

where $\frac{25}{1.33}$ is the real depth of the image.

From Eqs. (i) and (ii),

$$\frac{40f}{40 + f} = - \left[15 + \frac{25}{1.33} \right] = -33.79$$

$$\Rightarrow f = -18.31 \text{ cm}$$

80. (2) Critical angle from region III to region IV

$$\sin \theta_c = \frac{n_0 / 8}{n_0 / 6} = \frac{3}{4}$$

Now, applying Snell's law in region I and region III

$$n_0 \sin \theta = \frac{n_0}{6} \sin \theta_c$$

$$\sin \theta = \frac{1}{6} \sin \theta_c = \frac{1}{6} \left(\frac{3}{4} \right) = \frac{1}{8} \Rightarrow \theta = \sin^{-1} \left(\frac{1}{8} \right)$$

Therefore, correct option is (2).

Multiple Correct Answers Type

1. (3), (4)

As long as the object moves towards the mirror, image moves away from the mirror (for $u > f$) and $m = -v/u$ ($v > u$), so image size increases.

2. (1), (3)

(1) $n_3 < n_1$ so critical angle decreases. Therefore, TIR will take place at AB .

(2) $n_3 > n_1$, so critical angle increases. Therefore, TIR will not take place.

(3) If $n_3 > n_2$, then TIR will take place at CD and if $n_3 < n_2$, then TIR will take place at AB .

(4) TIR will not take place at face CD for $n_3 < n_1$ because ray is going from rarer to denser medium.

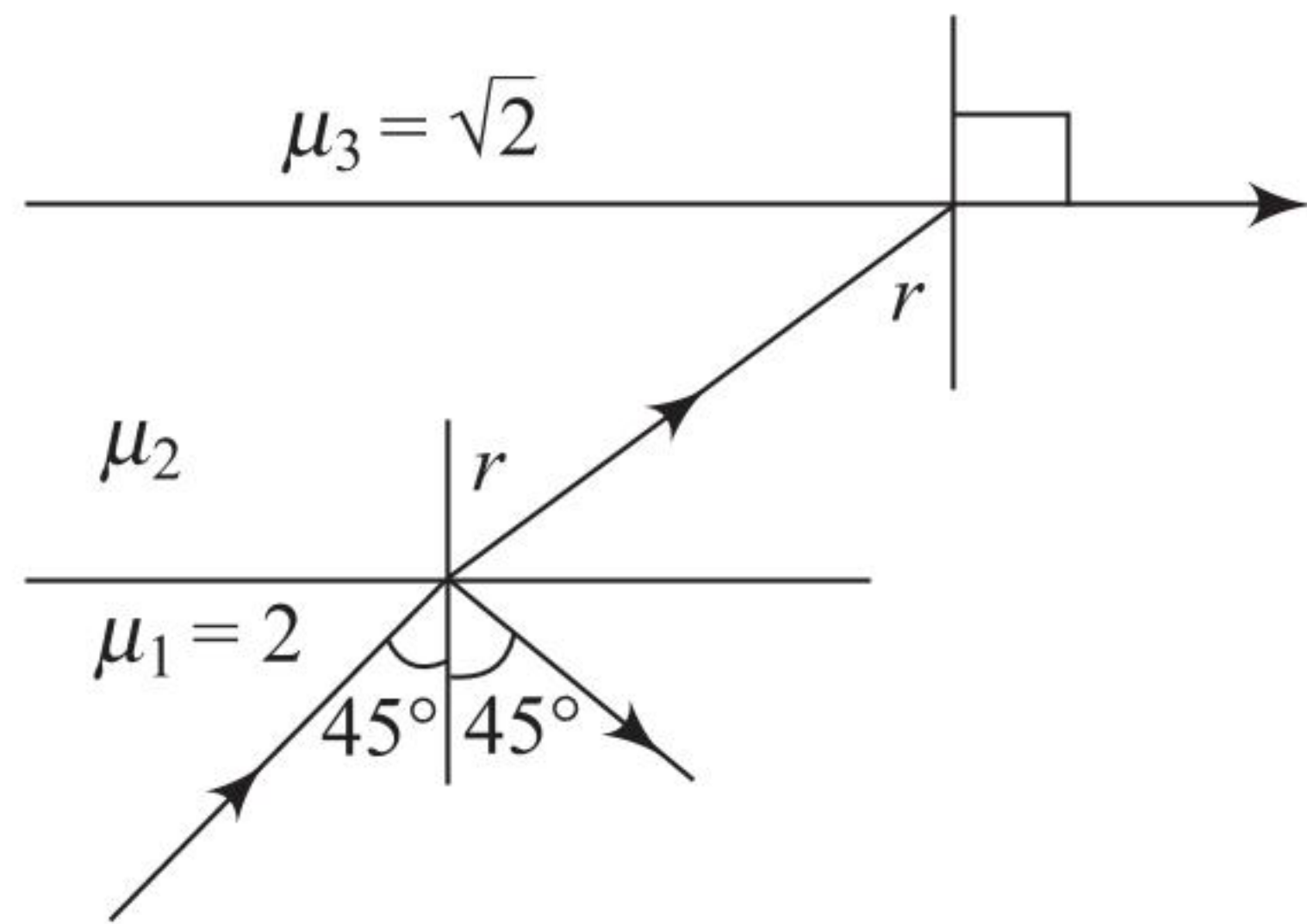
3. (1), (2)

For $\mu_2 > \sqrt{2}$ (TIR will not take place)

$$2 \sin 45^\circ = \mu_2 \sin r \text{ and } \mu_2 \sin r = \sqrt{2} \sin e$$

$$\Rightarrow e = 90^\circ$$

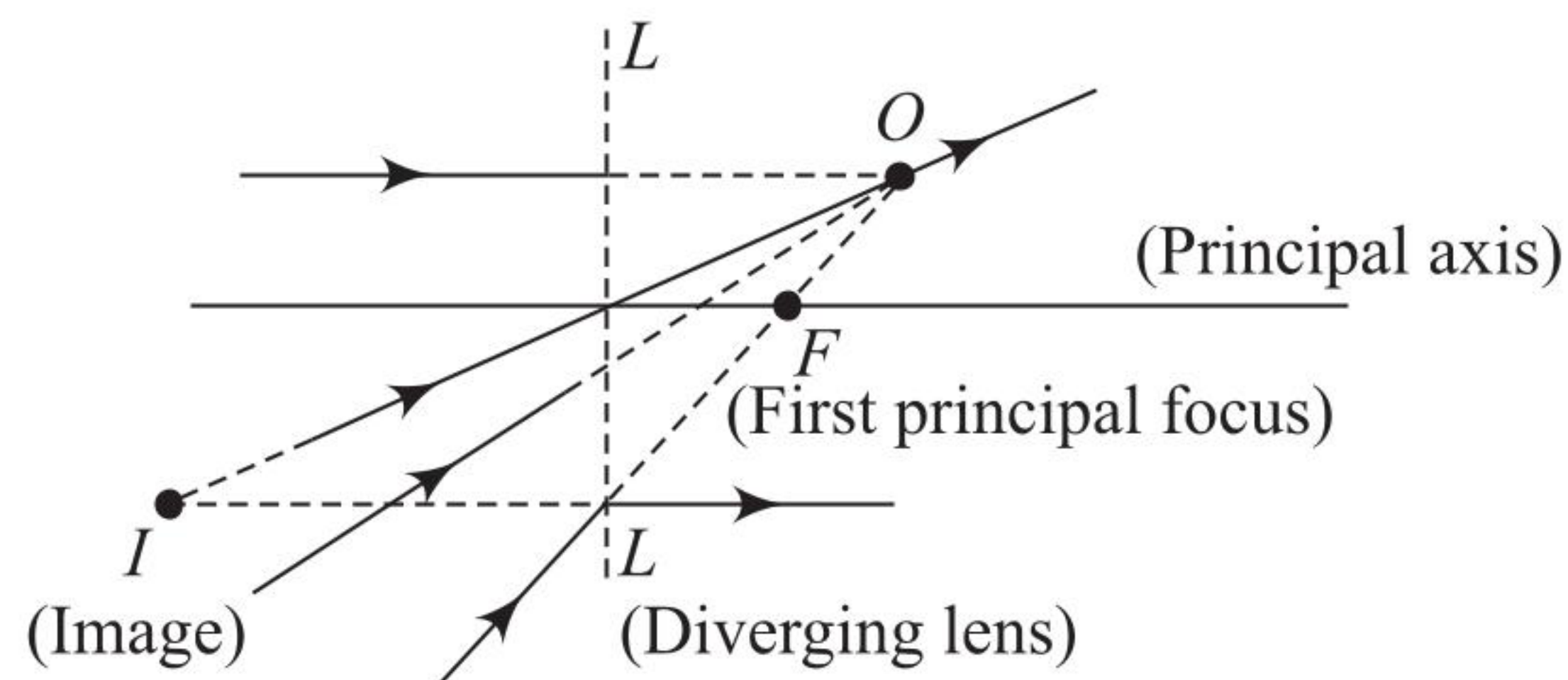
Hence, deviation is 45°



$\mu_2 < \sqrt{2}$. TIR will take place.

Deviation is 90° .

4. (1), (3), (4)



5. (1), (2), (3)

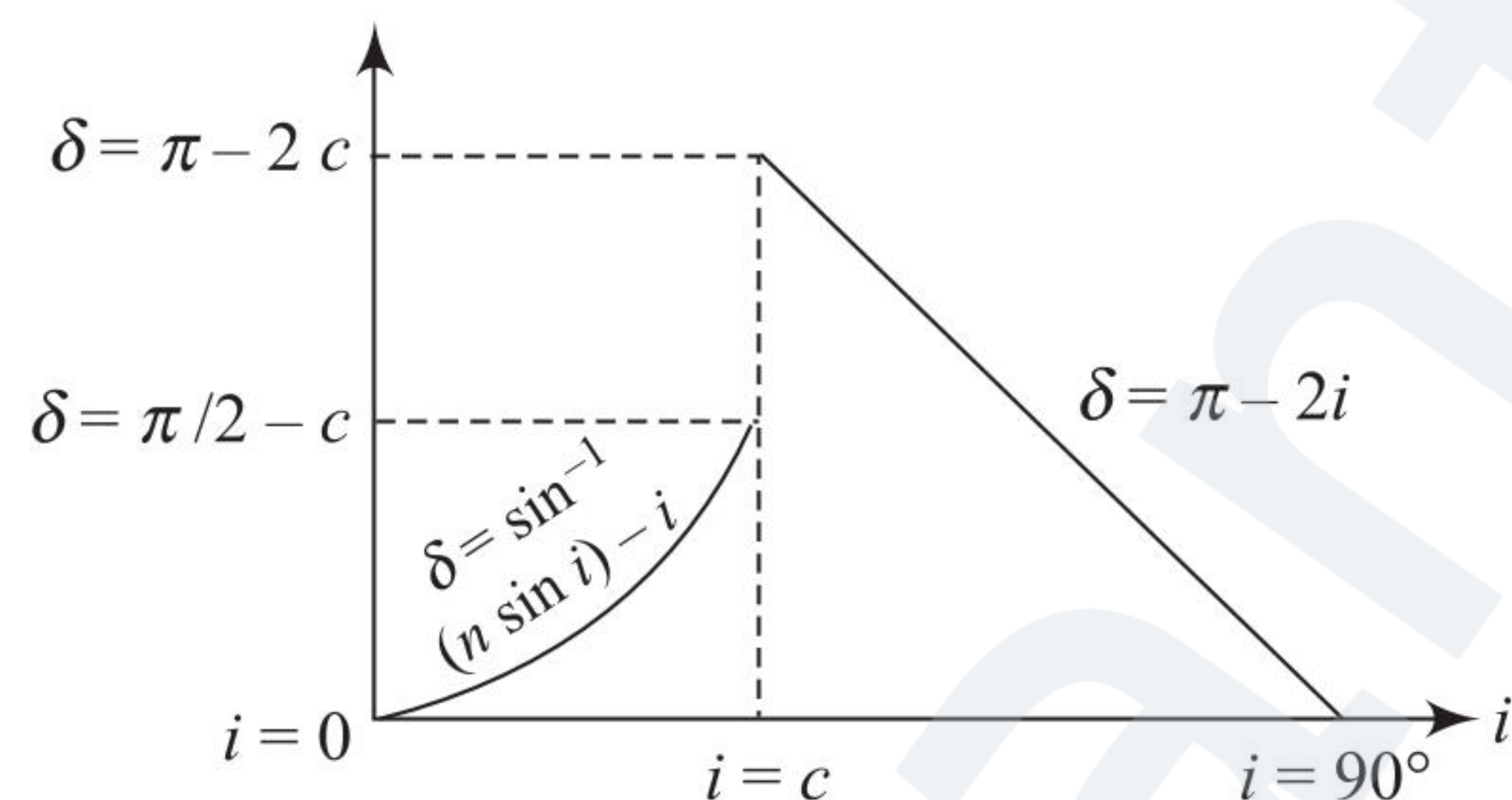
n for liquid = n for glass/yellow light

but n for liquid $< n$ for glass (red light), so it will deviate towards base, for blue light n (liquid) $> n$ (glass) so it will deviate towards vertex.

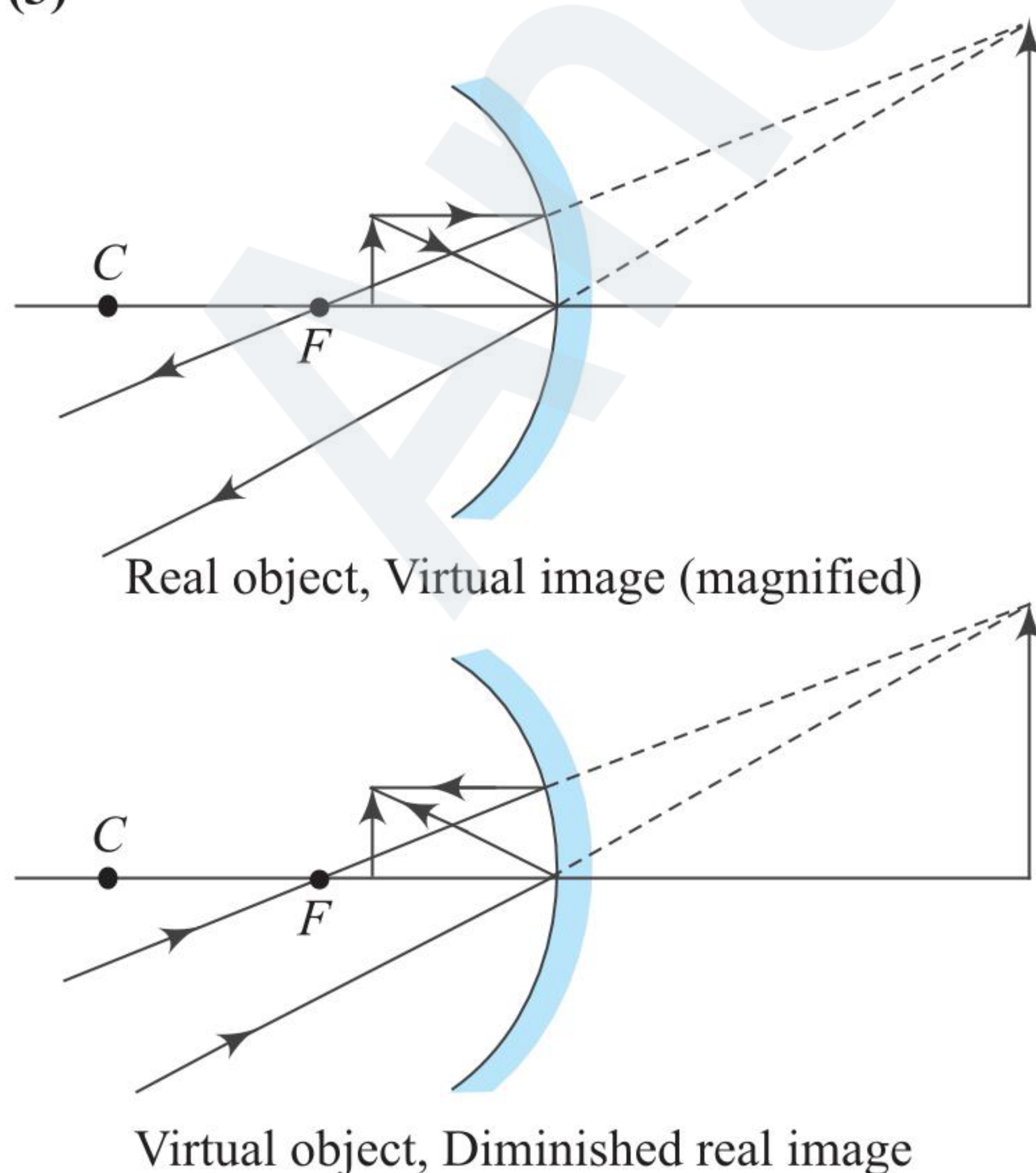
6. (1), (3)

The image of a point closer to the focus will be farther. As the transverse magnification of B will be more than A , the image of AB will be inclined to the optical axis.

7. (1), (2), (3), (4)

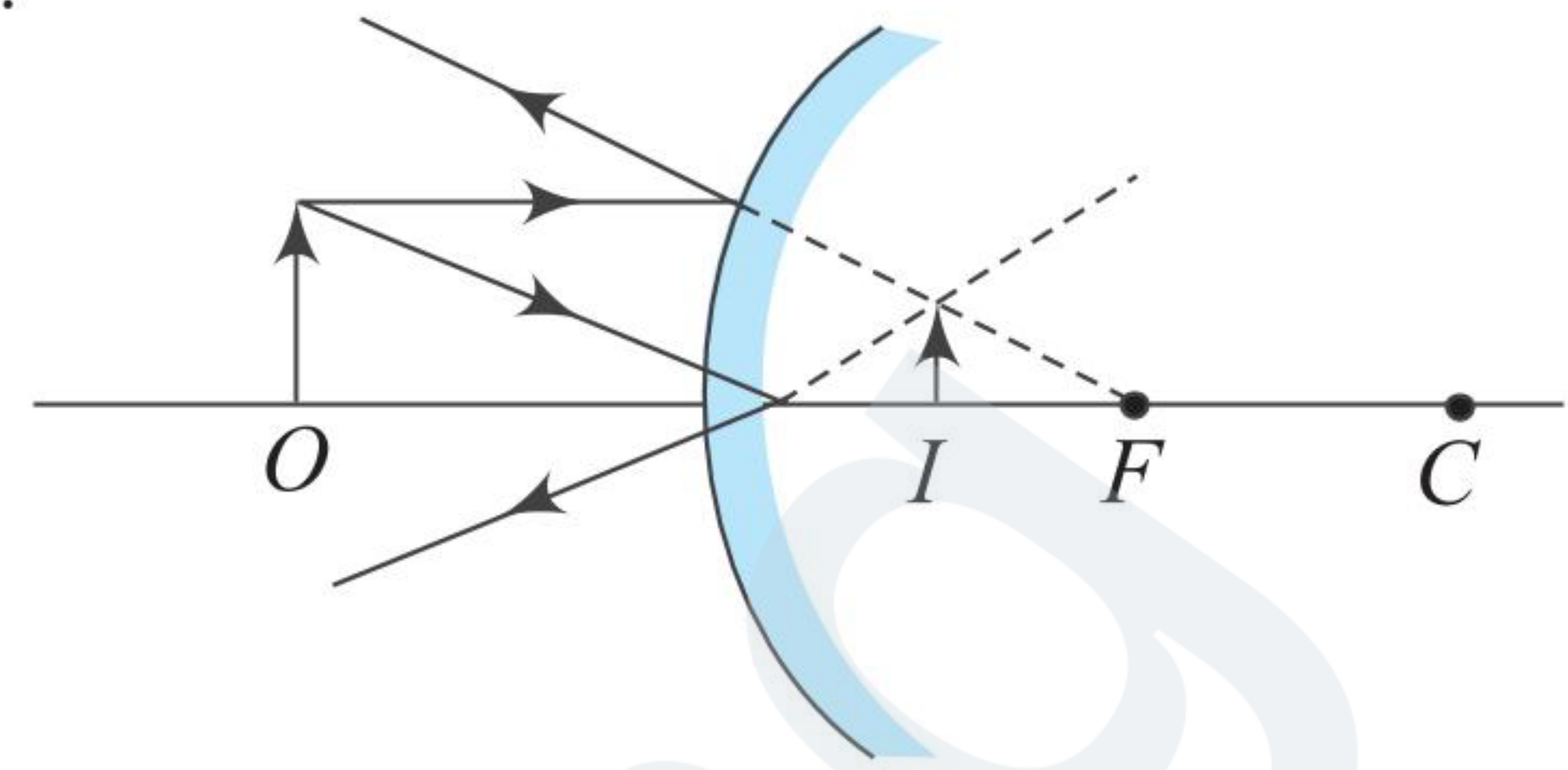


8. (2), (3)

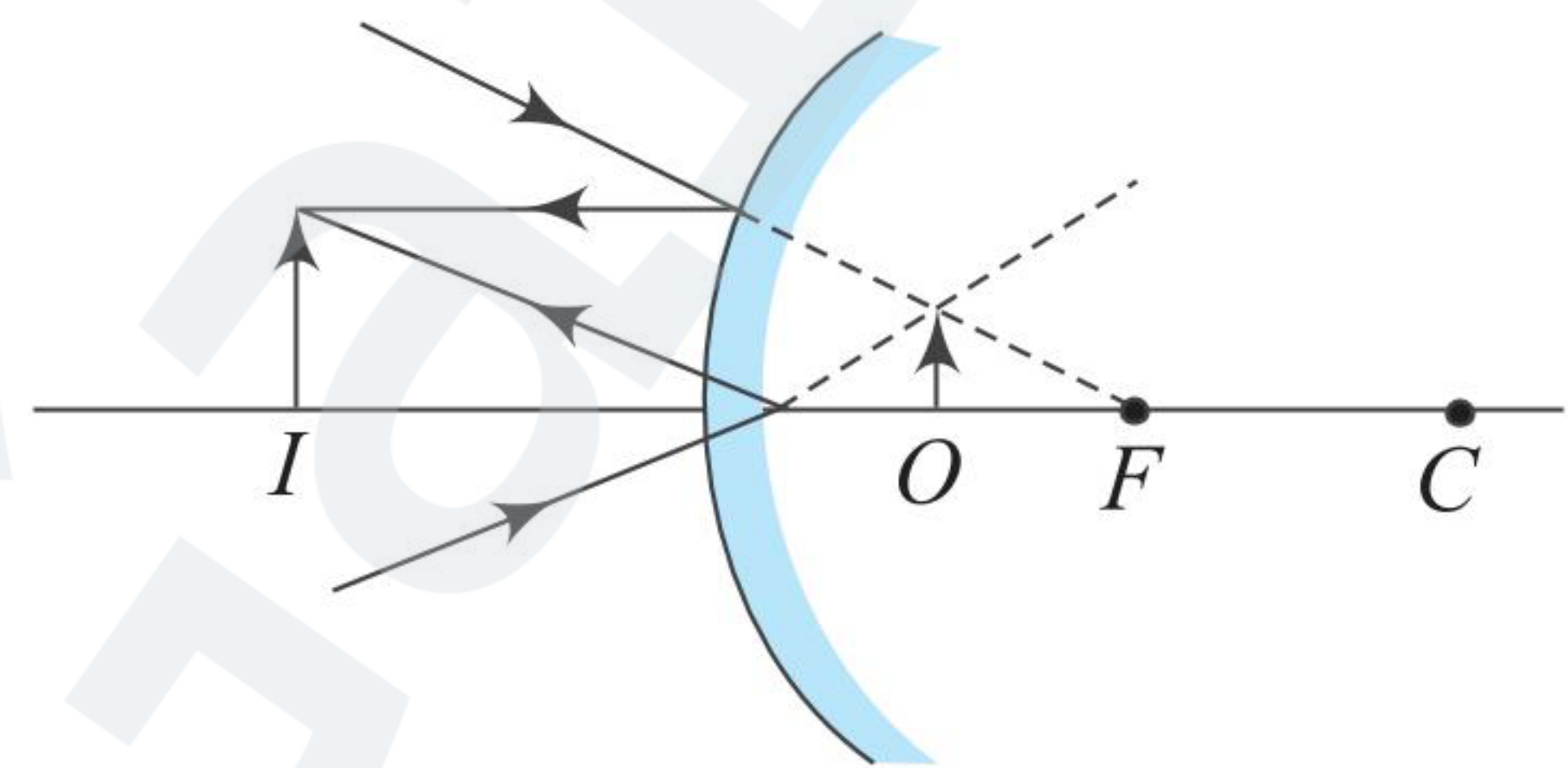


9. (1), (2), (3), (4)

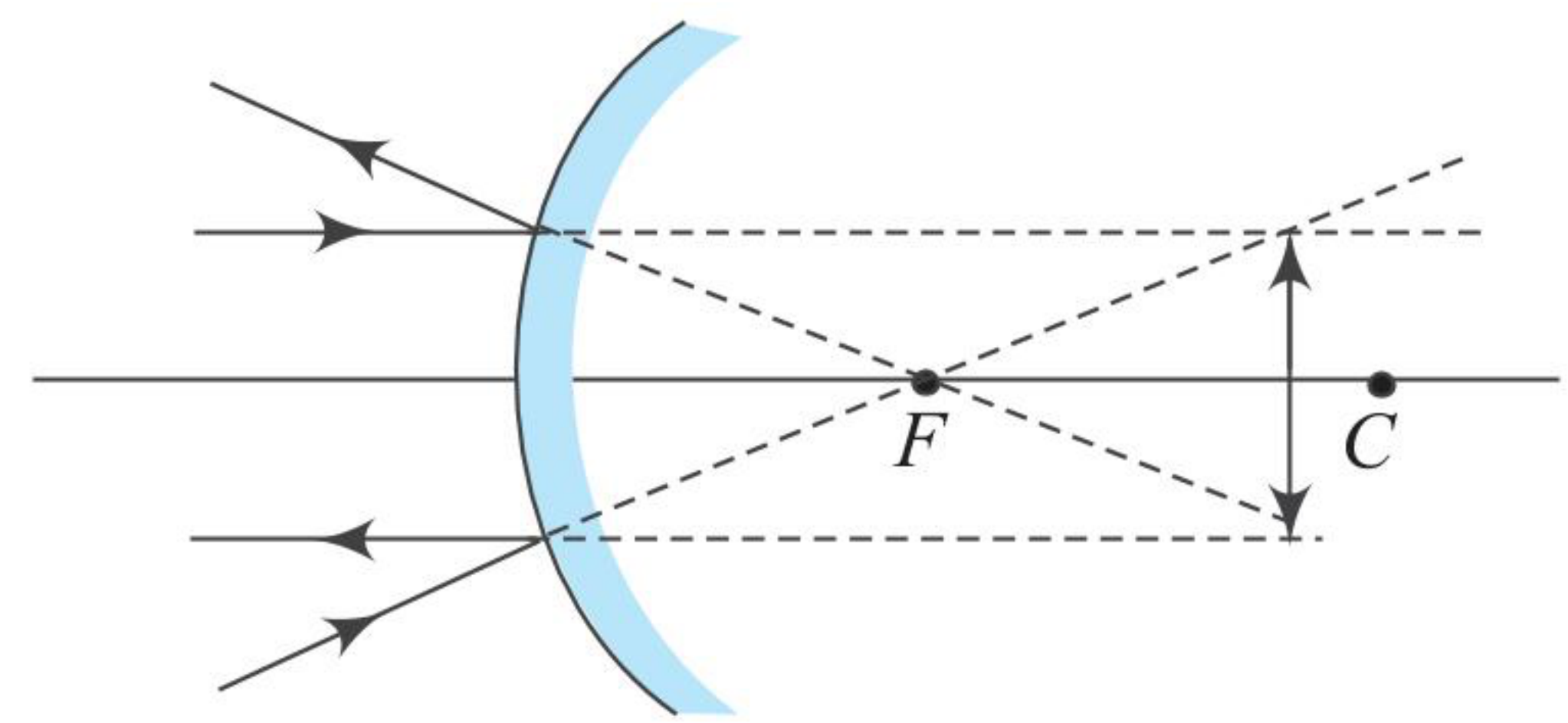
For a real extended object, image formed by convex mirror would be virtual, diminished, erect, and be lying between focus and pole.



For a virtual object, lying between focus and pole, image formed by a convex mirror would be real, magnified, erect, and lying in front of mirror.



For a virtual object beyond focus, image formed by a convex mirror would be virtual, inverted, magnified or diminished (depending on the location of object), and will lie on the same side of mirror.



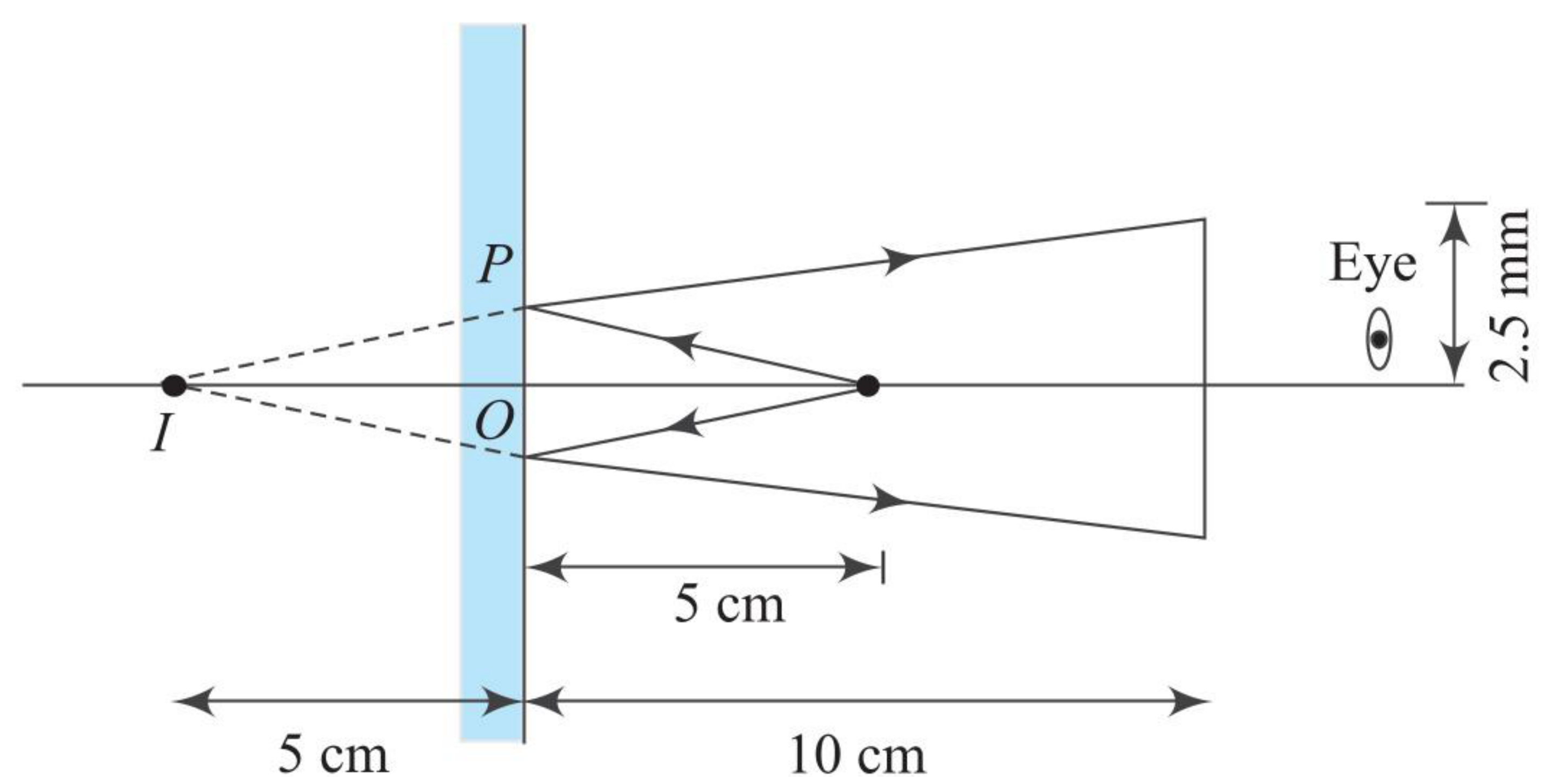
10. (2), (3), (4)

If the mirror is concave and a real object is approaching it, the image will move away from the mirror for object distance greater than focal length. If object distance is less than the focal length, virtual image will be formed which moves toward the mirror.

If mirror is convex, as object is approaching the mirror, image will move from focus to pole, i.e., toward the mirror.

11. (1), (4)

$$\text{Here, } \frac{OP}{5 \text{ cm}} = \frac{2.5 \text{ mm}}{15 \text{ cm}}$$



$$OP = \frac{5}{6} \text{ mm}$$

$$\text{Area of mirror used in reflection is } \pi \times OP^2 = \frac{25\pi}{36} \text{ mm}^2$$

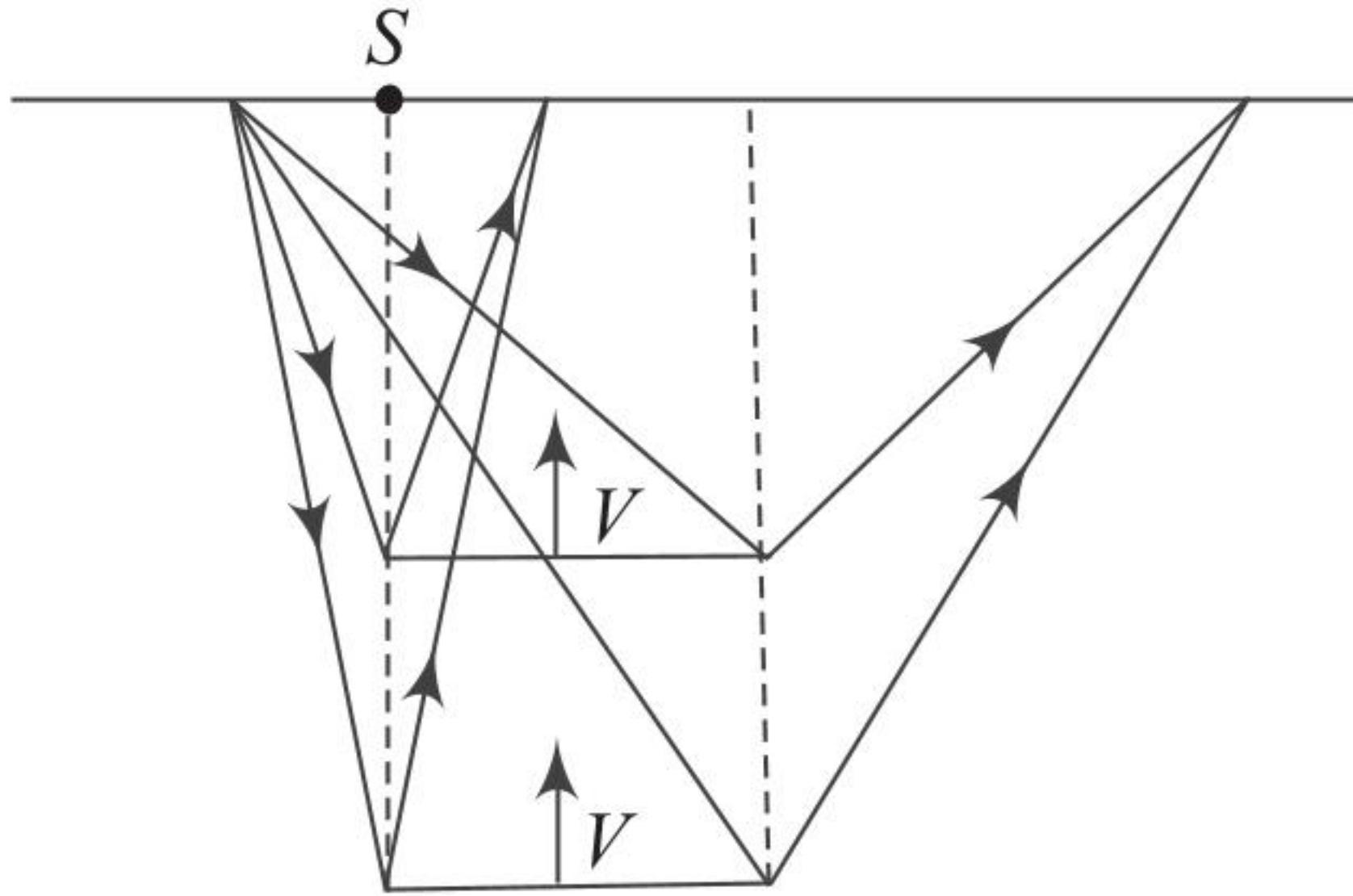
Let the power of source be P .

Intensity in absence of mirror is $I_1 = \frac{P}{4\pi \times (5)^2}$

Intensity in presence of mirror is $I_2 = \frac{P}{4\pi \times (5)^2} + \frac{P/2}{4\pi \times (5)^2}$

$$\frac{I_2}{I_1} = \frac{19}{18}$$

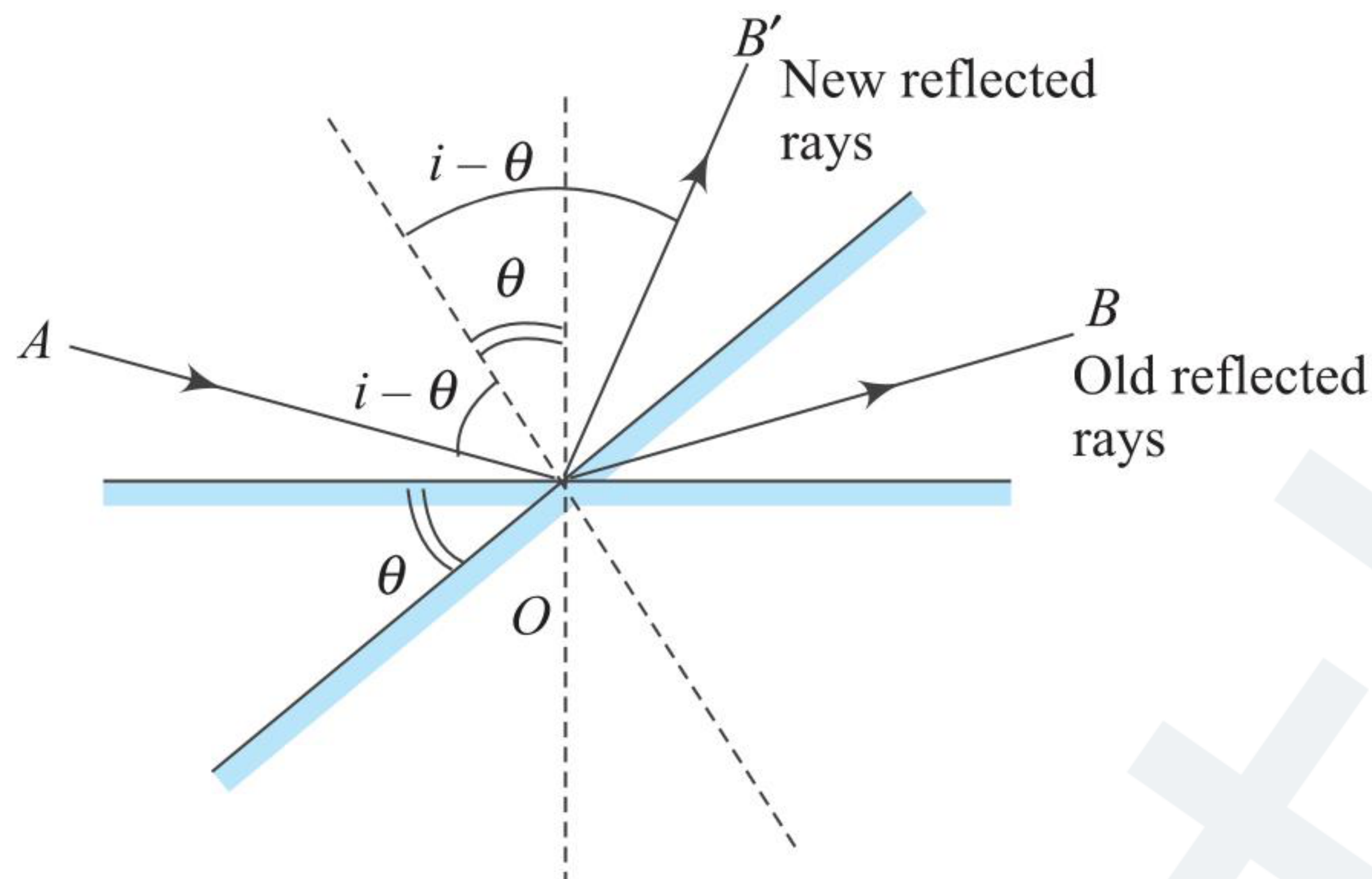
12. (2), (4)



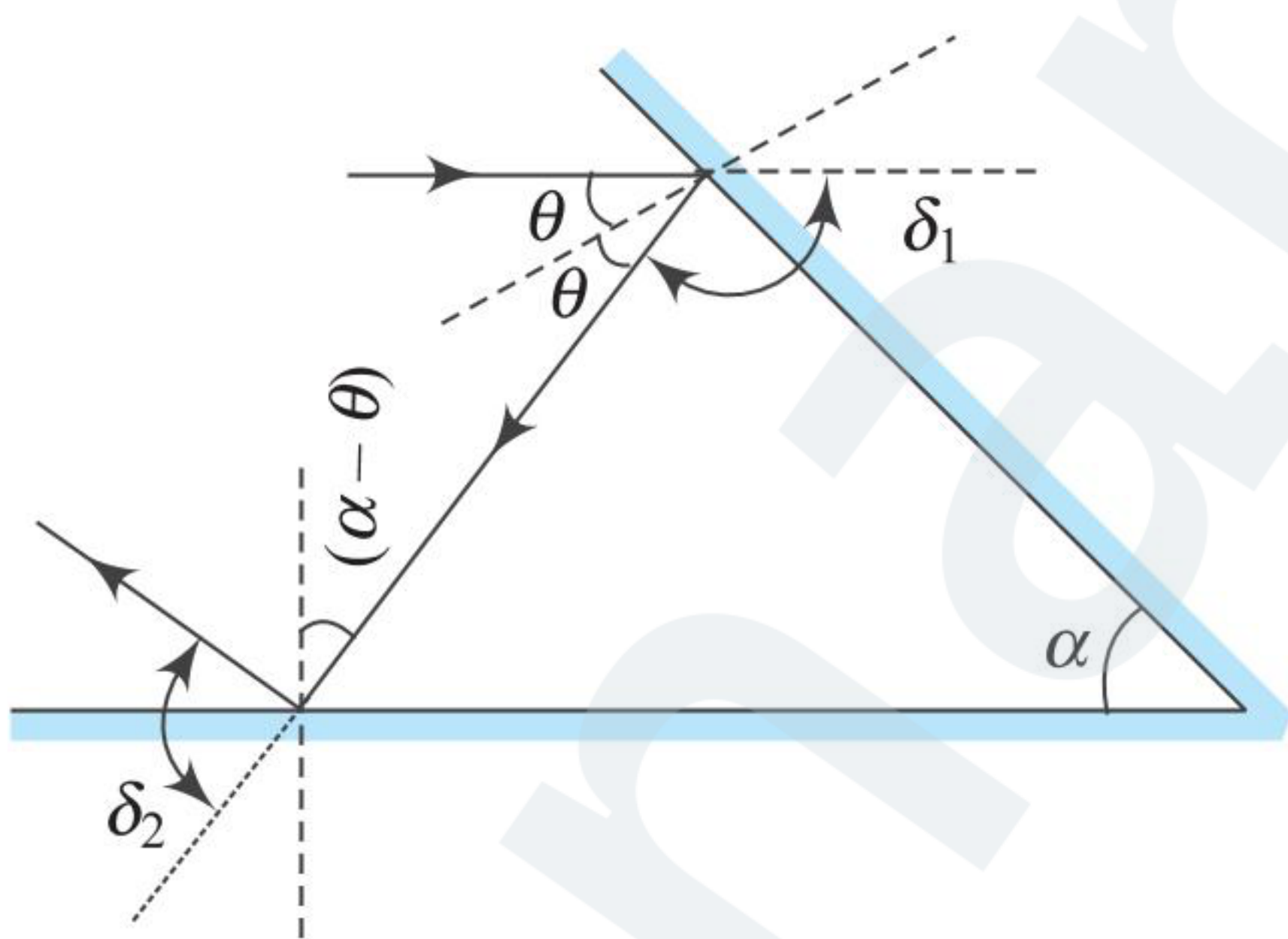
From the above ray diagram, it is clear that the options (2) and (4) are correct.

13. (1), (2), (3), (4)

(1) Angle $BOB' = \angle AOB' - \angle AOB = 2i - (2i - 2\theta) = 2\theta$



(2) Total deviation $\delta = \delta_1 + \delta_2 = (180^\circ - 2\theta) + 180^\circ - 2(\alpha - \theta) = 360^\circ - 2\alpha$



which is independent of angle of incidence.

(3) Power of a plane mirror is zero.

(4) Velocity of the image toward the object $= v + v = 2v$.

14. (2), (3)

If an object is at a distance x from a plane refracting surface and is viewed normally, then it appears at a distance x/μ from the surface, where μ is refractive index of that medium (in which it is situated) with respect to the medium in which observer is situated.

Suppose height of bird from water surface is x and depth of fish from the surface is y , then depth of fish, as observed by the bird, will be equal to

$$r_1 = x + \frac{y}{\mu} \quad \dots(i)$$

where μ is refractive index of water with respect to air.

$$r_1 = 6.60 \text{ m}$$

Hence, option (2) is correct.

Height of bird, as observed by the fish, will be equal to

$$r_2 = \frac{x}{\mu'} + y \quad \dots(ii)$$

where μ' is refractive index of air with respect to water.

Now, $\mu' = \frac{1}{\mu} = \frac{3}{4}$

$$\text{Hence, } r_2 = 8.80 \text{ m}$$

Hence, option (1) is wrong while option (3) is correct.

15. (2), (4)

In the previous question, velocity of fish, observed by the bird, will be equal to dr_1/dt . Therefore, differentiating Eq. (i), we get

$$\frac{dr_1}{dt} = \frac{dx}{dt} + \frac{1}{\mu} \frac{dy}{dt} = 4.50 \text{ m s}^{-1}$$

Hence, option (4) is correct. Similarly, velocity of bird, as

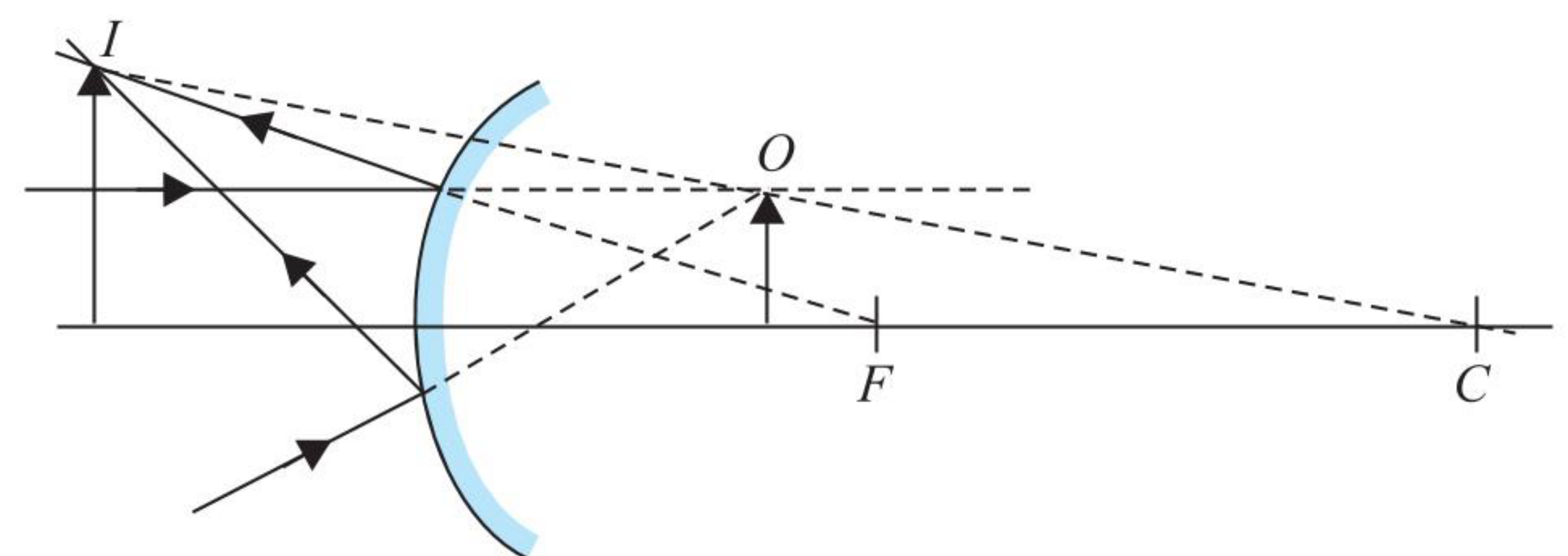
observed by the fish, will be equal to $\frac{dr_2}{dt}$.

$$\frac{dr_2}{dt} = \frac{1}{\mu'} \frac{dx}{dt} + \frac{dy}{dt} = 6.0 \text{ m s}^{-1}$$

Hence, option (2) is correct.

16. (1), (3)

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{y} + \frac{1}{x} = \frac{1}{f} \Rightarrow y = \frac{xf}{x-f}$$

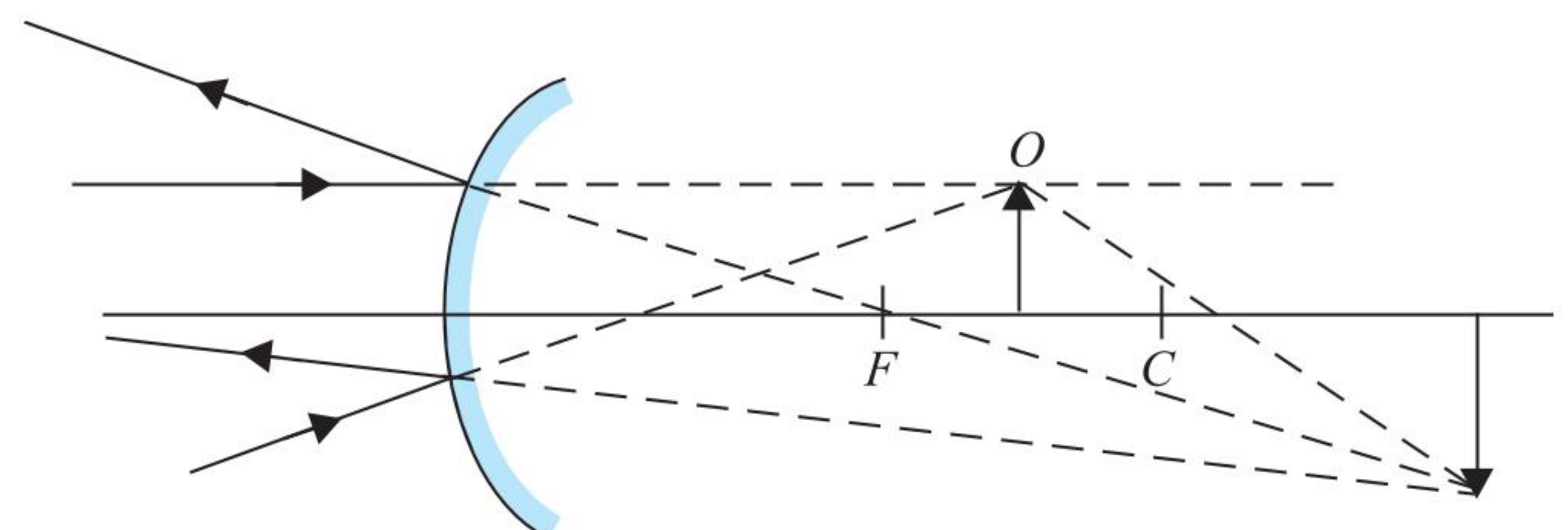


Since $x < f$, so y is negative and hence so image is real.

$$m = -\frac{v}{u} = -\frac{y}{x} \rightarrow \text{positive so image is erect.}$$

Image will be enlarged.

17. (2), (4)



$$y = \frac{xf}{x-f} \quad [\text{here } x > f, \text{ so } y \text{ is positive}]$$

Hence image is virtual

$$m = -\frac{v}{u} = -\frac{y}{x} \rightarrow \text{negative, so image is inverted.}$$

Image will be enlarged.

18. (2), (4)

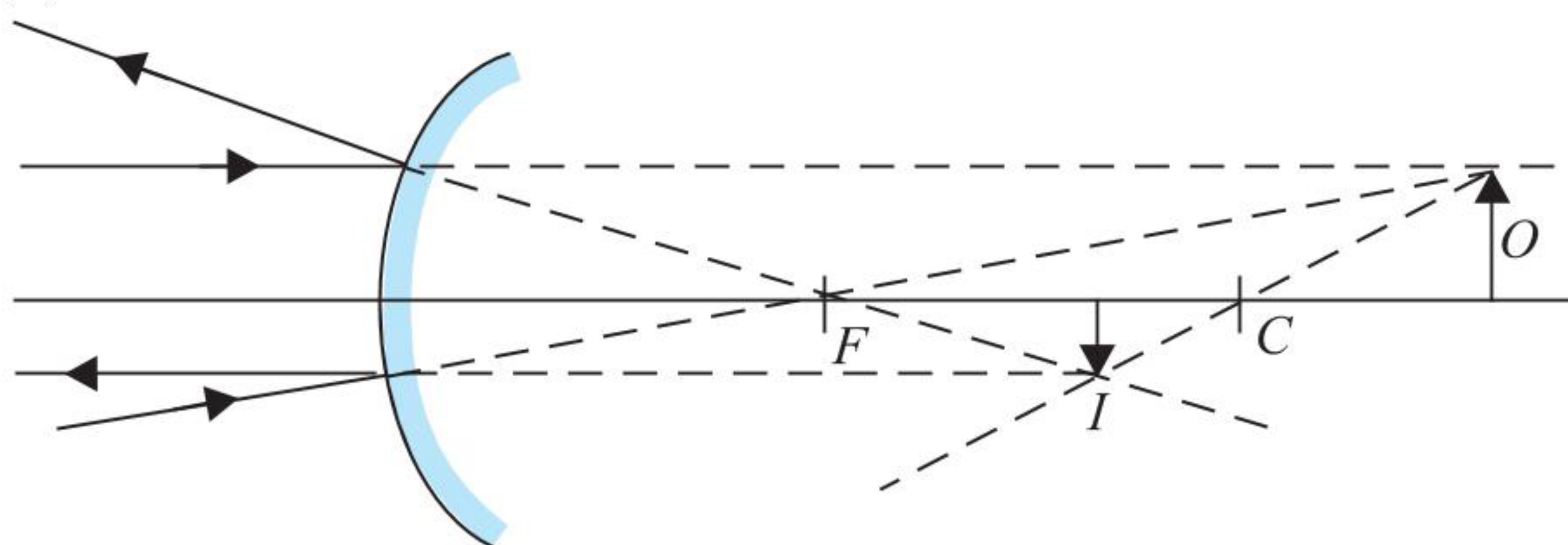


Image is virtual, inverted and diminished.

19. (2), (3)

Image of sun will be formed at the focus of mirror. Hence, focal length of mirror = 12 cm. To use it for shaving purpose, face should be between focus and pole, so that virtual image can be formed.

20. (1), (3)

$$\frac{1}{v} + \frac{1}{-u} = \frac{1}{f} \Rightarrow v = \frac{uf}{u+f} \quad (v < u \text{ and } v < f)$$

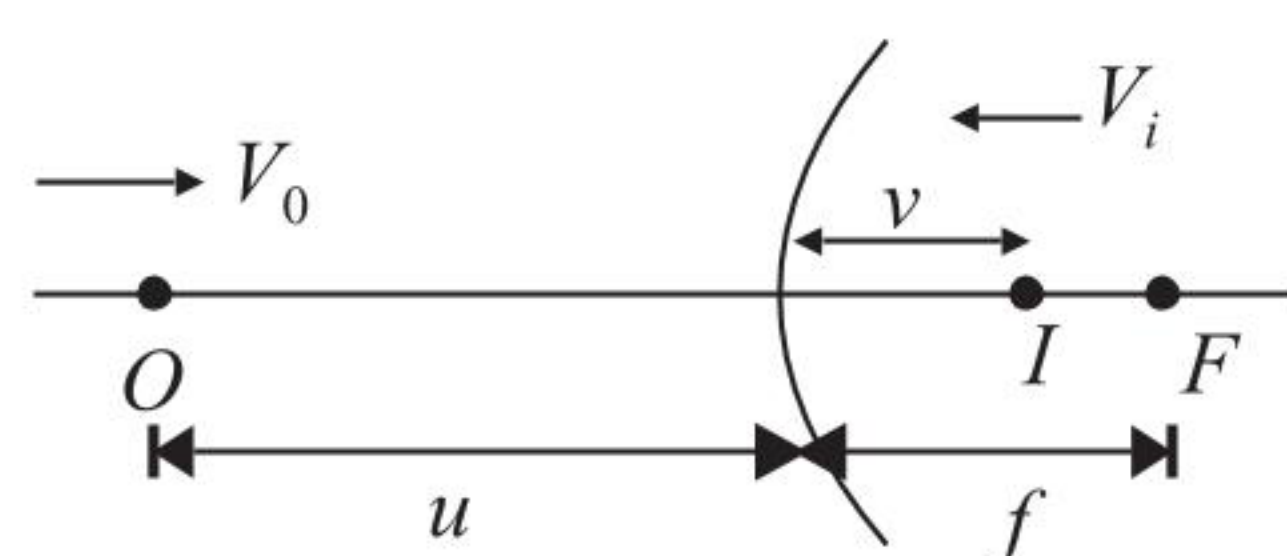
$$V_0 = -\frac{du}{dt}, V_i = -\frac{dv}{dt}$$

$$\text{Differentiating } \frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow -\frac{1}{v^2} \frac{dv}{dt} + \frac{1}{u^2} \frac{du}{dt} = 0$$

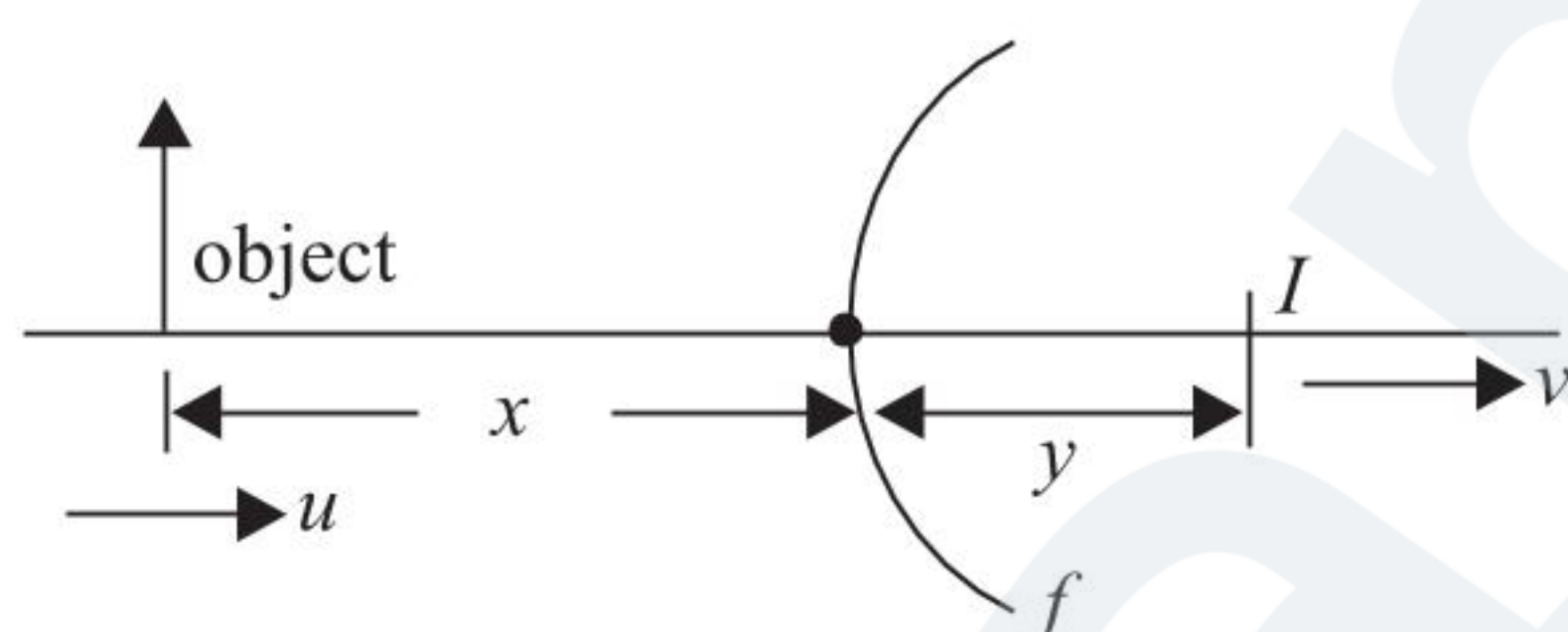
$$\Rightarrow -\frac{1}{v^2}(-V_i) + \frac{1}{u^2}(-V_0) = 0 \Rightarrow V_i = \left(\frac{v}{u}\right)^2 V_0$$

Since $v < u$, so $V_i < V_0$ for each case.



21. (1), (3)

$$\frac{1}{y} + \frac{1}{-x} = \frac{1}{f} \Rightarrow \frac{1}{y} = \frac{1}{x} + \frac{1}{f} \Rightarrow y = \frac{xf}{x+f}$$



$$\Rightarrow \frac{y}{x} = \frac{f}{x+f} \text{ as } x \text{ decreases, so magnification increases.}$$

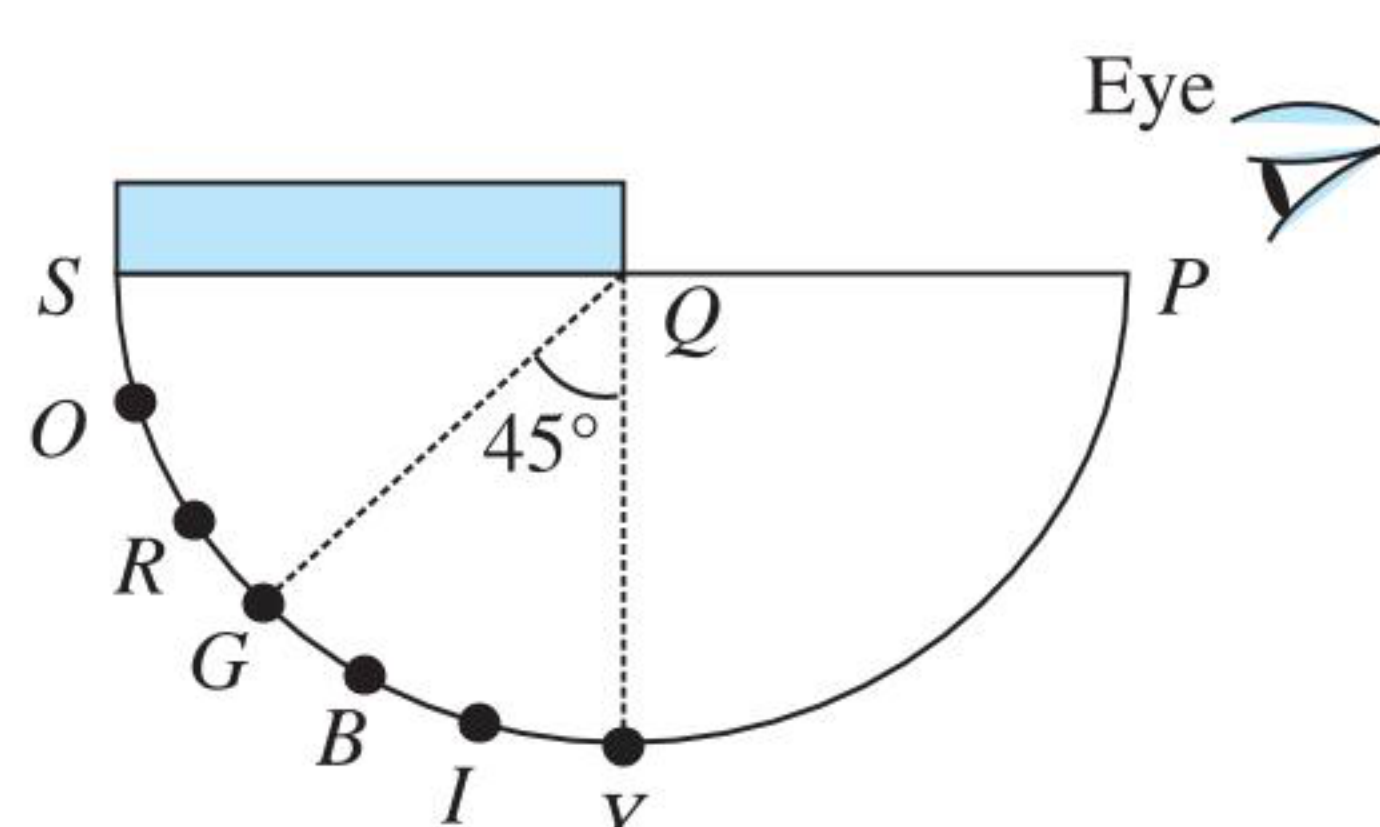
$$\text{Now, } \frac{dy}{dt} = \left(\frac{y}{x}\right)^2 \frac{dx}{dt} \Rightarrow v = -\left(\frac{y}{x}\right)^2 u$$

As $\frac{y}{x}$ increases, so v increases.

22. (2), (3)

$$\text{Let } C \text{ be the critical angle: } \sin C = \frac{1}{\mu} = \frac{1}{\sqrt{2}} \\ C = 45^\circ$$

For Y, I and B, angle of incidence at a point on QP can be less than C, so they will be refracted from QP and can be seen by person.



But for G, R and O, angle of incidence at a point on QP will be more than C. Hence, they will be internally reflected.

23. (1), (3)

$$\text{From } \mu = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin\frac{A}{2}}$$

We can see that $\delta_m = 30^\circ$ for ($\mu = \sqrt{2}$ and $A = 60^\circ$)

Further, at minimum deviation, $r_1 = r_2 = \frac{A}{2} = 30^\circ$

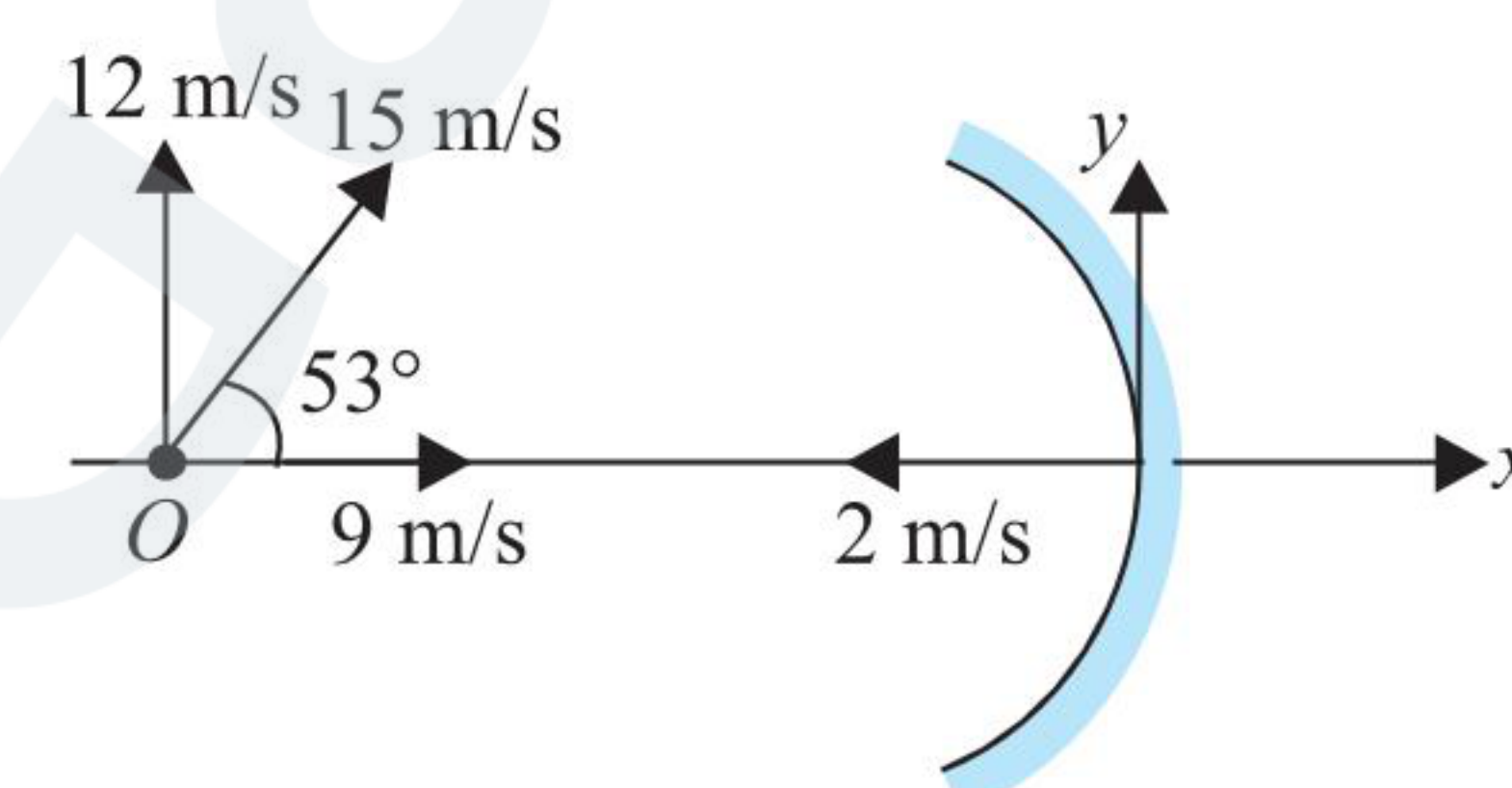
$$\sin i_1 = \mu \sin r_1 = (\sqrt{2}) \sin 30^\circ = \frac{1}{\sqrt{2}}$$

$$\Rightarrow i_1 = 45^\circ$$

24. (2), (3)

$$\vec{v}_0 = \text{velocity of object} = (9\hat{i} + 12\hat{j}) \text{ m s}^{-1}$$

$$\vec{v}_m = \text{velocity of mirror} = -2\hat{i} \text{ m s}^{-1}$$



$$m = \frac{f}{f-u} = \frac{-20}{-20 - (-30)} = -2$$

For velocity component parallel to optical axis,

$$(\vec{v}_{I/m})_{\parallel} = -m^2 (\vec{v}_{O/m})_{\parallel}$$

$$(\vec{v}_{I/m})_{\parallel} = -(-2)^2 11\hat{i} = -44\hat{i} \text{ m s}^{-1}$$

For velocity component perpendicular to optical axis

$$(\vec{v}_{I/m})_{\perp} = m (\vec{v}_{O/m})_{\perp} = -(-2) 12\hat{j} = -24\hat{j} \text{ m s}^{-1}$$

$$\vec{v}_{I/m} = \text{velocity of image with respect to mirror}$$

$$\vec{v}_{I/m} = (\vec{v}_{I/m})_{\parallel} + (\vec{v}_{I/m})_{\perp} = (-44\hat{i} - 24\hat{j}) \text{ m s}^{-1}$$

$$\text{Also, } \vec{v}_{I/m} = \vec{v}_I - \vec{v}_m$$

$$\text{or } \vec{v}_I = \vec{v}_{I/m} + \vec{v}_m = (-44\hat{i} - 24\hat{j}) - 2\hat{i} = (-46\hat{i} - 24\hat{j}) \text{ m s}^{-1}$$

25. (1), (2)

If image is virtual. Suppose $u = -x$, then $v = +2x$

From the mirror formula

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \text{ or } \frac{1}{2x} - \frac{1}{x} = \frac{1}{-f}$$

$$\text{or } x = \frac{f}{2}$$

If image is real, then suppose $u = -y$, then $v = -2y$

Again applying the mirror formula,

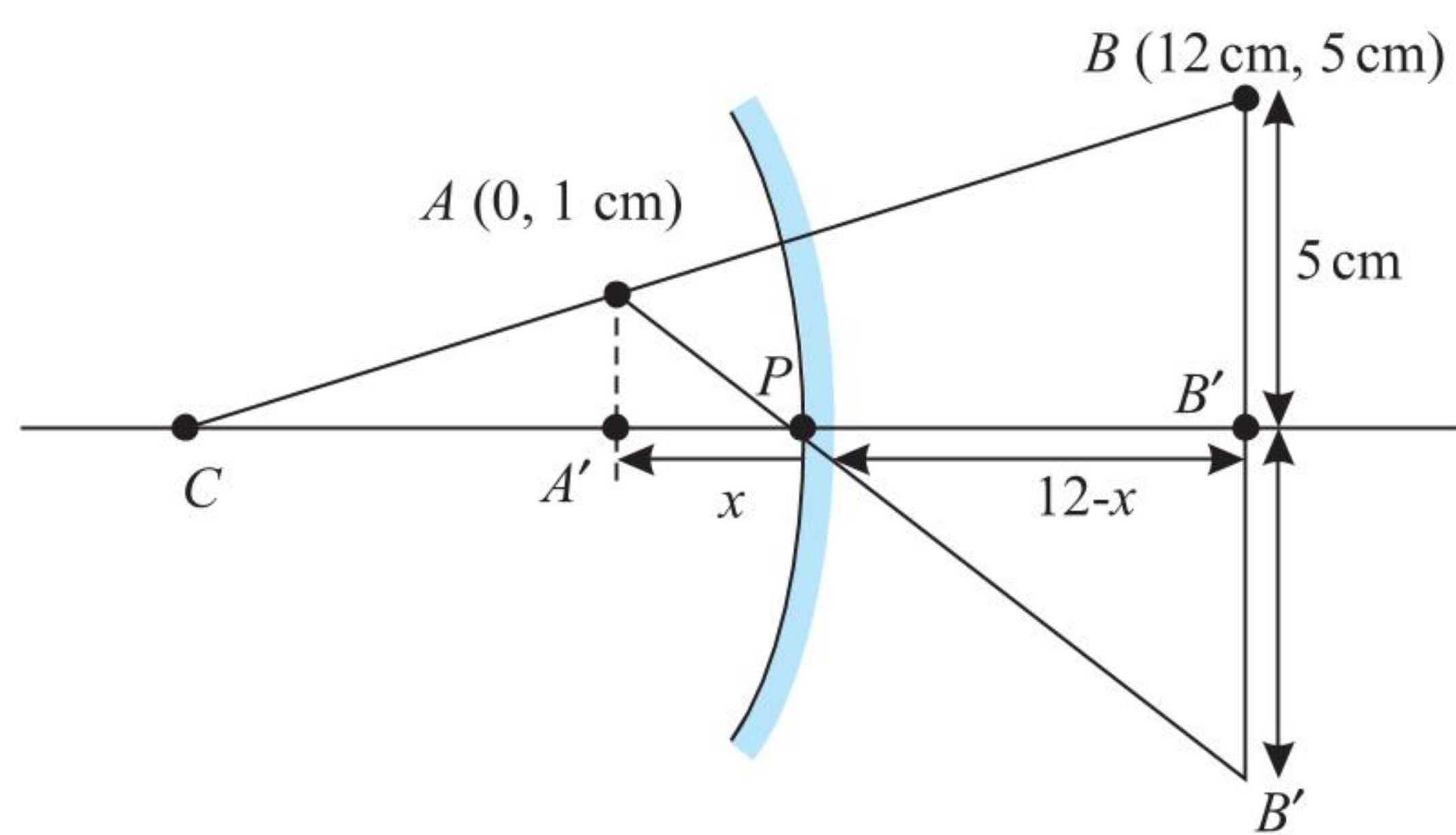
$$\frac{1}{-2y} - \frac{1}{y} = -\frac{1}{f} \text{ or } \frac{3}{2y} = \frac{1}{f} \Rightarrow y = \frac{3}{2}f$$

26. (1), (2), (3)

Join, object and image with a line that intersects the principal axis at point C. (centre of curvature of mirror)

From the similar triangles: $AA'P$ and $BB'P$,

$$\frac{1}{x} = \frac{5}{12-x} \Rightarrow x = 2 \text{ cm}$$



If, A is the object and B is the image then, concave mirror having its pole at $(2 \text{ cm}, 0)$ must be placed.

Apply mirror formula:

$$\Rightarrow \frac{1}{10} + \frac{1}{-2} = \frac{1}{f} \Rightarrow f = -2.5 \text{ cm}$$

If B is the object and A is the image then, convex mirror must be placed.

Apply mirror formula:

$$\Rightarrow \frac{1}{2} + \frac{1}{-10} = \frac{1}{f} \Rightarrow f = 2.5 \text{ cm}$$

27. (1), (2), (3), (4)

Using mirror formula, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$$\text{Hence, } f = -\frac{R}{2}$$

$$\Rightarrow \frac{1}{v} - \frac{3}{5R} = \frac{-2}{R}$$

$$\frac{1}{v} = \frac{-2}{R} + \frac{3}{5R} = -\frac{7}{5R} \Rightarrow v = -\frac{5R}{7}$$

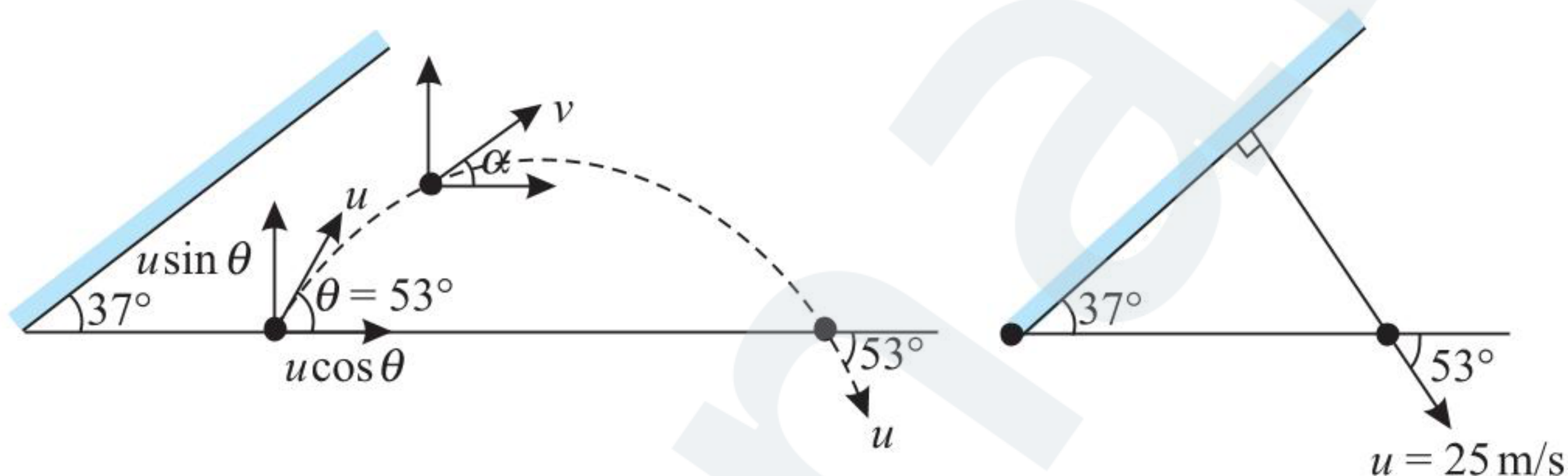
$$m = \frac{I}{O} = -\frac{v}{u} = -\left(\frac{-5R/7}{-5R/3}\right) = -\frac{3}{7}$$

Hence amplitude of oscillation of image = $\frac{3}{7} \text{ mm}$

As image is inverted hence phase difference of oscillation is π .

28. (1), (2), (3), (4)

Relative speed will be minimum (zero) when velocity of object will be parallel to plane mirror.



$$\tan \alpha = \frac{u \sin \theta - gt}{u \cos \theta}$$

$$\frac{3}{4} = \frac{25 \cdot \frac{4}{5} - 10t}{25 \cdot \frac{3}{5}}$$

$$\frac{3}{4} = \frac{20 - 10t}{15} \Rightarrow 45 = 80 - 40t$$

$$40t = 35 \Rightarrow t = \frac{7}{8} \text{ s}$$

At maximum relative speed the object moves normal to mirror. And it occurs when the particle hits the ground. Hence, maximum relative speed = 50 m/s .

29. (2), (3)

$$u = -\frac{R}{4}; f = -\frac{R}{2}; v = \frac{uf}{u-f} = +\frac{R}{2}$$

$\Rightarrow$ Image is at a distance $R/2$ to the right of the mirror. Its virtual and upright.

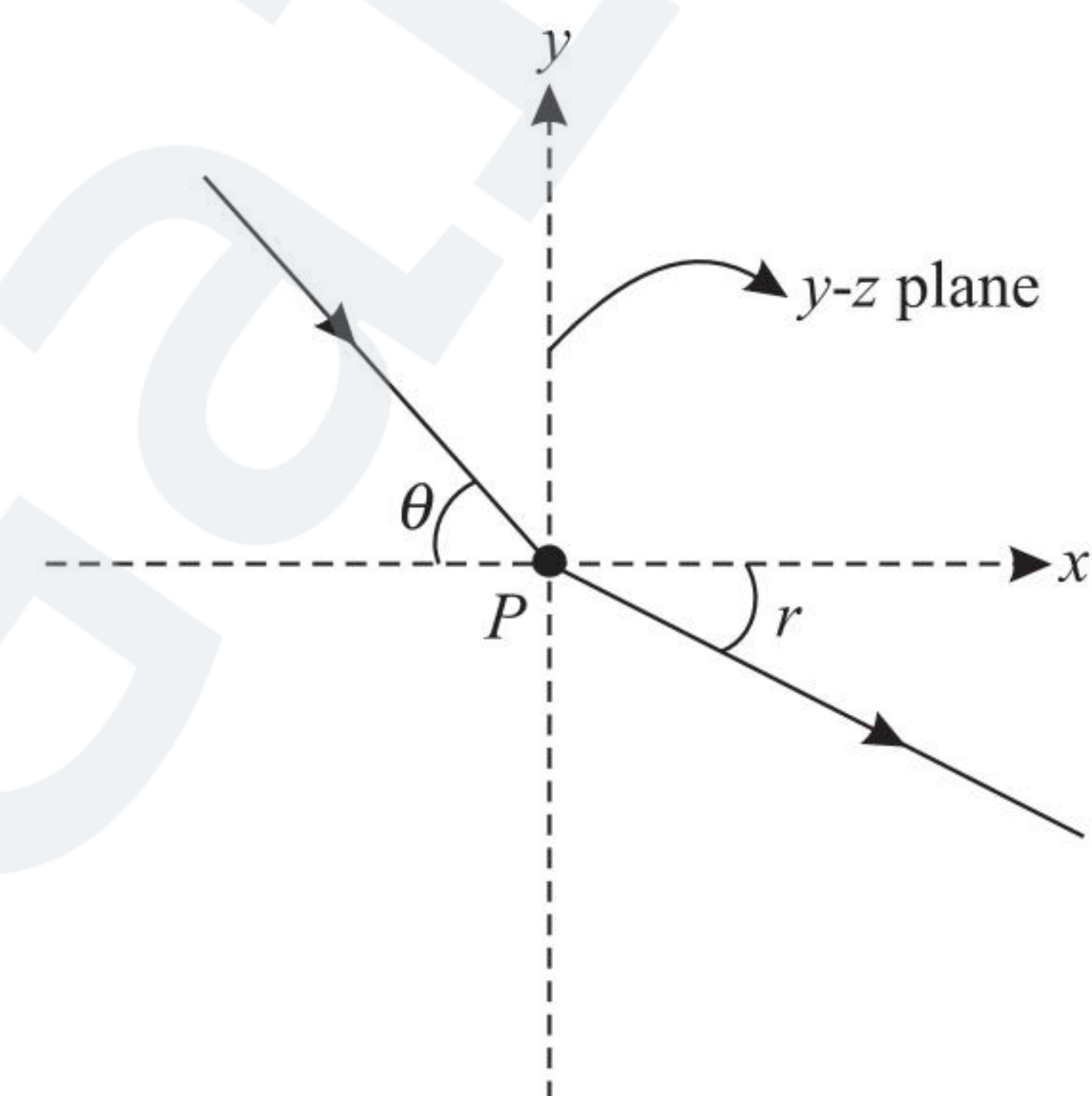
30. (2), (4)

$$|3\sqrt{2}\hat{i} - 3\hat{j} - 3\hat{k}| = \sqrt{(3\sqrt{2})^2 + (3)^2 + (3)^2} = 6$$

Angle between normal x -axis and incidence ray is

$$(3\sqrt{2}\hat{i} - 3\hat{j} - 3\hat{k}) \cdot (\hat{i}) = 6 \cdot 1 \cdot \cos \theta$$

$$\Rightarrow \frac{3\sqrt{2}}{6} = \cos \theta \text{ or } \cos \theta = \frac{1}{\sqrt{2}}$$



Applying Snell's law at P , $\sin \theta = \mu \sin r$

$$1 \cdot \frac{1}{2} = \sqrt{2} \sin r \Rightarrow \sin r = \frac{1}{2} \Rightarrow r = 30^\circ$$

Let the angle made by refracted ray with y -axis and z -axis is α .

Then, $\cos^2 \alpha + \cos^2 \alpha + \cos^2 r = 1$

$$2 \cos^2 \alpha = 1 - \frac{3}{4} \Rightarrow \cos \alpha = \frac{1}{\sqrt{8}}$$

$$\hat{u}_{\text{refracted}} = \cos r \hat{i} - \cos \alpha \hat{j} - \cos \alpha \hat{k}$$

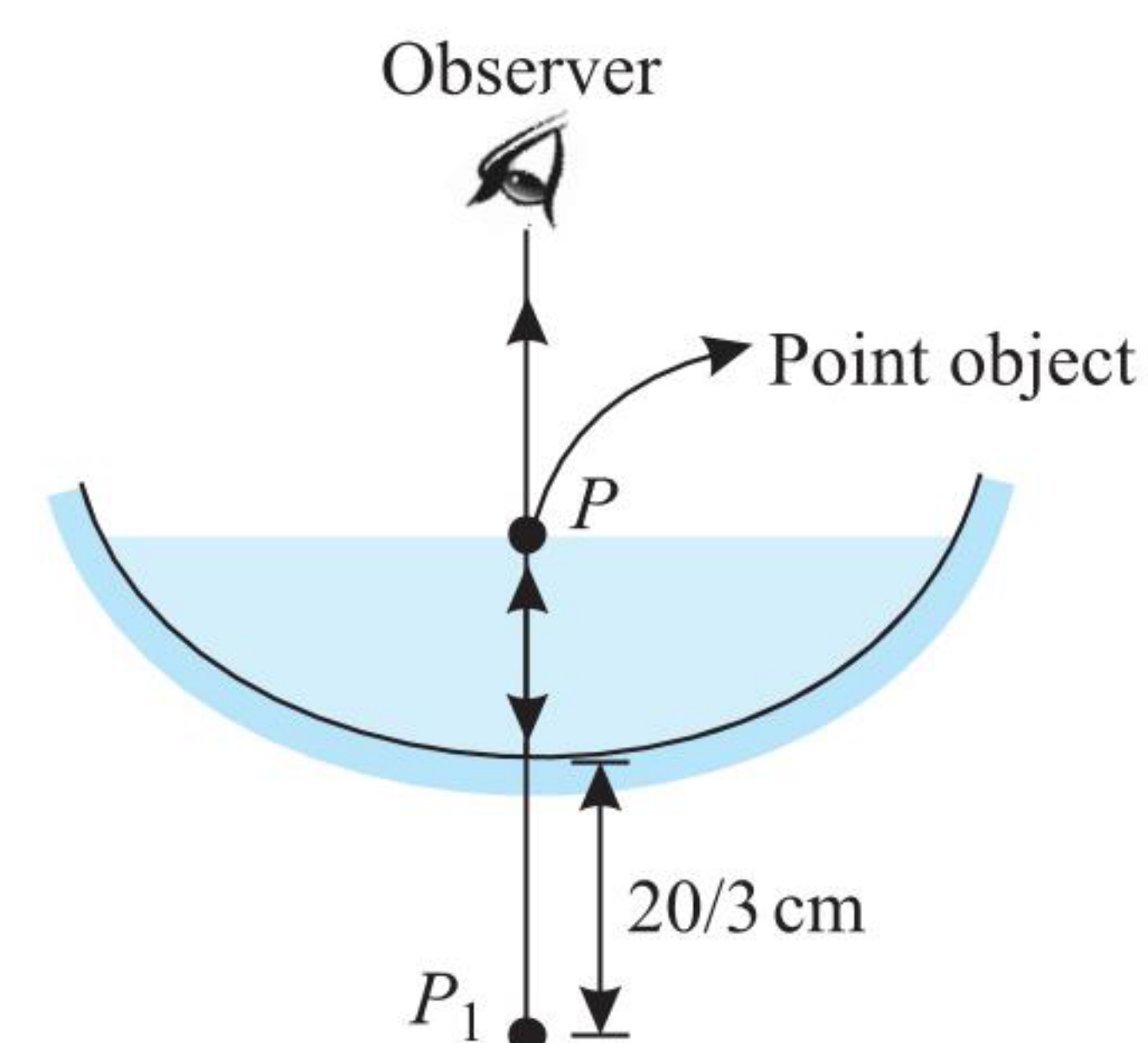
$$\hat{u}_{\text{refracted}} = \cos 30^\circ \hat{i} - \frac{1}{\sqrt{8}} \hat{j} - \frac{1}{\sqrt{8}} \hat{k} = \frac{\sqrt{3}}{2} \hat{i} - \frac{1}{2\sqrt{3}} \hat{j} - \frac{1}{2\sqrt{2}} \hat{k}$$

So unit vector along refracted ray, $\hat{u}_{\text{refracted}} = \frac{\sqrt{6}\hat{i} - \hat{j} - \hat{k}}{\sqrt{8}}$

31. (1), (3)

Image formation due to reflection from concave mirror

Here, $u = -5 \text{ cm}$, $f = -20 \text{ cm}$



Using mirror formula, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$$\text{or } \frac{1}{5} - \frac{1}{20} = \frac{1}{v} \Rightarrow v = \frac{20}{3} \text{ cm}$$

$\therefore v = \frac{20}{3} \text{ cm}$ i.e., behind the mirror.

The image P_1 formed due to reflection through concave mirror behaves as object for refraction through water surface. For water surface,

$$\mu_1 = \frac{4}{3}, \mu_2 = 1, u = -\left(5 + \frac{20}{3}\right) = -\frac{35}{3} \text{ cm}$$

$$\therefore \frac{\mu_2}{\mu_1} = \frac{v}{u}$$

$$\therefore v = \frac{\mu_2}{\mu_1} u = \left(\frac{1}{\frac{4}{3}}\right) \left(-\frac{35}{3}\right)$$

$$\therefore v = \frac{3}{4} \times \left(-\frac{35}{3}\right) = -\frac{35}{4} = -8.75 \text{ cm}$$

32. (1), (3)

The object shift due to slab is $x = t \left(1 - \frac{1}{\mu}\right) = 12 \left(1 - \frac{3}{4}\right) = 3 \text{ cm}$

This shift should be in the direction of ray travelling.

The image formed due to refraction through slab behaves as object for concave mirror.

Now image formation by concave mirror, $u = -60 \text{ cm}$, $f = -20 \text{ cm}$

According to mirror formula,

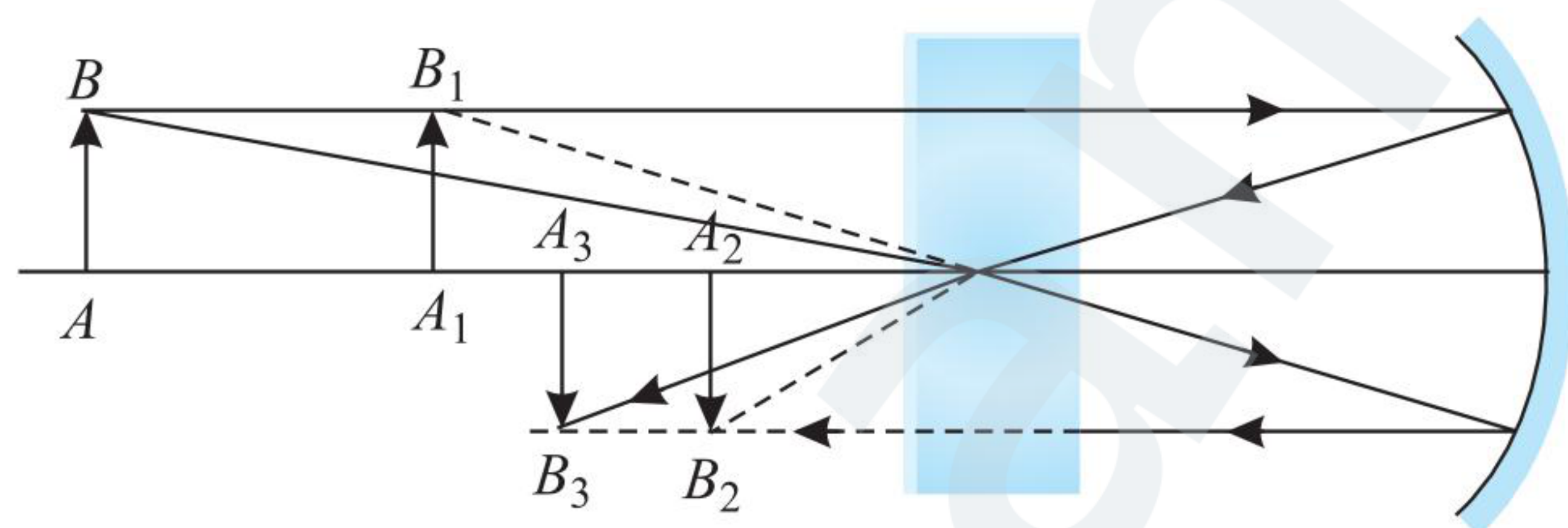
$$\therefore \frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\therefore v = -30 \text{ cm}$$

$$m = \frac{I}{O} = -\frac{v}{u} = -\frac{(-30)}{-60} = -\frac{1}{2}$$

$$\therefore I = -1 \text{ cm (i.e., inverted)}$$

This image (I) behaves as virtual object for second times refraction through slab. As shifting should be in the direction of ray travelling. Hence, the final image I' is 3 cm leftward from I due to second times refraction through slab. Thus, final image is virtual, 1 cm high inverted and 33 cm from the concave mirror.



33. (1), (3)

(1) From figure at D , $\theta + 30^\circ = 90^\circ \Rightarrow \theta = 60^\circ$

$$\therefore \theta = 60^\circ$$

Hence angle $\angle CDE = 90^\circ - \theta = 90^\circ - 60^\circ = 30^\circ$

$$\therefore \angle CED = 180^\circ - (30^\circ + 30^\circ)$$

$$\therefore \angle CED = 120^\circ$$

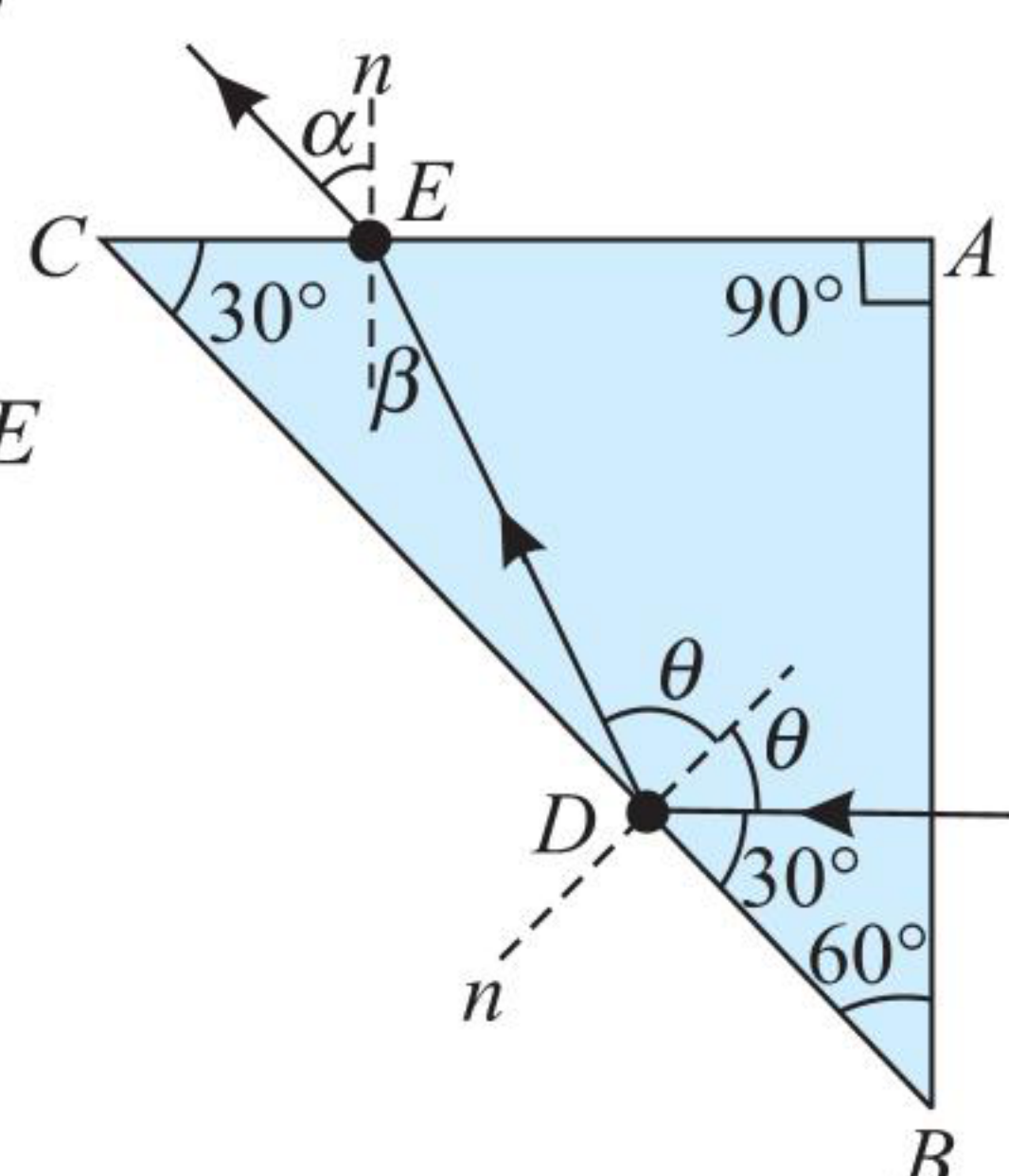
$$\therefore \beta = 120^\circ - 90^\circ = 30^\circ$$

According to Snell's law at part E

$$\frac{5}{3} \sin \beta = \frac{4}{3} \times \sin \alpha$$

$$\therefore \sin \alpha = \frac{5}{4} \times \frac{1}{2} = \frac{5}{8}$$

$$(3) \text{ Here, } \sin C = \frac{5}{2\sqrt{3} \times \frac{5}{3}} = \frac{\sqrt{3}}{2}$$



$$\therefore C = 60^\circ$$

$$\text{But } \theta = 60^\circ$$

Hence, total internal reflection does not take place.

34. (3), (4)

For total internal reflection to take place:

Angle of incidence, $i >$ critical angle, θ_c [where $\sin \theta_c = \frac{1}{n}$]

$$\text{or } \sin 45^\circ > \frac{1}{n} \text{ or } \frac{1}{\sqrt{2}} > \frac{1}{n} \text{ or } n > \sqrt{2} \text{ or } n > 1.414$$

Therefore, possible values of n can be 1.5 or 1.6 in the given options.

Linked Comprehension Type

For Problems 1–2

1. (2), 2. (1)

The normal shift produced by a glass slab is

$$\Delta x = \left(1 - \frac{1}{\mu}\right)t = \left(1 - \frac{2}{3}\right)(6) = 2 \text{ cm}$$

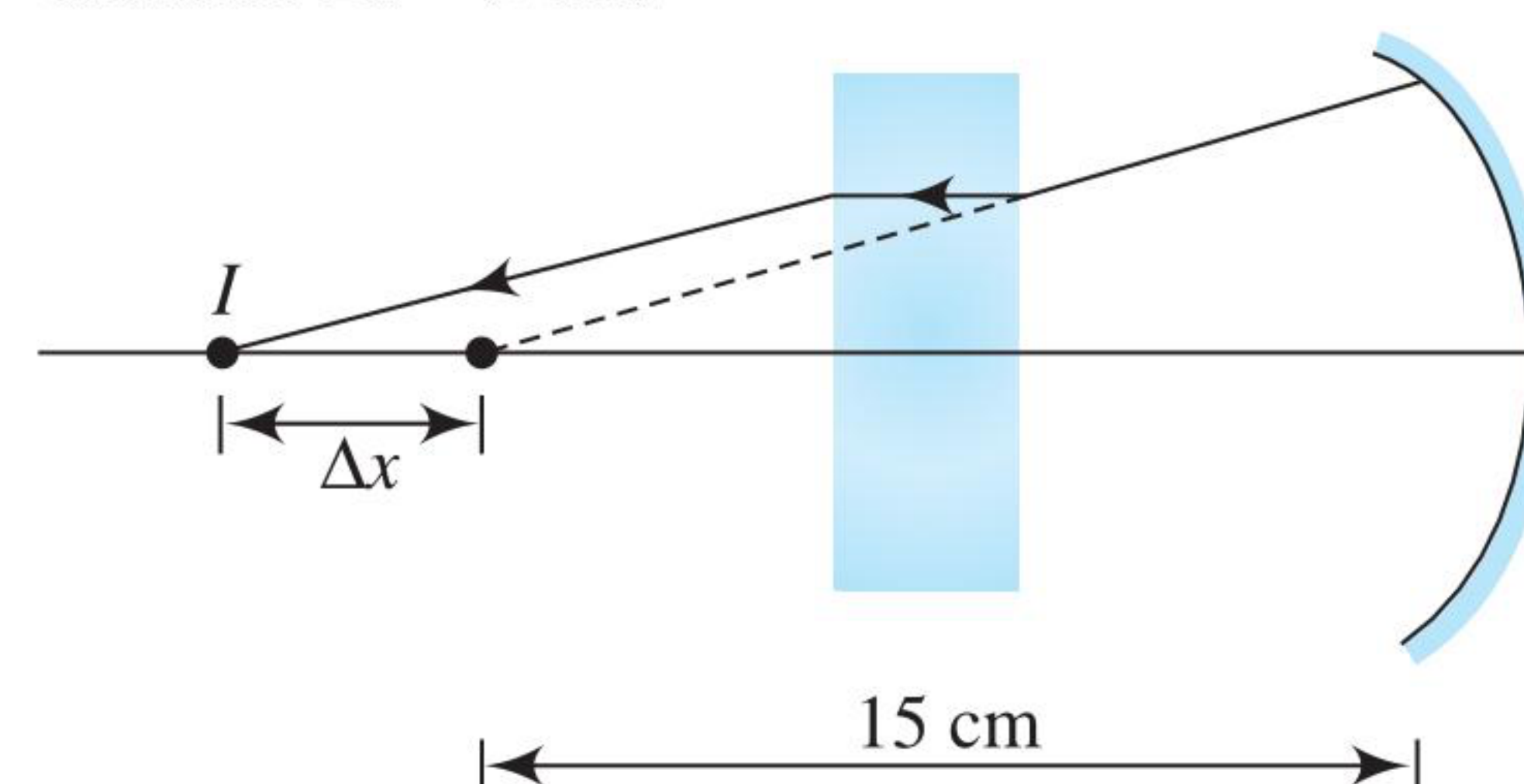
i.e., for the mirror, the object is placed at a distance $(32 - \Delta x) = 30 \text{ cm}$ from it.

Applying mirror formula,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \quad \text{or} \quad \frac{1}{v} + \frac{1}{30} = -\frac{1}{10}$$

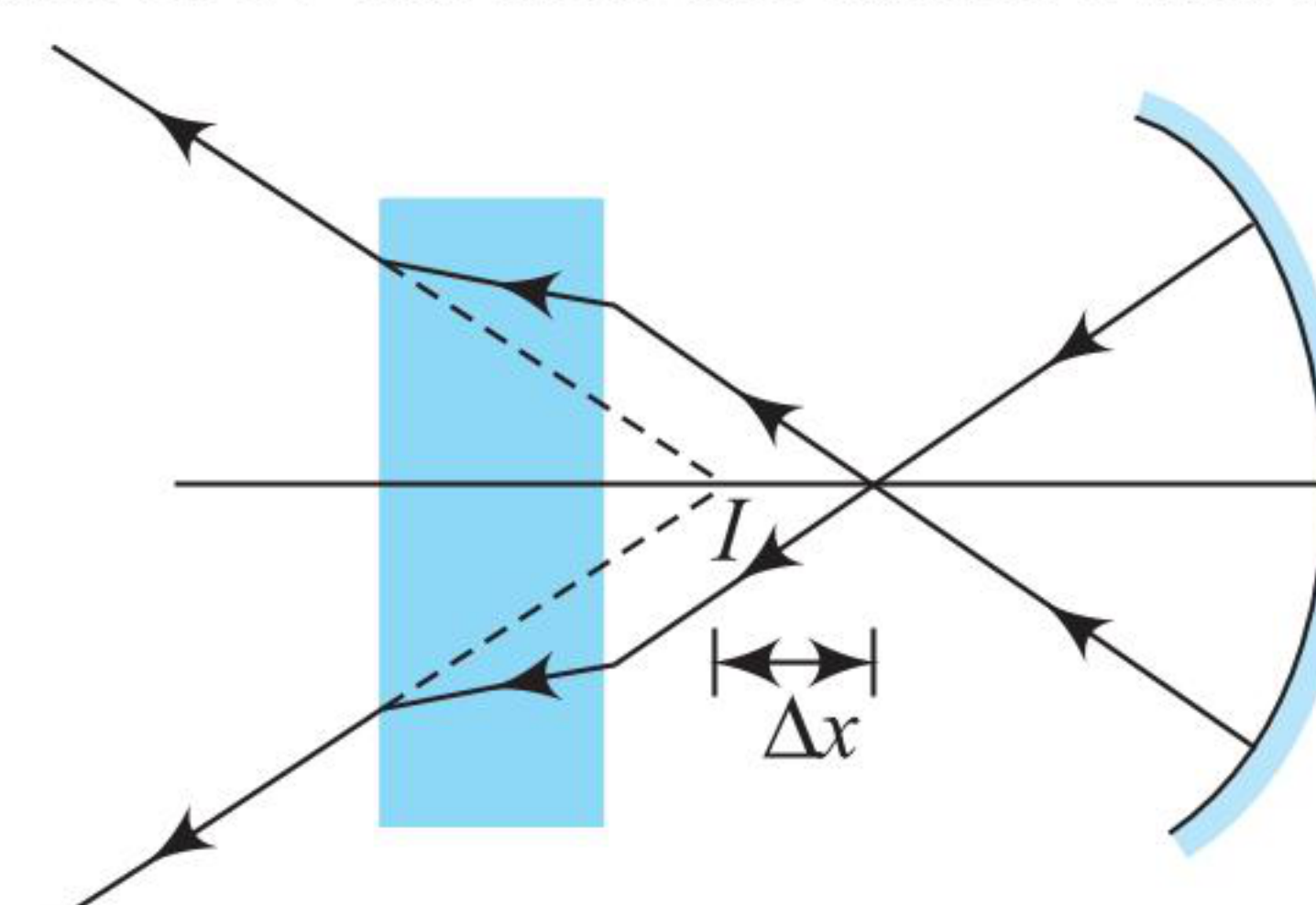
$$\text{or, } v = -15 \text{ cm}$$

(i) When $x = 5 \text{ cm}$: The light falls on the slab on its return journey as shown. But the slab will again shift it by a distance $\Delta x = 2 \text{ cm}$.



Hence, the final real image is formed at a distance $(15 + 2) = 17 \text{ cm}$ from the mirror.

(ii) When $x = 20 \text{ cm}$: This time also the final image is at a distance of 17 cm from the mirror but it is virtual as shown.



For Problems 3–5

$$3. (4) \quad \sin C = \frac{\mu}{\mu_0} \Rightarrow \sin[90^\circ - r] = \frac{\mu}{\mu_0} \quad [\because C + r = 90^\circ]$$

$$\Rightarrow \cos r = \frac{\mu}{\mu_0} \quad \dots(i)$$

$$\mu_0 = \frac{\sin i}{\sin r} \Rightarrow \sin r = \frac{\sin i}{\mu_0} \quad \dots(ii)$$

$$\text{From (i) and (ii)} \left(\frac{\mu}{\mu_0} \right)^2 + \left(\frac{\sin i}{\mu_0} \right)^2 = 1$$

$$\Rightarrow \mu = \sqrt{\mu_0^2 - \sin^2 i}$$

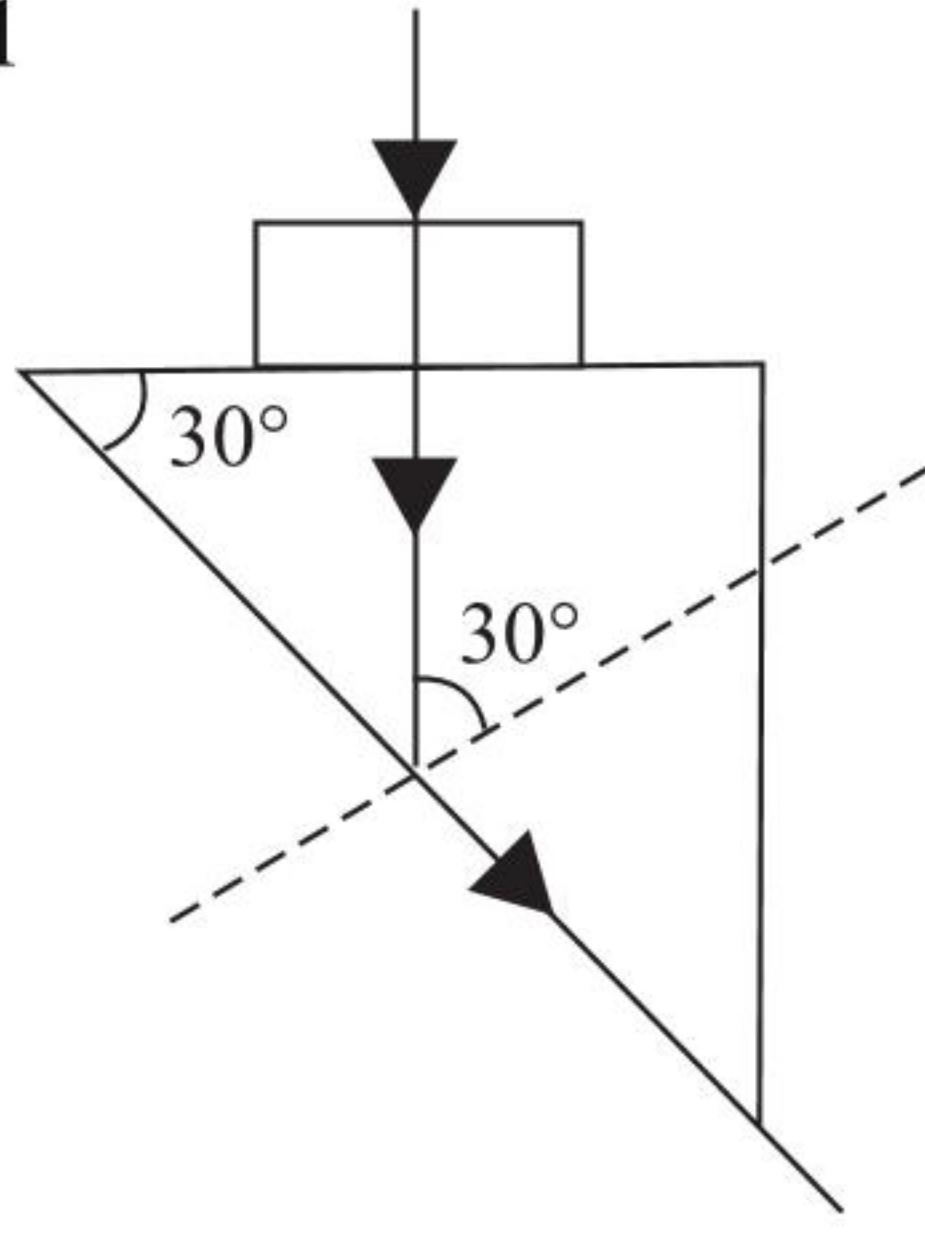
4. (3) If ray just fails to emerge, $i = 90^\circ$,
given $\mu_0 = \sqrt{2}$

$$\mu = \sqrt{(\sqrt{2})^2 - \sin^2 90^\circ} = 1$$

5. (3) Here $30^\circ > C$

$$\Rightarrow \sin 30^\circ > \sin C$$

$$\Rightarrow \frac{1}{2} > \frac{1}{\mu_0} \Rightarrow \mu_0 > 2$$



For Problems 6–8

6. (1) $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$$\Rightarrow \frac{1}{v} + \frac{1}{-30} = \frac{1}{-20} \Rightarrow v = -60 \text{ cm}$$

7. (4) Let $u = -x$, $v = -y$

$$\frac{dx}{dt} = -2 \text{ cm s}^{-1}; \frac{1}{-y} + \frac{1}{-x} = \frac{1}{f}$$

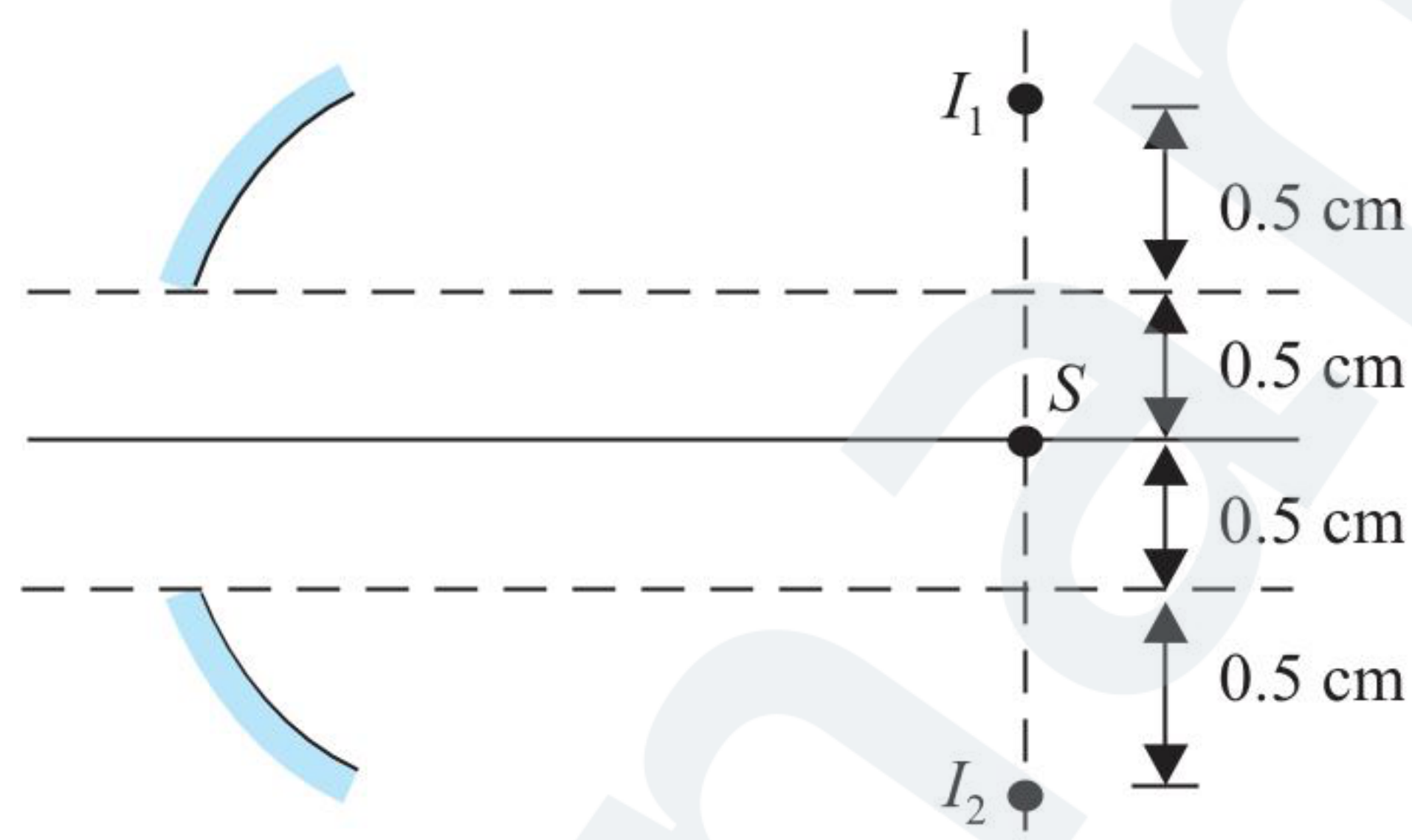
$$\Rightarrow \frac{1}{y^2} \frac{dy}{dt} + \frac{1}{x^2} \frac{dx}{dt} = 0$$

$$\Rightarrow \frac{dy}{dt} = -\left(\frac{y}{x}\right)^2 \frac{dx}{dt} \Rightarrow v_I = -\left(\frac{60}{30}\right)^2 (-2) = 8 \text{ cm s}^{-1}$$

8. (1) $m = -\frac{v}{u} = -\frac{-60}{-3} = -2$; so image will move twice distance in same time as compared to object. Hence, velocity of image will be double.

For Problems 9–11

9. (4) Using $\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \Rightarrow v = -50 \text{ cm}$



$$\text{and magnification} = \frac{-v}{u} = -1$$

So distance between two images = 2 cm.

10. (2) For half the mirror, due to reflection being half of the incoming rays, so the intensity of the image is obviously half. More the lateral separation, more will be the gap between the two images formed.
11. (4) Both the mirrors will be having the common object at their centre of curvature. Hence the image position also coincides.

For Problems 12–13

12. (3) 13. (4)

- (i) Given: $u = -30 \text{ cm}$, $f = 20 \text{ cm}$

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow v = 12 \text{ cm}$$

$$\text{Also, } \frac{dv}{dt} = -\left(\frac{v^2}{u^2}\right) \frac{du}{dt} = -0.16 \text{ cm s}^{-1}$$

- (ii) $H = -\frac{v}{u}h$, H is the height of image and h is height of object.

$$\frac{dH}{dt} = \frac{u \frac{dv}{dt} - v \frac{du}{dt}}{u^2} h = 0.008 \text{ cm s}^{-1}$$

For Problems 14–16

14. (3) For extreme position A, $u = -60 \text{ cm}$, $f = -20 \text{ cm}$

$$\therefore \frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\text{or } \frac{1}{v} - \frac{1}{60} = -\frac{1}{20}$$

$$\text{or } v = -30 \text{ cm}$$

For extreme position B,

$$u' = -50 \text{ cm}$$

$$f' = -20 \text{ cm}$$

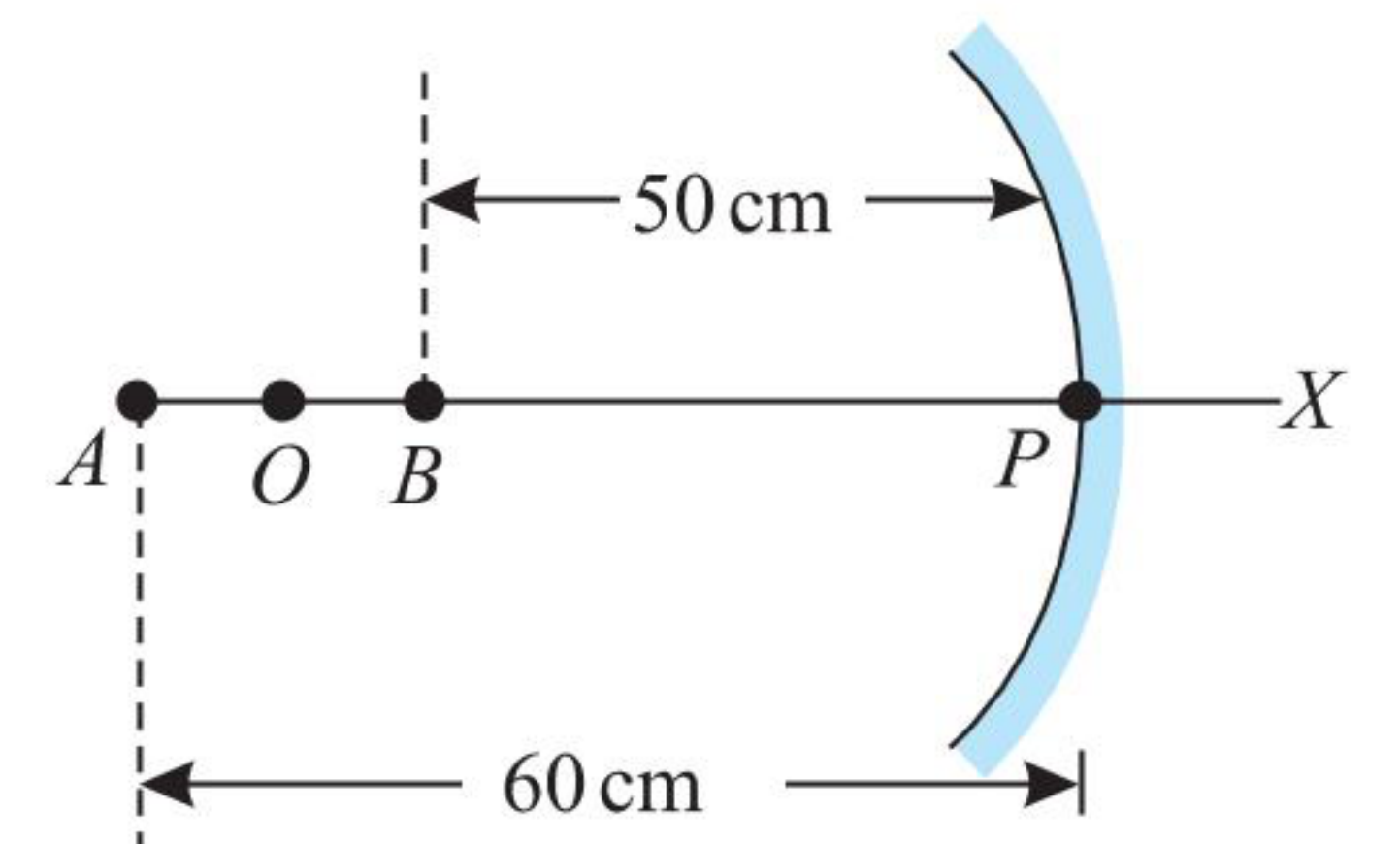
$$\therefore \frac{1}{v'} - \frac{1}{50} = -\frac{1}{20}$$

$$\text{or } \frac{1}{v'} = \frac{1}{50} - \frac{1}{20} = \frac{2-5}{100}$$

$$\therefore v' = -\frac{100}{3} \text{ cm} = -33.33 \text{ cm}$$

$\therefore$ The range of vibration of image is

$$A'B' = |v' - v| = (33.33 - 30) \text{ cm} = 3.33 \text{ cm}$$



15. (4) The maximum distance of image from mirror is at that time, when object is at extreme position B.

16. (2) The time period vibration of image = time period of vibration of object = $\frac{2\pi}{\omega} = \frac{2\pi}{10\pi} = 0.2 \text{ s}$

Matrix Match Type

1. i. $\rightarrow$ a.; ii. $\rightarrow$ c.; iii. $\rightarrow$ b.; iv. $\rightarrow$ d.

$$\mu_{\text{blue}} > \mu_{\text{green}} > \mu_{\text{yellow}} > \mu_{\text{red}}$$

2. i. $\rightarrow$ b.; ii. $\rightarrow$ c., d.; iii. $\rightarrow$ a.; iv. $\rightarrow$ c., d.

Here critical angle C;

$$\sin C = \frac{1}{\mu} = \frac{1}{2}$$

$$\Rightarrow C = 30^\circ$$

$$\text{Let } r_1 = C = 30^\circ,$$

$$r_1 = A - r_2 = A - 30^\circ$$

$$\mu = \frac{\sin i}{\sin r_1}$$

$$\Rightarrow \sin i = \mu \sin r_1 = 2 \sin(A - 30^\circ)$$

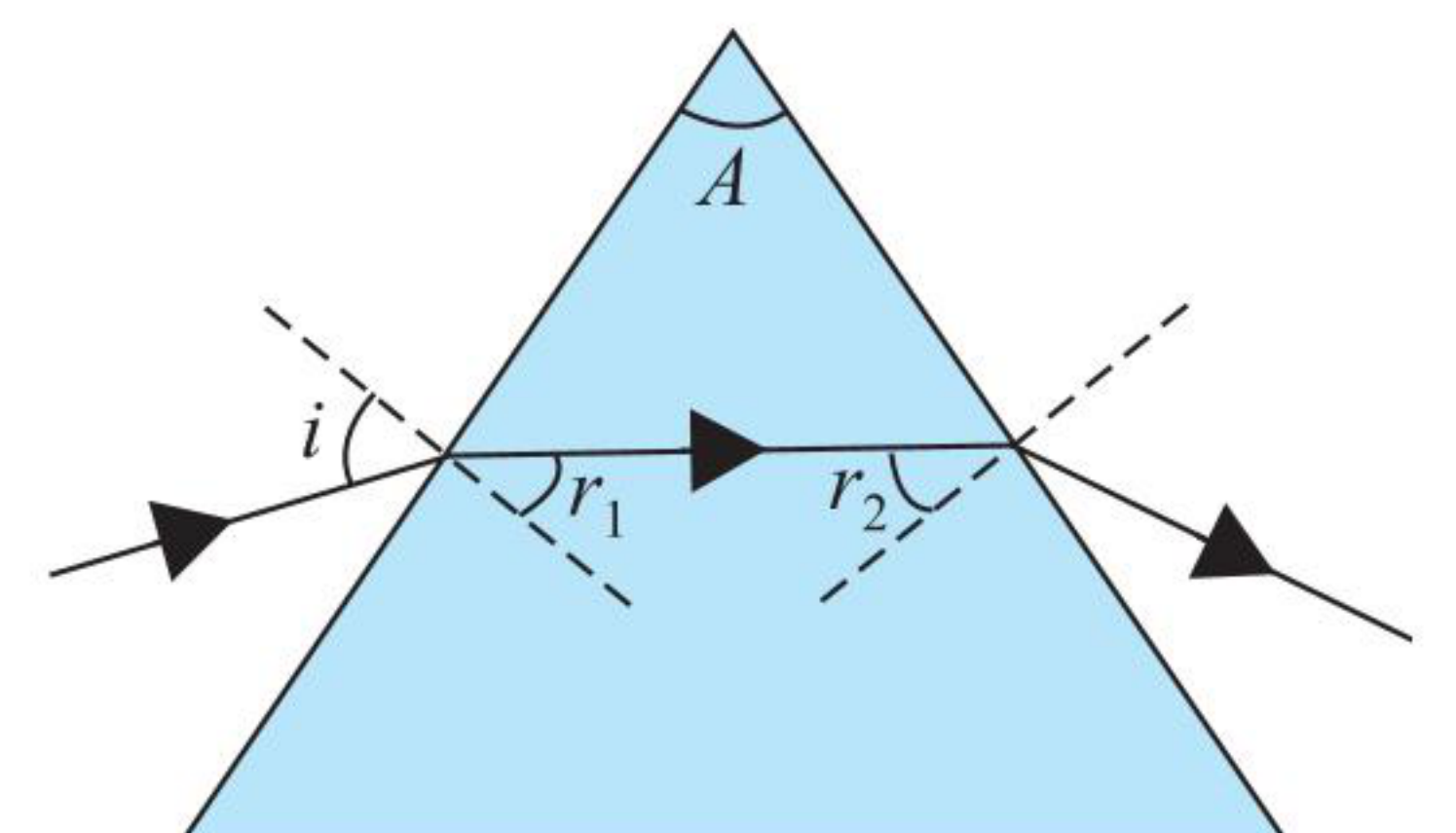
for $i = 90^\circ$, we get $A = 60^\circ$

So for $A = 60^\circ$: for $i = 90^\circ$, we have

$$r_2 = C = 30^\circ$$

But if $i < 90^\circ$, $r_2 > C$

Hence all the rays are reflected from second surface.



...(i)

For $A > 60^\circ$: for $i = 90^\circ$, we will get $r_2 = C$

and also for $i < 90^\circ$, we will get $r_2 > C$.

So, all rays will be reflected from second surface.

Similarly for $i = 0$, we get $A = 30^\circ$ and $r_2 = 30^\circ = C$

So, $r_2 < C$. Hence all the rays are refracted from second surface.

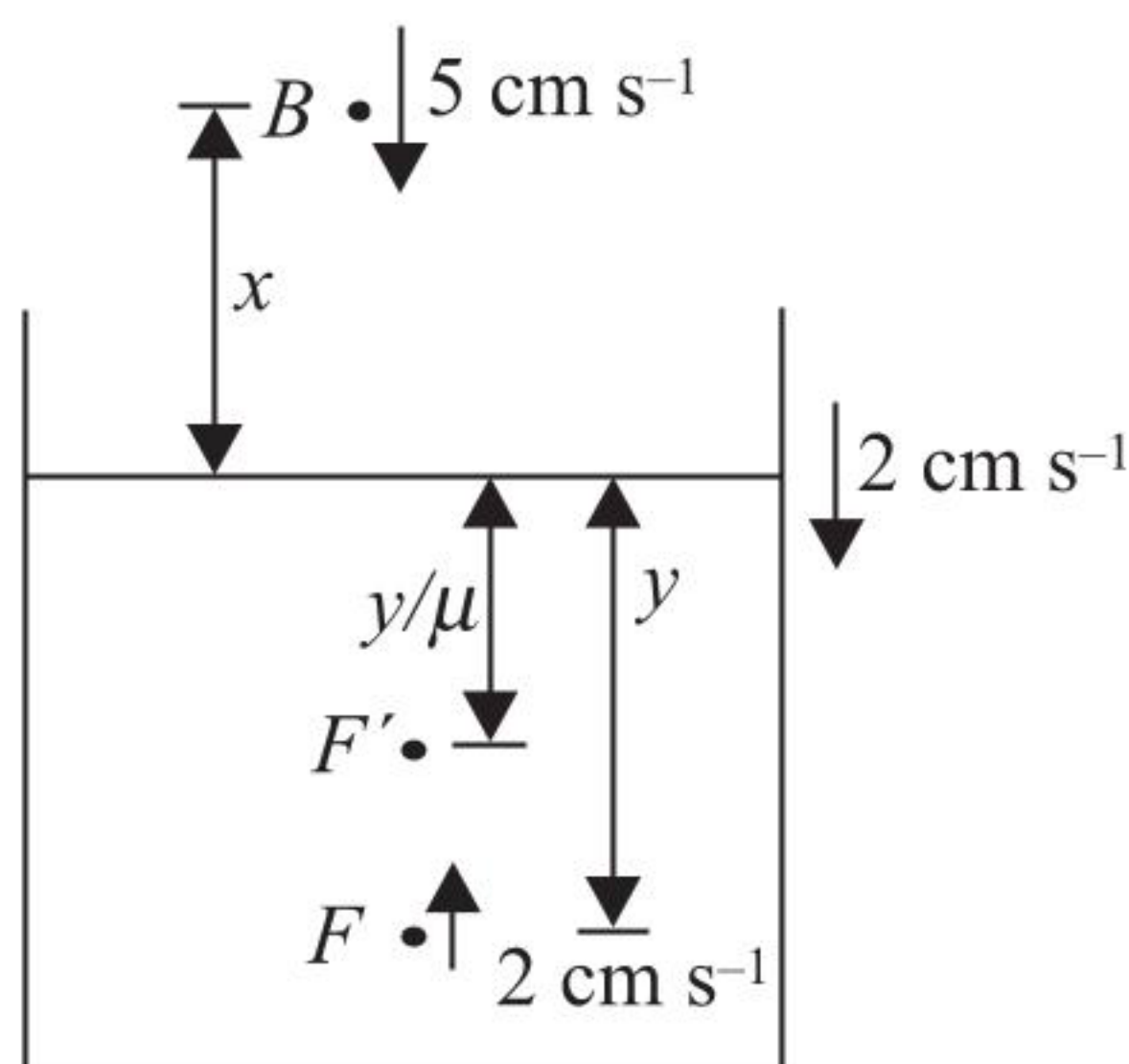
For $30^\circ < A < 60^\circ$, some rays will be reflected and some refracted.

3. i. $\rightarrow$ b.; ii. $\rightarrow$ c.; iii. $\rightarrow$ a.; iv. $\rightarrow$ d.

i. We can write

$$\frac{dx}{dt} = -(5 - 2) = -3 \text{ cm/s}$$

$$\frac{dy}{dt} = -(2 + 2) = -4 \text{ cm/s}$$

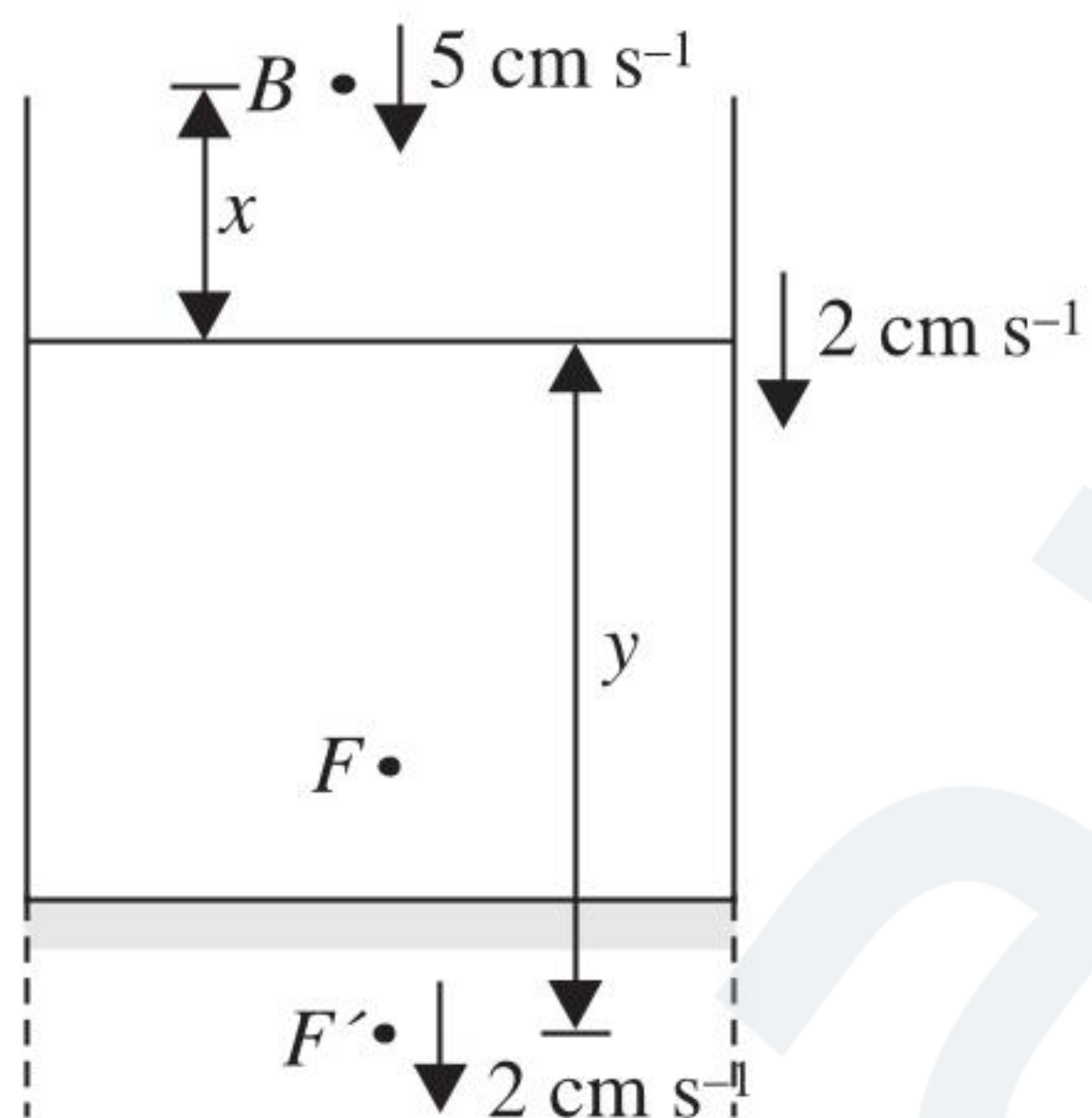


Speed of fish w.r.t. bird

$$= \frac{d}{dt} \left[x + \frac{y}{\mu} \right] = \frac{dx}{dt} + \frac{1}{\mu} \frac{dy}{dt} = -3 + \frac{1}{4/3}(-4) = -6 \text{ cm/s}$$

ii. Here $\frac{dy}{dt} = 0$, so speed of image of fish (F') w.r.t. bird will be

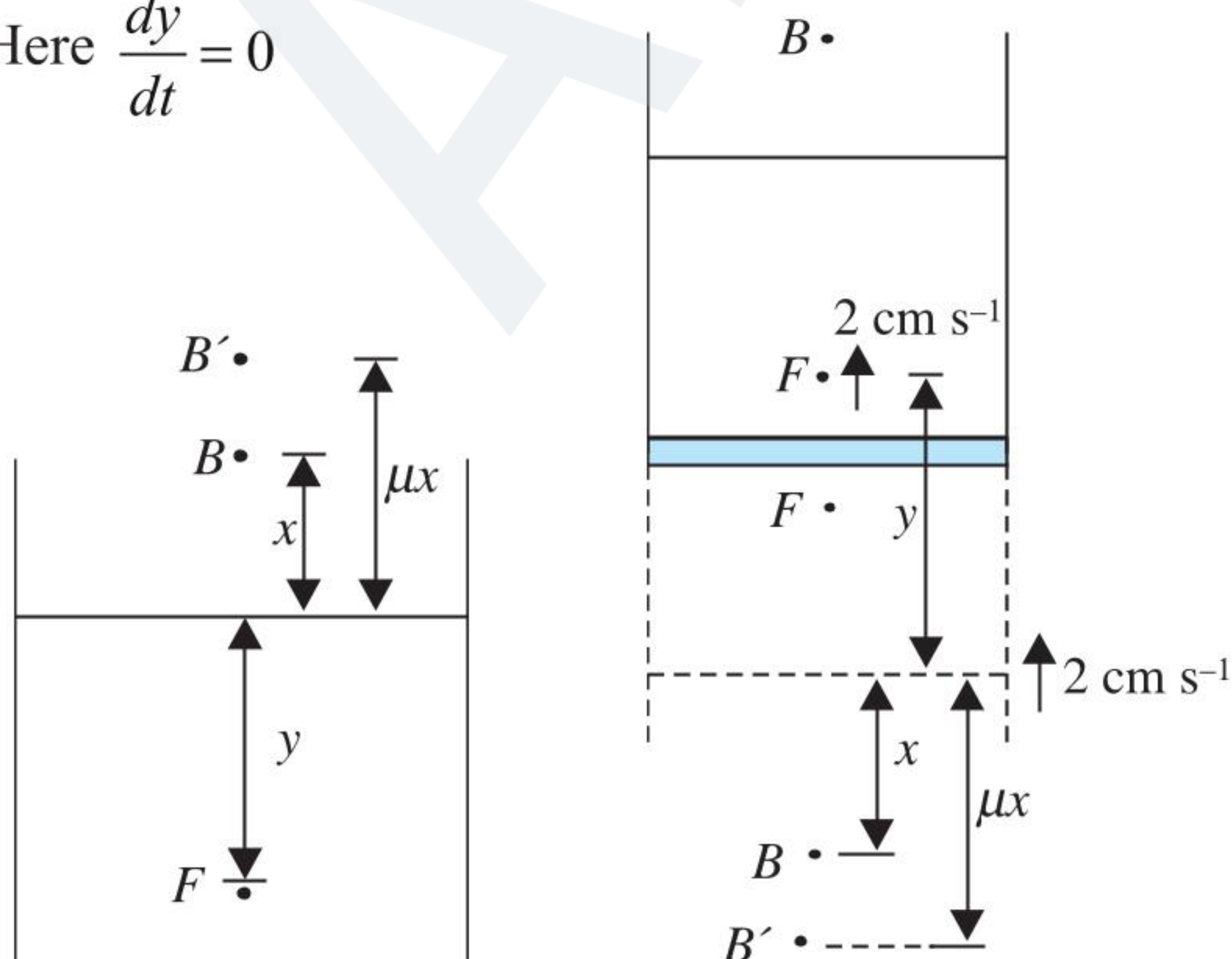
$$\frac{dx}{dt} = -3 \text{ m s}^{-1}$$



iii. Speed of bird relative to fish $= \frac{d}{dt}(\mu x + y)$

$$= \mu \frac{dx}{dt} + \frac{dy}{dt} = \frac{4}{3}(-3) - 4 = -8 \text{ cm s}^{-1}$$

iv. Here $\frac{dy}{dt} = 0$



$$\frac{d}{dt}(\mu x + y) = \mu \frac{dx}{dt} + \frac{dy}{dt} = \frac{4}{3}(-3) + 0 = -4 \text{ cm s}^{-1}$$

4. (1) Virtual object means initial rays converge behind the mirror for which u is positive. We have mirror formula as $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

For concave mirror, f is negative. So, v is always negative. Therefore, image is always real.

$$\frac{1}{v} = \frac{1}{(-f)} - \frac{1}{(+u)}$$

For convex mirror, f is positive. So, v may be positive or negative.

$$\frac{1}{v} = \frac{1}{(+f)} - \frac{1}{(+u)}$$

Hence, image may be virtual or real.

For plane mirror, $f = \infty$. So, v is always negative.

Hence, image is always real.

$$\frac{1}{v} = \frac{1}{(\infty)} - \frac{1}{(+u)}$$

5. (1) (i) and (ii)

$$v = \frac{\mu_2}{\mu_1} u$$

$$\Rightarrow |v| < |u| \text{ as } \mu_1 > \mu_2$$

i.e. v and u are of same sign.

Or they are on same side of plane surface.

From plane surface, if object is real, image is virtual and vice-versa.

(iii) and (iv)

$$v = \frac{\mu_1}{\mu_2} u \Rightarrow |v| > |u|$$

Other explanations are same.

6. (2)

(i) $f = -15 \text{ cm}, u = -30 \text{ cm}, V = -30 \text{ cm}$

$$\frac{du}{dt} = 0 \Rightarrow \frac{dv}{dt} = -\frac{v^2}{u^2} \frac{du}{dt} = 0$$

Velocity of image with respect to mirror = 0

Velocity of image with respect to mirror = 2 cm/s

(ii) $f = -15 \text{ cm}, \Rightarrow u = -30 \text{ cm}, v = -30 \text{ cm}$

$$\frac{du}{dt} = -4 \Rightarrow \frac{dv}{dt} = -\frac{v^2}{u^2} \frac{du}{dt} = +4$$

Velocity of image with respect to mirror = +4 cm/s

Velocity of image with respect to ground = +6 cm/s

(iii) $f = +20 \text{ cm}, u = -20 \text{ cm}, v = +10 \text{ cm}$

$$\frac{du}{dt} = +4 \Rightarrow \frac{dv}{dt} = -\frac{v^2}{u^2} \frac{du}{dt} = -1 \text{ cm/s}$$

Velocity of image with respect to mirror = -1 cm/s

Velocity of image with respect to ground = -1 cm/s

(iv) $f = +20 \text{ cm}, u = -20 \text{ cm}, v = +10 \text{ cm}$

$$\frac{du}{dt} = +4 \Rightarrow \frac{dv}{dt} = -1 \text{ cm/s}$$

Velocity of image w.r.t. mirror = -1 cm/s

Velocity of image w.r.t. ground = -1 cm/s

7. (2) The magnification in case of a mirror, $m = \frac{f}{f-u}$

(i) Convex mirror $u = -ve$, $f = +ve$

$$m = \frac{f}{f-(f)} = \frac{1}{2}$$

(ii) Concave $\rightarrow m = \frac{-f}{-f-(-f)} = \infty$

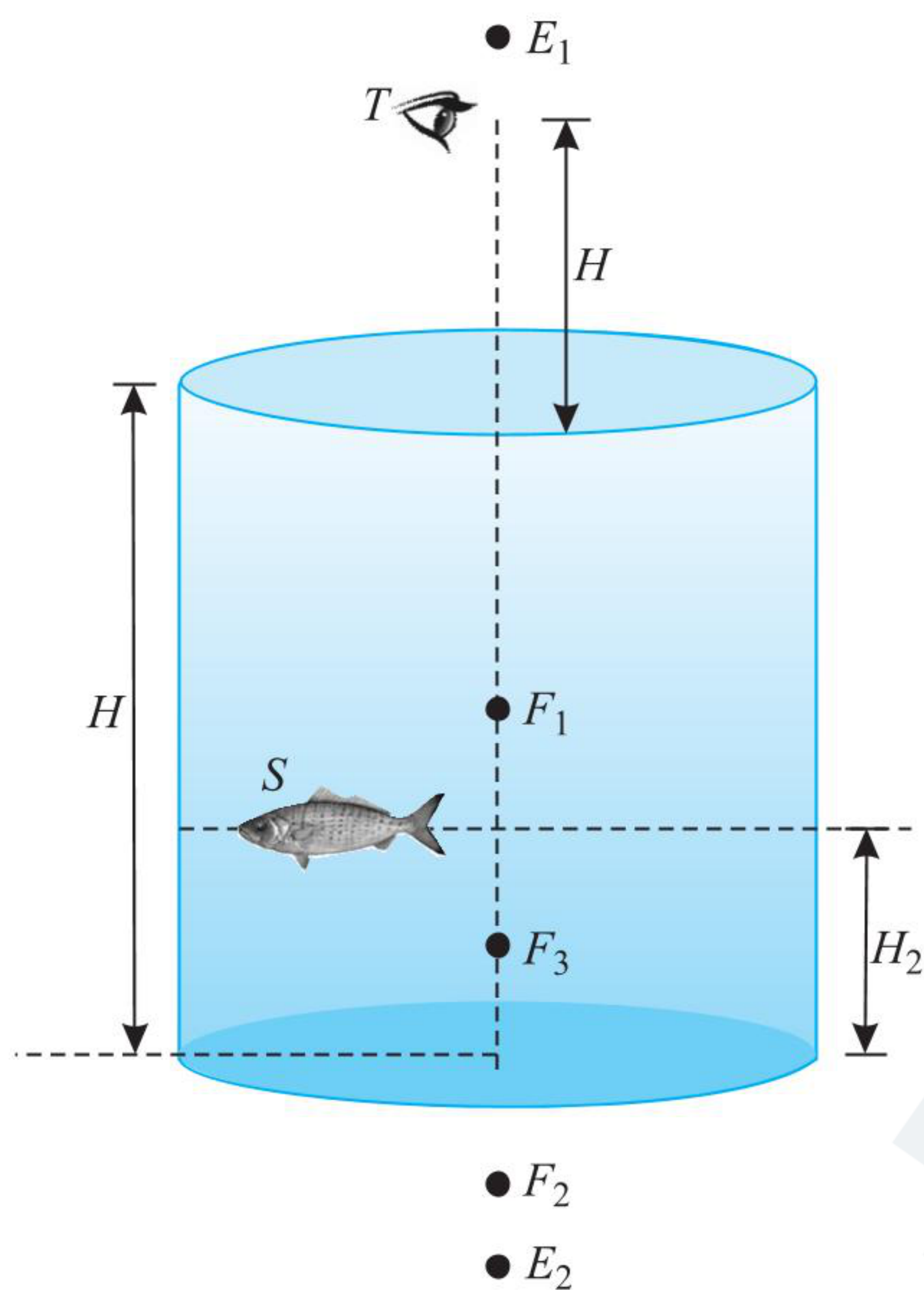
(iii) Concave mirror $u = -R$ ($2f$)

$$m = \frac{-f}{-f-(-2f)} = \frac{-f}{f} = -1$$

(iv) Convex mirror $u = -2f$

$$m = \frac{f}{f-(-2f)} = \frac{f}{3f} = \frac{1}{3}$$

8. (4)



Fish observing the image of eye:

$$\text{For } d_1: E_1 \text{ at } \left(\frac{H}{2} + \mu H\right) = H\left(\mu + \frac{1}{2}\right)$$

$$\text{For } d_2: E_2 \text{ at } \left(\mu H + H + \frac{H}{2}\right) = H\left(\mu + \frac{3}{2}\right)$$

Eye observing the image of fish:

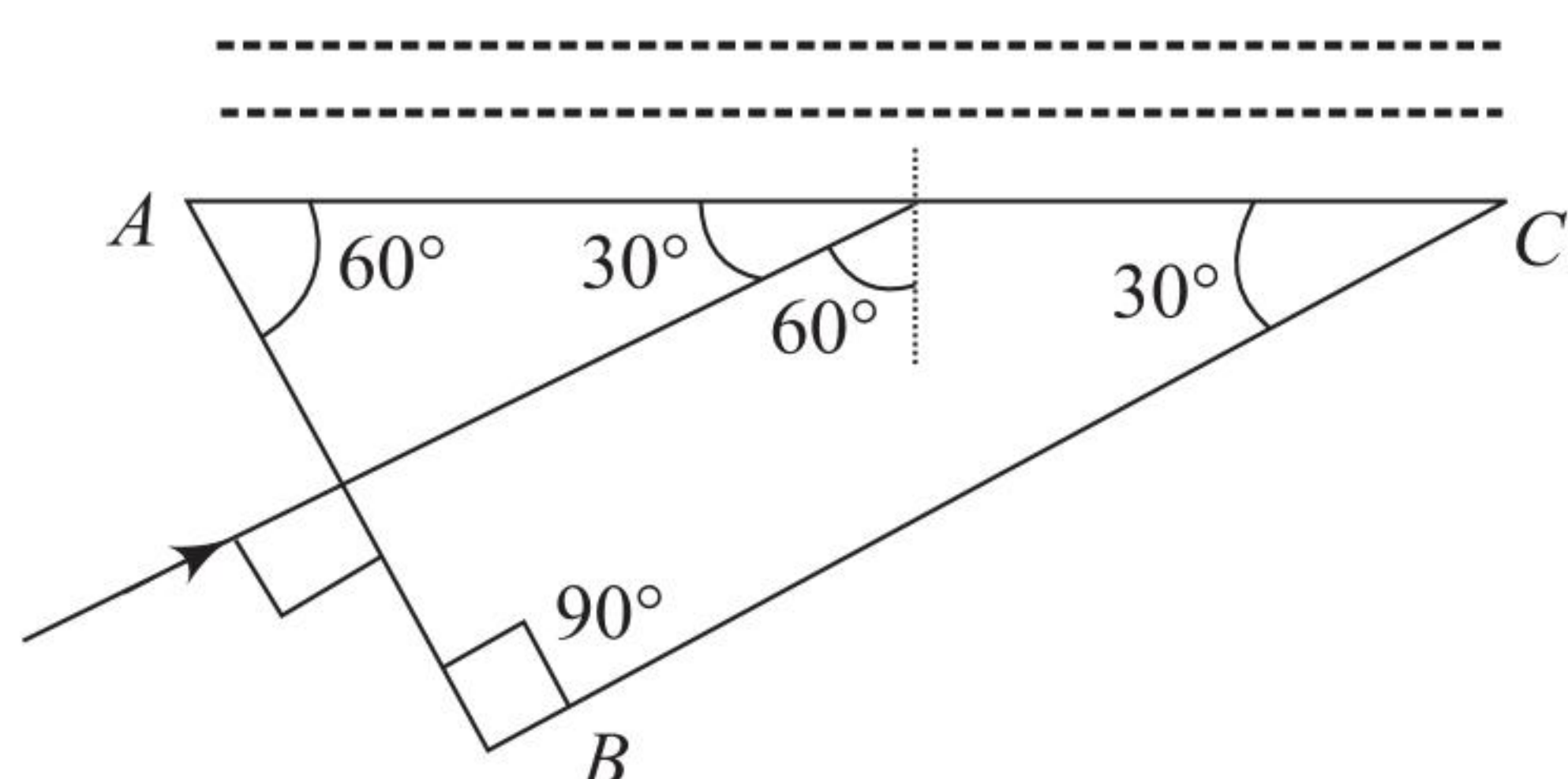
$$\text{For } d_3: F_1 \text{ at } \left(\frac{H}{2\mu} + H\right) = H\left(\frac{1}{2\mu} + 1\right)$$

$$\text{For } d_4: F_2 \text{ at } \left(\frac{3H}{2\mu} + H\right) = H\left(1 + \frac{3}{2\mu}\right)$$

Numerical Value Type

1. (3) Critical angle between glass and liquid.

$$\sin \theta_c = \frac{\mu}{(3/2)} = \frac{2\mu}{3}$$



Angle of incidence on AC = 60°

For TIR, $i > \theta_c \Rightarrow \frac{2}{3}\mu$

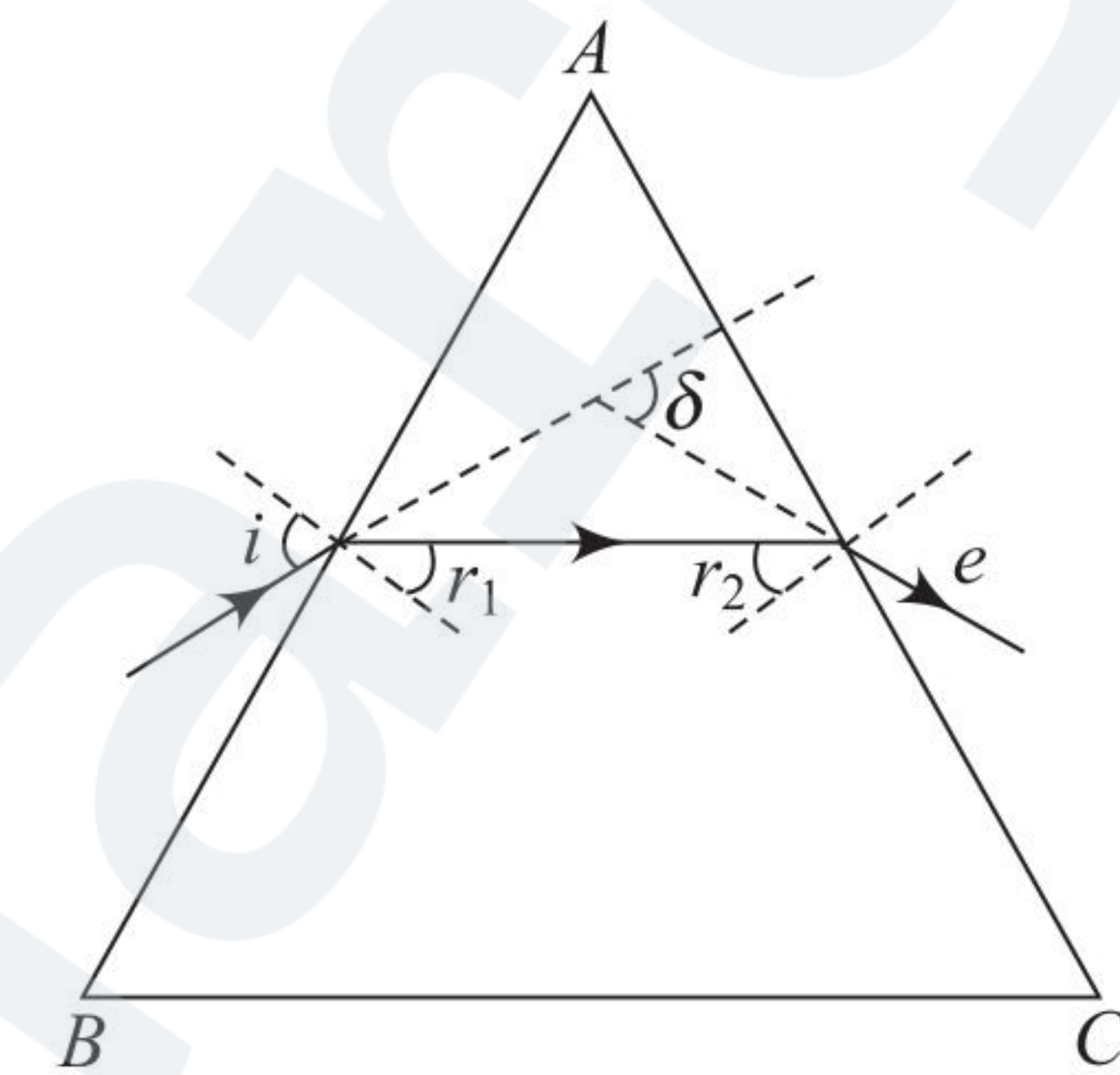
$$\mu < \frac{3\sqrt{3}}{4} = \frac{I\sqrt{3}}{4} \quad (\text{given}) \quad \text{So, } I = 3$$

2. (3) Given $i = 60^\circ$, $\delta = 30^\circ$ and $A = 30^\circ$.

We have $\delta = i + e - A$

From Eq. (i), we get $30^\circ = 60^\circ + e - 30^\circ$ or $e = 0$

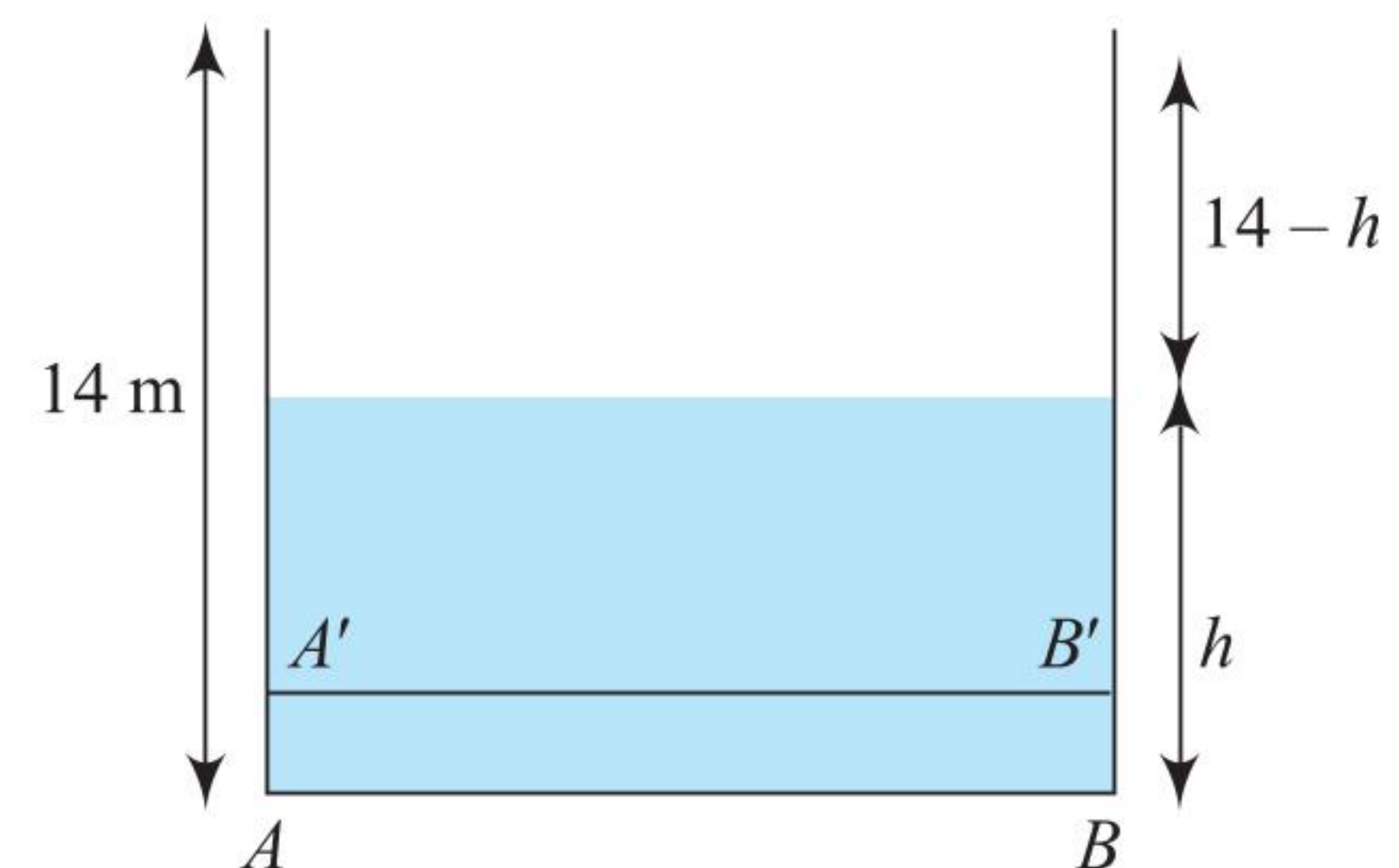
So r_2 is also zero, then $r_1 = A = 30^\circ$



$$\text{So } \mu = \frac{\sin i}{\sin r_1} = \frac{\sin 60^\circ}{\sin 30^\circ} = \sqrt{3}$$

Hence the value of $a = 3$.

3. (8) $A'B'$ is the apparent position of bottom of container, at a distance h/μ from water surface.

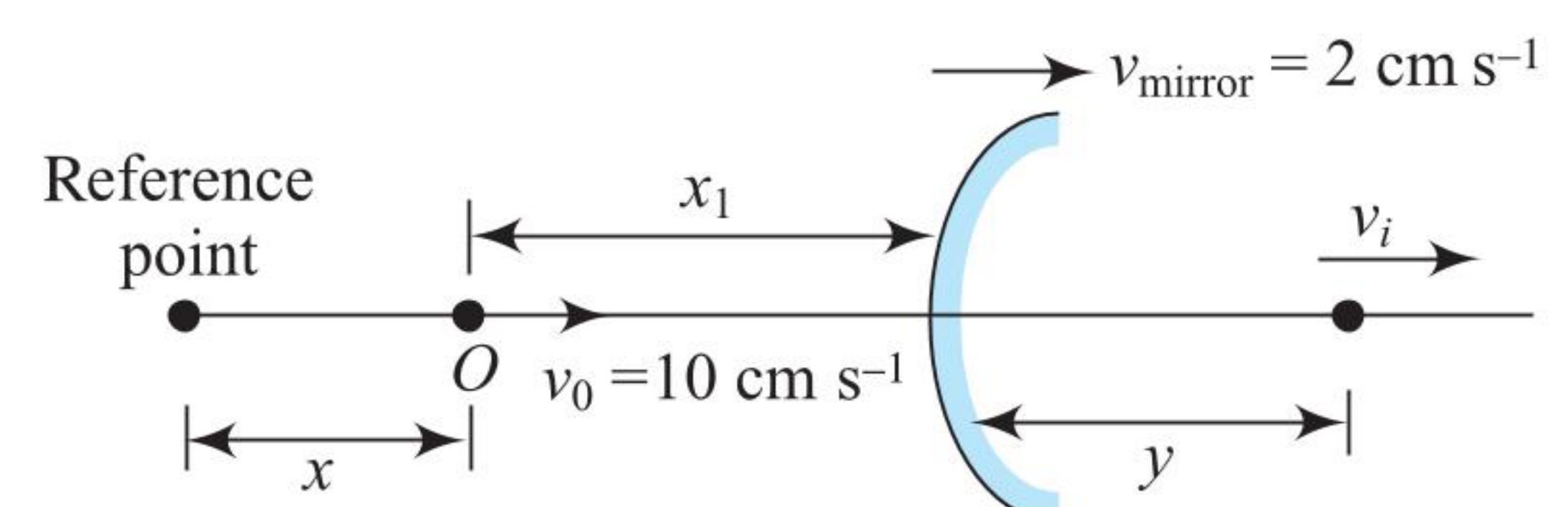


If the container seems to be half filled. Then

$$14 - h = \frac{h}{\mu} \Rightarrow 14 - h = \frac{3h}{4} \Rightarrow h = 8 \text{ m}$$

4. (0) $\frac{1}{y} + \frac{1}{-x_1} = \frac{1}{10}$... (i)

$$\frac{1}{y} - \frac{1}{10} = \frac{1}{10} \Rightarrow y = 5 \text{ cm}$$



$$\frac{dx}{dt} = 10 \text{ cm s}^{-1}, \quad \frac{d(x+x_1)}{dt} = 2 \text{ cm s}^{-1}$$

$$\frac{dx_1}{dt} = -8 \text{ cm s}^{-1}$$

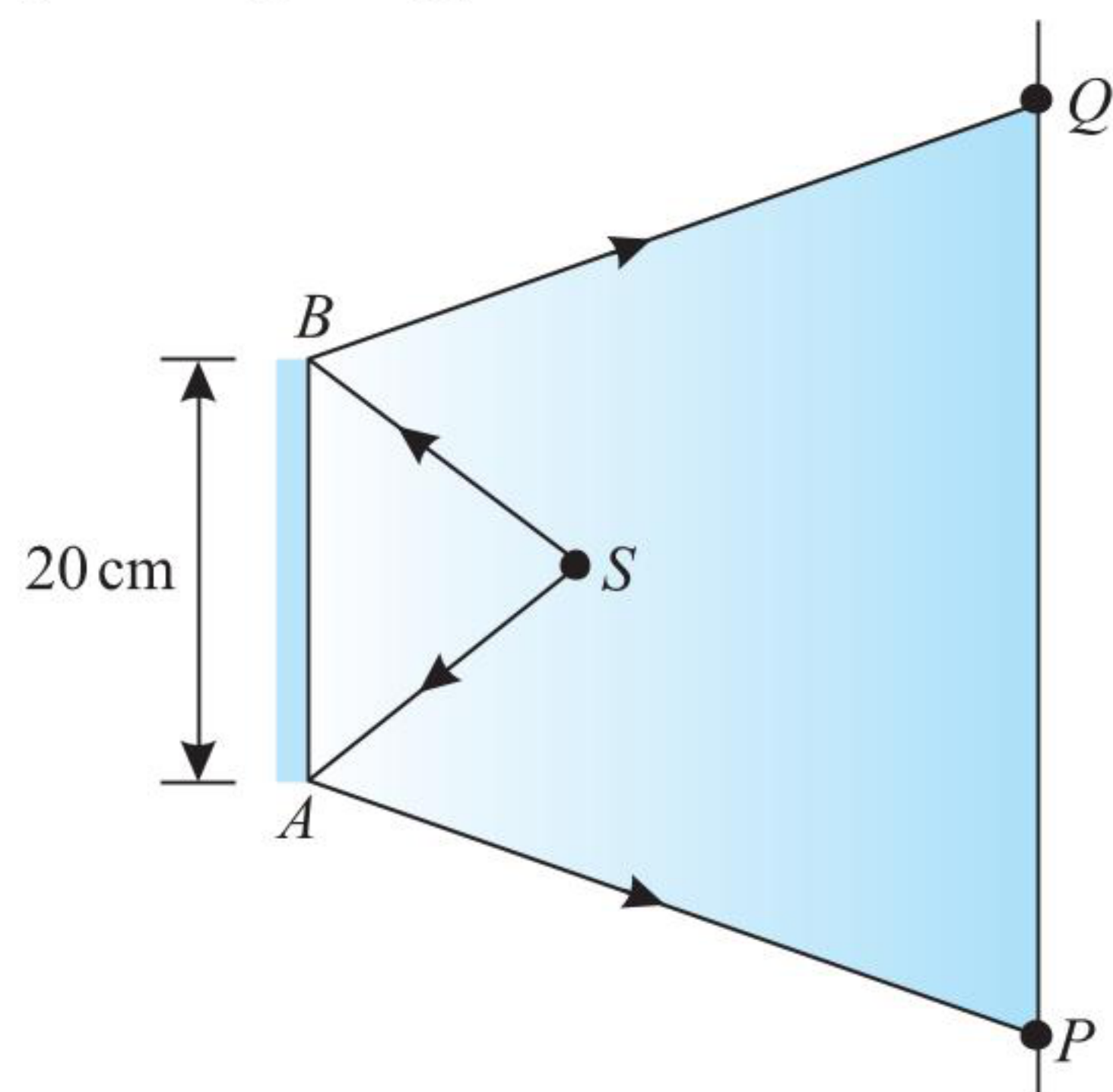
$$\frac{dy}{dt} = v_i = 2$$

$$\text{From (i): } \frac{dy}{dt} = \left(\frac{y}{x_2}\right)^2 \frac{dx_1}{dt}$$

$$\Rightarrow v_i - 2 = \left(\frac{5}{10}\right)^2 (-8) \Rightarrow v_i = 0$$

5. (6) Insect can see the image of source S in the mirror, so far as it remains in field of view of image overlapping with the road.

Shaded portion is the field of view, which overlaps with the road up to length PQ .



By geometry we can see that, $PQ = 3AB = 60$ cm

$$\therefore t = \frac{\text{Distance}}{\text{Speed}} = \frac{60}{10} = 6 \text{ s}$$

6. (21)

Applying $d_{app} = \frac{d}{\mu}$

$$d = (\mu)d_{app}$$

$$\therefore d_1 = (1.5)(6) = 9 \text{ cm}$$

$$d_2 = (1.5)(8) = 12 \text{ cm}$$

Therefore, actual thickness of the glass slab is

$$d_1 + d_2 = 21 \text{ cm}$$

7. (1.95)

For middle prism, $r + r = A$

$$2r = 60^\circ \Rightarrow r = 30^\circ$$

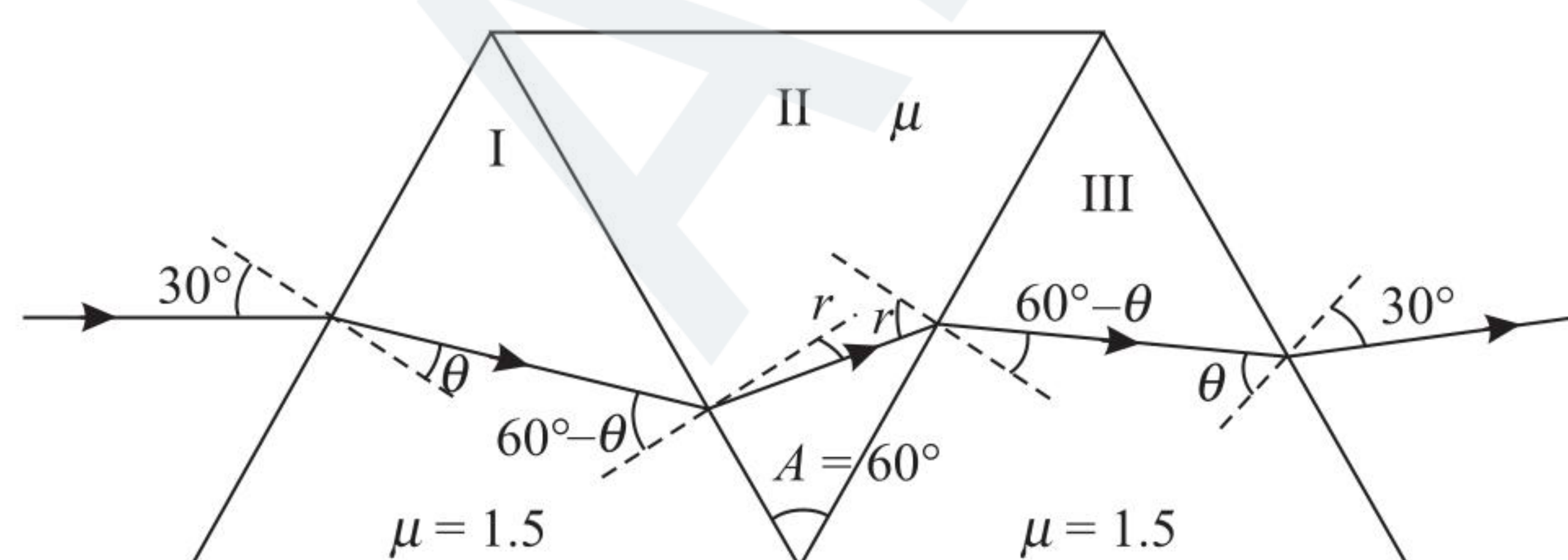
Refraction from Ist surface: $1 \sin 30^\circ = 1.5 \sin \theta$

$$\Rightarrow \sin \theta = \frac{1}{3} \text{ and } \cos \theta = \frac{\sqrt{8}}{3}$$

Refraction from 2nd surface: $1.5 \sin (60 - \theta) = \mu \sin 30^\circ$

$$1.5[\sin 60^\circ \cos \theta - \cos 60^\circ \sin \theta] = \mu \sin 30^\circ$$

$$\Rightarrow \mu = \sqrt{6} - \frac{1}{2} = \frac{2\sqrt{6} - 1}{2} = 1.95$$



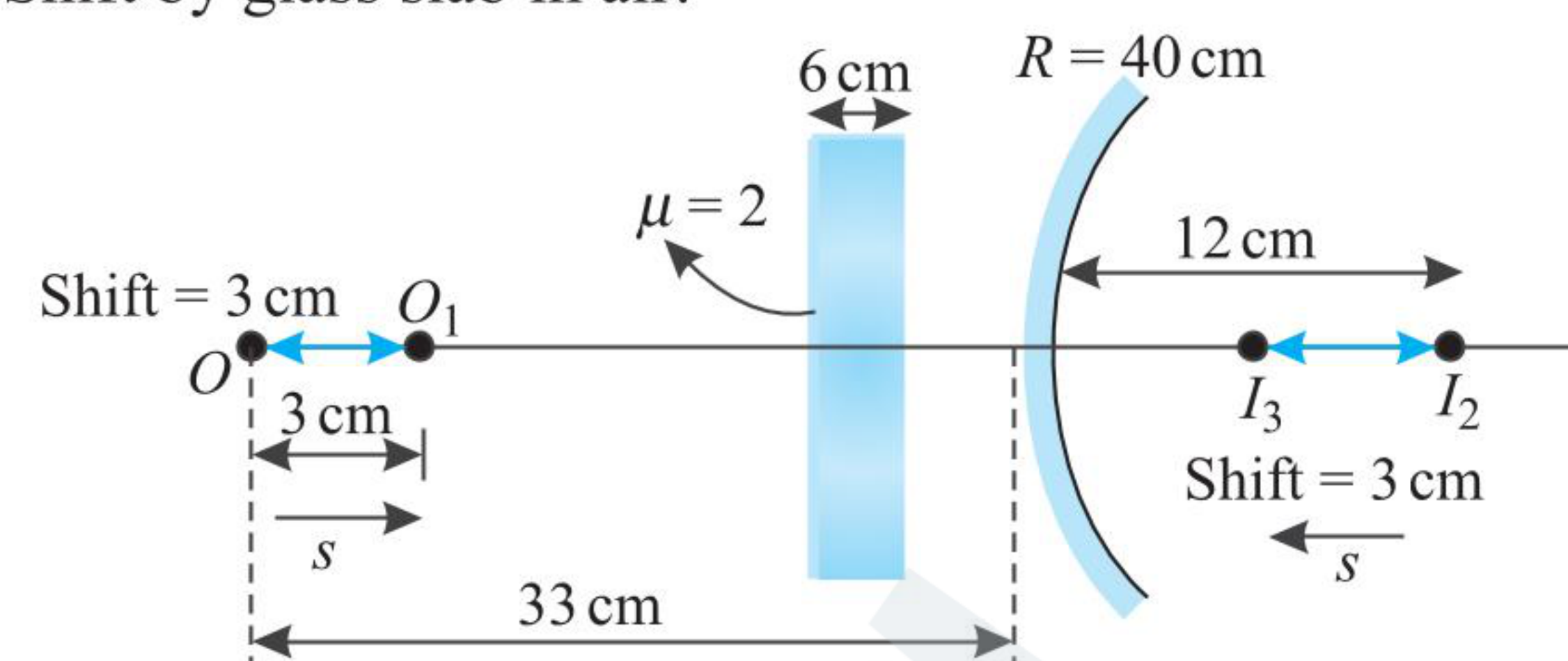
8. (42)

I_1 = image formed by the glass slab through the formula of shift

I_2 = image of I_1 formed by the reflection by the convex mirror.

= image of I_2 formed by the refraction through glass slab.

Shift by glass slab in air:



$$\text{Shift, } S = t \left(1 - \frac{1}{\mu}\right) = 6 \left(1 - \frac{1}{2}\right) = 3 \text{ cm}$$

Shift will be in the direction of ray travelling will be zero. Hence for mirror the object distance will be 30 cm.

Apply mirror formula $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ (For calculations of I_2)

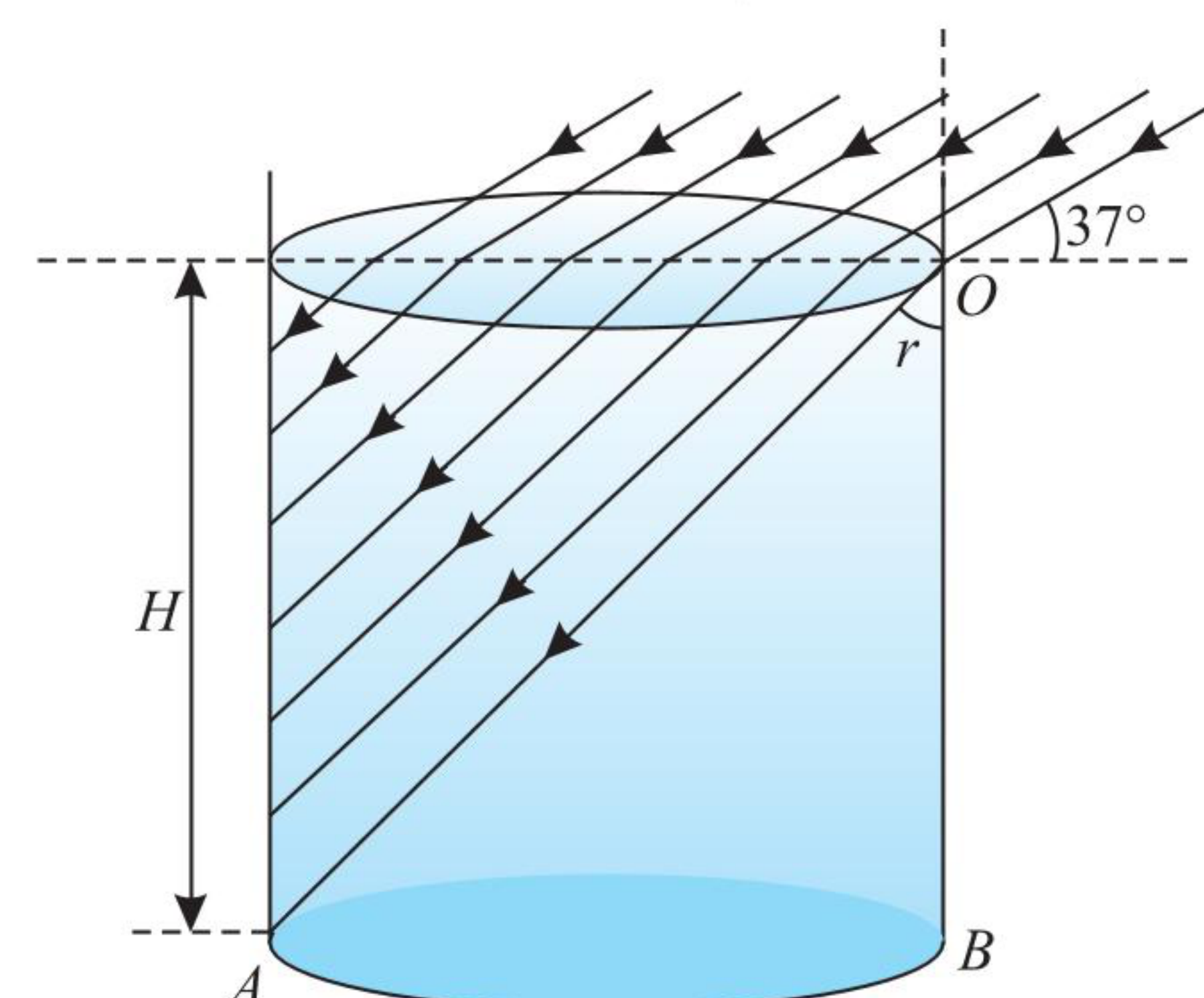
$$\frac{1}{v} + \frac{1}{-30} = \frac{1}{20} \quad (u = -30 \text{ cm; } f = 20 \text{ cm, convex mirror})$$

$$\Rightarrow v = +12 \text{ cm}$$

+12 indicates the position of I_2 as shown in figure. Again the rays will pass through slab and produce shift of I_2 towards left.

$$\text{So, } OI_3 = 33 + 9 = 42 \text{ cm}$$

9. (4) By Snell's law, $1 \sin 53^\circ = \frac{4}{3} \sin r$,



$$\frac{4}{5} = \frac{4}{5} \sin r, \sin r = \frac{3}{5} \Rightarrow r = 37^\circ$$

$$\text{In } \triangle OAB, \tan r = \frac{3}{H} = \tan 37^\circ \Rightarrow H = 4 \text{ metre}$$

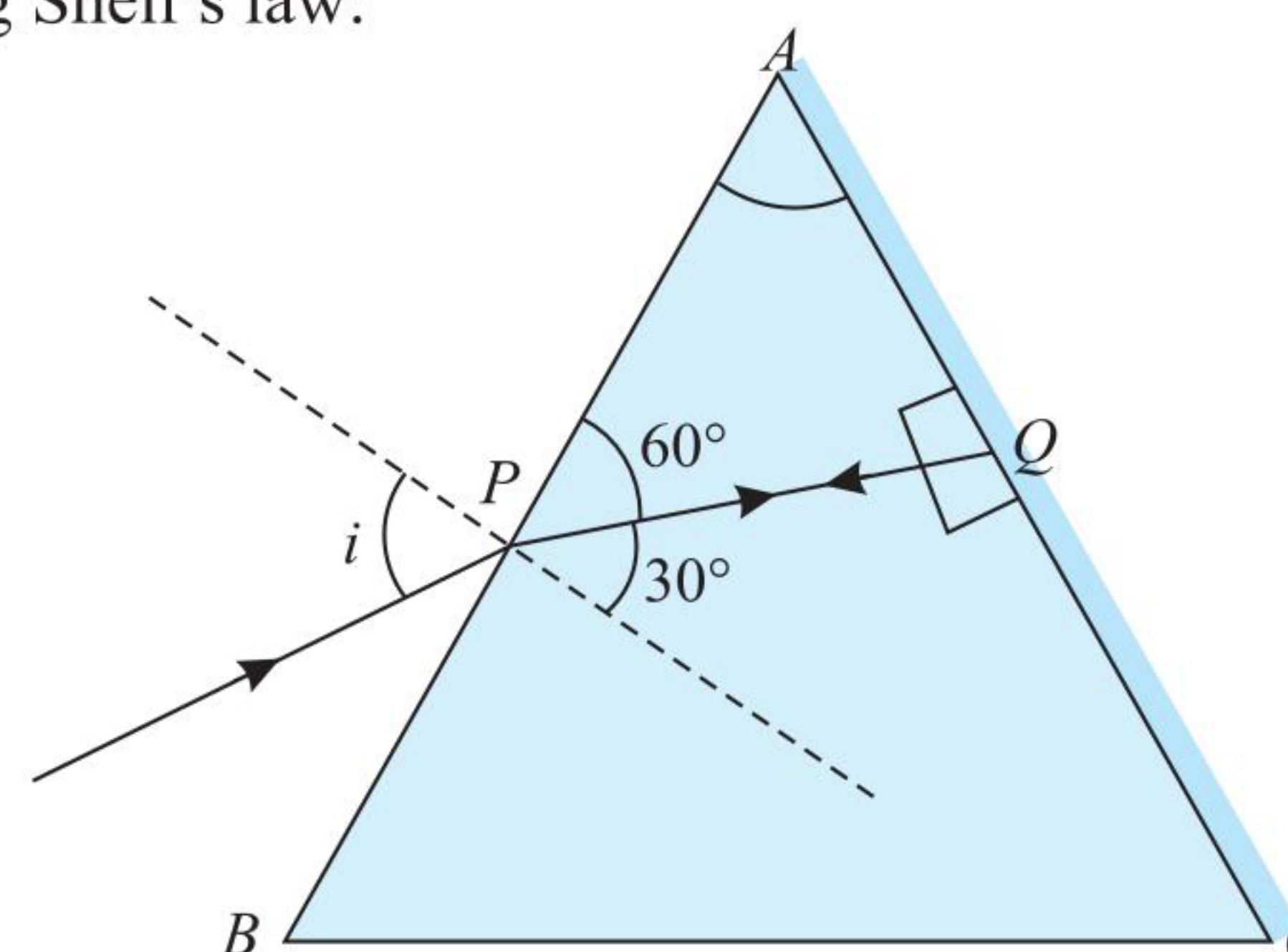
10. (45)

Refracting angle = angle of prism = 30° (given)

For any light ray to retrace its path, it must fall normally on the side AC (plane mirror).

By geometry, $\angle APQ = 60^\circ \therefore r = 30^\circ$ (by geometry)

Using Snell's law:



$$1 \times \sin i = \mu \sin 30^\circ \Rightarrow 1 \times \sin i = \sqrt{2} \times \frac{1}{2}$$

$$\sin i = \frac{1}{\sqrt{2}} \Rightarrow i = 45^\circ$$

11. (3) Magnification, $m = \frac{f}{f-u}$

Here $f = -f$ and $u = -\frac{3}{2}f$

Hence $m = \frac{-f}{-f + \frac{3f}{2}} \Rightarrow m = -2$

Taking the direction in the right-hand side

$$V_{im} = -m^2 V_{om}$$

Here: V_{im} = velocity of image w.r.t. mirror

V_{om} = velocity of object w.r.t. mirror

$$\Rightarrow V_{im} = -4u$$

As no external force acting on system 'man + platform' in horizontal direction the linear momentum of the system should always be zero.

$$m(u + V_m) + 2mV_m = 0$$

Velocity of platform (mirror)

$$V_m = -\frac{mu}{(m+2m)} = -\frac{u}{3} \text{ (or leftward)}$$

$$\vec{V}_{im} = \vec{V}_i - \vec{V}_m \Rightarrow \vec{V}_i = \vec{V}_{im} + \vec{V}_m$$

Hence, V_i = velocity of image w.r.t. ground

$$\Rightarrow \vec{V}_i = -4u - \frac{u}{3} = -\frac{13}{3}u \text{ (leftward)}$$

$$\Rightarrow V_i = 3 \text{ m/s}$$

12. (60)

If image is formed at position P , it means point P should be the centre of curvature of the mirror.

$$\text{Shift due to slabs } S = S_1 + S_2 = 6\left(1 - \frac{2}{3}\right) + 5\left(1 - \frac{4}{5}\right) = 3 \text{ cm}$$

The shift should be in the direction of ray travelling, hence the ray should conserve $17 + 3 = 20$ cm from the right face of the slab (2).

$$\text{Hence } l = 17 + 3 + 2 \times 20 = 60 \text{ cm}$$

13. (12.5)

Initially observer is mirror for image formation of object O . As O is kept at the bottom, then it sees somewhat above the bottom.

The distance of the object as seen from plane mirror

$$d_1 = 5 + \frac{10}{\mu} \Rightarrow d_1 = 5 + \frac{10 \times 3}{4} = 12.5 \text{ cm (in front)}$$

Hence image from the mirror after reflection is also at 12.5 cm (behind the mirror).

14. (2) Using mirror formula, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$$\text{Here } u = -10 \text{ cm, } f = -20 \text{ cm}$$

$$\text{Hence } \frac{1}{v} + \frac{1}{(-10)} = \frac{1}{(-20)} \Rightarrow v = 20 \text{ cm}$$

$$\text{The magnification, } m = \frac{I}{O} = -\frac{v}{u}$$

$$\text{From } M_1: O = -1 \text{ mm, } u = -10 \text{ cm, } v = 20 \text{ cm}$$

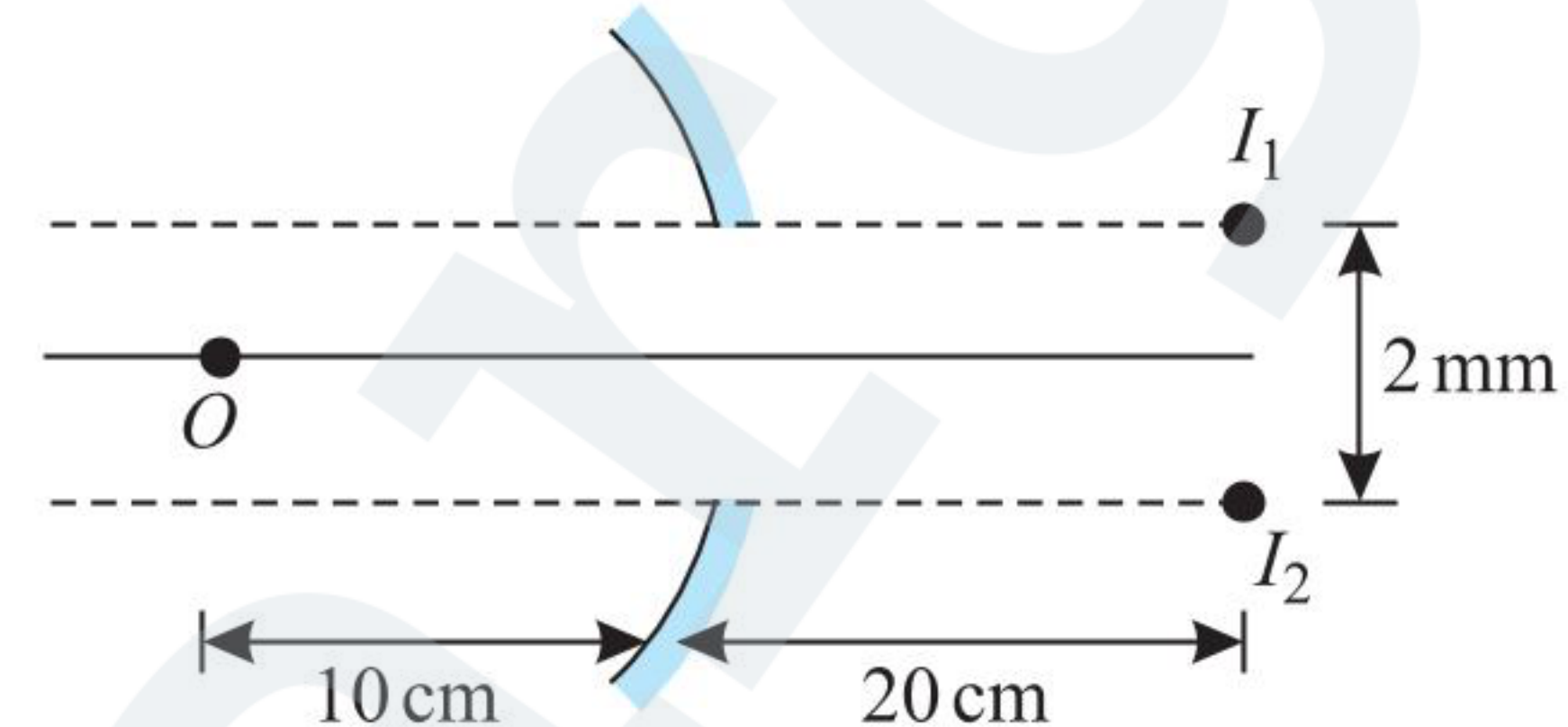
$$\frac{I_1}{(-1 \text{ mm})} = -\frac{(+20 \text{ cm})}{(-10 \text{ cm})} \Rightarrow I_1 = -2 \text{ cm}$$

Hence the image from M_1 will form 2 mm below principal axis behind the mirror.

Similarly, for M_2

$$\frac{I_2}{(+1 \text{ mm})} = -\frac{(+20 \text{ cm})}{(-10 \text{ cm})} \Rightarrow I_2 = +2 \text{ cm}$$

Hence the image from M_2 will form 2 mm above the principal axis.



Hence separation between the images will be 2 mm.

15. (80)

Position of the ball 4 sec after dropping,

$$h = 15 + 20 \times 4 - \frac{1}{2} \times 10 \times 4^2 = 15 \text{ m}$$

It means at this time the ball is at height 15 m from ground and moving down with speed 20 m/s.

Now using mirror formula, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$$\text{Here } u = -15 \text{ cm, } f = -10 \text{ cm}$$

$$\frac{1}{v} + \frac{1}{-(15)} = \frac{1}{(-10)} \Rightarrow v = -30 \text{ m}$$

The velocity of image, $V_{\text{image}} = -\left(\frac{v}{u}\right)^2 V_{\text{object}}$

$$\Rightarrow V_{\text{image}} = -\left(\frac{-30}{-15}\right)^2 \times (-20) = 80 \text{ m/s}$$

16. (22.50)

The apparent distance (x) of the fly from the interface of the slab and water

$$\text{Using: } \frac{h_1}{\mu_1} = \frac{h_2}{\mu_2} \Rightarrow \frac{45}{3/2} = \frac{x}{4/3} \Rightarrow x = 40 \text{ cm}$$

Hence the distance of the fly from the mirror,

$$x_{\text{fly}} = 20 + 40 = 60 \text{ cm}$$

Using mirror formula: $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$$\frac{1}{v} + \frac{1}{(-60)} = \frac{1}{(-20)} \Rightarrow v = -30 \text{ cm}$$

Hence the image will form at a height 30 cm from mirror, or 10 cm from lower surface of the slab.

Now considering separation from lower surface of the slab.

$$\text{Using, } \frac{h_1}{\mu_1} = \frac{h_2}{\mu_2} \Rightarrow \frac{10}{4/3} = \frac{x_1}{3/2} \Rightarrow x_1 = \frac{45}{4} \text{ cm}$$

Hence the image will form at a distance $\frac{45}{4}$ cm above the lower surface of the slab.

Now finally considering refraction from upper surface of the slab

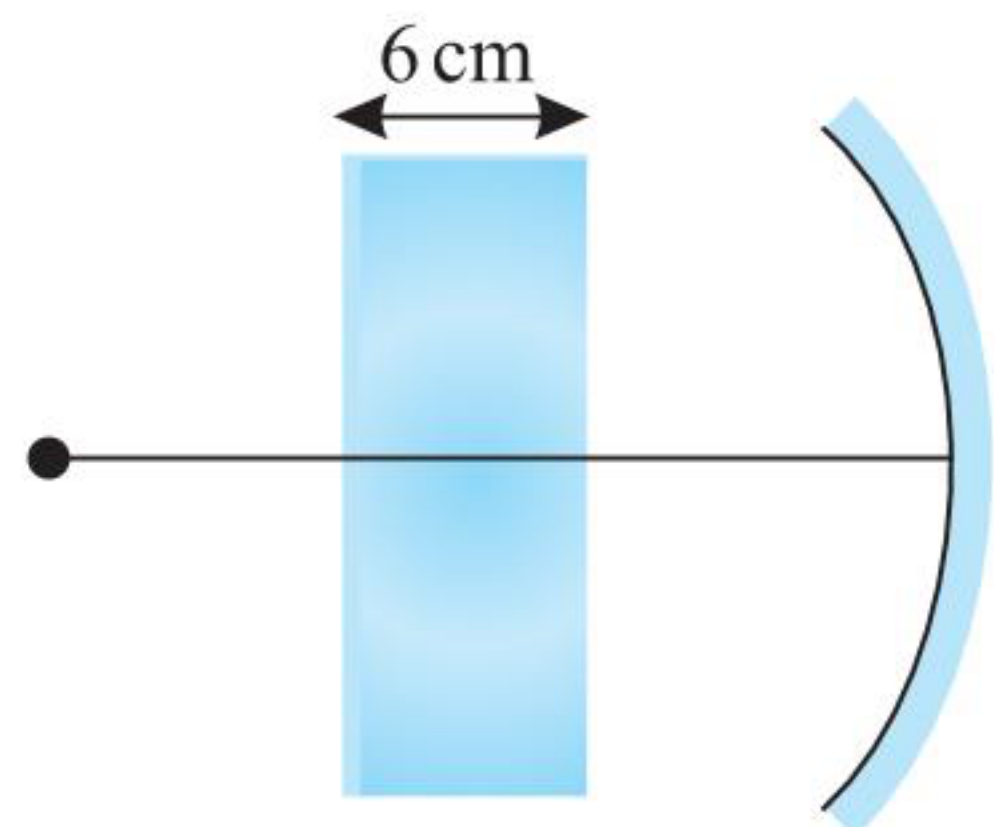
$$\text{Using, } \frac{h_1}{\mu_1} = \frac{h_2}{\mu_2} \Rightarrow \frac{\left(45 - \frac{45}{4}\right)}{3/2} = \frac{x_1}{1}$$

$$\Rightarrow x_2 = 45 \times \frac{3}{4} \times \frac{2}{3} = \frac{45}{2} \text{ cm} = 22.50 \text{ cm}$$

17. (42)

Shift due to glass slab,

$$S = 6 \left(1 - \frac{1}{3/2}\right) = 6 \times \frac{1}{3} = 2 \text{ cm}$$

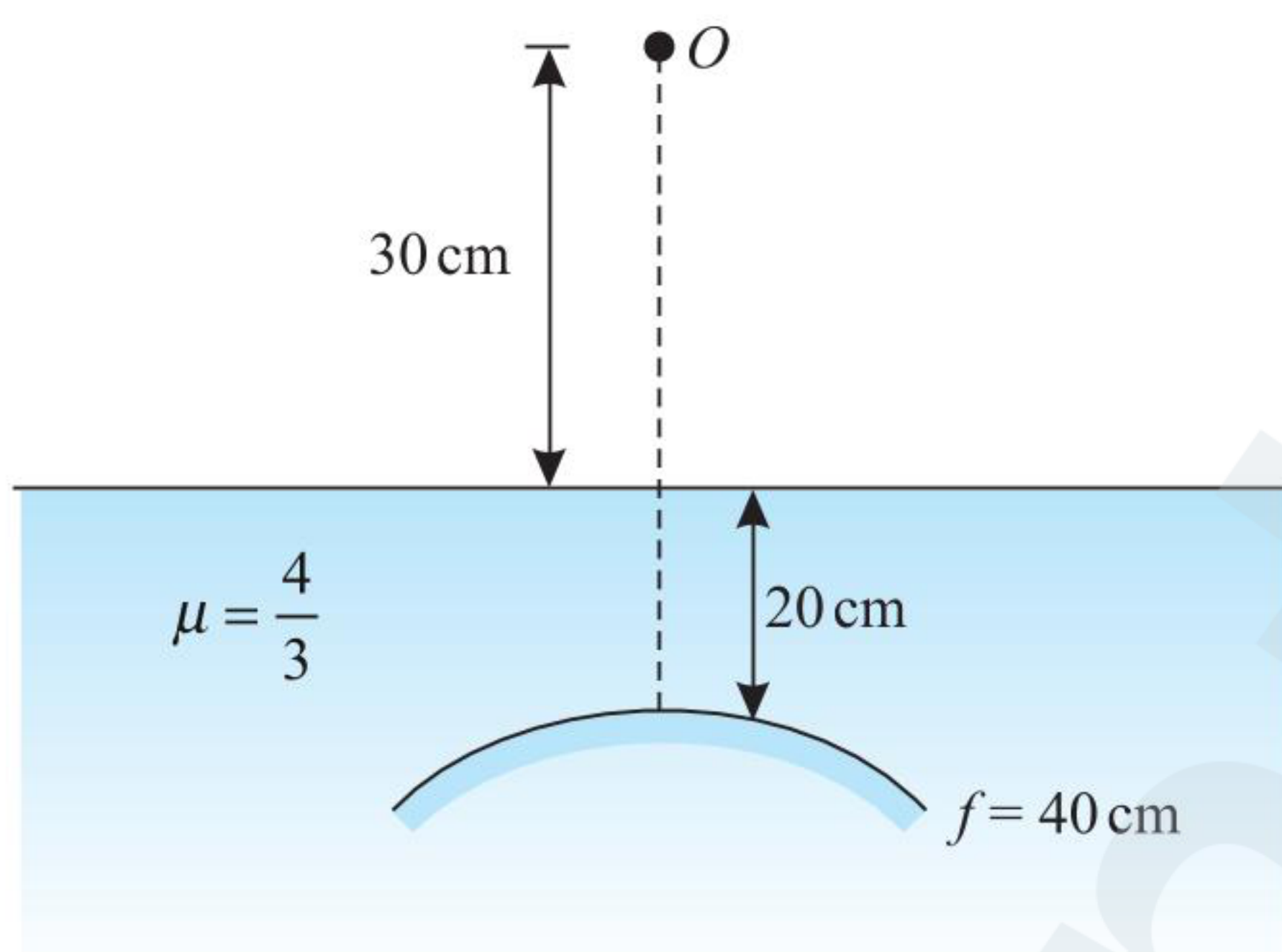


This shift will be in the direction of ray travelling or in rightward direction.

Reflected final image coincides with the object when the effective distance of the object is equal to radius of curvature of mirror $u = 40 + 2 = 42 \text{ cm}$.

18. (33)

Image formed by partial reflection from water surface at a distance 30 cm below the air water surface.



Secondly image formed by refraction from water surface followed by reflection from mirror and again refraction out of the water surface.

$$\therefore \text{Distance of the object from mirror} = u = 20 + \frac{4}{3} \times 30 = 60 \text{ cm}$$

Now considering reflection from the mirror.

$$\text{Using, } \frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

Here $u = -60 \text{ cm}$, $f = +40 \text{ cm}$

$$\frac{1}{v} + \frac{1}{(-60)} = \frac{1}{(-40)} \Rightarrow v = +24 \text{ cm}$$

Now considering refraction from water surface

$$\text{Using } \frac{h_1}{\mu_1} = \frac{h_2}{\mu_2} \Rightarrow \frac{(20+40)}{4/3} = \frac{h_2}{1}$$

$$\Rightarrow h_2 = 44 \times \frac{3}{4} = 33 \text{ cm}$$

Hence the final image from water surface will form at a depth 33 cm from water surface.

Hence separation between both the images

$$\Delta x = 33 - 30 = 3.00 \text{ cm}$$

19. (5)

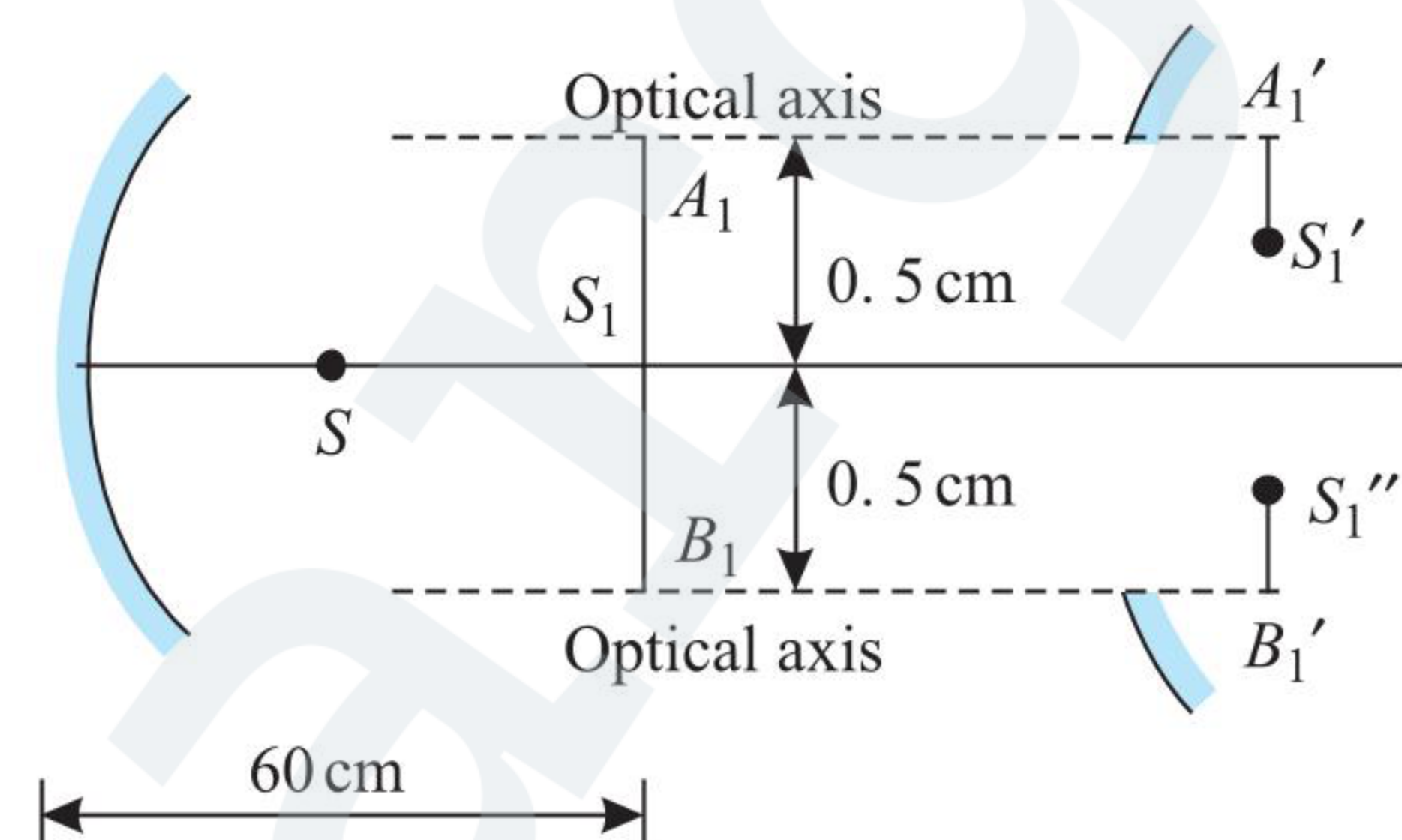
For mirror M_1 , $u = -30 \text{ cm}$, $f = -20 \text{ cm}$

$$\text{Using, } \frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow v = -60 \text{ cm}$$

The image S_1 behaves as object for two halves.

For convex mirror, $u' = -10 \text{ cm} \Rightarrow f' = +10 \text{ cm}$

$$\frac{1}{v'} + \frac{1}{u'} = \frac{1}{f'} \Rightarrow v' = +5 \text{ cm}$$



$$\therefore m' = -\frac{v'}{u'} = \frac{-5}{-10} = \frac{1}{2}$$

$$\text{Thus, } A_1' S_1' = \frac{1}{2} \times 0.5 = 0.25 \text{ cm}$$

$$\text{and } B_1' S_1'' = \frac{1}{2} \times 0.5 = 0.25 \text{ cm}$$

Hence separation between the images

$$S_1' S_1'' = 0.5 + 0.5 - 0.25 - 0.25 = 0.5 \text{ cm} = 5 \text{ mm}$$

Archives

JEE Advanced

Single Correct Answer Type

1. (1) At minimum deviation ($\delta - \delta_m$):

$$r_1 = r_2 = \frac{A}{2} = \frac{60^\circ}{2} = 30^\circ \text{ (for both colors)}$$

$\therefore$ Correct answer is (1).

2. (3) $V_{\text{ball}}^2 = 2 \times 10 \times 7.2 \Rightarrow v = 12 \text{ m s}^{-1}$

$$X_{\text{image of ball}} = \frac{4}{3} X_{\text{ball}}$$

$$V_{\text{image of ball}} = \frac{4}{3} V_{\text{ball}} = \frac{4}{3} \times 12 = 16 \text{ m s}^{-1}$$

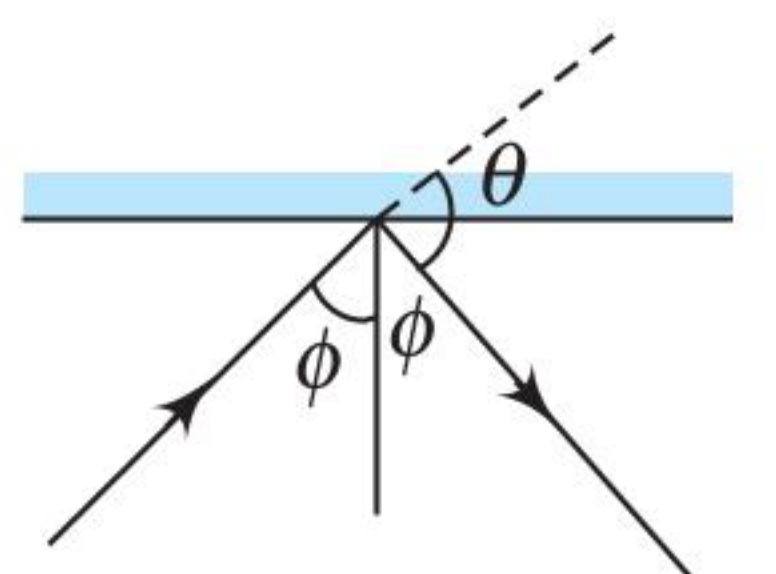
3. (1) Let angle between the directions of incident ray and reflected ray be θ

$$\cos \theta = \frac{1}{2} (\hat{i} + \sqrt{3}\hat{j}) \cdot \frac{1}{2} (\hat{i} - \sqrt{3}\hat{j}) = -\frac{1}{2}$$

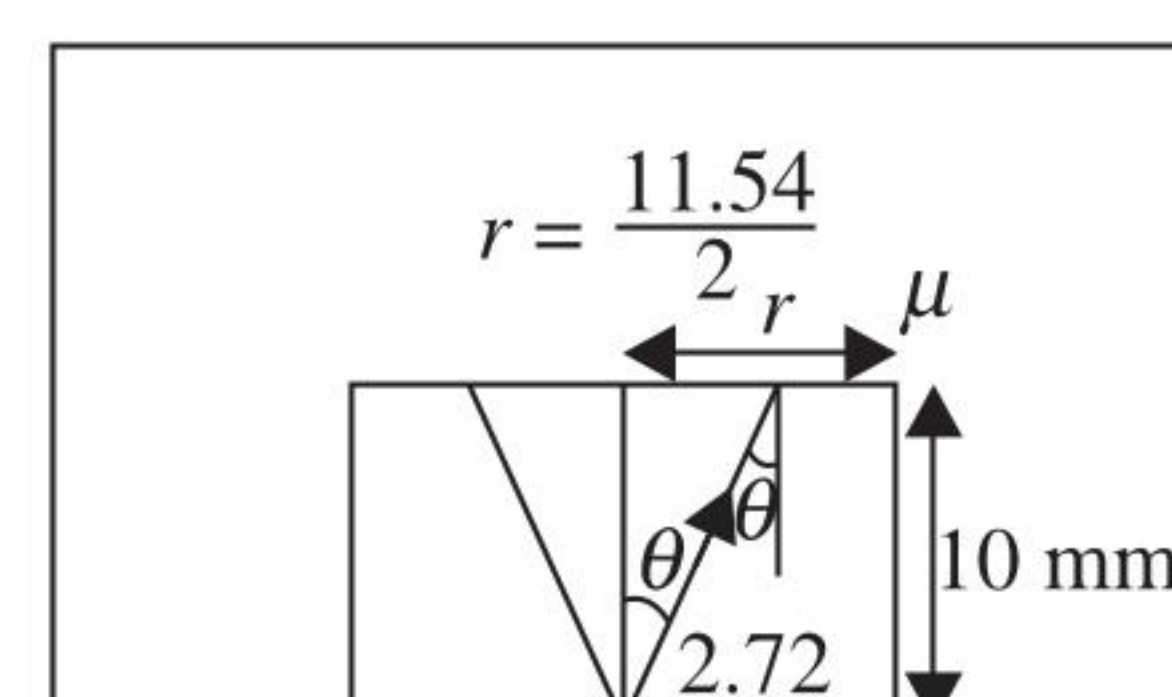
$$\theta = 120^\circ$$

$$2\phi + \theta = 180^\circ$$

$$\phi = 60^\circ/2 = 30^\circ$$



4. (3) $\tan \theta = \frac{11.54}{2 \times 10}$



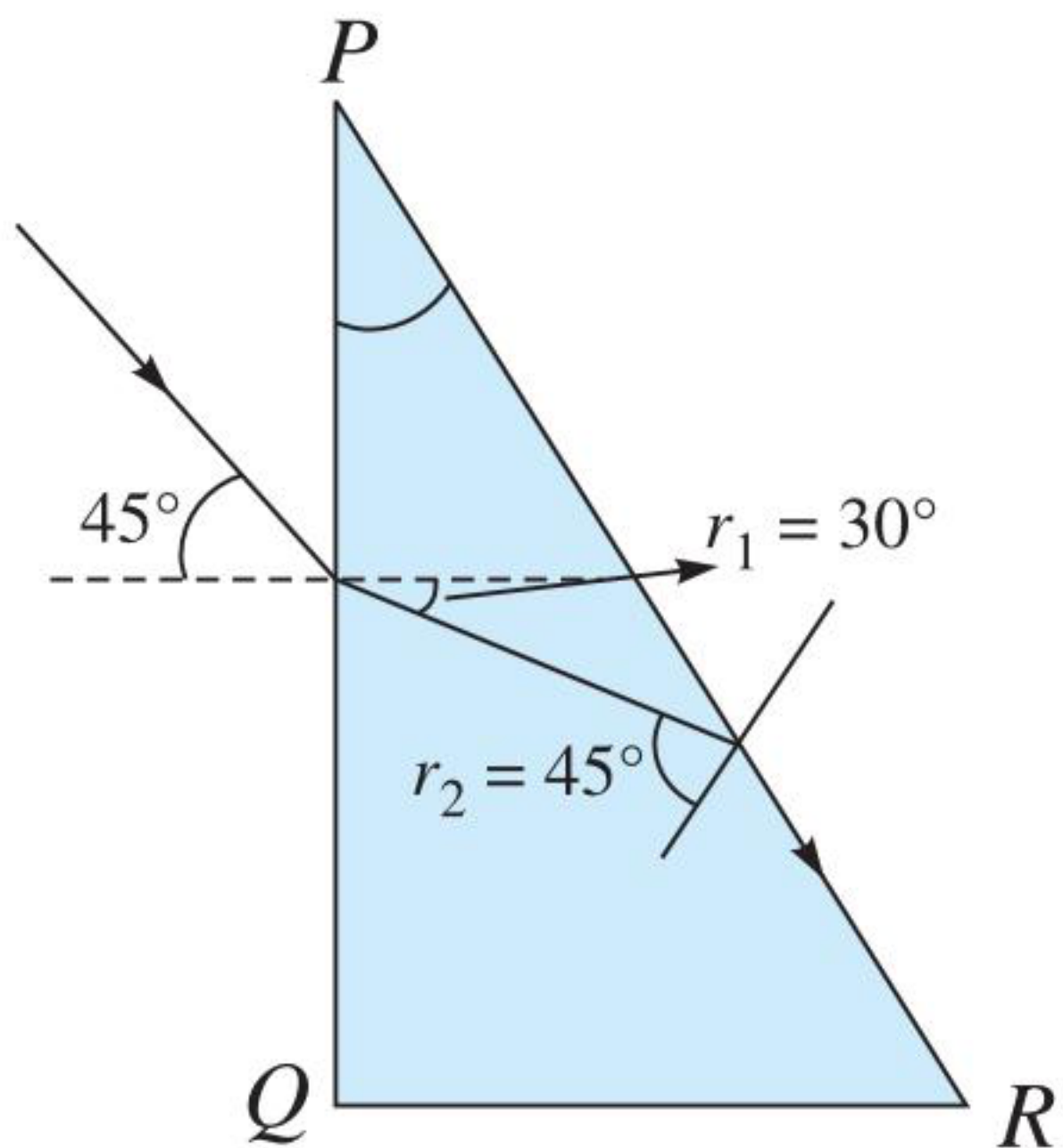
$$\sin \theta = \frac{11.54}{\sqrt{(11.54)^2 + (400)}}$$

$$2.72 \sin \theta = \mu \times \sin 90^\circ$$

$$2.72 \times \frac{11.54}{\sqrt{(11.54)^2 + 400}} = \mu$$

$$\mu = 1.36$$

5. (1)



$$1 \sin 45^\circ = \sqrt{2} \sin r_1$$

$$r_2 - r_1 = \theta$$

$$\theta = 45^\circ - 30^\circ$$

$$\Rightarrow \theta = 15^\circ$$

6. (4) Distance of point A is $f/2$, the nature of image of the AB will be virtual and behind the mirror. Let A' is the image of A from mirror, for this image

$$\frac{1}{v} + \frac{1}{-f/2} = \frac{1}{-f}$$

$$\frac{1}{v} = \frac{2}{f} - \frac{1}{f} = \frac{1}{f}$$

$$\text{or } v = f$$

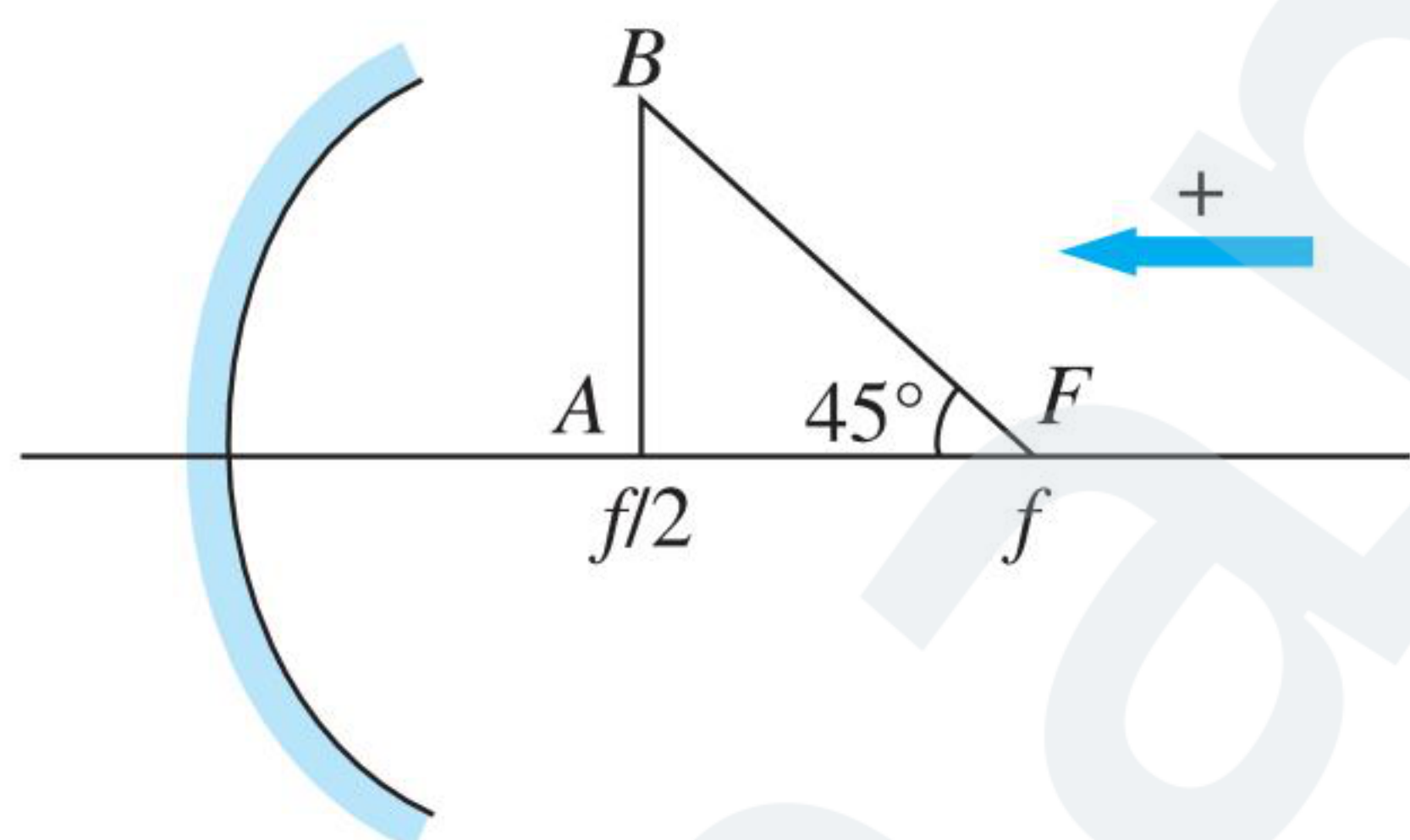
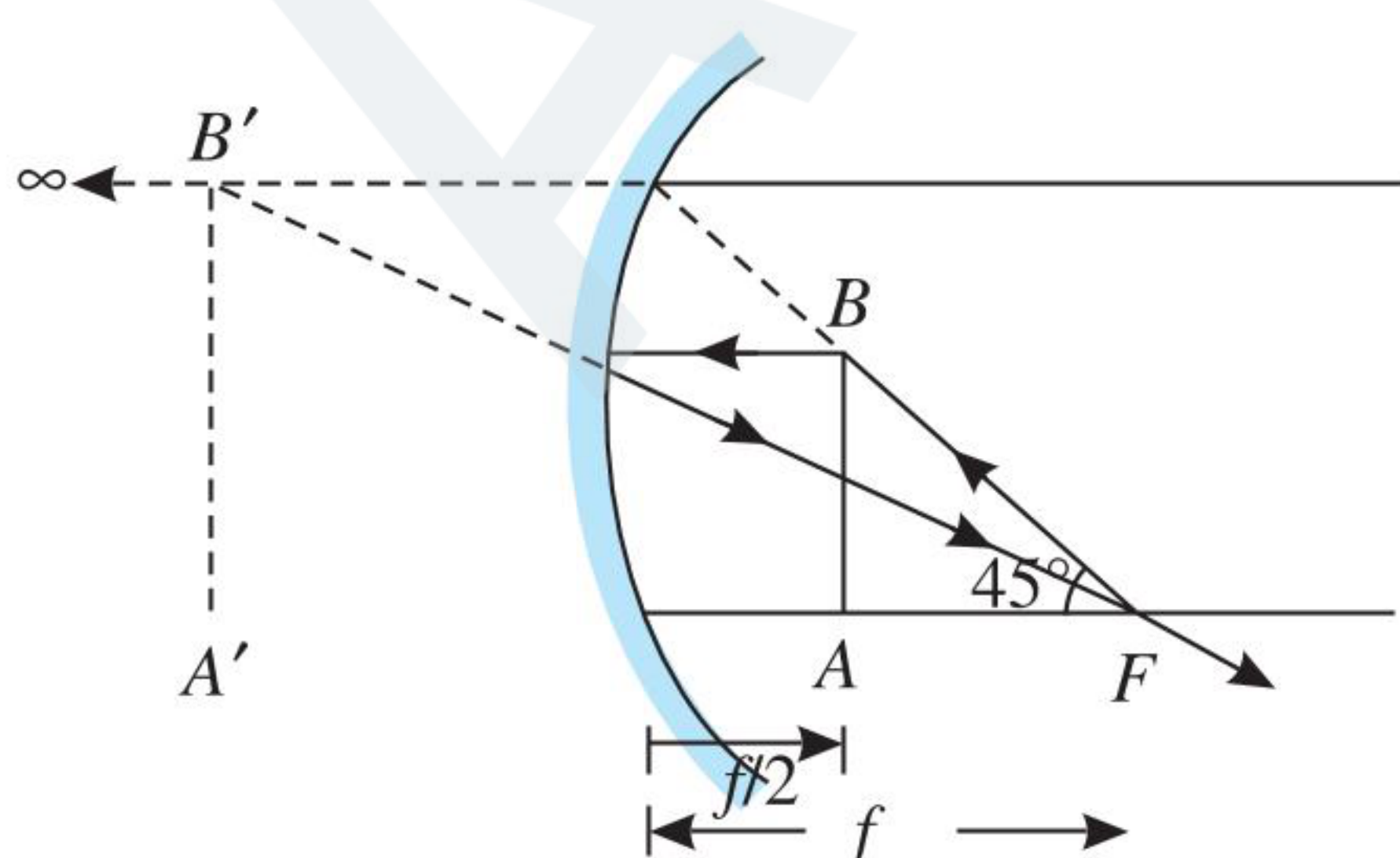


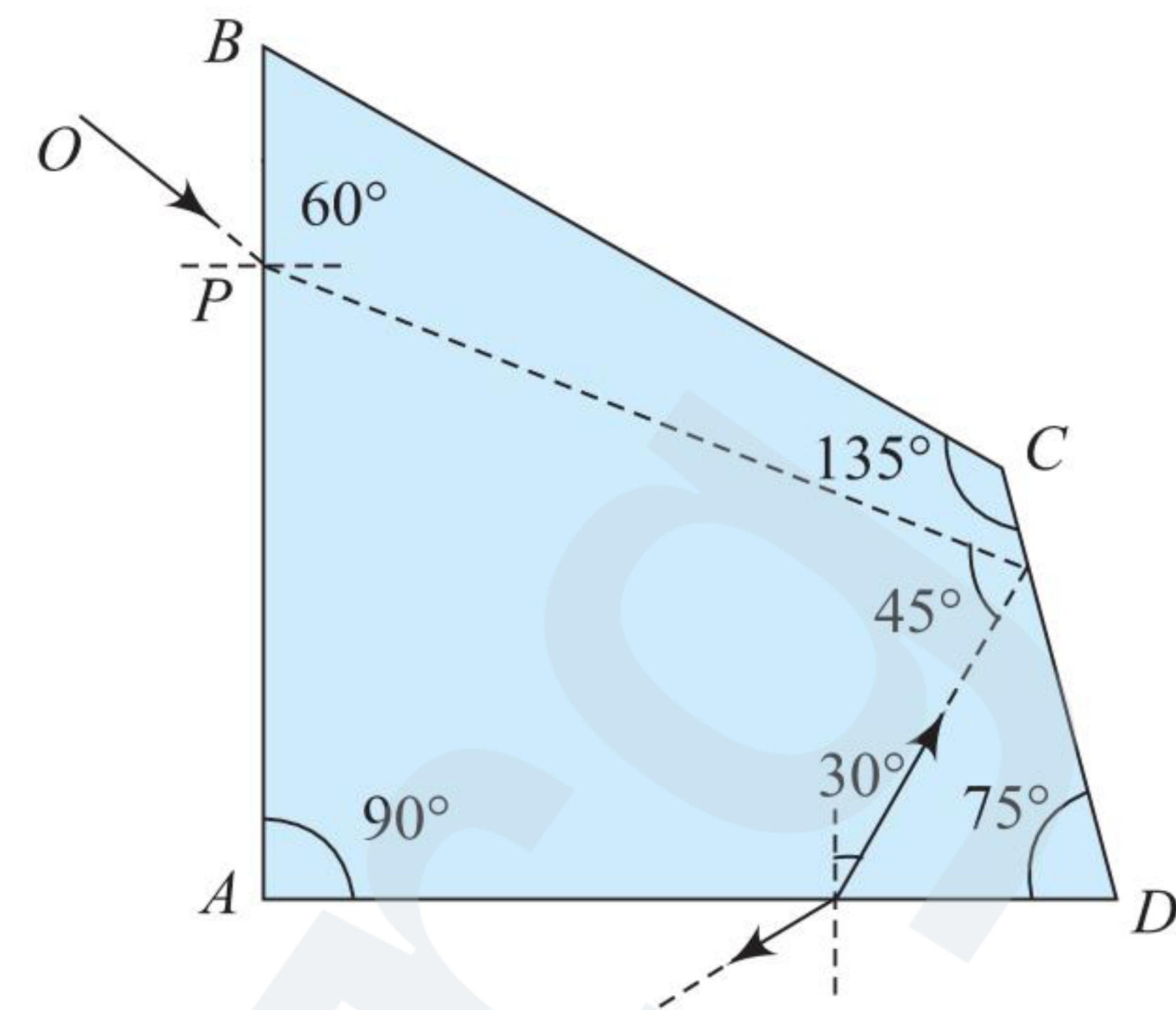
Image of line AB should be perpendicular to the principal axis and image of F will form at infinity, therefore correct image diagram is



The image of part BF will form at infinity. Hence option (4) is correct.

Multiple Correct Answers Type

1. (1),(2),(3)



$$\text{Using Snell's law, } \sin^{-1} \frac{1}{\sqrt{3}} < \sin^{-1} \frac{1}{\sqrt{2}}$$

Net deviation is 90°

2. (1),(2),(4)

For parallel slab, $n_1 \sin \theta_i = n_2 \sin \theta_f$

and l depends on refractive angle in slab

$\therefore l$ depends on refractive index of slab and independent of n_2

3. (1),(3),(4)

Angular deviation, $\delta = i_1 + i_2 - A$

For minimum deviation, $i_1 = i_2$ so, $i_1 = A$... (i)

Also, $r_1 + r_2 = A$

$$\text{i.e., } r_1 = \frac{A}{2}$$

... (ii)

$$\therefore r_1 = \frac{i_1}{2} \text{ (option (1) is correct)}$$

By Snell's law, $\sin i_1 = \mu \sin r_1$

$$\text{i.e., } \sin A = \mu \sin \left(\frac{A}{2} \right)$$

$$\Rightarrow 2 \cos \left(\frac{A}{2} \right) = \mu$$

$$A = 2 \cos^{-1} \left(\frac{\mu}{2} \right)$$

Hence option (2) is incorrect. Option (3) is obviously correct. For tangential emergence, $i_2 \rightarrow 90^\circ$.

So, $\mu \sin r_2 = 1$

$$\cos r_2 = \sqrt{1 - \frac{1}{\mu^2}}$$

Also, $r_1 = A - r_2$

$$\sin r_1 = \sin A \cos r_2 - \cos A \sin r_2 = (\sin A) \frac{\sqrt{\mu^2 - 1}}{\mu} - (\cos A) \frac{1}{\mu}$$

By Snell's law on 1st surface, $\sin i_1 = \mu \sin r_1$

$$\Rightarrow \sin r_1 = \sin A (\mu^2 - 1)^{1/2} - \cos A$$

$$\Rightarrow i_1 = \sin^{-1} \left(\sin A \sqrt{4 \cos^2 \frac{A}{2} - 1} - \cos A \right)$$

i.e., option (4) is correct.

4. (2), (3), (4)

$\sin \theta$ is slightly larger than $\frac{1}{n_1}$

$$\sin \theta > \frac{1}{n_1} \Rightarrow n_1 \sin \theta > 1$$

$$(1) \text{ if } n_1 = n_2 \sin C = \frac{1}{n_1}$$

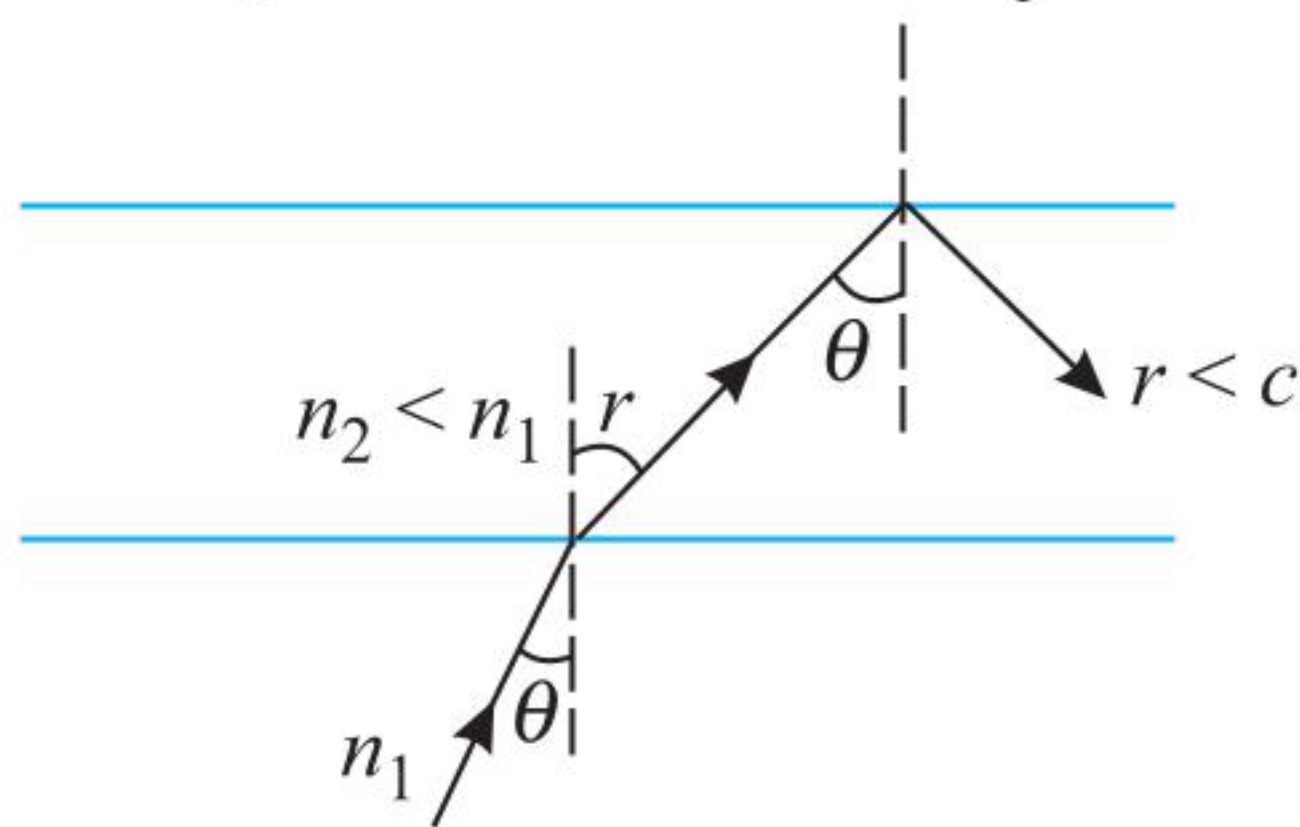
$$\sin \theta > \sin C \Rightarrow \theta > C$$

Total internal reflection, occurs

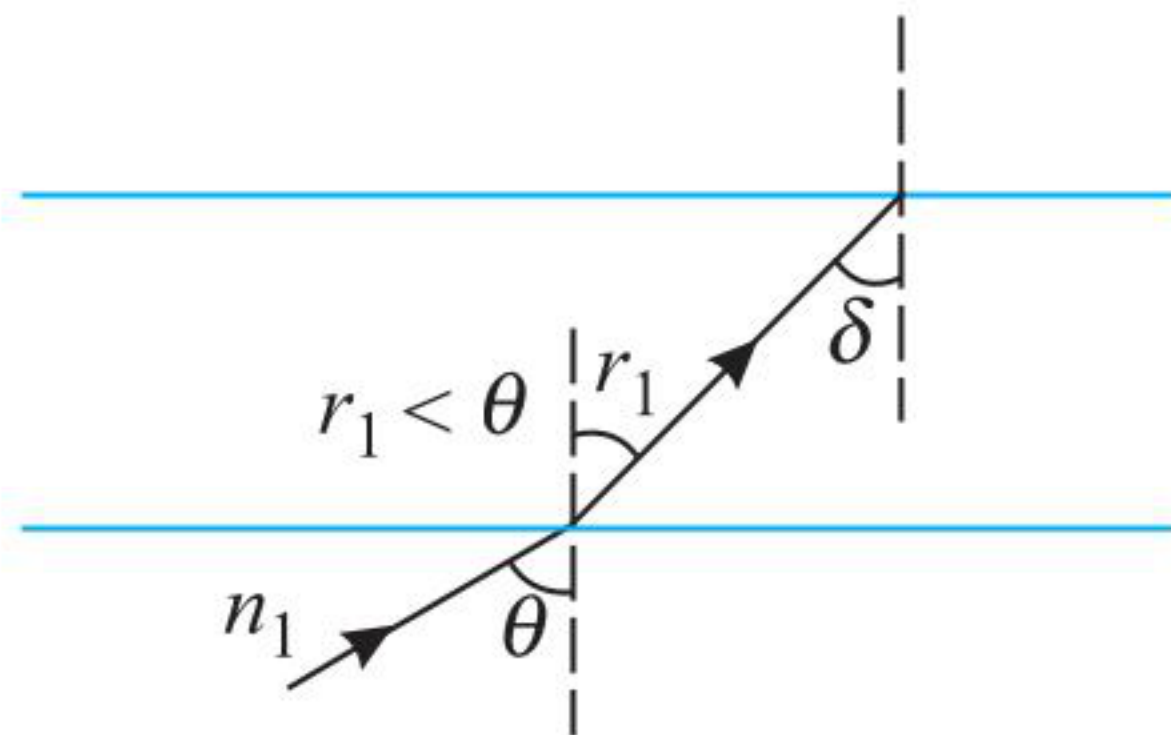
It means light will not emerge in air.

$$(2) n_2 < n_1$$

The light ray finally reflects back in n_1



$$(3) n_2 > n_1$$

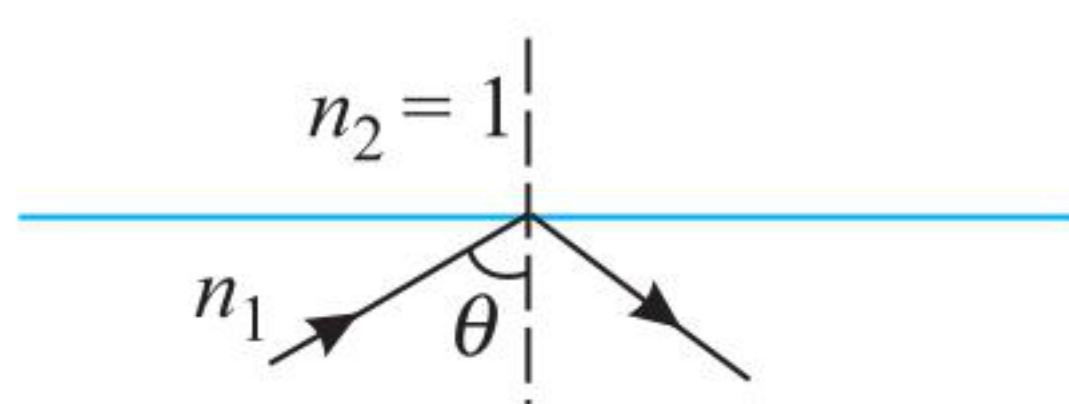


$$n_1 \sin \theta = n_2 \sin r_1 = 1 \sin c \Rightarrow \sin r_1 = \frac{n_1 \sin \theta}{n_2}$$

$$\sin r_1 > \frac{1}{n_2} \text{ and } \sin c = \frac{1}{n_2}$$

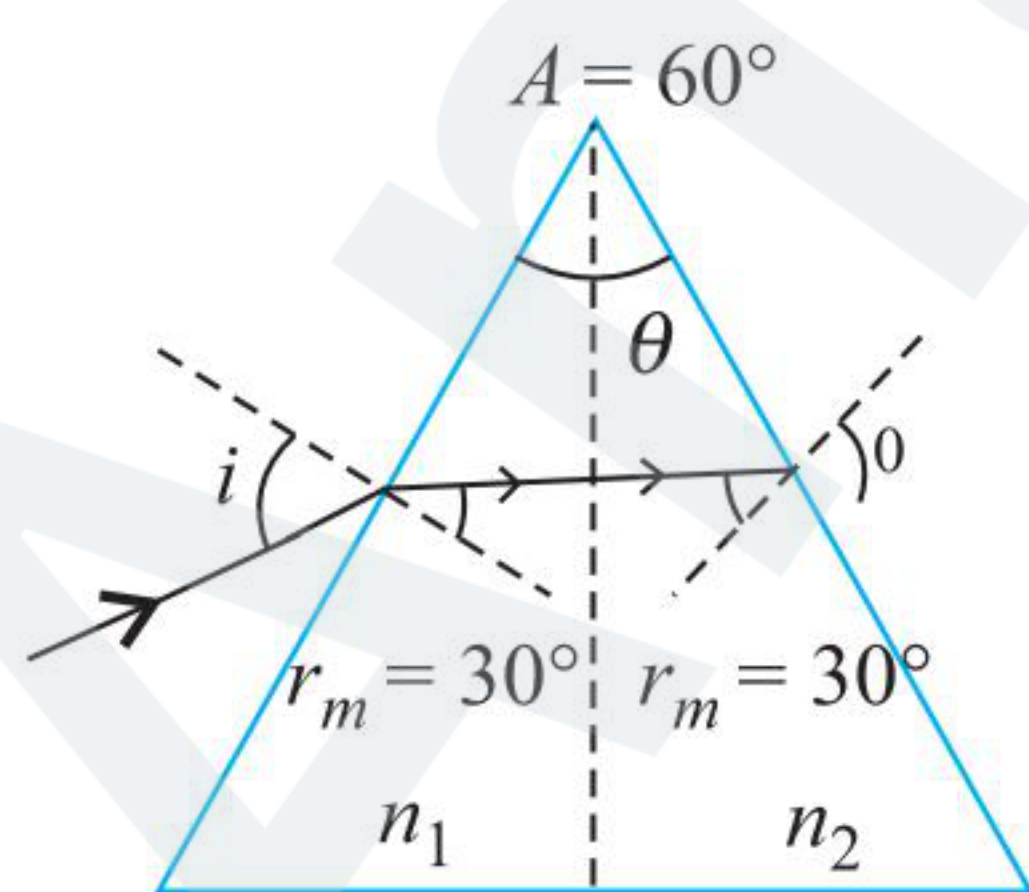
$$\sin r_1 > \sin c \Rightarrow r_1 > c$$

$$(4) \frac{1}{n_1}$$



$$\sin \theta > \frac{1}{n_1} \text{ and } \sin c = \frac{1}{n_1} \Rightarrow \theta > c$$

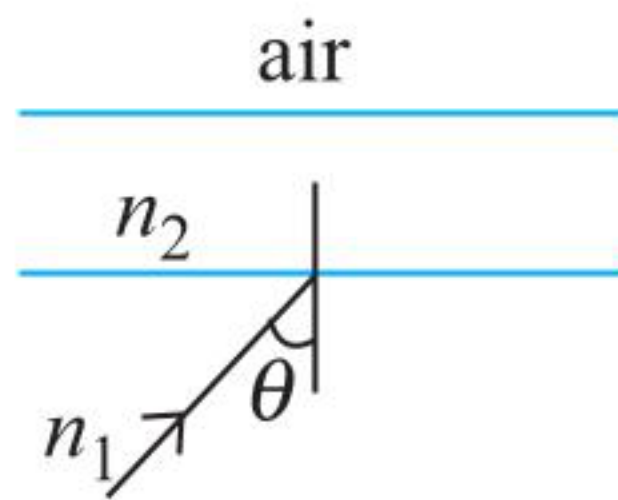
5. (2), (3)



$$n_1 = n_2 = n = \frac{3}{2} \text{ for minimum deviation}$$

$$r_m = \frac{A}{2} = \frac{60}{2} = 30^\circ \text{ and } i = e = \theta$$

$$n = \frac{\sin\left(\frac{\delta_{\min} + A}{2}\right)}{\sin\left(\frac{A}{2}\right)} \Rightarrow \frac{3}{2} = \frac{\sin(\theta)}{\sin\left(\frac{60}{2}\right)}$$



$$\Rightarrow \sin(\theta) = \frac{3}{4}, \cos \theta = \frac{\sqrt{7}}{4}$$

$$\text{If } n_1 = n = \frac{3}{2} \text{ and } n_2 = n + \Delta n$$

$$e = \theta + \Delta \theta$$

$$(1) \sin \theta = n \sin 30^\circ$$

$$(n + \Delta n) \sin 30^\circ = (1) \sin(\theta + \Delta \theta)$$

$$\text{Solving } \frac{1}{2}(\Delta n) = \sin(\theta + \Delta \theta) - \sin \theta$$

$$\frac{\Delta n}{2} = \frac{\sin(\theta + \Delta \theta) - \sin \theta}{\Delta \theta} \times \Delta \theta$$

$$\frac{d(\sin \theta)}{d\theta} = \cos \theta$$

$$\frac{\Delta n}{2} = \cos \theta \Delta \theta \Rightarrow \frac{\Delta n}{2} = \frac{\sqrt{7}}{4} \Delta \theta \Rightarrow \frac{\Delta n}{\Delta \theta} = \frac{\sqrt{7}}{2} = \frac{2.64}{2}$$

$$\frac{\Delta n}{\Delta \theta} = 1.34 > 1 \Rightarrow \Delta n > \Delta \theta, \text{ so option (1) is incorrect}$$

$$\Delta n = (1.34)\Delta \theta \Rightarrow \Delta \theta \propto \Delta n \Rightarrow \text{option (2) is correct.}$$

$$\frac{\Delta n}{\Delta \theta} = \frac{\sqrt{7}}{2} \Rightarrow \frac{2.8 \times 10^{-3}}{\Delta \theta} = \frac{\sqrt{7}}{2}$$

$$\Delta \theta = \frac{5.6 \times 10^{-3}}{\sqrt{7}} = \sqrt{7} \times 0.8 \times 10^{-3}$$

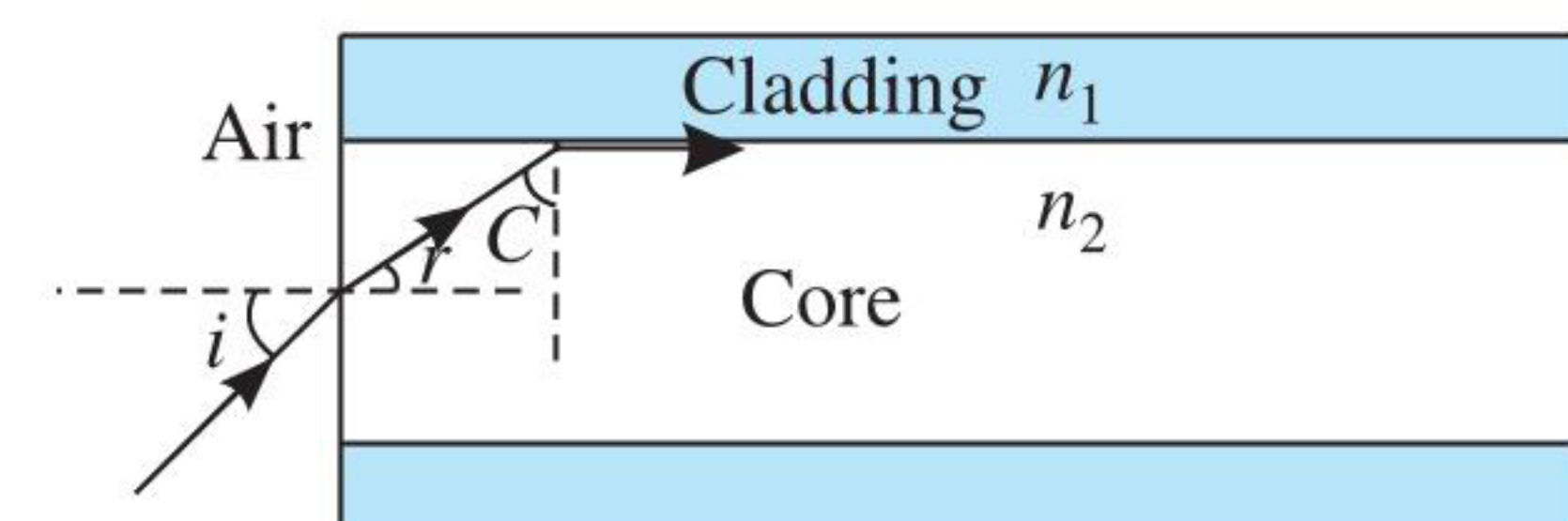
$$\Delta \theta = (2.64 \times 0.8) \times 10^{-3} = 2.11 \times 10^{-3} \text{ rad}$$

$$\Rightarrow \text{option (3) is correct.}$$

Linked Comprehension Type

1. (1), (3)

i_m is the angle of incidence for which $r = 90^\circ - C$



$$\sin C = \frac{n_2}{n_1}$$

$$\Rightarrow n_0 \sin(i_m) = n_1 \sin(90^\circ - C)$$

$$\Rightarrow N_A = \sin(i_m) = \frac{n_1 \cos(C)}{n_0} = \frac{\sqrt{n_1^2 - n_2^2}}{n_0}$$

$$\text{For } S_1, N_{A1} = \frac{\sqrt{\frac{45}{16} - \frac{9}{4}}}{n_0} = \frac{3}{4n_0}$$

$$\text{For } S_2, N_{A2} = \frac{\sqrt{\frac{64}{25} - \frac{49}{25}}}{5n_0} = \frac{\sqrt{15}}{5n_0}$$

$$\text{For (1) } N_{A1} = \frac{3 \times 3}{4 \times 4} = \frac{9}{16}; N_{A2} = \frac{\sqrt{15} \times 3 \sqrt{15}}{5 \times 16} = \frac{9}{16}$$

Hence option (1) is correct.

$$\text{For (2) } N_{A1} = \frac{3 \times \sqrt{15}}{4 \times 6} = \frac{\sqrt{15}}{8}; N_{A2} = \frac{\sqrt{15} \times 3}{5 \times 4} = \frac{\sqrt{15} \times 3}{20}$$

Hence option (2) is incorrect

$$\text{For (3)} \quad NA_1 = \frac{3}{4 \times 1} = \frac{3}{4}; \quad NA_2 = \frac{\sqrt{15} \times \sqrt{15}}{5 \times 4} = \frac{3}{4}$$

Hence option (3) is correct.

$$\text{For (4)} \quad NA_1 = \frac{3}{4 \times 1} = \frac{3}{4}; \quad NA_2 = \frac{3}{4 \times 4} = \frac{9}{16}$$

Hence option (4) is incorrect.

2. (4) It is given $NA_2 < NA_1$

$$\Rightarrow i_{m_1} > i_{m_2}$$

Hence if the combination can be placed both ways i.e. 1st structure and then 2nd structure and then reversed also, then the condition of TIR is satisfied for lower i_m then it can be satisfied for all other less angle as well.

Hence NA_2 will be the numerical aperture of the combined structure.

Matrix Match Type

1. (4) (i) → (b); (ii) → (c); (iii) → (d); (iv) → (a)

i. $\mu_2 > \mu_1$... (towards normal)

$\mu_2 > \mu_3$... (away from normal)

ii. $\mu_1 = \mu_2$... (no change in path)

$\angle i = 0 \Rightarrow \angle r = 0$ on the block.

iii. $\mu_1 > \mu_2$... (away from the normal)

$\mu_2 > \mu_3$... (away from the normal)

$$\mu_1 \times \frac{1}{\sqrt{2}} = \mu_2 \sin r \Rightarrow \sin r = \frac{\mu_1}{\sqrt{2}\mu_2}$$

Since $\sin r < 1 \Rightarrow \mu_1 < \sqrt{2}\mu_2$

iv. For TIR: $45^\circ > C \Rightarrow \sin 45^\circ > \sin C$

$$\Rightarrow \frac{1}{\sqrt{2}} > \frac{\mu_2}{\mu_1} \Rightarrow \mu_1 > \sqrt{2}\mu_2$$

Numerical Value Type

1. (6) $\sin \theta_c = 3/5$

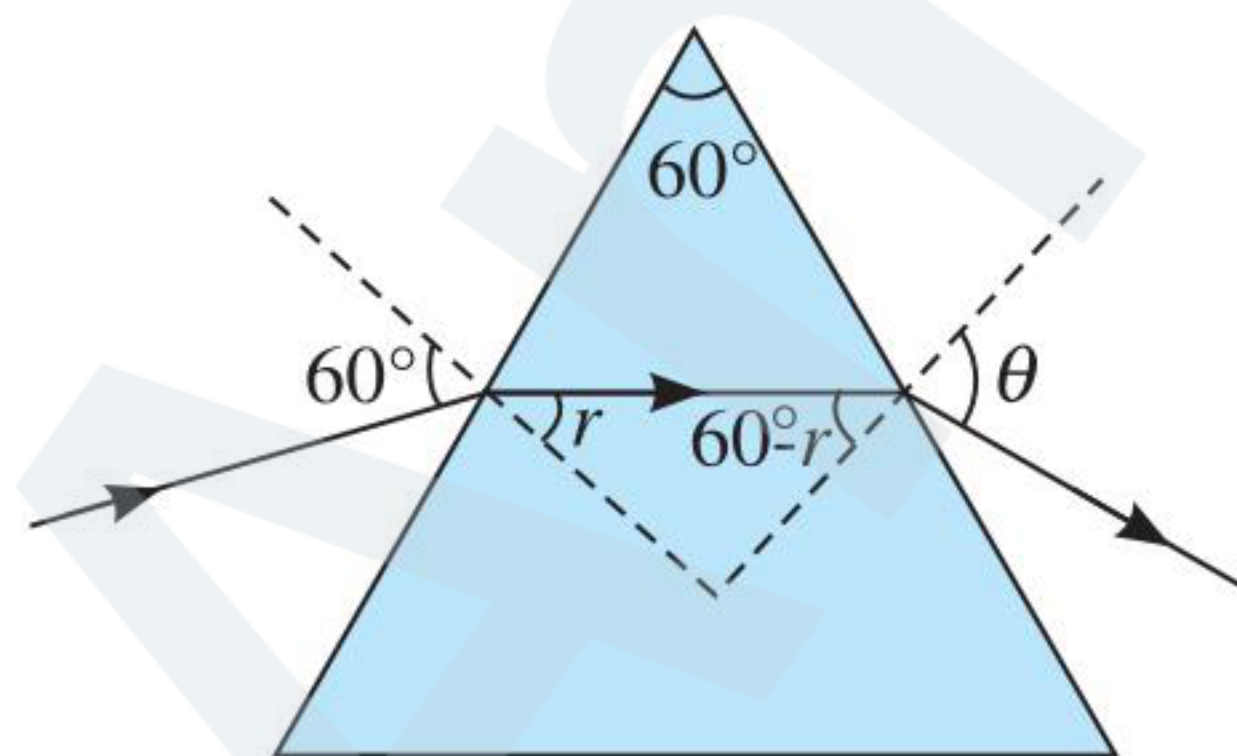
$$\therefore R = 6 \text{ cm}$$

2. (3) For $v_1 = 50/7 \text{ m}$, $u_1 = -25 \text{ m}$, $v_2 = 25/3 \text{ m}$, $u_2 = -50 \text{ m}$

$$\text{Speed of object} = \frac{25}{30} \times \frac{18}{3} = 3 \text{ km h}^{-1}$$

3. (2) $\sin 60^\circ = n \sin r$... (i)

$$\sin \theta = n \sin (60^\circ - r) \quad \dots (ii)$$



Differentiating Eq. (ii).

$$\cos \theta \frac{d\theta}{dt} = -n \cos(60^\circ - r) \frac{dr}{dt} + \sin(60^\circ - r)$$

Differentiating Eq. (i).

$$n \cos r \frac{dr}{dt} + \sin r = 0$$

$$\cos \theta \frac{d\theta}{dt} = -n \cos(60^\circ - r) \left(\frac{-\tan r}{n} \right) + \sin(60^\circ - r)$$

$$\frac{d\theta}{dt} = \frac{1}{\cos \theta} [\cos(60^\circ - r) \tan r + \sin(60^\circ - r)]$$

$$\begin{aligned} \frac{d\theta}{dt} &= \frac{1}{\cos 60^\circ} (\cos 30^\circ \times \tan 30^\circ + \sin 30^\circ) \\ &= 2 \left(\frac{1}{2} + \frac{1}{2} \right) = 2 \end{aligned}$$

4. (8) Considering Snell's law between first layer and m th layer.

$$n \sin \theta = (n - m\Delta n) \sin 90^\circ$$

$$\text{So, } 1.6 \sin(30^\circ) = (n - m\Delta n) \sin 90^\circ$$

$$\text{i.e. } 0.8 = n - m\Delta n \quad \text{Solving, } m = 8$$

Note: Mathematical answer is 8, but this value of refractive index is physically impossible.

5. (1.50)

When angle of incidence on first face of the prism is 60° the angle of incidence on the other surface of the prism will be slightly greater than critical angle.

At $\theta = 60^\circ$ ray incidents at critical angle at second surface

$$\text{So, } \sin \theta = \sqrt{3} \sin r_1$$

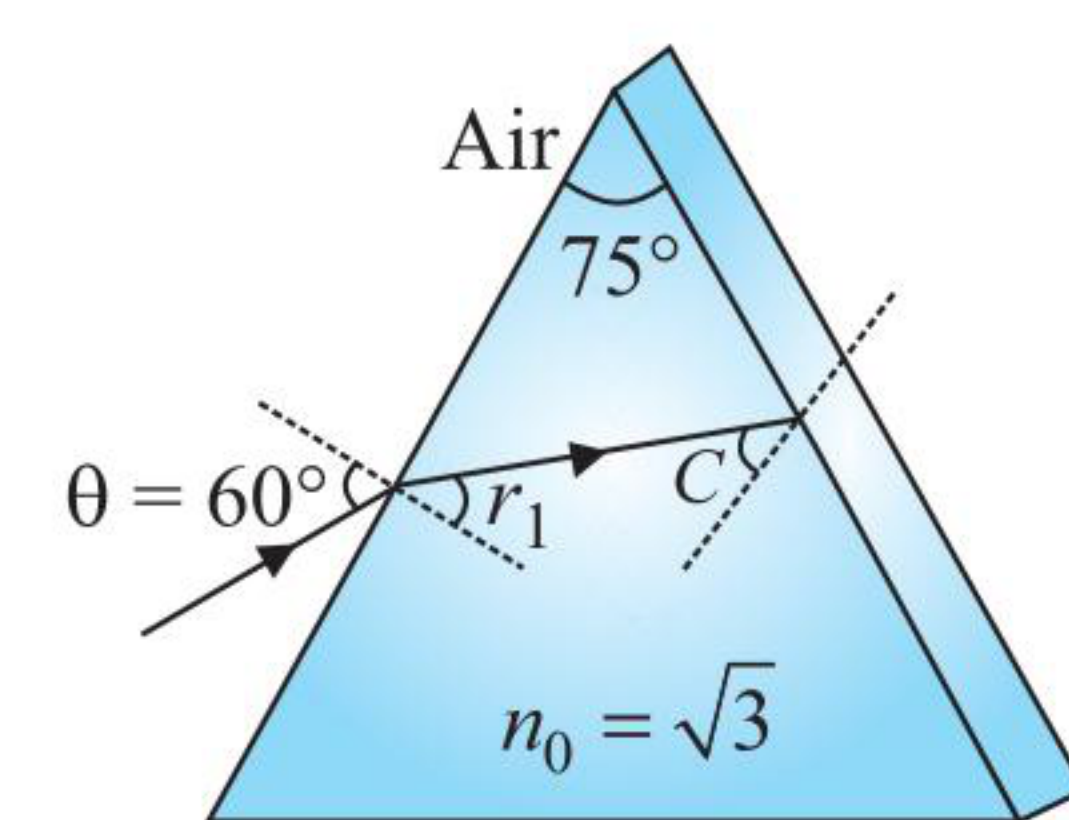
$$\frac{\sqrt{3}}{2} = \sqrt{3} \sin r_1$$

For second surface, $r_1 = 30^\circ$

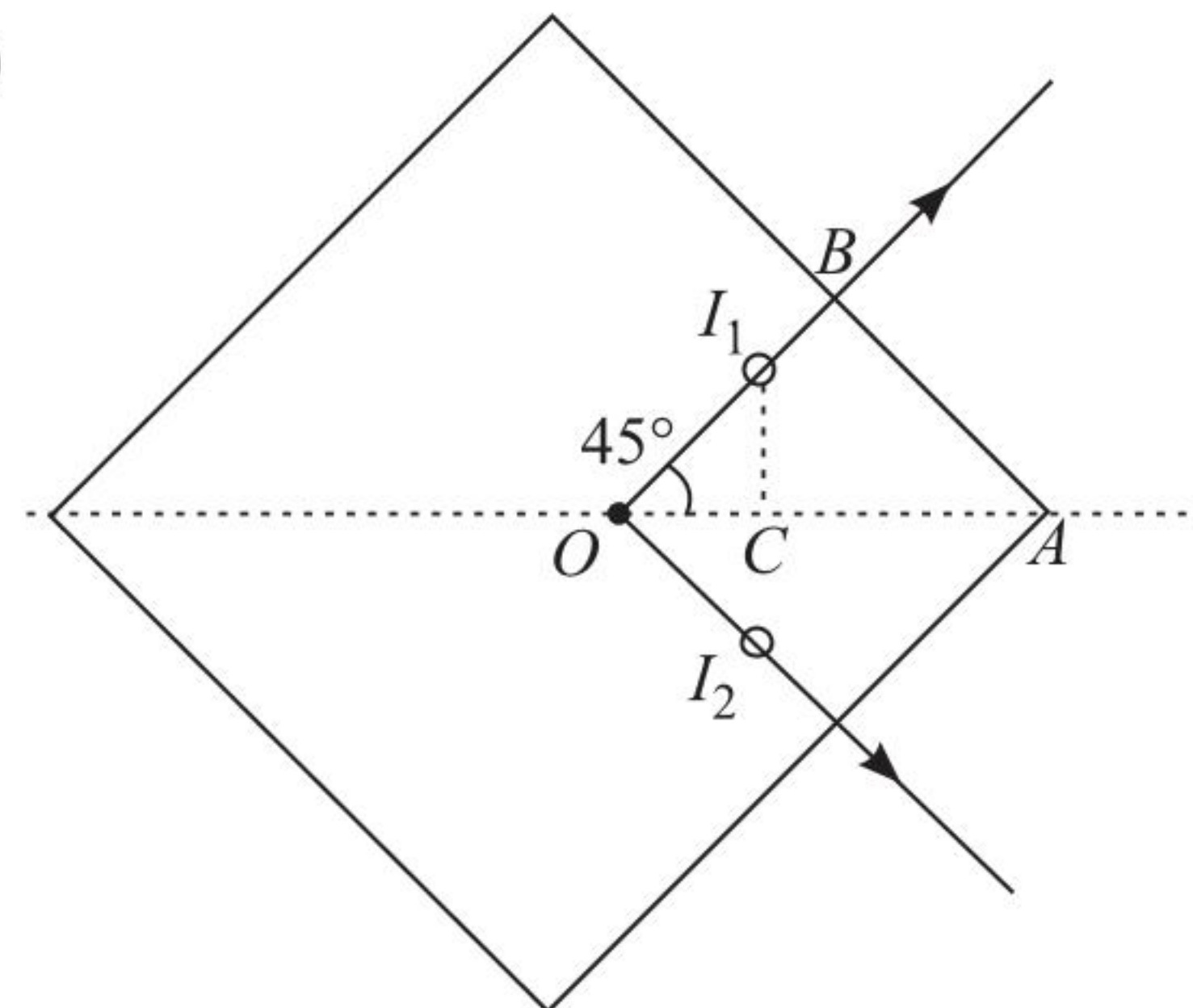
$$r_2 = 45^\circ = C$$

$$\sqrt{3} \sin 45^\circ = n \sin 90^\circ$$

$$n = \sqrt{\frac{3}{2}} \Rightarrow n^2 = \frac{3}{2}$$



6. (2)



Let us consider normal incident through faces

$$OB = OA \cos 45^\circ = 12 \times \frac{1}{\sqrt{2}} \text{ cm} = 6\sqrt{2} \text{ cm}$$

OB is the real depth while I_1B is the apparent depth

$$I_1B = \frac{OB}{\mu} = \frac{6\sqrt{2}}{4/3} \text{ cm} = \frac{9}{\sqrt{2}} \text{ cm}$$

$$OI_1 = \frac{12}{\sqrt{2}} - \frac{9}{\sqrt{2}} = \frac{3}{\sqrt{2}} \text{ cm}$$

$$\text{Hence, } CI_1 = OI_1 \sin 45^\circ = \frac{3}{\sqrt{2}} \times \frac{3}{\sqrt{2}} \text{ cm} = \frac{3}{2} \text{ cm}$$

The separation between the images $I_1I_2 = 3 \text{ cm}$

Note: In this solution we are assuming normal incident of ray through the faces, which is not mentioned in the problem. In official answers released by JEE Advanced bonus marks were awarded to students.